delta-k

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Volume 56, Number 1 April 2021

Mathematical Connections

Guidelines for Manuscripts

delta-K is a professional journal for mathematics teachers in Alberta. It is published twice a year to

- promote the professional development of mathematics educators and
- stimulate thinking, explore new ideas and offer various viewpoints.

Submissions are requested that have a classroom as well as a scholarly focus. They may include

- personal explorations of significant classroom experiences;
- descriptions of innovative classroom and school practices;
- reviews or evaluations of instructional and curricular methods, programs or materials;
- discussions of trends, issues or policies;
- a specific focus on technology in the classroom; or
- a focus on the curriculum, professional and assessment standards of the NCTM.

Suggestions for Writers

1. *delta-K* is a refereed journal. Manuscripts submitted to *delta-K* should be original material. Articles currently under consideration by other journals will not be reviewed.
2. If a manuscript is accepted for publication, its author(s) will agree to transfer copyright to the Mathematics Council of the Alberta Teachers' Association for the republication, representation and distribution of the original and derivative material.
3. Peer-reviewed articles are normally 8–10 pages in length.
4. All manuscripts should be typewritten, double-spaced and properly referenced. All pages should be numbered.
5. The author's name and full address should be provided on a separate page. If an article has more than one author, the contact author must be clearly identified. Authors should avoid all other references that may reveal their identities to the reviewers.
6. All manuscripts should be submitted electronically, using Microsoft Word format.
7. Pictures or illustrations should be clearly labelled and placed where you want them to appear in the article. A caption and photo credit should accompany each photograph.
8. References and citations should be formatted consistently using *The Chicago Manual of Style*'*s* author-date system.
9. If any student sample work is included, please provide a consent form from the student's parent/guardian allowing publication in the journal. The editor will provide this form on request.
10. Letters to the editor, description of teaching practices or reviews of curriculum materials are welcome.
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MCATA Mission Statement

Providing leadership to encourage the continuing enhancement of teaching, learning and understanding mathematics.

delta-K

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From the Editor's Desk

Lorelei Boschman

Dear math teachers,

You are appreciated! I want you to read it and feel it, because it is true. I debated on what to write about in this editorial. Do I talk about the hurdles, the challenges and the losses we've experienced, or do I talk about what's really on my heart these days? The heart won hands down.

Each one of you reading this are important to so many people. You are needed and valued. You have a circle of belonging travelling around you, whether it be family, students, parents, colleagues or friends. You are still a part of all these lives. It may look different and feel even more different, but teachers are a part of something much bigger. We always talk about the impact you can have on your students and families, but it has been made very clear now that you are needed. I have seen so many teachers collectively rise to what is in front of them. As I teach preservice teachers, what stands out to me is that many of them are entering the profession to have an impact on students. In fact, many of them have been significantly impacted by a past teacher. Well we now have a real situation where that impact can truly be a light shining forth. It's a real thing to be managing the demands of teaching in this time.

How can we measure the ripples of effect we have on others? How can we sense that our seemingly small action contributes to and makes a difference to those around us, including, of course, our students? How do we consider the implications of our care and attention toward the lives of students, even on a daily basis?

I just can't not mention this (sorry logic people). We as a math council have heard about so many encouraging, brave, impactful, caring, helpful and thoughtful teachers. Thank you. The ripples in your circle are surrounding you and reaching out. We want you to know we are thankful for you!

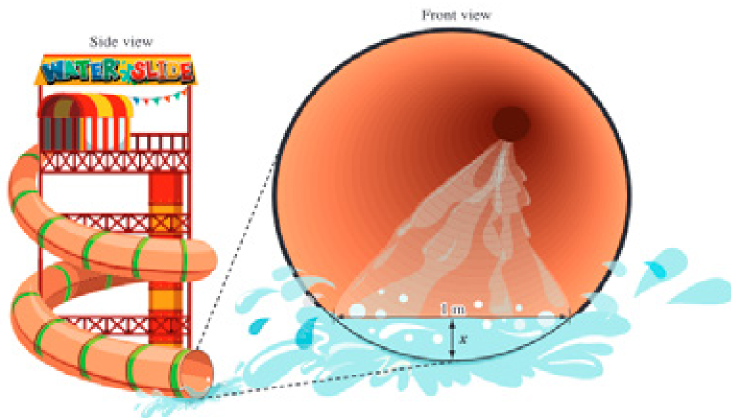
The Problem with Proxies

Steven Khan

A proxy is something (for example, a document) or someone (for example, an attorney) that is used as a stand-in or substitute for another. In scientific research or professional practice, when a phenomenon or object can't be measured directly one often resorts to an indirect measure, indicator or proxy until more direct measures can be devised. A classic example is estimating the height of a tall object by accurately measuring the length of the shadow cast by the object, measuring one's own height and shadow length at the same time, assuming that the angle between you and the sun and the object and the sun remains the same in the time it takes to do the measurements, and using similar triangles to calculate an approximation of the height. In this case the system is simple, the mathematical relationships well established and valid with an acceptably small error (remember the surface of the earth is curved though experienced by us as flat) and the use of shadow length as a proxy for an unavailable direct measure is legitimate. But this is not always the case.

Take the digestive system for example. The human gut is not simply a biochemical or electromechanical system. Our guts contain a variety of living microorganisms whose activities contribute to our well-being and health. These constitute the gut microbiome. Traditionally, fecal samples, or scat, have been used as a proxy for the gut microbiome and to infer health and functioning. This makes sense. Like the shadow cast by the tree, we can get a rough indication of what is happening in the gut by identifying and measuring

The diameter of a cylindrical waterslide is 1.5 m. The water running along the bottom of the slide measures 1 m across, as shown in the diagram below.



5. To the nearest hundredth of a metre, what is the depth of the water, x, at its deepest point?

A. 0.19 m

B. 0.25 m

C. 0.40 m

D. 0.50 m

Alberta Education, Provincial Assessment Sector 21 Mathematics 9

FIGURE 1. *Item from Alberta* Grade 9 Assessment Highlights Mathematics Provincial Achievement Test 2017–2018 (*Alberta Education 2018*)

which organisms come out of the gut. The math makes sense, right? In a beautifully designed, executed and reported study (Zmora et al 2018), immunologist Eran Elinav, of the Weizmann Institute of Science in Israel, and colleagues, put this belief and common research practice to the test in studies examining probiotic (products marketed as containing healthy microorganisms claimed to restore gut health in everyone) colonization patterns using a more direct method involving colonoscopic and endoscopic sampling. In brief, they found that the microbiome represented in the fecal samples was not representative of what was in the gut and that the pattern of colonization was "highly individualized" and not predictable based on scat analysis; that is, you couldn't tell whose gut had responded in what way or even whether they had received placebos based on the microbes in their poo! Their findings show quite strongly that, "solely relying on stool sampling as a proxy of mucosal [gastrointestinal] composition and function may yield limited conclusions" (p 1400). Or as bluntly reported in an interview with *New Scientist*, "Relying on faecal samples as an indicator of what goes on inside the gut is inaccurate and wrong" (George 2018).

We cannot tell by looking at a single score how much "value" an individual teacher added for an individual learner or class, nor can we tell by looking at that score what particular pedagogical approaches were used (or not), with what fidelity and to what effect.

The complexity of what goes on in classrooms at any level is probably, at least, of the same order as that occurring in our guts (and likely higher). A standardized common achievement test score is the traditional single proxy often used by education reporters and activists and upon which multiple claims regarding what happened or didn't happen in classrooms is based. They are probably as valid proxies as the fecal samples that have been traditionally used as indicators of what happens in the gut—behaving less like shadows in a well-defined physical system underpinned by well-understood and valid mathematical models and more like . . . a gross index of overall diet. As with Elinav's study, we cannot tell by looking at a single score how much "value" an individual teacher added for an individual learner or class, nor can we tell by looking at that score what particular pedagogical approaches were used (or not), with what fidelity and to what effect. Standardized common achievement test scores are poor proxies for these activities. Such popular and populist claims that rely primarily on such scores are likely to be of limited practical value at best, are most likely inaccurate and wrong, and dangerous contributors to truth decay (Kavanagh and Rich 2018) in public discourse at worst. Like probiotic colonization, the effects of teaching and what "sticks" to be "measured" on the traditional paper-based tests are likely to be highly individualized and dependent on a vast array of factors.

What goes on in classrooms and into learning across the lifespan is complex.

Some of those factors are identified in a pair of recent companion articles (Cantor et al 2019 and Osher et al 2020) by researchers from the Harvard Graduate School of Education and the American Institutes for Research Pamela Cantor, David Osher and colleagues. They identify and synthesize the foundational knowledge across multiple disciplines of relevance to education of how human beings develop in contexts (learn). They zero in on features of relationships and contexts that are drivers and delimiters of human development and articulate the strong convergences among the sciences studying learning. These ideas are reiterated and elaborated on in the most recent (2018) National Academies of Sciences, Engineering and Medicine consensus report, *How People Learn II: Learners, Contexts and Cultures* (HPL II) a follow-up to the hugely influential report *How People Learn: Brain, Mind, Experience, and School: Expanded Edition* published in 2000. Below are the key findings of HPL II:

1. Human development depends on the ongoing, reciprocal relations between individuals' genetics, biology, relationships, and cultural and contextual influences.
2. Each individual's development is a dynamic progression over time.
3. The human relationship is a primary process through which biological and contextual factors mutually reinforce each other.
4. All children are vulnerable. In addition to risks and adversities, micro- and macroecologies provide assets that foster resilience and accelerate healthy development and learning.
5. Students are active agents in their own learning, with multiple neural, relational, experiential and contextual processes converging to produce their unique developmental range and performance. This holistic, dynamic understanding of learning has important implications for the design of personalized teaching and learning environments that can support the development of the whole child.

These findings are drawn from syntheses across multiple domains of research involving contexts ranging from homes to schooling from early years onward and involve such constructs and phenomena as epigenetics, neural malleability and plasticity; integrated complex skill development, human variability, relationships and attachment; self-regulation; science of learning; dynamics of stress, adversity and resilience; relational patterns, attunement, cognitive flexibility, executive function, working memory, sociocultural context, constructive development, self-organization, dynamic skill development, neural integration, relational pattern making, adverse childhood experiences, poverty, racism, families, communities, schools and peers in supporting and/or undermining the healthy development of children and youth; relationships with parents, siblings, peers, caregivers and teachers; chronic stress, institutionalized racism, stereotype threat and racial identity.

In short, what goes on in classrooms and into learning across the lifespan is complex. Standardized common assessment tests are likely not a good measure of and a poor proxy for this complexity and contribute little to shifting public discourse toward more reasoned, informed and evidence-based discussions about what is happening, not happening, should be happening and might be possible in classrooms.

Closer to home here in Canada, Eizadirad (2019), in *Decolonizing Educational Assessment,* has argued that the EQAO standardized common achievement tests administered in Ontario are "culturally and racially biased as it promotes a Eurocentric curriculum and way of life privileging white students and those from higher socio-economic status while simultaneously lacking relevant connections to the identity and lived experiences of racialized students and families from lower socio-economic backgrounds" (p 205). In Khan (2020) I have argued that standardized common achievement tests are over-represented as a genre among forms of assessment that limit human and multispecies' flourishing as ends for education. They are not just poor proxies; they are demonstrably harmful to an increasingly large number of students.

Eizadirad (2019, 205) goes on to argue for equity in addition to equality framing his recommendations through decolonization. He argues that such an approach must begin by

> assessing the needs of students and the local communities; how students socio-emotionally and culturally enter the learning environment and the power dynamics embedded in the community and the learning space. This involves practices such as validating the histories and lived experiences of students as a form of valuable knowledge; recognizing their interests, passions…; and recognizing what resources are available and what new resources can be secured through cooperation and synergic collaborations with external organizations. Above all, a decolonized education focuses on providing support to teachers in numerous ways in order for them to assess in multiple ways and in different contexts the potential and competencies of students in relation to their unique needs … and personalities.

This work has taken on new significance and urgency during the COVID-19 pandemic with many school jurisdictions locally and internationally temporarily suspending the use of grades and standardized common achievement tests (AAC 2020; Vernell 2020) upon recognizing and prioritizing students' and teachers' well-being and health. The inability to provide a secure and "standardized" testing environment, the irrelevance of such tests during a period where survival, dignity, belonging to community and managing everyday challenges during a period of mass vulnerability provides an opportunity to carefully reconsider and design alternatives for a future that may be radically different.

To translate the recommendations through the digestive system analogy used earlier: We need to ensure that all learners are provided appropriate, nourishing meals; we need to support those who grow, procure, transport and sell the necessary ingredients needed to prepare those meals; we need to support and value those essential workers whose labour brings them to the table; we need to observe the short- and long-term effects of these meals, not on stool composition and probiotic colonization alone but on individual/collective physical and psychical well-being and see if our practices also are increasing the overall well-being of our communities and all the members of the multispecies world (that is, all inspirited beings) therein.

Note: A shorter version of this article originally appeared on the Medium platform on November 6, 2018. https://medium.com/@skkhan_87562/the-problem-with-proxies-1bd6f9c569f1 (accessed December 17, 2020).

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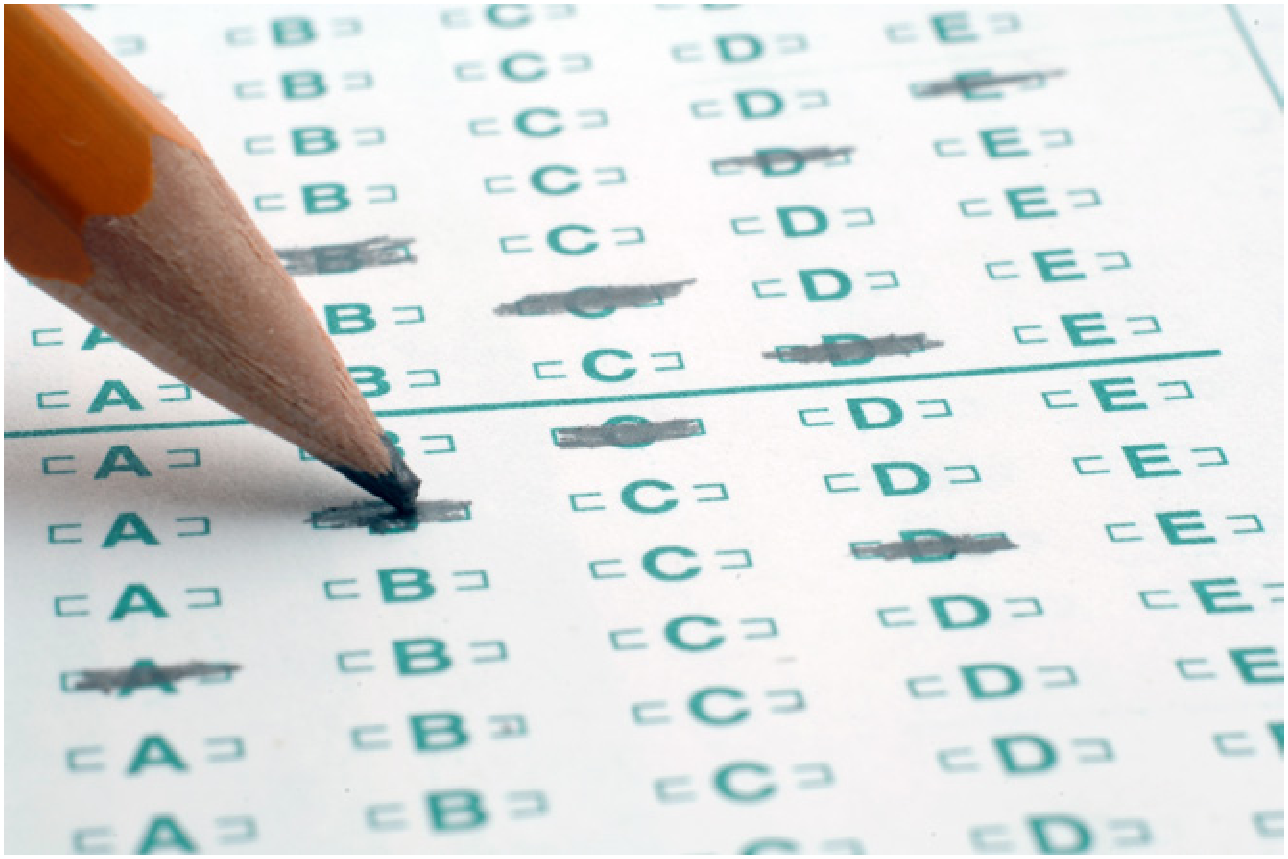
The Test with No Questions

Cherra-Lynne Olthof



My Grade 8 students posed an interesting question in math class during the probability unit: Could you pass a multiple-choice test without knowing any of the content?

Our theory was that since you have a one-quarter chance of selecting a correct answer, you should at least likely score 25 per cent. So here was the set-up: I created a 10-question test with randomly selected answers (through Google Forms). The students took the test, and then we looked at their scores. The class average was 25.2 per cent (yay for probability!) with a range of 0–6 questions correct (yes, someone passed!).

Each iteration of the experiment seemed to generate more ideas with different ways to try and find new answers.

This generated a flurry of questions from the students. Could they do better if they took the test again? (Spoiler alert: they did not). Since they were unable to figure out which questions they had correct and which they had wrong, the second test results were comparatively the same as the first (class average of 26.7 per cent). So, what were some of their follow-up questions?

1. What if they could look back at previous answers and adjust theirs over multiple trials?
2. If they could keep track of their answers to see which iterations of the answer key led to a more successful score, could they figure out which questions were right and which were wrong?

3. Could they compare their answer keys with the keys of those students who had higher scores and from there figure out a better score?
4. If they knew which questions were wrong and could select another answer, would their results increase in a statistically significant way?

The quality of the questions they were generating and their desire to develop a test for each hypothesis were quite stunning. Each iteration of the experiment seemed to generate more ideas with different ways to try and find new answers.

For the most part this obsession of how to score better led to a complex set of trial-and-error methods that took up probably more time than it should have. The only one we fully tested with any success was to see what would happen if they went back for a second time and changed just the answers they knew they got wrong. They weren't allowed to cheat and compare (which they acknowledged would be an easy way to score very close to 100 per cent). So, in allowing them to correct wrong answers (now choosing from only three potential answers, having eliminated one) and resubmit, they discovered that their scores did indeed go up. But only 22 per cent of the class now had a passing mark, and the range of correct answers was 1 to 8 out of 10. They acknowledged that this was because their probability of scoring higher the second time was because they now had a one-third instead of one-quarter chance of selecting a correct answer.

What students ultimately concluded was that there was no reliable way to beat the system, and that, really, they should just study the course content. Oh. And read the question. :)

Cherra-Lynne Olthof has spent the majority of her career teaching Grades 6–9. Having "taught it all," she has primarily been a math and language arts teacher for the majority of her career. She has spent all 20 of her teaching years with Chinook's Edge School Division, and you can currently find her in a Grade 8 classroom in Didsbury, Alberta, at Westglen School. She makes her home in Carstairs with her husband, also a teacher, and her two teenage children.

Measuring the Giant

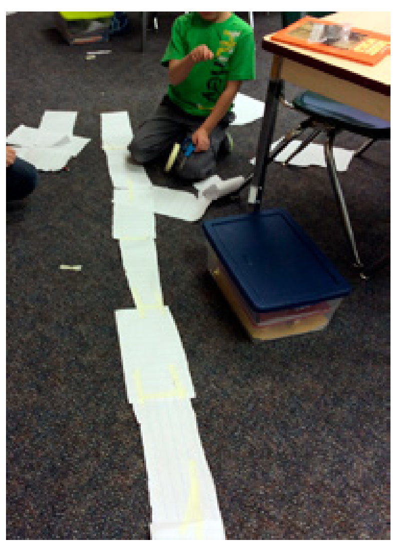
Barbara O'Connor

Measuring the Giant is a K–6 activity that uses ancient units of length based on body measurements, the span and the cubit, to determine the height of a giant. The students' only instruction, although a story could be made up with it, is the number of spans and cubits. Descriptions or demonstrations of a span (distance between the little finger and thumb when the hand is spread out) and a cubit (distance between the elbow and tip of the middle finger) are given.

The problem can be tailored to age by the number of cubits and spans. For example, how tall is the giant? It was said to be 12 cubits and 5 spans tall. Descriptions of a cubit and span are given.

This activity results in a great deal of problem solving because students likely don't know where to start. They often need lots of time to figure it out. Let them persevere, observe, connect with other groups and struggle somewhat. They may create the full length of the giant using paper, or you may see the beginnings of iterations. Some will end up using both partners and their cubits (even though the lengths are different). All giants will likely end up a different size. What you will also notice is their ability to use a meter stick. I have done this activity with Grade 2s and 4s. The activity can last several days with great discussions about the differences in measurements and why standard tools were created. Grade 2s could make their own rulers. Further extensions and comparisons are highly probable for exploration.

With 12 years' experience as an elementary educator, Barbara O'Connor is moving to support educators in their professional learning needs by pursuing further education in a doctoral program. She is passionate about hands-on learning for both students and educators, with supports to make that both meaningful and connected to their contexts, especially in mathematics. Barbara has used this activity in her classroom and found it to be engaging for students.



Legitimizing the Obscured Experiences of One Another

Darcy James W House

The high school completion rate and socioeconomic position of Indigenous Peoples indicate that we continue to be excluded from or choose to discontinue our involvement in academic programs, and that the forces of discrimination and meritocracy still heavily influence our academic choices within the available mathematics programs. This paper demonstrates the need for continued critical conversations regarding the current instructional practices and identifies exclusive forces at work in classrooms despite sincere efforts to develop programs that help more students succeed. I explore my own experiences as a student and educator of mathematics and suggest that we mathematics educators take responsibility for the evidential educational debt rather than laying the blame at the feet of our Indigenous students. The difficulties that Indigenous students encounter are not their own; they are ours. Though fraught with hardship, it is no longer unthinkable to challenge long-held assumptions about mathematics education and provide alternatives to the Platonist, Euro-American mathematical thinking that has usurped conversations for so long. In providing instruction of a wider breadth, we can journey toward a student population that is more interested in participating in our instructional programs. Thankfully, the work has already begun by educators and researchers who act as beacons of light as we familiarize ourselves with the needs of our students and consider new directions for our practices.

Our students deserve
a real chance
to discover the subject of mathematics.
Our history demands
that the legitimacy of mathematics
employed by Indigenous Peoples
be recognized,
that the activities,
thinking, and
practices
of Indigenous Peoples
be valued,
and that we participate in
conversations about
mathematics
with Indigenous Peoples
that are wholly dedicated to
Indigenous Peoples
without subjecting them to a colonial
educator's transformative lens.

A Call for Action

Tensions seem to have mounted here in Alberta; complexity defines the various arguments that I've heard that defend the perceived economic prosperity of Albertans and those of recent economic reports that submit that Albertans are being condemned to unskilled positions, low-wage jobs, poverty, struggle and the dredges of society. There is now evidentiary support for the claims that many Albertans have become dependent on social programs, government assistance, second jobs and payday loans (Cloutier 1997; Howe 2013). In particular, Indigenous Peoples experience reportedly lower rates of obtaining their high school diplomas when compared to non-Indigenous people at a cost to Alberta of over a quarter of a trillion dollars (Howe 2013). While the education provided to me has evidentially fallen short of successful for many students and others in the province of Alberta, I feel it is not necessary to lay the blame at the feet of those in need or to continue the cycles of meritocracy, poverty and structural violence (Cloutier 1997; Howe 2013). Howe's (2013) report implies that our educational institutions subject Indigenous Peoples to learning programs that fail to address their needs. It seems to me that students, parents, community members and, quite frankly, the economy, deserve better from all those interested in the welfare of Albertans.

Admittedly, many students, like myself, find success during their 12 years spent in traditionally structured classrooms that choose to adopt Euro-American programming and participate in the practices of decontextualization and meritocracy (Aikenhead 2017; Cloutier 1997). I'm afraid that my personal success and positive educational experiences have clouded my perception. Have I been so blinded to the wider mathematical experience? Have I been a poor witness to students having difficulties? Have the educational institutions I've participated in really failed to make room for Indigenous Peoples? Might I consider carving out the room needed to explore Indigenous mathematizing in my classroom given the amassing evidence demonstrating the failures of traditional Euro-American programming? Is programming that is restorative and regenerative of Indigenous cultural identity and cultural

Yet, all our students will benefit when we explore and participate in Indigenous communities of mathematics.

communities necessary, and, if so, is it accessible? Do I need to familiarize myself with Indigenous culture, history and current events in light of Howe's (2014) report and the Truth and Reconciliation Commission of Canada 2015 report? How can I participate in regenerative educational experiences with all my students? What might regenerative educational experiences look like? These questions now permeate my being and have sparked a personal search for identity. As someone of Métis descent, I feel responsible and can offer no concrete solutions, instead suggesting that we must take heed. For in my experience, there are no ultimate authorities nor are there any "legitimate" Indigenous resources that might finally satisfy the need for participation in the mathematizing of Indigenous cultures. Yet, all our students will benefit when we explore and participate in Indigenous communities of mathematics.

A Developing Practice

My own practice stems from years of studying formal mathematics. Beginning at home, my dad handed me a measuring tape and asked me to measure and cut boards to various lengths; together we built everything from pens and sheds to stairs and decks.

My first memorable academic success occurred in Grade 9 when my teacher claimed I would likely be unsuccessful in mathematics. My 12-year-old self disagreed, tackling difficult problems, suggesting various solutions and completing the course with a recommendation for the academic stream from the same teacher. I encountered classical mathematical streams while attending the University of Alberta such as geometry, calculus, statistics and algebra as well as less practised mathematical streams rarely found in high school mathematics such as combinatorics, ring theory and group theory. While largely successful, my interest in mathematics was brought to a swift close by the fields of complex variables and tensor analysis until I was able to explore these fields in a more visual manner. At the time, I was told that the complex field holds no similarities to the real numbers and that constructed proofs required memorization of procedure and analysis of code-like mathematical scripture. My later explorations of the complex field revealed many similarities to vector multiplication and rotation, but to this day I have been unable to imagine tensor analysis in everyday terms, a fact I attribute to the algorithmic pedagogy and rote procedural learning favoured by my instructors in the place of sense-making.

While some of my students find success in class through algorithm and memorization, many find success through alternative activities that more completely reflect the field of mathematics. For example, to find a set of multiples of two you can choose to multiply a set of individual numbers by two. This results in pieces of memorabilia highly valued in inclusive education and often the focus of individualized educational plans in my own practice despite my own efforts (personal communication, January 24, 2018). Alternatively, thinking about multiplication in the real numbers as a mapping or an elastic-like stretching of the entire line by various amounts allows one to apply the metaphor to the multiplication of complex numbers; what was once a linear elastic becomes a latex glove stretching and rotating around the origin. Using such an interpretation, the memorization of single-, two- and three-digit multiplication problems and algorithms become noncontributory pieces of trivia like those valued by game show contestants. The years I spent convinced of the inaccessibility of various mathematical streams were impactful. Today, my classes are filled with measurement and number sense, visual and mental descriptions of various dynamic ideas, and explorations of the unknown.

The Complexity of Mathematics Education

The results from PISA, an internationally acclaimed standardized exam, have been used as fuel for campaigns in support of a mathematics curriculum filled with "literacy, numeracy, higher standards, [and] student testing" that "just a decade ago, was the envy of the world" (Wente 2014). Similar polling projects, such as TIMSS, have been employed to create political leverage and justify authoritative educational structures "mostly at the expense of mathematics and science teachers" (Aikenhead 2017). Misinterpreted rankings become ammunition for media who proliferate the message that the content taught and pedagogy used in our educational system does not meet international standards (Rodney, Rouleau and Sinclair 2016; Chorney, Ng and Pimm 2016). In addition, private interests use the results to suggest that their own pedagogy and content are more effective in meeting these standards (Stockard et al 2018). These unquestioned assessment devices create a barrier—a "mechanical, detached, emotionless, value-free, and morally neutral" gateway into high-paying careers and other STEM programs (Aikenhead 2017, 82). In Alberta, these barriers have induced such extreme anxiety that they have chased children away streaming in tears from STEM programs (Burdess 2019). In such a way, Platonist, Euro-American mathematics is assessed at the expense of other cultural content, worldviews and students to provide a smokescreen for socially privileged families to maintain their position in the societal hierarchy (Cloutier 1997).

A more complex picture is one of Albertans who have suffered, are suffering and may continue to suffer for a long time. Cloutier (1997, 3) recalls a particularly memorable instance in a classroom that anecdotally demonstrates the conditions we have been imposing on our Indigenous youth:

> The teacher of this Grade 5 class managed the arithmetic lesson in what I thought was an unusual but at the same time familiar way. She sat at her desk and when students had a question, they were directed to bring their work up to her. This routine was broken by the teacher's disciplinary comments shouted at the students.

Another instance, a young Métis man:

> Lorne:
>
> I walk in with a note telling them why I had missed three days of school. [The teacher said] "Oh it's the disappearing Mr. Lorne. Was there like, a Pow

Wow in town that no one knows about? Were you drunk or passed out somewhere?"

Joe:

Is that what the teacher said?

Lorne:

Yeah, the Math teacher in front of everybody, and you know how those classes are: twenty, thirty people in a class and they're all sitting there laughing. (Cloutier 1997, 164)

Dominating themes of low self-esteem, undermined self-confidence, marginalization and powerlessness in mathematics classrooms are reproduced in programming that flagrantly disregard psychological theories of motivation and the development of youth. No student should ever feel that they can't "do anything right. [That they are] a failure at everything and [can] do nothing at all" (Cloutier 1997, 1). Memories flood back to me. The increasingly smaller class size as my classmates and I progressed through school together, the note taking from a chalkboard or words copied off a projected image on a wall and the hours of assigned problems from outdated textbooks. By the time I reached the 30-level mathematics courses, all that was left of my class was my brother, my cousin and one additional student. The education system I grew up with was driven by meritocracy (Goodman and Kaplan 2018). It created "feelings of inadequacy and low self-esteem" that shaped the future of the young adults of today (Cloutier 1997, 2; Goodman and Kaplan 2018). I, by pure coincidence, survived and flourished for years in our educational system, and it is a privilege to be sharing my story with you now. But programs that attend to the needs of the few, that our children attend from the time they are old enough, fail many students. As someone who has won the educational lottery, I acknowledge my survivors' privilege.

The experience of generations has convinced whole communities that their efforts will be unfruitful.

Conditions have not improved for many in Alberta since Hagey, McBride and Larocque (1989) published results demonstrating that 37 per cent of First Nations youth choose to end their academic careers in junior high (Cloutier 1997). In 2006, measures indicated that First Nations and Métis individuals ages 15–69 had obtained their high school diplomas at rates of only 48.5 per cent and 64.5 per cent, respectively, a statistic that contains "a great deal of heterogeneity" (Howe 2013, 20). Though experiencing education parallel to their non-Indigenous counterparts, young Indigenous people encounter racism, marginalization, structural violence and social destruction to such an extent that quantitatively significant differences have persisted despite decades of effort by educators to minimize differences in educational outcomes (Cloutier 1997). The experience of generations has convinced whole communities that their efforts will be unfruitful, that the time they spend in classrooms is worthless despite their struggle, and that any attempt to fight, kick and scratch from elementary to high school will be pronounced deficient.

Traditional mathematics programming does not provide a neutral ground but rather excuses participants from engaging in meaningful explorations because they feel uncomfortable legitimizing other ways of knowing.

Altering the Axioms

At the heart of this all, a nearly invisible "hidden agenda," a false dichotomy. It is an "elite aristocracy [fighting] for decontextualized content [sustained by] artisans and slaves" who have been convinced of the acontextual nature of mathematics that can be demonstrated through relative truths and imagined certainty (Aikenhead 2017, 89). The assumptions, limitations and restraints that cleverly disguise content as absolute and allocate authority have dictated what is and what is not valuable in our mathematics programs and disempowered those whose mathematics does not fall neatly among the subscribed axioms (Aikenhead 2017). Altering these axioms and exploring the resulting systems is reserved until late in the practice of mathematics when students of mathematics are convinced of absolutist ideologies and have spent years vigorously studying these ideologies (Aikenhead 2017). Concurrently, practitioners like myself easily mistake the popular Platonist, Euro-American ideologies as essential because of the demands that are placed on our programs through prescribed curriculums. Teachers administer "content, materials, descriptions, and representations" that favour Euro-American cultural content in the limited time available to them (Cloutier 1997, 39). The "voluminous" program of studies that teachers are responsible for delivering prevents us from engaging in cultural responsiveness; even my own mathematics classroom engages in a large amount of decontextualization, linearity and stratification (Aikenhead 2017, 9).

The move from dominantly Euro-American mathematics programs to culturally restorative programs necessarily disrupts the notions believed by the dominant group. Among the barriers teachers face when disrupting traditional classroom pedagogies are the efforts to support students and provide scaffolding in traditional programs, which Rubel (2017, 90) asserts are "not equivalent to changing the game." Traditional mathematics programming does not provide a neutral ground but rather excuses participants from engaging in meaningful explorations because they feel uncomfortable legitimizing other ways of knowing. We rationalize our impetuous participation in traditional mathematics programming and present our own cultural knowledge as representative of mathematics because we are afraid of "[crossing boundaries and confronting fears]." In doing so, we absolve ourselves from our responsibility "to connect to [our] students' experiences" (Rubel 2017, 86). I too have fallen prey to these representations and have left, unexamined, the influence that my own activities lend while my students are building their understanding of the field of mathematics. Every time my students ask, Where's the measuring tape? or Do these correspond? or Is this right? these moments and the activities from which they have stemmed must be examined for purpose and intentionality. It may be easy to examine the impressions left by the odd questions on page 15, but it is much more difficult to do so when building a birdhouse, exploring the radioactive decay of various isotopes or sewing a ribbon dress. I must ask myself what ways of knowing I am helping students to explore.

Among their discussions, both Cloutier (1997) and Rubel (2017) share their concerns about meritocracy. Rubel argues that "teacher's views about students' mathematical capabilities play a central role in their task selection and in the mathematical opportunities they provide," and Cloutier adds that these views perpetuate "the [unequal] transfer of social and economic position from generation to generation" (Rubel 2017, 70; Cloutier 1997, 42). Meritocracy suggests that it is not the educator's responsibility to examine the barriers that exist within their current programming, it is merely a matter of effort on the students' part (Goodman and Kaplan 2018). The teacher can then "act as a missionary" saving the students from their struggles and transforming them into successful mathematicians (Rubel 2017).

According to Rubel (2017), educators must challenge the belief that effort will always result in reward, that students' experiences are largely similar to our own and that mathematics distributes power equally to all active participants.

Teachers play superhero everyday and choose who needs their attention and who doesn't. I recall a scene from early in my career: I encountered a student struggling with the tasks assigned by the supervising teacher. The student's progress was much slower than the rest of the class, and the skills that the student had demonstrated during one-on-one coaching were an order of magnitude behind other students in the class. As a young professional, I was encouraged by more experienced teachers to supervise and entertain the students who had already completed the assigned tasks rather than concentrate my efforts on strengthening the mathematical processes of the student in need.

Thus, addressing the difficulties experienced within our mathematics programs could not only serve Indigenous Peoples but could improve the learning experiences of many other students as well.

It seemed as though it was important to these teachers that the students who were stronger mathematically be given every opportunity to become stronger, even at the expense of others in the classroom. I found myself needing to defend the choice I made to spend five minutes with this student despite the student's academic needs, which I felt were high priority. In my opinion, I should have never needed to act as a saviour, but in my experience as an educator, the gift of mathematical understanding is reserved for those students believed by educators to be capable; educators bestow mathematical understanding on those who are subjectively interpreted as having applied themselves.

According to Rubel (2017), educators must challenge the belief that effort will always result in reward, that students' experiences are largely similar to our own and that mathematics distributes power equally to all active participants. Facilitated by superficial efforts from educators, the lack of representation makes it difficult for students from marginalized populations to imagine themselves as participators in the field of mathematics. This "psychic disequilibrium" leads to crises of identity among students who begin to describe their world without consideration of their place in it and to clash with those who exercise authoritative control over those students (Rubel 2017). But this narrative is difficult to validate. Cloutier (1997) and AADAC (1991) reveal just how notoriously difficult it is to capture information from these students.

Exacerbating the problem, in Alberta, the diploma examinations and other summative assessments offer no solutions to the crisis nor do they offer any additional information. Diplomas are only written by the small percentage of students who have experienced a large degree of success within the field of mathematics. Through these examinations we gather information needed to strengthen existing curriculums, programs and lessons, but no data regarding those students whom the system has failed nor how we would address the difficulties they were experiencing prior to their exit. Our conversations here are therefore limited to the theoretical as we have not received feedback from the students we have failed. Thus, addressing the difficulties experienced within our mathematics programs could not only serve Indigenous Peoples but could improve the learning experiences of many other students as well. In the interest of servicing these missing voices, I offer a metaphor.

Ellenberg (2014) describes a wartime story of planes returning from enemy airspace. Fighter planes full of fuel, loaded with ammunition and flown by experienced pilots would return from the battlefield, or they wouldn't. Every single one of those planes was skillfully prepared for battle in anticipation of the difficult flight ahead. In much the same way, students are skillfully prepared to participate in mathematics. Yet, Indigenous students have found significantly less success as measured by the completion of their high school diplomas. Professor Wald suggested we might find additional pilots returning home if we bolster the armour of their planes in the areas observed to be most affected by the enemy fire. Like Professor Wald, I suggest that armour must be built that addresses the loss represented by the choices of young Indigenous Peoples to not continue in the field of mathematics. This armour must be neither too heavy, preventing them from successfully navigating in academic environments, nor too light, furthering the marginalization of these populations through tokenism (Ellenberg 2014).

An Emerging Field of Study

Wiseman, Glanfield and Borden (2017) share their summary of this subfield of mathematics education. With only 195 academic and grey literature (news media and social media) sources of "Indigenous perspectives, knowledges, [and] worldviews" within the science and mathematics educational context, this subfield is in its infancy (2017, 16). The lack of longitudinal studies and in-depth policy analyses of mathematics curricula contributes to the crises faced by Indigenous youth. Much of the existing literature focuses "upon theoretical considerations or reflections related to program implementation" rather than demonstrating these programs in action (p 16). Their work demonstrates the difficulties encountered by teachers when implementing culturally responsive programs and pedagogy and the difficulties of access faced when teachers seek information about the original studies. In addition, Wiseman, Glanfield and Borden (2017) also illustrate that there has been little effort to include student voice as a part of these discussions. Of most interest to me was their concession that "there are very few studies which examine senior high school (Grades 10–12) levels of mathematics" (p 20).

However, there is an unmistakable shift in mathematics education toward listening to the voices of First Nations, Métis and Inuit Peoples.

It's not just academia that is lacking in resources. Classrooms are using the same textbooks now as I remember using in 2003. Teachers are teaching the same lessons they received from their instructors. Change is slow. As a teacher, I am tired of developing lessons at breakneck speed with no consideration for the cultural and economic impact of my lessons and delivering the same exercises as my predecessors simply because there are no other resources readily available for use in my high school mathematics courses. I often inherit my resources from other teachers, format a few small details and put them to use the very next day. The problem is heightened in semesters when I'm teaching courses I've not taught before and am scrambling to put together lessons with some semblance of organization. I imagine that beginning teachers feel similarly overwhelmed. And to pile on the problem, we've been using the same tools and technology since 1996 despite massive improvements simply because the technologies approved for the Alberta Government assessments are narrowly defined as calculators without including other technologies.

However, there is an unmistakable shift in mathematics education toward listening to the voices of First Nations, Métis and Inuit Peoples. In Canada, educators and researchers of both Indigenous and non-Indigenous descent have been busy building relationships with Indigenous populations to provide informal and formal research opportunities that prioritize the needs of Indigenous students within our education system. Wiseman, Glanfield and Borden (2017) consult a Circle of Advisors in performing a systematic search of media outlets and academic journals. Their work highlights that current efforts to collaborate, connect and conversate have been moving away from deficit-based language, exposing foundational assumptions and forging open-ended inquiry projects. Of note are the works being produced in Indigenous communities; "Inuit, Mi'kmaq, Cree, Blackfoot, and Haida communities" have committed themselves to creating educational opportunities for their youth (2017, 15). Aikenhead (2017) examines a wide body of research across the Northern Hemisphere related to Indigenous mathematics. His publication summarizes the work occurring in British Columbia, Alaska, Hawaii, Ontario, Norway, Sweden, Atlantic Canada and Alberta.

Together, these researchers demonstrate that programs addressing the concerns of Indigenous communities are being implemented even though much of the published literature excludes detailed descriptions of these mathematics programs. Excellent exceptions stand out among the rest; a teacher of Blackfoot ancestry in Alberta invited her community to engage her Grade 9 students in explorations of the housing commitments made by the reserve's committee and discussed the tax structure and band policies in class. This same teacher collaborated with an "Elder, an archeologist, a reflective writing instructor, and a Blackfoot cultural teacher" to arrange a field trip to a sacred indigenous site" and to supervise the subsequent reflections on their visit (Aikenhead 2017, 107). The strength-based approaches demonstrated by these programs are consistent with that of Indigenous Elders and Knowledge Holders in my own educational community.

The strength-based approaches demonstrated by these programs are consistent with that of Indigenous Elders and Knowledge Holders in my own educational community.

Engaging in Reconciliation

The pressure is enormous even for me, as one who identifies as Métis and specializes in mathematics education. I feel anxious thinking about the responsibility of seeking out Elders, facilitating culturally appropriate activities, participating in cultural exercises and advocating for alternative educational experiences. I wasn't taught that our family lifestyle was one of the Métis people of Canada; my siblings and I were simply growing up. To me, nothing was inherently special about the way of life of the people in our community. Even so, I am afraid of engaging in or being accused of cultural appropriation. There is little risk in paying for access to existing programs, adopting activities from colleagues and using tasks from my own educational experience. Using curriculum material that I am already comfortable with is much easier than building relationships with Knowledge Keepers, obtaining the required permissions and "engaging in [an] in-depth exchange of [...] people's worldviews, language-laden cognition, and values" (Aikenhead 2017). My concerns are only heightened by the risk associated with the marginalization of Indigenous students. For example, ethnomathematics suggests that "mathematics educators [should] draw upon their professional constructed forms or images [and] superimpose their mental forms or images on Indigenous group's mathematizing" thus enabling educators to claim they are "[blind] to the cultural nature of Platonist mathematics" (p 104). Aikenhead (2017) insists that these programs dilute Indigenous perspectives only to superimpose a Platonist process on Indigenous mathematizing and place these perspectives in a hierarchical relationship. Thus, ethnomathematics recognizes other culture's mathematizing as cultural, but not Euro-American mathematics.

Gutiérrez (2017) attempts to bring to consciousness the "the ways mathematics can dominate" (p 4). Her research blurs the lines between formal and informal mathematics. She encourages us to explore other knowledges and to recognize that these explorations will cause "tensions and contradictions" that require educators to act as politicians in the legitimization process. The intention, she states, is not to "make humans better or into fuller versions of themselves" but rather to "help us recognize our place in this world" (p 6). She embraces three Indigenous ways of being that resist the dominance asserted by western ideologies: In Lak'ech, Reciprocity and Nepantla. Lak'ech celebrates our unique identities while simultaneously acknowledging the similarities among us. Reciprocity enacts our need for each other; we become more than we could be individually when acting together. Nepantla carves the space reserved for exploring multiple consciences and existing "in tension long enough to birth new knowledge" (p 12).

In place of "mathematics," Gutiérrez (2017) uses "mathematx" to validate the multiplicity of the practice of mathematics and proposes that mathematics programs should be "intricately tied to what is pleasing and rewarding in a connected way" (p 12). She invites educators of mathematics to engage in political processes and intervene meaningfully in reality. In doing so, the work of educators can imitate that of professionals who embrace such consequential undertakings as the reclamation of the historical range of bison, the investigation of the AIDS epidemic and the optimization of distance-based transit fares, among other projects (Parks Canada 2018; Parsons et al 2003; Hoshino and Beairsto 2017).

Thankfully, there are already those who have been searching for rich cultural activities that are filled with learning opportunities for students and who are sharing their methods of curriculum creation. Aikenhead (2017) suggests a process that can be used to select these activities for the classroom. First, develop a relationship with an Indigenous Knowledge Holder. Next, familiarize yourself with the alternative ways of "counting, locating, measuring, designing, playing and/or explaining" that might be encountered (Bishop 1988). Third, identify any peripheral concepts that you may have unnecessarily stripped away and in doing so acknowledge the cultural impact that your educational experience has on your own practice. And finally, teach your students in a wholistic manner and integrate cultural transparency into your lessons. In addition, familiarizing ourselves with the work of authors and researchers of Indigenous descent provides insight into the worldviews of and teaching methods advocated by Indigenous communities. Michell (2005), of Nēhîthâwâk descent, suggests teaching methods that the Woodlands Cree worldview encourages, including "sustained contact with natural environments […]; traditionally developed technologies […]; utilising Cree resource people and elders in curriculum planning, development, implementation and evaluation processes; experiential learning and hands-on activities […]; trial and error; experimentation […]; peer mentoring […]; sharing circles […]; making models; [… and] traditional games;" among others (p 38).

Familiarize yourself with the alternative ways of "counting, locating, measuring, designing, playing and/or explaining" that might be encountered.

Future Direction of My Practice

Even my recent teaching assignments, paired with friends and educators of Indigenous descent, have provided little progress toward a classroom filled with diverse learning opportunities. My own efforts have thus far been limited to amending available resources, experimenting with fair assessment practices and providing spaces where my Indigenous students feel welcome. My classroom continues to lack mathematically rich cultural explorations despite my best intentions. Instead, I wade through textbooks of preconstructed material searching for instruction and assessment activities that will fulfill the requirements of the program of studies best in an effort to fill the 125 hours of study that all Grades 10–12 mathematics courses in Alberta consist of. In fact, for much of my career I have unknowingly aided in the propagation of Platonist, Euro-American mathematics by filling the hours using my lengthy experiences from the University of Alberta. I continue to provide programs disproportionately depleted of activities from Indigenous cultures without applying significant effort or recruiting aid from community members. I am likely to continue using resources that contribute to the struggle and stagnation of marginalized populations until a more complete mathematics program that accounts for Indigenous learners has been developed. As such, I find myself caught in a race against time. Valuable years pass by as educators develop contributions to the field of mathematics education, but in the meantime, the learning of my students is limited in cultural breadth to the exclusion of Indigenous populations. I feel as though my own engagement with the existing field of Indigenous mathematizing has thus far been noncontributory.

Groups that are knowledgeable in mathematics, both Euro-American and Indigenous, must work together to affect a long-term mass adoption of mathematics programming that is more reflective of the practice of mathematics. Those interested in impacting the historical narrative of Indigenous communities might begin by exploring Indigenous knowledge and mathematizing, focusing on the "counting, locating, measuring, designing, playing, and explaining" (Gutiérrez 2017, 11) of Indigenous people, the "images, artifacts, and symbols" of Indigenous communities, and the personal ways of knowing and being of Indigenous community members (Russell and Chernoff 2012, 114). I feel as though there are so many unanswered questions, so much so that educators may find it easier to adopt the mathematics programming of traditional mathematics curriculums. But for educators like me who want to do some heavy lifting, thankfully, we aren't alone. Indigenous Knowledge Holders and Elders are busy sharing their stories and history, educators are busy compiling resources, and researchers are busy kick-starting conversations. I've found this community to be friendly and inviting. I sincerely hope that you find the same.

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Building Mathematical Confidence: Using Math Talk in a Second Language Context

Allison Robb-Hagg

This paper evaluates the effectiveness of Math Talk to communicate mathematical reasoning in French. This high-impact strategy built student confidence and fact fluency in a French immersion context. This action research study, conducted with 19 Grade 4 students, used a mixed-methods approach to examine changes in student confidence as they communicated their mathematical thinking and reasoning. Findings suggested that to increase student confidence and communication of mathematical reasoning, students must collaboratively construct their knowledge. Language frameworks, targeted mental math strategies and established norms were required to cocreate knowledge. Growth in self-confidence in mathematical discourse was $p<0.001$, indicating a statistically significant time effect, signifying a positive impact on student self-confidence in their ability to communicate their mathematical reasoning in French. This research suggested Math Talk is effective in helping students communicate mathematical thinking and reasoning in a second language learning context.

Punaro and Reeve (2012) found a significant relationship between worrying about math and problem solving in nine-year-old students.

Math trauma and math anxiety are real. As a teacher and as someone who suffered math trauma as a child, I see and feel the impact of mathematical anxiety every day in my Grade 4 French immersion classroom. Students who were once described as confident and competent mathematicians are becoming tentative and reluctant, saying that they are "bad at math." Punaro and Reeve (2012) found a significant relationship between worrying about math and problem solving in nine-year-old students (as cited in Hill et al 2015). An increase in subject-specific mathematical anxiety results in a lack of confidence among students, which has also been tied to a lack of motivation, and students who have low self-concept are not confident in their mathematical abilities, resulting in

Did their lack of ability to express their mathematical thinking and reasoning affect their mathematical confidence and computational skills?

poor performance (Organization for Economic Co-operation and Development [OECD] 2014). Mathematical anxiety affects not only conceptual understanding but also students' social-emotional well-being and mathematical confidence. Student mental health is at risk with the absence of foundational knowledge, jeopardizing their mathematical futures (Dowker, Sarkar and Looi 2016). It was this growing lack of confidence and observed anxiety in the math classroom that led me to conduct an action research project with my students. Action research is a teacher-driven inquiry that responds to a problem of practice. In this case, why are students becoming less confident in communicating their mathematical thinking and reasoning? What can I do about it?

As a language teacher, the tension between wanting to have students express their mathematical thinking and reasoning while correcting their grammar and vocabulary in the target language is a constant battle. Language learners do not have the necessary vocabulary at their fingertips the way they do in their first language, nor do they always have the correct sentence structure. Teachers are encouraged to prompt and recast to correct mistakes (Lyster and Izquierdo 2009) to avoid solidifying errors; however, this has the potential to interrupt the flow of mathematical thinking, resulting in students getting flustered and giving up.

My school has been focusing on the use of the neurolinguistic approach (NLA) to language acquisition. NLA is a pedagogical approach to acquiring and teaching a second language that focuses on the ability to communicate, with an emphasis on the use of oral language. As a second language school, our students are strong in the use of oral language. I began to wonder how we could leverage this strength in using oral language in math class. Did their lack of ability to express their mathematical thinking and reasoning affect their mathematical confidence and computational skills? I wondered if there was a way to combine our current work in NLA and the high-impact strategy of Math Talks as a way for students to build their mathematical confidence. It was my theory that if you can talk about a concept in the target language, then students will have some level of engagement and understanding, hence building confidence. "The core of mathematics is reasoning—thinking through why methods make sense and talking about reasons for the use of different methods" (Boaler, Williams and Confer 2015).

Barwell (2020) reminds us to consider the added layer of complexity when learning a second language while developing the language of mathematics.

Parker (2019) also suggests students are more eager to learn subject-specific vocabulary when it helps them communicate their thinking more efficiently. A way to do this is with a Math Talk. A Math Talk is a short routine (10–15 minutes) that intentionally designs oral language math tasks that engage students in metacognitive thinking, empowering a culture where learning is cocreated by the students and results in the automaticity of their basic facts (Schoenfeld 1992; Veenman, Van Hout-Wolters and Afflerbach 2006, 148, as cited in Hattie, Fisher and Frey 2017). Research shows that past pedagogical practices such as timed tests and procedural learning create conditions for subject-specific anxiety to flourish. This results in a child's inability to become an autonomous, confident and competent mathematician (Sun 2018); thus, it is important to look at learning conceptually. Barwell (2020) reminds us to consider the added layer of complexity when learning a second language while developing the language of mathematics. This is reflected in my research as this study suggests that engaging in Math Talks builds student confidence by giving them the tools necessary to communicate their mathematical thinking and reasoning in the target language. This research argues the use of Math Talk in a French immersion classroom will increase student confidence, resulting in a decrease in observed mathematical anxiety during math class.

Review of the Literature

Decreased Mathematical Self-Confidence as a Result of Mathematical Anxiety

Mathematical anxiety, a subject-specific anxiety, is a strong physiological and psychological aversion to mathematics, which causes functional changes in the brain including declines in working memory and processing speed (Hembree 1990; Maloney and

Beilock 2012; Richardson and Suinn 1972, as cited in Gunderson et al 2018). Mathematics anxiety has been defined as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in ... ordinary life and academic situations" (Dowker, Sarkar and Looi 2016). Mathematical anxiety in the classroom can lead to withdrawal, inattention, fixed mindset and negative attitude resulting in academic decline, but where does the math anxiety come from?

The OECD (2014) results show that math anxiety is influenced by several factors including gender, ethnicity, culture, parents' views of math, teachers' anxiety and ineffective pedagogy. All of these variables play a role in identifying possible roots. It is important to identify possible causes and what can be done to potentially mitigate the effects for students.

Math Talk is a high-impact strategy currently being used in many English classrooms to build flexibility, fluency and automaticity with numbers.

Math Talk as an Intervention

Math Talk is a high-impact strategy currently being used in many English classrooms to build flexibility, fluency and automaticity with numbers (Boaler, Williams and Confer 2015). First developed by Ruth Parker and Kathy Richardson, it is intended to allow "every student to find their voice in math class … the beauty of it is that this routine can fully engage every student and every teacher at every level" (Parker 2019, 3). Math Talk returns to the basics of number sense, establishing the foundation for all subsequent work, while providing a psychologically safe environment. Metacognition research shows that students achieve more when they engage in reasoning about their thinking (Schoenfeld 1992; Veenman, Van Hout-Wolters and Afflerbach 2006, 148, as cited in Hattie, Fisher and Frey 2017). Using metacognitive strategies during Math Talk facilitates the students' ability to struggle productively. For example, students will explain as many ways as possible for mentally solving the problem of 18×5 (such as $10 \times 5 = 50$ and $8 \times 5 = 40$ then $50 + 40 = 90$ or $5 \times 5 + 5 \times 5 + 5 \times 5 + 5 \times 3 = 25 + 25 + 25 + 15 = 90$ and so on). Students discuss, compare, agree, disagree and analyze the different solutions provided by their peers. Value is placed on the process rather than the answer, which supports productive struggle. A productive struggle is a habit of mind that encourages perseverance and flexibility. Emphasis is put on the process of arriving at the answer (not the product) and the idea that to struggle is productive. This way, for students it becomes about the journey of arriving at the answer and less about finding the right answer. Math talks can "help students learn to embrace the idea of confusion or cognitive dissonance and come to understand that, with mathematics, a state of confusion can be an important and productive place to hang out" (Parker 2019, 6).

When students explain their mathematical reasoning verbally, others understand their process, and all students benefit (Hattie, Fisher and Frey 2017). Research suggests that Math Talk is an effective teaching strategy to increase fact fluency and flexibility to build confidence (Humphreys and Parker 2015). It is yet unknown if this research can be generalizable to second language learning, in this case French immersion, where language is an additional factor.

Implementation of Math Talk in the French Immersion Classroom

Successful implementation of Math Talk requires a classroom community with a mathematical growth mindset where mistakes are valued and productive struggle is embraced. Norms are to be established collaboratively facilitating the positive and supportive culture where students have the psychological safety to be vulnerable. Norms are the set of principles that the classroom community agreed on, which allows a group to work collaboratively. These principles are based on sharing, respect, trust and belonging. Teachers help students to learn to persevere, seeing mistakes as an opportunity in learning rather than a failure (Furner and Berman 2012). One simple yet effective norm, putting up a thumb close to their body rather than putting up their hand, gives students time to think, allowing for all students to participate, regardless of ability. Students who are quick to answer are encouraged to remain silent and to come up with additional strategies, adding a finger up to their thumb for each additional strategy, levelling the playing field between students (Parrish 2011). To facilitate discourse, language frameworks can also be useful as a help to prompt the type of language we want to hear from the students (Hattie, Fisher and Frey 2017). In French immersion, sentence frameworks and posted vocabulary can function as a lifeline for students who otherwise might not participate, serving as a jumping-off point for thinking or discussions.

Humphreys and Parker (2015) advise using caution when focusing too intently on the correct sentence structure and use of vocabulary, warning that it may be counterproductive. Focusing on the correct sentence structure, vocabulary and pronunciation, the expression of mathematical reasoning may be hindered, which is especially problematic in a French immersion context. Attention must be given to the balance between intentionality of the lesson (expression of mathematical reasoning) and correction of the language structure.

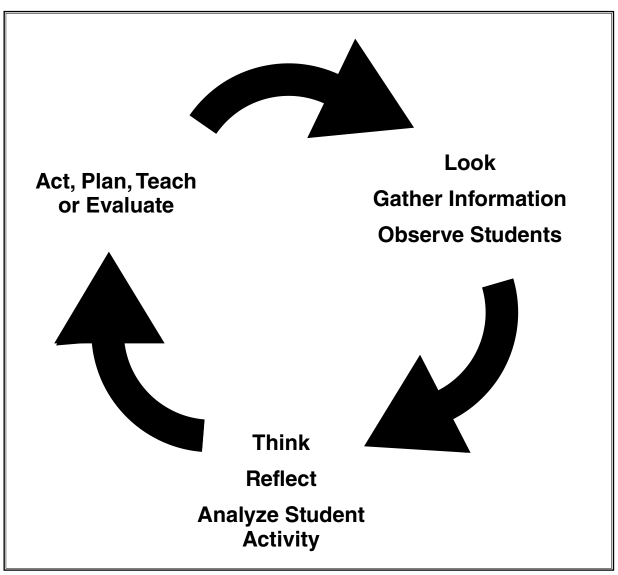
Research has demonstrated that learning mathematics in a second language is a factor in mathematical performance.

Neurolinguistic Approach (NLA) in Support of Mathematical Discourse

Language acquisition research allows for making a plausible connection between the pedagogical application of Math Talk and teaching math in the target language. "Research has demonstrated that learning mathematics in a second language is a factor in mathematical performance" (Bournot-Trites and Reeder 2001; Clarkson 2007; Ní Ríordáin and O'Donoghue 2009; Secada 1991, 1992, 151, as cited in Barwell 2020). The body of literature focuses primarily on using math talks with English learners, so examining research on language acquisition aids in understanding how mathematical language is acquired.

Netten and Germain's (2012) NLA paradigm can be examined, and generalizations can be based on what is currently known of the brain and how it processes the acquisition of language. This paradigm explains that language acquisition occurs through implicit and explicit competencies. Implicit competence is the ability to use language spontaneously. Explicit competence is the conscious awareness of how the language works and the structures and frameworks necessary to successfully have knowledge of how the language works. Most important, language acquisition occurs first through oral competence followed by reading and writing (Netten and Germain 2012).

FIGURE 1. *Stringer's Action Research Cycle*



While all of the NLA principles apply, principle five, the need for interplay between students in the classroom, is notable when examining the use of Math Talk in the classroom. NLA research supports frequent use and reuse of oral language in authentic spontaneous communicative situations to build neural pathways to create procedural memory (Netten and Germain 2012). Teachers facilitate language acquisition through modelling and correction of the proper sentence structure and vocabulary. Proper language structures should be encouraged for language to solidify in specific contexts. Using sentence structures and subject-specific vocabulary are essential to student success (Hattie, Fisher and Frey 2017). For example, when students explained their strategy for addition, they used the correct vocabulary (quand j'additionne, la somme est ___ parce que… when I add, the sum is ____ because) instead of the vague descriptions (this and that is…), which is common when students lack the necessary vocabulary to express their process or reasoning.

Methodology and Theoretical Framework

Action research is a systematic method that addresses problems of practice. Grounded in the pragmatist paradigm, action research is iterative in nature and therefore has to be flexible to respond to student needs in the classroom (Parsons et al 2013). This study used both quantitative and qualitative data to evaluate the impact of Math Talks on student confidence and computation. As an action research, this study was teacher-driven and addressed a problem of practice in my Grade 4 classroom. I adhered to Stringer's (1999, as cited in Parsons et al 2013) simple, yet flexible methodology for conducting classroom-based action research (see Figure 1).

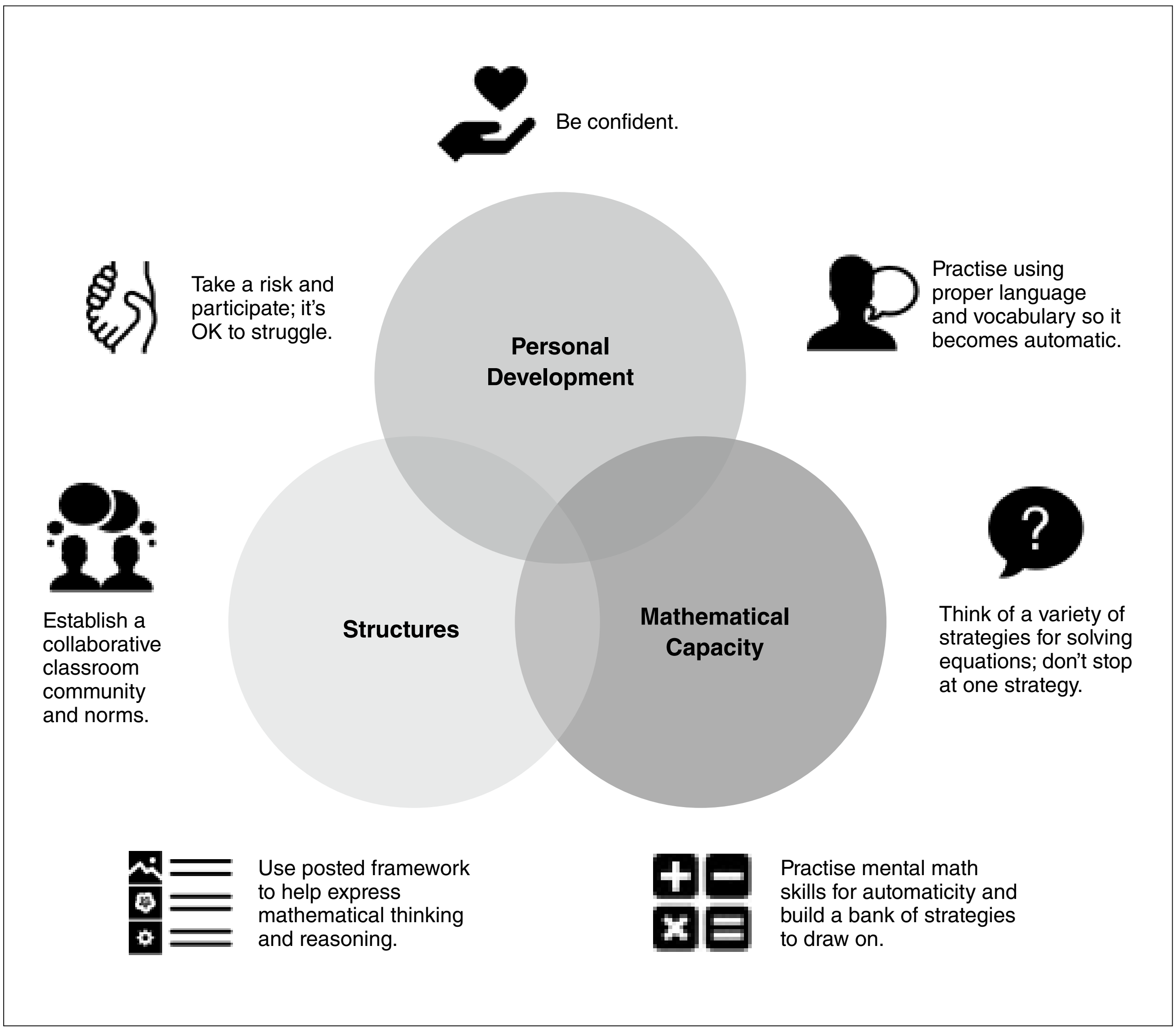
Quantitative and qualitative types of data were useful, adding to the strengths of each perspective while at the same time mitigating their limitations (Creswell 2014). This helped to triangulate the data, enhancing its validity. Quantitative data was collected by having students self-reflect and answer a standard paper confidence questionnaire and self-assessment, which statistically indicated growth of students' self-confidence over the course of the study. Qualitative data was collected through standard paper student self-assessments, including a rubric asking students to rate their confidence in specific areas such as when talking about math out loud and their thinking and reasoning skills. Students responded with one of three choices: I am there, I am getting there or I am not there yet. Students also commented on things that they did well and suggested an area for growth. Comments provided insight into students' opinions of their own growth as well as a deeper understanding of the implications for Math Talks in a French immersion classroom and were used to adjust teaching strategies when planning subsequent lessons.

Method

Setting and Participants

This study was conducted with students from a western Canadian Grade 4 French immersion classroom. All students had attended French immersion since kindergarten and were conversational in the French language. All mathematical instruction was conducted in French, while assessments were administered in English. Of the possible 25 students, 19 gave consent; in addition, parental consent was obtained. Thirteen girls and six boys with a mean age of 9.3 constituted the study. No student had participated in a Math Talk prior to participating in this study.

FIGURE 2. *Student conditions for success of Math Talk in French immersion.*



Data Collection

Students were the exclusive source of data collection. The study comprised three phases. All data was collected in students' first language: English.

Phase 1: Confidence questionnaire was administered to students. Questions included subcategories of motivation, value, self-confidence and enjoyment of math. Students indicated their feelings based on a four-point Likert scale.

Phase 2: Four Math Talk lessons were administered over two weeks. Students communicated mathematical thinking, reasoned and used mental math strategies. Students completed self-assessments of their level of confidence, which were analyzed after each Math Talk and used to plan subsequent tasks as well as to adjust teaching. Qualitative data and student comments were coded, then grouped and themed. Emerging themes identified by students were grouped into overarching categories of personal development, mathematical capacity and structures.

Phase 3: Confidence questionnaires were administered a second time to compare with the baseline. To meet ethics requirements, a third party collected the data while a school board employee ensured student safety and confidentiality.

All data was anonymized prior to analysis. Limitations of this study include the limited amount of time to conduct the research study resulting in a limit in scope and small sample size, which varied between 8 to 19 participants depending on the Math Talk.

Discussion

Based on the findings of this study and the literature, key conditions are necessary for the successful implementation of Math Talk in a French immersion or language learner context. Highlighted are the three intentional steps to take to successfully impact student self-confidence, resulting in a decrease in mathematical anxiety. In the data collected, students commented that they were building their confidence by developing a growth mindset and engaging in productive struggle. Using the structures provided as well as building their mathematical capacity in computation allowed students to feel more confident in sharing their mathematical thinking and reasoning. The surrounding figures represent the conditions that students indicated were the necessary conditions for success when engaging in a Math Talk in French (see Figure 2).

Beginning by establishing classroom norm and culture that is safe and caring is paramount to the successful implementation of a Math Talk.

Norms and Psychological Safety

A consistent theme throughout the literature, which was supported by the study, is that students require a psychologically safe environment that allows them to take risks and engage in productive struggles. "A situation in which a young language learner feels subject to the ridicule of his or her friends has a strong potential to divert attention to self-protection motives because social acceptance is one of the most salient motives for adolescents" (Olthof and Goossens 2008; Ullrich-French and Smith 2007, as cited in MacIntyre, Burns and Jessome 2011).

Beginning by establishing classroom norm and culture that is safe and caring is paramount to the successful implementation of a Math Talk. An analysis of the students' self-assessment data after the Math Talks suggested positive growth between the pre and post self-assessment questionnaire for question one: "I feel comfortable talking about math in class." A statistically significant time effect indicated growth in student confidence over the course of the four Math Talks. These results were supported by the qualitative findings that students had developed mathematical confidence through the use of Math Talks. Students were asked for specific examples of things that they had done well after each Math Talk. Comments for the question, "Things I did well" included, "I told Madame my answer," "I did well at contributing to the discussion," "I feel comfortable talking about math in front of the class" (student comments from self-assessment 2018). A review of students' comments allowed for monitoring of developing self-confidence and fact fluency as well as gained insight into student perseverance (Kersaint 2017) between each Math Talk. The students' comments also allow for teaching adjustments as well as implementing the required supports. The most important adjustment made during the study was the addition of the posted Math Talk sentence structure framework and posted vocabulary.

Language Framework

As part of the action research cycle, students' comments were analyzed after each Math Talk for possible improvement and teaching adjustments. The greatest statistical significance during the study was indicated between time two and four, yet there was a statistically significant time effect between the beginning and conclusion of the Math Talk study. This could be attributed to students becoming accustomed to the norms and structures as they were beginning to reason using the French language. By Math Talk four, students were beginning to not only explain their mathematical reasoning in French but were also engaged in a debate about the validity and efficacy of their claims with their peers.

These findings are supported by the research of Humphreys and Parker (2015) that indicates establishing a way of communicating about mathematical thinking not only grows student confidence but also helps them make sense out of mathematics and mathematical relationships. Building a classroom culture based on agreed on norms is essential in providing students with the psychological safety necessary to fully engage in discourse, struggle productively and feel safe enough to publicly contest others.

Following the first Math Talk, students' comments indicated a lack of confidence when having to discuss and defend their answers. Specifically, students mentioned that they did not have the necessary French vocabulary or sentence structure to express what they were wanting to communicate mathematically. As a result, students coconstructed the visual sentence framework and listed the strategies possible using the correct vocabulary.

FIGURE 3. *Sentence structure framework for « la discussion mathématique ». Collaboratively designed framework and vocabulary used to facilitate Math Talk in French, specific to the concept of two-digit by two-digit addition and its possible strategies.*

In Figure 3, the student will begin by stating the answer. "La somme est ___ parce que… (the sum is… because…). The word *because* is the most important part of this framework as it requires students to explain how and why. In particular, students will explain why they used a particular strategy and will to justify it. The students will then defend their answer using the yellow strips; for example, I used the strategy of…, I rounded, I estimated, I decomposed ____ so that I could____. Once the student has finished their statement, their peers are invited to agree or disagree using the green strips: I agree because or I disagree because…, as well as a way to specifically address a strategy used by a peer. This framework allows students to use the correct sentence structure and vocabulary necessary to express their mathematical thinking and reasoning. Using the principles of NLA, it is through the explicit teaching of the mathematical vocabulary and sentence structure that the implicit use of vocabulary and correct sentence structure will arise. Building common mathematical language aids in constructing conceptual understanding as a whole class. It was instrumental to the success of the subsequent Math Talks, and similar frameworks are still coconstructed with students at the beginning of each new mathematical concept.

When examining the pairwise comparison on time effect between times one and three, there was a positive impact on student confidence. However, student comments identified that "using the math vocabulary" and "trying to remember the strategy in my head to talk about it" were still areas for growth in the subsequent self-assessments. Students continued to require the teacher to pay attention to the specific French vocabulary and sentence structure while learning to reason mathematically.

Traditionally, no visible vocabulary or language frameworks are present during a Math Talk or NLA. In the context of this study, however, it was determined that students required additional visual supports that were coconstructed to meet their emerging needs. Once readily available, the students actively used the visual supports to confidently express their mathematical thinking in French. Students indicated through comments that these frameworks were necessary, yet practising using the correct sentence structure and vocabulary was still an area for growth. This was supported by the preliminary data that indicated that greater efficacy could have been achieved with more Math Talks over a longer time.

Do note that "more talk in classrooms does not necessarily enhance student understanding. Better understanding is dependent on particular pedagogical approaches purposefully focused on developing a discourse culture that elicits clarification and produces consensus within the classroom community" (Walshaw and Anthony 2008, 522).

Encourage Productive Struggle and Mathematical Growth Mindset

A mathematical growth mindset is paramount for a successful Math Talk. Self-confidence is an important factor in mathematics self-concept, which is necessary for a mathematical growth mindset. The mathematical growth mindset is a belief that with hard work and perseverance, anyone, regardless of ability, can succeed in mathematics. Fostered intentionally through effective strategies, meaningful discourse occurs when a classroom culture of participation and equitable learning experiences are valued (Bennett 2014). In contrast, fixed mindset, students are unable or unwilling to make the changes necessary to grow, resulting in low self-confidence (Dweck 2006). Implications of low self-confidence could include avoiding mathematical activities and situations, which leads to reduced fluency (Dowker, Sarkar and Looi 2016). Risk-taking acquiesces mistakes and facilitates growth mindset, thereby increasing student confidence. "We now know that when students make mistakes in math, their brain grows, synapses fire and connections are made" (Boaler, Williams and Confer 2015, 5). Students need to know that struggling is productive, and it facilitates deeper learning, which helps students to make connections among mathematical relationships. Establishing a culture where mistakes equate to learning is the foundation of having a mathematical growth mindset (Dweck 2006).

Research shows increasing students' conceptual understanding in math is essential to math-anxious students' success (Beilock et al 2004; Ashcraft and Krause 2007; Turner et al 2002, as cited in Boaler 2014; Parrish 2011; Ramirez et al 2013). Although conceptual understanding can be achieved in different ways, one of the more effective methods is through open-ended tasks and practices that value different ways of thinking such as number talks, also known as Math Talks (Parrish 2011; Boaler 2014). Scholars have also observed that math anxiety was higher in classrooms that valued performance over mastery (Turner et al 2002; Ashcraft and Krause 2007, as cited in Boaler 2014).

Over the course of the study, students who self-assessed their confidence using Math Talk suggested positive growth in self-confidence in the area of communicating their thinking and reasoning. Impact on self-confidence was indicated by analyzed data collected from the self-assessment rubric, goal setting and self-confidence subcategory of the pre and post questionnaire. As my position as a researcher was emic, from inside the classroom, I witnessed students increase their use of mathematical vocabulary, mental math, strategies and use of the framework as well as a decrease in observed mathematical anxiety.

Conclusion

Talking about math "can lure students into the world of mathematics and builds their confidence and their beliefs in themselves as sense makers" (Parker and Humphreys 2018, 81). Building self-confidence is the first step to reducing observed mathematical anxiety in the classroom as research has shown that mathematics anxiety impacts basic computation and fluency (Ashcraft and Ridley 2005; Maloney et al 2010, as cited in Young, Wu and Menon 2012) impacting students' self-confidence in their abilities and therefore their performance.

Math talks are now a daily routine in my Grade 4 French immersion classroom as well as many other classrooms in my school. Supported by visual sentence frameworks and posted vocabulary, students are able to reason mathematically in the target language with confidence and ease. Important to remember is the purpose of the Math Talk by correcting sentence structure or vocabulary in French only if it will not hinder the flow of the mathematical thinking.

After the conclusion of the study, other teachers at my school began to implement Math Talks in their classroom using the planning tools and frameworks developed during this study. We are beginning to see an impact as a school, for example, in the level of excellence achieved by the Grade 6 students during their provincial achievement test (PAT) on the fact fluency component. Post PAT, students reported feeling confident and scored much higher than they had on the MIPI (Mathematical Intervention/Programming Instrument) conducted in September since beginning the daily routine of Math Talks. We will continue to use Math Talks and monitor students for long-term effects on confidence and computation.

This study is a beginning for evaluating the impact of Math Talk in a second language context, specifically in French, on student mathematical confidence.

More research is needed to study the long-term effects of using Math Talk supported by NLA principles and frameworks. A longitudinal study with more participants is required to examine its full impact and to assess the long-term effects on increasing student confidence in combating mathematical anxiety. Observationally, its efficacy is evident as my students grow in mathematical confidence, mindset and fluency. They engage with rigour and enthusiasm in the daily Math Talks, shining with confidence. I highly recommend you give it a try!

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Crossing the Divide: Responding to Parent Criticism Through Mutual Concern

Kylie Palfy, Janelle McFeetors and Lynn McGarvey

It's 3:30 PM, when a parent of one of my students pokes her head into my classroom and asks, "Um, can I talk to you about math?" This is not unexpected. I just began three-digit addition with the Grade 3 students and they are exploring different strategies. I know that some strategies are completely foreign to parents, so I've been anticipating the possibility of a parent visit and have had similar discussions in the past. In our conversation, the parent is frustrated because her child is regrouping after adding instead of "carrying the one like you're supposed to." She tells me she wants me to teach her child the "proper way" to add. How can I communicate with this parent, so she feels empowered to support her child's mathematical learning?

It's often hard for educators to communicate reforms and advantages to parents who experienced a more traditional mathematics education.

This scenario has likely been repeated thousands of times in elementary classrooms in Alberta and more broadly across North America for at least two decades. In mathematics, the difference between parents' experiences of learning and their perceptions of their children's learning has been an ongoing source of conflict. Current curriculum and teaching methods can be mysterious and confusing for parents (Marshall and Swan 2010). Additionally, it's often

Research also shows that parent involvement in children's educational experiences increases achievement, improves motivation and reduces anxiety.

hard for educators to communicate reforms and advantages to parents who have experienced a more traditional mathematics education. Unfortunately, without understanding these approaches and the underlying rationale, parents may feel unable to support their children's learning at home and become critical of curriculum and pedagogy. Social media is rife with parents' critiques of personal strategies, estimation and use of manipulatives.

Despite these criticisms, research continues to demonstrate that reform-based curriculum and pedagogy have significant advantages over traditional approaches (for example, Boaler 2002; Henry and Brown 2008). Research also shows that parent involvement in children's educational experiences increases achievement, improves motivation and reduces anxiety (Patall, Cooper and Robinson 2008). Parents are critical partners in education (Remillard and Jackson 2005). Therefore, we must continue teaching using research-informed, reform-based pedagogy and engaging productively with parents to enlist their support in their children's mathematics education.

To more productively engage with parents, we endeavour to understand their critiques of current approaches to mathematics and identify expectations they hold for their children's mathematical learning. We chose to listen to parents as they shared instances of their children's and their own mathematics learning and why they valued mathematics learning in school. Rather than fielding parents' comments at the end of the school day rush, our aim was to focus on parent voice and understand the nature of their concerns. To do so, we carried out a qualitative study where we hosted two-hour focus groups in 10 communities in north central Alberta with 38 parents of elementary school-aged children. The focus groups also allowed parents to consider others' experiences and more fully develop their own stances during discussions around the table. To delve deeper, we also conducted individual follow-up interviews with 12 parents who had a range of strongly held perspectives on their children's mathematics learning.

We view parents as intellectual resources and important education partners. In identifying parent expectations, we aim to acknowledge parent experiences as essential for fruitful dialogue and seek to find mutual concerns through which parents and teachers can support children's learning. In this article, we identify nine parental expectations that were developed from analysis of comments shared during the focus groups and individual interviews. We believe that the expectations identified will help mathematics teachers in Alberta to more productively dialogue with parents and engender their support in mathematics learning.

Parental Expectations

Parents' feelings toward mathematics ranged from enjoyment to acknowledgement of struggles and avoidance. Through the process of contrasting their own learning experiences with those of their children and sharing their goals for their children's mathematics learning, the nine parental expectations outlined in Table 1 arose. Each parental expectation often relates to one or more of the three interconnected areas—expected goals, essential supports and communication—and were often framed by parents' past experiences, their current realities and their hopes for their children's future.

Reframing parental criticism of mathematics learning by identifying the underlying expectations will allow teachers to engage in thoughtful conversations, which respect parents' personal experiences and empower parents to realize their agency in their children's learning. To amplify parent voice, we share parents' specific comments so that as a community of mathematics teachers we can reflect on the potency of listening closely to the words of parents who seek to support their children's learning. We hope that the parents' comments will enable reflection on the meaning of parents' concerns removed from the pressure of particular circumstances and will highlight common hope among parents and teachers of children's mathematics learning.

Expected Goals

Parents and teachers believe that students need the opportunity to reach expected goals of mathematics learning. While teachers' goals for students are carefully outlined in the mathematics program of studies (Alberta Education 2016) and supported by other pedagogical documents (for example, NCTM 2000, 2014), parental goals and involvement have been less clearly delineated. Examples of parent comments illustrate each of the parental expectations related to expected goals.

TABLE 1. *Three categories of parental concerns capturing nine parental expectations.*

Categories of parental concerns	Parental expectations
Students need the opportunity to reach expected goals of mathematics learning.	Students to feel confident and successful at home and at school, yet parents feel current approaches are more confusing than traditional ones.
	Students to develop essential skills they perceive as necessary for success in school and beyond.
	Students to enjoy math and to not be turned off or checked out, so in the future they have the option to pursue a career in a mathematics-related field.
Essential supports must be in place for students to reach goals of mathematics learning.	To help their children at home, but they tend to encounter difficulties in understanding current approaches to computation.
	Students to be taught essential skills in school, rather than through tutoring agencies or extensive efforts of parents.
	Curriculum and teaching resources to be clear to parents and compatible with how students learn.
	Teachers to be sufficiently prepared to meet the demands of the current curriculum.
Communication between school and home is necessary for parental understanding and engagement.	To be informed about the mathematics content studied and their child's progress even if their child is confident and successful in mathematics.
	Evidence justifying changes to the curriculum as they felt evidence, rationale and justification for curriculum changes had not been successfully communicated.

Goal One

Parents expect their children to feel confident and successful at home and at school, yet they feel current approaches are more confusing than traditional ones.

> I think because she was so frustrated, she thought she was bad at math when she was in Grade 3 and so I, you know, made extra effort to build her up again and give her the confidence and make her realize that she is really good at math. (Linh)

In this quote, Linh expresses anxieties about her daughter's low self-confidence in mathematics due to perceived frustration with learning multiple strategies for computation. Her concerns mirror the expectation outlined in the mathematics program of studies (Alberta Education 2016) to ensure that students "use mathematics confidently to solve problems … appreciate and value mathematics" (p 2). Approaching Linh's worries by letting her know that her daughter's confidence in mathematics is also a priority for her teacher allows for a starting point from which to address her concerns and opens the possibility for Linh and the teacher to partner together to support the daughter. Additional pedagogic approaches could be used to incorporate number talks (Parrish 2010) as these help students develop confidence and fluency in computational strategies or low-floor/high-ceiling tasks (Gadanidis and Hughes 2011).

Goal Two

Parents expect their children to develop essential skills they perceive as necessary for success in school and beyond.

> If we gave them time to understand that math is not just numbers, math is not just adding or subtracting, math is shapes and math is patterns and math is—it's actually all these other things and get a real sense of an appreciation for that. (Leah)

> I still think it's really important that you memorize your times tables. Because in the future when you're doing bigger math, I mean, you really need to know that stuff just off the top of your head. (Patricia)

> That's math to me, it's taking whatever real-world situation you're in and using math to get the answers that you need. So, it's not about just memorizing the facts, it's how are you going to apply them in a real-world situation! (Leslie)

As the previous quotes illustrate, what parents perceive as essential skills for mathematics learning varies greatly and may be based on their past and present experiences, or goals for their children's academic future. However, recognizing that parents and teachers hold a mutual expectation of skill attainment allows for a common ground from which to discuss curriculum. Through a parent survey in the fall, teachers could determine what parents define as important skills and highlight opportunities for different skills development in monthly classroom newsletters.

Goal Two

Parents expect their children to enjoy math so that in the future they have the option to pursue a career in a mathematics-related field.

> Because he loves every other subject except math, and I know that it closes a lot of the doors if you check out at the earliest that you can, because you need to have math all the way to Grade 12, if I'm not mistaken, to graduate. (Mandy)

Although Mandy's son is only in Grade 4, she already expresses fears that he will not continue to pursue mathematics academically due to his dislike for the subject. Her anxieties revolve around his need to graduate high school, but many other parents of elementary schoolchildren had future fears that extended beyond high school graduation to postsecondary studies and future career opportunities.

> I want them to appreciate, respect math and to understand it and be able to use it in whatever. Whether it's your social sciences and they're using it in terms of statistics and doing research, or whether it's because they're an engineer. (Rachel)

Rachel directly connects appreciation for and enjoyment of mathematics to career potential. Possible career paths were mentioned over 60 times during the interviews and represent an important expectation for parents. Parents did not hold the same level of concern in relation to other academic subject areas. The possible reason for this was expressed succinctly by Susan: "I think math is really the basic fundamental scale for the kids' future." Susan's assertion seemed to be the motivation for many parents' career-related concerns. Teachers must consider the possibility that this belief may be underlying parental criticism and share their own hopes and dreams for their students' futures to more fully address concerns raised by parents.

While approaches to address goal one could be relevant in addressing goal three, teachers could consider more explicit connections between mathematics and careers. In partnering with parents, teachers could invite parents to share with the students in short presentations where they use mathematics in their specific careers. Or, if teachers are incorporating board games (McFeetors and Palfy 2018) in their mathematics classes, they could spotlight mathematicians as games designers in career possibilities (for example, Hamilton 2016).

Essential Supports

Parents' second area of concern centred on the supports they perceived as necessary for children to attain the expected goals, including supports parents could provide themselves as well as those provided by the teacher, school, governing educational bodies or teacher preparation programs. Parents expressed concerns that some of these essential supports were not present in their children's mathematics education.

Support One

Parents expect to help their children at home, but they encounter difficulties in understanding current approaches to computational strategies.

> When math homework comes home, I find out that they're learning differently than I learnt. And it's sometimes frustrating to try to help them at home when we're teaching it different or try to help them. And it's different than what the teachers are teaching them. So, it makes it a little difficult to help. (Wanda)

Many parents expressed similar frustrations. They want to help their children feel confident and successful in their mathematics learning, but they themselves are unsure of the methods and strategies being taught in the classroom. Parents who received such additional supports as teachers' explanations of homework, examples of calculation or estimation strategies, or parent-information sessions felt more capable to support their children with homework.

Other parents stated that their children rarely receive math homework. These parents still identified many activities and strategies that they implement at home to support their children's mathematics learning. These include playing card and board games; calculating payments and change; practising measurement, multiplication and fractions while baking; calculating angles while playing sports; analyzing sports statistics; identifying patterns in their environment; calculating car speed during family trips; measuring fabric; and many other informal conversations related to mathematics. When teachers engage in discussion with parents about mathematics, it is important to recognize that parents expect to help their children formally and informally at home. Through productive dialogue with parents, teachers can leverage some of the activities that already occur at home to align with curricular objectives covered in the classroom. Additionally, more clearly articulating homework expectations and computation strategies can relieve some of the parents' frustration when they attempt to support their children at home.

Support Two

Parents expect children to be taught essential skills in school, rather than at tutoring agencies or through extensive efforts of parents.

> I have to supplement at home since Grade 1, 2, to feel like I'm keeping up with the other kids, the other parents who are all going to [tutoring agencies] apparently. But amongst my network of people, which is quite a lot of different schools … everybody seems to be supplementing. (Natalie)

While parents expect to support children at home, they do not expect to be primarily responsible for their child's mathematics education. Some parents felt that the essential skills they had identified in goal two were not being adequately addressed in the classroom, thus requiring extensive intervention. When parents raise concerns of this nature, elementary school teachers must understand what essential skills parents feel are being missed. Once these have been identified, teachers can engage in discussion to find common goals and enlist parental support.

Support Three

Parents expect curriculum and teaching resources to be clear and compatible with how children learn.

> I have difficulty reading it. It's written in such a language, in such a strange convoluted language that I have trouble reading it. (Viktor referencing curriculum)

> It was absolutely baffling to us. It was just—it made no sense whatsoever the way that it was written. (Faith referencing a textbook)

> It's frustrating as a parent to not be able to look at a workbook or whatever or look in the curriculum and understand what it's saying so that I can help with something that I should know. (Meghan)

Parents often had difficulty deciphering curriculum and resources and found the subject-specific vocabulary challenging. Meghan shared a personal frustration with the word *regrouping*, which in her experience had always been referred to as *carrying*. In discussion with parents, it is important for elementary teachers to be cognizant of how much unfamiliar vocabulary is contained in curriculum and teaching resources. Teachers could try to translate mathematical terms into language that parents can more readily understand. In anticipation of a new Alberta mathematics curriculum (Learn Alberta 2018), teachers could use school-based professional development time to simultaneously create parent resources—like one-page overviews for each essential understanding per grade—to help parents understand new learning outcomes while planning for new expectations of learning in the classroom.

Support Four

Parents expect teachers to be sufficiently prepared to meet the demands of the current curriculum.

> Well I think that there needs to be more education for teachers on how to get the outcomes that they want. (Dan)

> I don't think that most elementary teachers of math know what math is about. Kinds of things that are encompassed by the phrase mathematics. (Tom, parent and mathematician)

Several parents expressed concerns about elementary schoolteachers' level of mathematics education and understanding. While individual classroom teachers cannot be responsible for teacher preparation programs, they can ensure that they themselves are adequately prepared to tackle mathematics curriculum in their classroom. At the school and district levels, administrators can organize professional development activities in mathematics to enhance teacher capacity and improve student performance. Teachers could request Alberta Education to continue financial investment in initiatives like the math bursary to enrol in university courses to learn and grow a network of colleagues provincially as resources. Additionally, elementary teacher education programs could also implement mathematical knowledge for teaching courses for future teachers (for example, Stylianides and Stylianides 2010).

Kilpatrick, Swafford and Findell (2001, 132) believe "it is critical that they [students] encounter good mathematics teaching in the early grades." We believe that good teaching requires well-prepared and confident classroom teachers. Taking steps to ensure this at the individual-teacher, school, district, state or provincial and university levels will not only assuage parents' fears but will also improve students' learning opportunities.

Communication

Effective communication is the key to ensuring that many of the essential supports parents identify are present. Parents specifically identified two expectations related to communication that they felt would expand their understanding of curriculum and improve their ability to support their children.

Communication One

Parents expect to be informed about the mathematics content studied and their child's progress even if their child is confident and successful in mathematics. Information about the topics their children were currently learning would help parents have conversations at home, and information about progress would help parents know when they need to provide more specific help for their child to succeed.

> I need the support of the communication. I need to know what unit he's on. I need to know what level he's on. I know they can measure things in terms of, you know, expected grade level outcomes. (Kristine)

Although some parents felt that classroom communication was lacking, others expressed satisfaction with the level of home–school communication. Some communication methods that were repeatedly referred to as helpful were year-plan outlines, monthly newsletters, parent–teacher interviews, e-mail, classroom websites and regular homework. Teachers would benefit by determining the most effective form of communication with their parent community. Overall, parents felt that consistent communication was the key to supporting their children at home.

Communication Two

Parents expect evidence justifying changes to the curriculum as they felt evidence, rationale and justification for curriculum changes had not been successfully communicated.

> I think one thing that was lacking for me as a parent was feeling that a solid rationale had been provided to me for why this is the better way to do math. (Elise)

Quite frequently, parents asserted that they would feel more comfortable supporting curriculum expectations and pedagogical strategies if they understood why the methods they experienced in school had changed. Elementary school teachers and administrators should take steps to ensure that evidence, rationale and justification for curriculum changes are successfully communicated to parents to garner their support. Many research-based professional articles can be used to develop an evidence-based rationale, especially for computational strategies (for example, Baek 2006; Crespo, Kyriakides and McGee 2005; Kamii, Lewis and Livingston 1993). This could be achieved through direct teacher-to-parent communication or through more formal school mathematics information sessions.

Conclusion

A primary goal of mathematics education is to ensure that "students have the opportunity to become proficient with mathematical knowledge and confident in their ability to learn and make sense of mathematics" (NCTM 2014, 110). Understanding parental expectations and enlisting their support in their children's education is essential in ensuring that this goal is achieved.

FIGURE 1. *Suggestions for teacher–parent interactions responding to parent expectations.*

Expected Goals	• Find out what parents see as essential skills in math through a home questionnaire or online survey. • Ask parents to identify three goals they have for their child's math learning. • At parent–teacher interviews, ask parents about their future aspirations for their child and how it may or may not relate to math.
Essential Supports	• Hold a parent math information session, outlining current curriculum and teaching approaches. • Take photos of math strategies students use in class and post on classroom blog. • Anticipate parental difficulties to understand textbooks and provide "translation." • Take advantage of professional development in math.
Communication	• Send home a monthly newsletter with upcoming math topics and strategies. • Regularly send home completed work for consistent, specific progress updates. • Listen carefully to parental concerns and identify underlying expectations. • Ensure you understand current curriculum approaches and explain advantages to parents.

We suggest that teachers consider the three areas of parental expectations outlined in this article as they plan for their school year. Figure 1 provides some examples of actions that teachers could take to engage with and better understand the parents of their students. These common-sense actions along with an understanding of the nine parental expectations outlined in this article create a foundation from which teachers can begin to assuage parental fears, identify common goals and enlist parental support.

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Math Focus 7 and English Language Learners

Monique Wilson

English language learners, or ELLs, are present in every classroom in Alberta. While some schools may offer differentiated reading or English classes to accommodate these students, often they are mainstreamed into other core subjects, including math. Therefore, not only do math teachers teach math and its language to students but also take on the additional task of teaching reading to students who may speak fair to poor English. If teachers are expected to use a textbook as the primary resource in the classroom, they must choose one that cannot only convey mathematical language and concepts but does so in a way that is inclusive of these learners. This paper explores the use of the textbook *Math Focus 7* (2007) in the context of ELL students and why it is an inappropriate choice for many reasons, including lack of diversity, lack of cultural context and poor use of written language. While the book does have some merits, including a bilingual (French and English) glossary, the book does not meet the linguistic needs of ELL students who do not speak French. To successfully meet the needs of ELLs in the math classroom, math teachers cannot rely solely on any textbook and must be prepared to adjust their pedagogy and material.

If teachers are depending on textbooks to facilitate math instruction, they must be sure these books are effective in reaching all learners.

"The more things change, the more they stay the same" may be an apt aphorism to describe the use of textbooks in school classrooms. Students and teachers use these books every day, and their use is pushed not only by governments that develop curricula but also by textbook publishers who make considerable amounts of money by creating and developing resources. Despite the evolution of the classroom, such as the increased use of technology, flexible seating arrangements, open-concept spaces and the differences in which material is taught to students (for example, direct instruction versus project-based learning), the textbook and the expectation that students will use one, either a hard copy or online, have not changed (for example, the textbooks currently approved for use in Grade 7 math in Alberta were all copyrighted in 2007). However, have the textbooks themselves changed? One focus for the analysis of textbooks and the ways in which they have changed must be considered in the face of the complexities of our classrooms, especially with regards to English language learner (ELL) students. These students may not only be new to a teacher's classroom or school but also, perhaps, even to Canada. If teachers are depending on textbooks to facilitate math instruction, they must be sure these books are effective in reaching all learners. With this in mind, I turn my attention to *Math Focus 7* (2007) and its effectiveness for ELLs. I chose this book because it is used in my own school and in many schools across Alberta.

Math Focus 7

The primary author of this textbook is Marian Small, in addition to a committee of other authors and consultants. Small is a professional development math consultant who works with school districts, mainly helping them to improve K–12 math education and differentiated math instruction. She is the dean and professor emerita at the University of New Brunswick; in the summer of 2012 she was a guest professor at the University of Alberta (Small 2019).

Math Focus is a series of curriculum materials, including student textbooks, workbooks, teacher's guides and other products (such as game packs, poster packs and online material) for Grades 1–9. The language speaks directly to the student (including the use of imperatives, such as calculate or estimate), and the book is full of photographs of children. The book is very colourful and includes photos, drawings and illustrations, and graphs and charts, which may be more attractive to children, as opposed to a textbook with a more factual, minimalist approach. In *Math Focus 7*, there are 11 chapters, and each chapter follows the same pattern: first, there is a Getting Started activity. The chapter is broken down into subtopics (indicated using header numbers such as 1.2 or 9.8), which go more in-depth, followed by questions designed to take students through the solving process step by step. Finally, several questions (usually broken down further, such as 1a, 1b, 1c and so on) are included. Each chapter also has a section called Curious Math, which extends the chapter topic with a short game or experiment. There is a mid-chapter review and a self-review in each chapter. Chapters also have a longer math game, which often asks students to use dice or other toys. There is also a chapter task that asks students to apply math to contextual situations, such as planning a party. Finally, there are cumulative reviews every three or four chapters.

They may favour using self-generated materials and regularly reviewing or updating them to meet the needs of new and diverse student populations.

An experienced teacher could take a lot of freedom with this book, because they would be aware of which questions would be more relevant to the students and which could be skipped or removed. They would also know how best to organize the chapters or if following the order of the textbook is the best approach. For example, some teachers at our school have seen students struggle with fractions, so teachers introduce the chapter on fractions earlier than can be expected based on the book so they can be revisited throughout the year. Experienced teachers use the book only as a supplemental tool; instead, they may favour using self-generated materials and regularly reviewing or updating them to meet the needs of new and diverse student populations.

This book is written for a narrower segment of Canadian students. There is a lack of diversity represented in the book (some names demonstrate a bit of diversity; instead of Fiona calculating a problem, it might be Yan or Pavlo). There is very little Indigenous representation and what there is, is rather token (a non-Indigenous student making a dream catcher needs to find the branches necessary; students need to use the divisibility rules to calculate groups of people in a Métis dance; students use the Quilt of Belonging, made up of squares representing Indigenous and immigrant groups to determine common factors). At least half of the children in the photographs in the book appear to be White. These questions in particular would be an excellent place to not only feature Indigenous students, but incorporate the culture behind the activities (for example, What is a dream catcher for? How or why did it originate? Why is the quilt called a Quilt of Belonging?)

No provision appears to be in the book for students of varying abilities although some questions, because of the way they are written, are harder than others. Although there are recommendations in the teacher's guide for how to differentiate instruction, but nothing specific to ELLs. There is no section labelled Challenge or similar for students of higher ability. Within the student textbook, there are no questions or exercises specifically tailored to ELL students. (In fact, the word *English* only appears twice in the student book, both times in an exercise.)

Teachers who use Math Focus 7 *in their classes not only teach ELLs who were not born in Canada and do not speak English at home but also teach ELLs who may have experienced trauma as a result of displacement.*

ELL Students in Alberta

As of April 2006, the year before *Math Focus 7* was published, the majority of ELLs in Alberta schools were foreign-born. Only in K–3 were the majority of ELLs Canadian-born students, and this number steadily decreased with grade level increase (Alberta Education 2006, 4). In 2006, 5 per cent of ELLs were refugees, with the majority in older grade categories. Teachers who use *Math Focus 7* in their classes not only teach ELLs who were not born in

Canada and do not speak English at home but also teach ELLs who may have experienced trauma as a result of displacement.

According to the *Canadian Magazine of Immigration*, between 2000 and 2015, 10 per cent of immigrants to Canada moved to Alberta. The top five source countries for immigrants to Alberta (and therefore for ELLs in Alberta schools) were the Philippines, India, China, the UK and its colonies, and Pakistan (*Canadian Magazine of Immigration* 2016). Of these immigrant students, perhaps the 5 per cent from the UK and its colonies will have a strong command of English. The rest will be ELLs. From 2000 to 2015, the number of immigrants to Alberta has steadily increased year over year. However, approved math textbooks for Grade 7 in Alberta have not been updated since 2007. Although the curriculum in Alberta was updated in 2016, materials were not, despite new and emergent issues in Alberta classrooms, including increased numbers of ELLs.

Teachers who are math specialists find themselves not only teaching math but also reading and interpreting text by using gestures, drawings, manipulatives and more, so their students can understand tasks.

According to Ron Schreiber, chair of Christ the Redeemer Catholic Separate School Division in southern Alberta, in 2017 110,000 students were considered ELL in Alberta (CBC/Radio-Canada 2018). These students may have parents who do not speak English well enough themselves to help with homework. As well, according to Alberta Education's 2016/17 *Results Analysis*, 74 per cent of ELLs completed high school within three years, which is 4 per cent lower than the rate overall (78 per cent) of students who completed high school within three years (Alberta Education 2017, 56–57). Likewise, 37 per cent of students enrol in postsecondary studies within four years of their Grade 10 year, but only 34 per cent of ELLs make the same transition (p 59). Math teachers could (and should) be playing a role to narrow this gap, meeting the needs of all students including ELLs, for instance, increased use of visuals, allowing students to annotate notes in a familiar language and having students work together so they can support each other, especially in classes with many students from similar communities.

Challenges for ELL Students in the Mathematics Classroom

Moschkovich (as cited in Kersaint, Thompson and Petkova 2012, 5) notes two principles teachers of ELLs need to consider as they plan their instruction: "Treat students' language as a resource, not a deficit; Address more than vocabulary, and support ELLs' participation in mathematics discussion as they learn English." Kersaint, Thompson and Petkova (2012, 7) themselves go on to say that even native English speakers may not be versed in mathematical language, and so all students should be considered mathematics language learners, while ELLs will be learning English at the same time. Teachers who are math specialists find themselves not only teaching math but also reading and interpreting text by using gestures, drawings, manipulatives and more, so their students can understand tasks.

Several challenges ELLs have in the math classroom are identified by Kersaint, Thompson and Petkova (2012, 46–47). Among them ELLs seldom encounter the passive voice, yet mathematics books often use it to write questions: "In a diagram, perpendicular line segments are indicated by a little square" (Small 2007, 309). In 1984, this was noted by Shuard and Rothery (1984), who, among other things, recommend the present tense only and the avoidance of passive sentences. The use of different determiners makes terms look as though they are different when they are referring to the same thing: "Jake ate $^3/_8$ of *a* [emphasis added] pan of lasagna, and his dad ate ¼ of *the* [emphasis added] pan" (Small 2007, 73). These conclusions are substantiated by O'Keeffe and O'Donoghue, who did considerable research in textbook analysis from a linguistic standpoint. Among their ideas, they state that "Consistency is also vital with the introduction of new vocabulary, and the introduction of new words should be planned so only a limited number are introduced at a time" (O'Keeffe and O'Donoghue 2015, 613).

Given that students and classrooms are more diverse than ever before, a modern math teacher can expect to teach mathematical and instructional language to a class of students with Tagalog, Punjabi, Hindi, Spanish, Mandarin and more as first languages. The common language they share in the classroom is English. In his 2007 article, Winsor details his own challenges as a math teacher who happened to speak a second language shared by many of his students (Spanish). Because of the shared languages, he was able to come up with a way to reach his students, which he called MSL or mathematics as a second language (Winsor 2007, 373). For a teacher in a diverse classroom, this may be one feasible approach.

In addition to the lack of English language skills ELL students bring to a classroom, another challenge teachers face is that math textbooks tend to be either quite Eurocentric, or even very Canadian-centred. Newcomers to Canada may lack a cultural context to help make sense of problems presented in a textbook. For example, in many countries (such as in eastern and central Europe or South America), commas are used instead of periods to represent decimals. Students who are unfamiliar with Canadian money may find such questions challenging since there is very little correlation between value and the size of coins or the colour of bills as in other cultures. Some countries represent algorithms differently; Kersaint, Thompson and Petkova (2012, 61) provide an excellent example of the structure of long division in different cultures.

Math Focus 7 and the ELL Student

The language in *Math Focus 7* tends to follow a pattern: first, there is a little story or sentence (often accompanied by a photo or illustration) about a person and what they are doing. Next, numbers are provided in context. Finally, either a question or imperative verb (or both) is used to signal what the student is to do.

The first concern for the ELL student using *Math Focus 7* is the wording of some of the questions. For example:

> Kyle is filling his little brother's wading pool. The pool holds 180L of water, and the hose supplies water at 22.5L for each minute. How long will it take to fill the pool? Answer to the nearest minute. (Small 2007, 127)

For many ELLs, especially recent newcomers, a problem like this may be too wordy. Is it relevant to know what Kyle is doing, and why? Or is the problem written this way simply to make it more interesting and engaging (which are subjective terms) to the student? An ELL student who cannot read a lot of English is likely to get frustrated with the amount of text. The background information about Kyle, his brother and pool is unnecessary, when the actual task is "If 1 minute = 22.5L, *x* minutes = 180L. Solve for *x*."

A common grammatical error with ELLs is the confusion of prepositions. Problems that rely on specific prepositions to dictate a desired result might produce something else. For example: "Fiona is doing a project for the science fair. She is recording the rise and fall of the water level in a pond. One day, she had this result" (Small 2007, 248). The illustration beneath this text shows a rod in a pond, with an arrow indicating the normal level of the water, and the water itself at 35 cm below the normal level. The text continues: "The next time Fiona measured the water level, it had risen by 40 cm. What was the new reading on the rod?" (Small 2007, 248). Grammatically speaking, the most important word in this problem is *by*, which implies the requirement of addition. However, nowhere in the question does it explicitly tell the student to add. A weaker ELL student may miss the preposition entirely; for such a student, a logical answer might be 40 cm (the number is, after all, in the question). The use of the word *risen* should tell the student the water increased (assuming they know that risen is the past participle of rise, which means to increase; considering this is an irregular past participle, many ELL students may be unfamiliar with its use). In addition to the grammatical issues, the problem is, again, wordy and filled with unnecessary information. Does the student need to know why Fiona is measuring pond water? This problem is about adding integers far from zero. Essentially, the problem is: -35 + 40 = ___.

For an ELL, this is a lot of information to sift through.

Another problem uses words that are short and fairly easy to understand. Unfortunately, there are so many of them:

> Sarah cares for 24 cats at the local animal refuge. Six of these cats have short tails, 12 are black, and 15 have long hair. All the cats in the refuge have at least one of these features. One cat is black and has a short tail and long hair. Two cats are black and have short tails, but do not have long hair. Two cats have short tails and long hair, but are not black. How many cats are black and have long hair, but do not have short tails? (Short 2007, 229)

This is a logic problem, and it is designed to be circular in its wording. However, in addition to being long and wordy, this problem uses both words and digits to express numbers (six versus 12 and 15). The expression *at least* indicates a minimum of one feature, but the possibility of more. The use of the conjunction *but* means that the feature following it is not present, whereas the conjunction *and* indicates that both features are present. Finally, the question in the last sentence asks students to sort between three features, identify which are present and which are not, and identify the number of cats indicated.

For an ELL, this is a lot of information to sift through. It requires knowledge that words are used in a grammatical sense (what conjunctions do and how they are used; the use of limiting expressions such as *at least*) and also a mathematical sense, as such words are used as logical connectors, which indicate a relationship. It requires students to be able to understand that words and numbers can be used to express the same idea, such as six = 6, which some ELLs may not have connected. In short, this question seems designed to trip up an ELL at multiple points. On further analysis, this question can be solved by using a Venn diagram, which would be a useful strategy for the teacher. Not only are Venn diagrams a strong visual tool for making connections, which means students without a strong grasp of English can use them, but they are also common in other core subjects, such as comparing and contrasting characters in language arts, or organizing data in science.

The cultural context in the book skews heavily Canadian, and western in general, which potentially alienates students who are unfamiliar with ideas their classmates can easily reference. Several questions deal with hockey: the area of a hockey rink, the plus and minuses (and other statistics) of a hockey player. There are pictures of children wearing hockey jerseys in the books. Some questions use French words: "Louise wants to integrate the Franco-Albertan flag into the flag for La Conférence de la Francophonie" (Small 2007, 113). Using words like this in isolation adds another layer of confusion for ELLs, who would have to wade through extraneous information in two languages to identify the task at hand.

What Works? What Needs to Be Changed?

The book does employ some strategies that make it more user friendly, not just for students in general, but for ELLs in particular. One interesting inclusion is the glossary, which not only defines instructional and mathematical terms (and distinguishes them) but also includes the word in French. For example, the first entry under Instructional Words is "calculate [calculer]: figure out the number that answers a question; compute" (Small 2007, 494). As a student who took math in French from kindergarten to Grade 10, and then struggled in Grade 11, I appreciate this inclusion. The majority of the instructional words, which are verbs, may be inferred from their English counterparts, such as determine (déterminer) or validate (valider). However, a few of the verbs and many of the mathematical words are very different in English and in French; it may be difficult for a native French speaker to get "sketch" from "esquisser" or "sample" from "échantillon." While I might not understand what my teacher means if they tell me to sketch a geometric shape, a quick glance in the glossary would tell me what I need to do. This glossary, however, is only of use to French speakers.

The book also uses strategies to draw students' attention to particular words, to help them understand the action they need to undertake to solve the task or problem. In some questions, words are bolded: "Write an **equivalent** decimal" (Small 2007, 152), although this is only useful if a student knows the meaning of the bolded word. In other questions, words are highlighted to draw attention to the definition in the margin. In a question about calculating the area of wallpaper required, the term *order of operations* is highlighted and defined in a box in the margin.

The *Mathematics Kindergarten to Grade 9 Program of Studies* says both that "Students learn by attaching meaning to what they do" (Alberta Education 2016, 1) and "Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding" (2016, 5). Both can (attempt to) explain why math textbooks write questions with so much background information in them. As well, being able to sort important information while setting the rest aside is an important skill for all students. But the use of language in *Math Focus 7* could be clarified and standardized. While many students (including but not limited to ELLs) may appear to have good second language skills in a social setting, their academic language is not as strong, and an improvement in textbook language should strike a better balance between the two.

While math is a language, it is not an international language.

As well, the inclusion of more "international" names is pseudo-contextual at best. While some students may appreciate seeing themselves reflected in the text, unfortunately the children in the textbooks (both math and in other subjects) continue to engage in activities that are very Canadian or western-centred. If the example activities are more culturally realistic, they are often presented without any context as to their importance, which can be more harmful than excluding them altogether. This does not meet the outcome on contextualization in the program of studies, and in fact is ridiculous from a mathematics instruction perspective. While math is a language, it is not an international language, and the attempt to culturally contextualize math without appropriate support and background knowledge is useless at best and offensive at worst.

Finally, questions that seem deliberately obfuscating or unrealistic, even to the native speaker, should be eliminated. One example would be the question cited previously about Sarah and the cats. Another would be the following, from the chapter on fractions, which is unrealistic mainly because of the inclusion of $\frac{1}{7}$, which is a fraction not common in everyday use:

> Leanne put some of her allowance into her bank account to save for a bicycle. After making the deposit, she had $\frac{2}{5}$ of her allowance left. At the end of the week, she still had $\frac{1}{7}$ of her allowance left. What fraction of her allowance did she spend during the week? (Small 2007, 73)

There are opportunities for a cross-curricular approach to some of these questions. For instance, calculating fractions of land covered by forests or prairies can tie in with geography in social studies and include the encounters all students have with new or unfamiliar words in a variety of contexts.

Conclusion

The book makes some strides toward accommodating different learners, but with regards to ELL students, it rather misses the mark. A teacher who chooses this textbook for their Grade 7 classes would do well to use a variety of teaching methods to convey information, give students strategies to help them sort out essential information in longer word problems, focus on questions that clarify what the student is actually expected to do and supplement with additional realistic tasks to allow students to demonstrate they are meeting the outcomes and can complete the skills required. This is also an excellent opportunity to collaborate with other professionals within a school, including other teachers, reading specialists, and other interventionists who can support ELLs and mathematics teachers.

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Mapping Grade 12 Mathematics to University Mathematics

Richelle Marynowski and Landry Forand

The Alberta K–12 mathematics program of studies includes three sequences at the secondary level (“-1,” “-2” and “-3”), each intended as prerequisites for specific postsecondary programs or direct entry into the workforce (Alberta Education 2008). Students progressing into Grade 11 are tasked with choosing the mathematics course sequence that will best fit their future goals. Choosing between sequences can be stressful for both students and parents. Confusion exists with parents about the goals and purposes of the mathematics sequences as many of them experienced a different set of mathematics courses and, for some of them, a different structure in high school in general (Palfy, McFeetors and McGarvey 2020).

There seems to be a lack of options for students progressing to university who take the “-2” sequence.

Each sequence is designed to provide students different pathways to suit their future goals and their strengths in mathematics. The implementation of these sequences initiated in 2008 to replace the Pure and Applied Mathematics sequences, which had been in place since 2000 (Alberta Learning 2002a, 2002b). Unlike Pure and Applied Mathematics, the new sequences would not vary on difficulty, but on content. Specifically, the Mathematics “-1” and “-2” sequences would be of similar difficulty; one course would not be easier than the other, and differentiation of content would define each sequence. The “-1” and the “-2” sequences were specifically developed for progress into postsecondary mathematics, while the “-3” was for entry into the workforce or as a gateway into trades programs (Alberta Education 2008). Additionally, unlike the previous Pure and Applied Mathematics sequences, the structure of the content in the mathematics "-1" and "-2" sequences allows for students to move between sequences if their plans for postsecondary change.

The focus of this study is on the "-1" and "-2" sequences and their mapping onto first-year university courses. The intentions for the two sequences linking to postsecondary are as follows:

> "-1" Course Sequence
> This course sequence is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of calculus. Topics include algebra and number; measurement; relations and functions; trigonometry; and permutations, combinations and binomial theorem.
>
> "-2" Course Sequence
> This course sequence is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of calculus. Topics include geometry, measurement, number and logic, logical reasoning, relations and functions, statistics, and probability. (Alberta Education 2008, 57)

Both the "-1" and the "-2" sequences include a variety of specific outcomes aimed to fulfill the description above. The "-1" sequence is specifically for programs that require calculus in postsecondary, and "-2" is for studies that do not require calculus. The choice to focus on university courses and not other postsecondary options was specifically due to the nature of the change in the Alberta program of studies from 2002 to 2008 in intending "-2" to be a viable entry to university studies.

Previous research showed that the "-2" sequence is not represented as a common program prerequisite when students are transitioning to university studies (see Marynowski and Forand 2016, 2017a, 2017b, 2017c). This study was developed to delve deeper into what specific outcomes from the "-1" and "-2" program of studies were relevant to specific first-year university-level courses. Information gathered from five Alberta universities (University of Lethbridge [U of L], University of Calgary [U of C], University of Alberta [U of A], Mount Royal University [MRU] and MacEwan University [MU]) found that Mathematics 30-2 does not provide as much access to faculties, programs or courses as was potentially intended. Knowing this, even for a student who shows no interest in programs containing calculus, that student may find it logical to choose Mathematics 30-1 to maintain a higher range of options for a university career. Additionally, 40 first-year math courses from the 2016/17 academic year were identified from the same five Alberta universities where Mathematics 30-1 was identified as a prerequisite option for all 40, and Mathematics 30-2 for only 11 of those courses. Only 15 of the 40 courses were calculus based, implying that the other 25 courses could potentially accept Mathematics 30-2 as a prerequisite or require no prerequisite (Marynowski and Forand 2017b).

What specific content from the Grade 12 mathematics courses in Alberta are required for first-year mathematics courses in universities in Alberta?

Despite the intentions of Alberta Education, there seems to be a lack of options for students progressing to university who take the "-2" sequence even though the number of students taking Mathematics 30-2 is increasing. From 2014 to 2018, the average number of students who wrote the Mathematics 30-1 diploma examination was 20,670, and the average number of students who wrote the Mathematics 30-2 diploma examination was 13,385. The number of students writing the Mathematics 30-1 diploma examination has been reduced by approximately 1,200 students over that five-year period, and the number of students writing the examination for Mathematics 30-2 has increased over the same time period by approximately 2,300 students (Alberta Government 2021).

The questions that arose from the previous research, and that this study strives to answer, are What specific content from the Grade 12 mathematics courses in Alberta are required for first-year mathematics courses in universities in Alberta? and Are the current first-year university mathematics course prerequisites appropriate for the skills that students need in that course?

To respond to these questions, a study was undertaken to map the specific outcomes from the Alberta Mathematics 30-1 and 30-2 program of studies to the first-year mathematics courses at five major universities in Alberta. The study consisted of surveying mathematics faculty and instructors at the same universities (U of L, U of C, U of A, MRU and MU) that were included in the previous study on course prerequisites.

Research Process

An electronic survey was developed and sent to the mathematics faculty and instructors at the five universities. Five unique surveys were created: one for each institution. Each survey included the specific course names and numbers that were offered at that institution and were sent to each institution's respective mathematics department faculty and instructors. Faculty and instructors from each institution were invited to select specific outcomes from the Grade 12 mathematics courses that mapped directly to a specific first-year course. Each respondent selected what first-year classes they were taught and then chose the outcomes from the "-1" and "-2" sequence that fit best as a prerequisite for their selected courses.

To avoid redundancy of outcomes being shared between Mathematics 30-1 and 30-2, the following outcome from Mathematics 30-1 was not included in the survey: "Demonstrate an understanding of logarithms" (Alberta Education 2008, 22), and the following six outcomes from Mathematics 30-2 were removed from the survey:

- "Solve problems that involve permutations." (p 29)
- "Solve problems that involve the fundamental counting principle." (p 29)
- "Solve problems that involve combinations." (p 29)
- "Solve problems that involve rational equations." (p 30)
- "Demonstrate an understanding of logarithms and laws of logarithms." (p 30)
- "Solve problems that involve exponential equations." (p 30)

These outcomes are identified as falling under "both" sequences as they appear in both the "-1" and "-2" sequence. Additionally, the outcomes from the "-1" and "-2" courses were listed in no particular order and were not identified as being from a specific sequence so that there was no bias in the mathematics faculty or instructors selecting outcomes from one particular sequence or another.

The surveys also included open-response questions that invited mathematics faculty and instructors to provide comments on the following:

- Topics you would like to see added or removed from secondary mathematics courses in Alberta.
- Strengths or weaknesses of students in introductory mathematics courses taught before, during or since 2012.
- Other comments pertaining to secondary mathematics education in Alberta.

Grade 12 "-1" and "-2" mathematics curriculum in Alberta were implemented in 2012, following changes to Grade 10 in 2010 and Grade 11 in 2011.

Invitations to complete the survey were sent to 212 mathematics faculty and instructors whose names and e-mail addresses were publicly available on university websites. Invitations were sent in July, and reminders were sent in mid-August. Table 1 shows the the number of invitees, respondents and courses that were mapped per institution. Not all of those invited taught first-year classes, which may partly account for the response rate.

Out of 48 respondents, 43 answered some or all of the open-response questions. Some taught more than one first-year course, so the total number of courses mapped was more than the number of respondents.

TABLE 1. *Survey Response Patterns*

Institution	Invitations Sent	Survey Responses	Response Rate	First-Year Courses	Courses Mapped
U of A	86	16	18.6%	10	24
U of C	66	18	27.3%	10	29
U of L	11	6	54.5%	7	9
MRU	25	4	16.0%	9	6
MU	24	4	16.7%	9	7
Totals	212	48	22.6%	45	75

Survey Responses

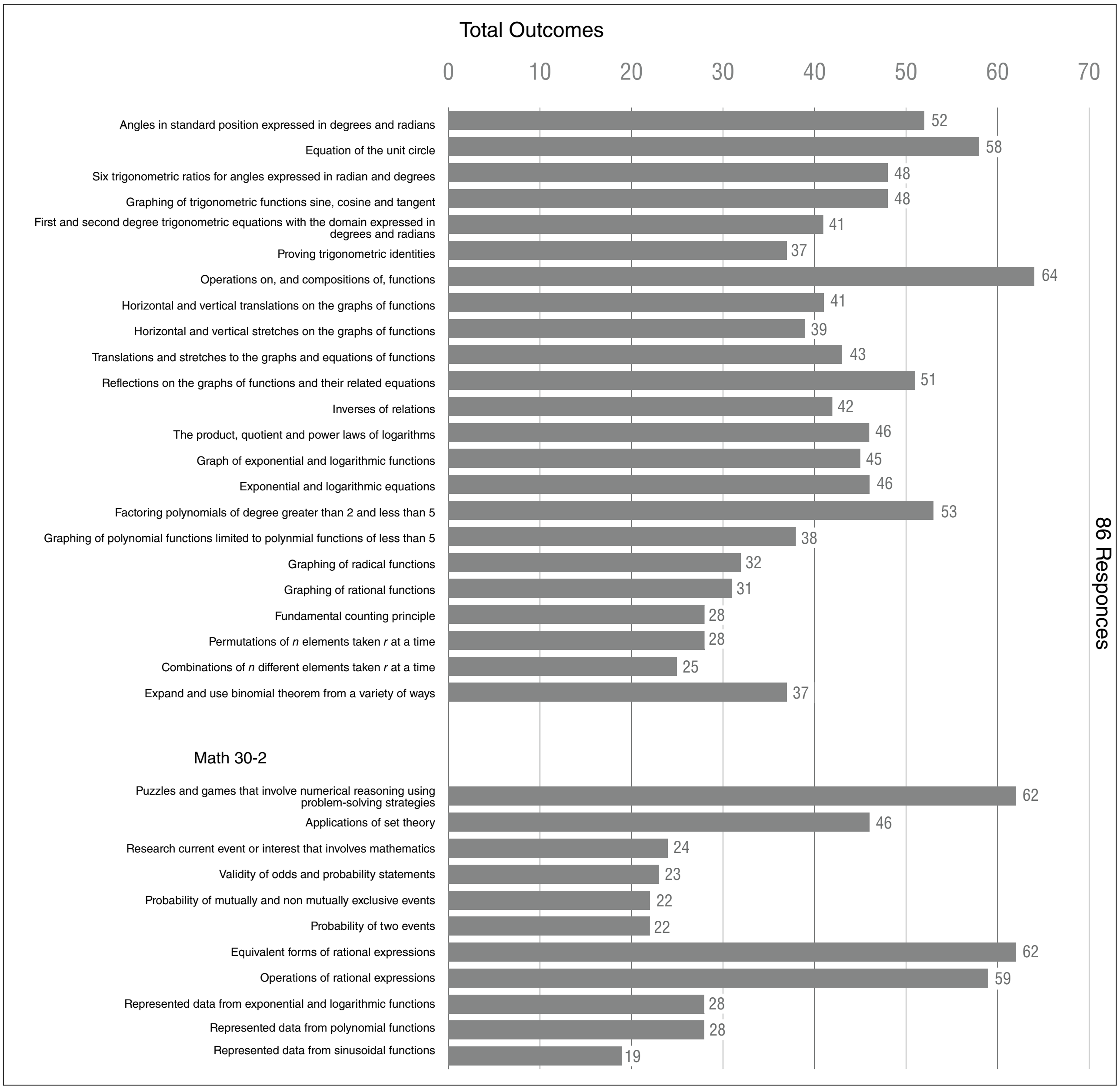
Data was summarized in terms of the number of times a specific outcome was selected. Responses to open questions were analyzed using thematic analysis (Braun and Clarke 2006); themes were identified within questions, and responses across questions were analyzed for global themes. Additionally, comments that were particularly poignant or noted an alternative perspective were highlighted for comparison. The quantitative data is presented first and the qualitive data is presented second.

Outcome Frequency

Figure 1 illustrates the number of times each outcome was selected as an appropriate prerequisite for a first-year university mathematics course.

As illustrated in Figure 1, the five outcomes that were most frequently selected as important for first-year mathematics courses were equation of the unit circle, operations on and compositions of functions, puzzles and games involving numerical reasoning, equivalent forms of rational expressions and operations on rational expressions. Three of these five are

FIGURE 1. *The frequency of selection for each outcome in the Mathematics 30-1 and Mathematics 30-2 courses.*



only addressed in the Grade 12 "-2" course, though rational expressions outcomes also occur in the Grade 11 "-1" course, implying that the students completing the "-1" Grade 12 course would have also been introduced to those outcomes.

For purposes of analysis, the first-year university courses were split into four categories: calculus, statistics, reasoning and linear algebra, allowing us to relate outcomes to specific course style to determine if the outcomes from the "-1" or "-2" courses were mapped as intended by Alberta Education. Tables 2 through 5 illustrate the top selected outcomes for each first-year university course type.

In Table 2, we can see that the outcomes from Mathematics 30-1 map strongly to first-year calculus courses. Thus, it appears that "-1" is an appropriate prerequisite for calculus.

Identified prerequisites for linear algebra (Table 3) were primarily found in Mathematics 30-1, possibly due to the abundance of trigonometric outcomes in the course. Linear algebra is not a calculus course, but currently Mathematics 30-2 does not offer necessary background. Adding additional trigonometry to the "-2" course might help it align more clearly with linear algebra.

In reasoning courses (Table 4), students learn higher arithmetic, discrete math and proofs. Here, the majority of outcomes were chosen from Mathematics 30-2; the U of A and MU are the only institutions of the five that take Math 30-2 as a prerequisite for their reasoning courses.

All but 2 of the 12 top outcomes for statistics align with Mathematics 30-2, which is consistent with the intention that the "-2" sequence be a prerequisite for courses not involving calculus (Table 5). However, many statistics courses require Mathematics 30-1.

TABLE 2. *Calculus Courses—Top Selected Outcomes*

Outcome	Course	Responses (41 total)
Equation of the unit circle	"-1"	41
Graphing of trigonometric functions sine, cosine and tangent	"-1"	41
Angles in standard position expressed in degrees and radians	"-1"	37
The product, quotient and power laws of logarithms	Both	37
Equivalent forms of rational expressions	"-2"	37
Six trigonometric ratios for angles expressed in radians and degrees	"-1"	36
Factoring polynomials of degree $x \geq 2$ and $x \leq 5$ with integral coefficients	"-1"	36
Operations on, and compositions of, functions	"-1"	35
Graphing of exponential and logarithmic functions	"-1"	35
Exponential and logarithmic equations	Both	35

TABLE 3. *Linear Algebra Courses—Top Selected Outcomes*

Outcome	Course	Responses (20 total)
Operations on, and compositions of, functions	"-1"	16
Puzzles and games that involve numerical reasoning using problem-solving strategies	"-2"	16
Equation of the unit circle	"-1"	15
Angles in standard position expressed in degrees and radians	"-1"	13
Factoring polynomials of degree $x \geq 2$ and $x \leq 5$ with integral coefficients	"-1"	12
Applications of set theory	"-2"	11
Six trigonometric ratios for angles expressed in radians and degrees	"-1"	10
Reflections on the graphs of functions and their related equations through the x, y, axis and $x = y$	"-1"	8
Proving trigonometric identities	"-1"	8
Equivalent forms of rational expressions	"-2"	8

TABLE 4. *Reasoning Courses—Top Selected Outcomes*

Outcomes	Course	Responses (8 total)
Puzzles and games that involve numerical reasoning using problem-solving strategies	"-2"	8
Applications of set theory	"-2"	7
Equivalent forms of rational expressions	"-2"	6
Fundamental counting principle	Both	5
Operations of rational expressions	"-2"	5
Permutations of *n* elements taken *r* at a time	Both	4
Combinations of *n* different elements taken *r* at a time	Both	4
Research current event or interest that involves mathematics	"-2"	4
Operations on, and composition of, functions	"-1"	3
Inverses of relations	"-1"	3
Expand and use binomial theorem from a variety of ways	"-1"	3
Validity of odds and probability statements	"-2"	3

TABLE 5. *Statistics Courses—Top Selected Outcomes*

Outcomes	Course	Responses (18 total)
Fundamental counting principle	Both	17
Permutations of *n* elements taken *r* at a time	Both	17
Combinations of *n* different elements taken *r* at a time	Both	17
Validity of odds and probability statements	"-2"	17
Probability of mutually and nonmutually exclusive events	"-2"	17
Probability of two events	"-2"	17
Puzzles and games that involve numerical reasoning using problem-solving strategies	"-2"	16
Expand and use binomial theorem from a variety of ways	"-1"	12
Applications of set theory	"-2"	12
Reflections on the graphs of functions and their related equations through the x, y, axis and $x = y$	"-1"	11
Equivalent forms of rational expressions	"-2"	11
Operations of rational expressions	"-2"	11

Qualitative Data

At the end of the survey we asked a series of open-ended questions about specific topics they wanted included in or removed from the secondary mathematics program, and about strengths and weaknesses of students before and after the curriculum changed. General response questions also allowed them to comment on other matters they deemed relevant. A total of 43 university faculty and instructors representing each of the five universities responded to the open-ended questions. There were eight open questions that garnered 152 comments. Comments ranged from one word to paragraph responses. Regardless of the prompt, comments centred on the following themes: specific mathematics content, specific mathematics skills, and student attitudes and behaviours. Themes are presented below along with illustrative examples of specific comments.

Specific Math Content

The prompt that received the most comments regarding specific math content was the one asking participants what topics should be included in the secondary mathematics curriculum. Of the 152 comments, 16 focused on logical reasoning and proof, noting this as either a weakness or as something that needs to be added and stressed in secondary mathematics. One respondent commented:

> Logical reasoning is the main thing. Basic understanding of propositional logics: negations of statements, the logical connectors "AND" and "OR" as well as the extremely important "IF… THEN…." Ideally also basic exposure to statements with quantifiers. Basic use of logical deductions. Familiarity with valid arguments, and with fallacies (invalid arguments).

Even on the questions that asked for strengths of students, the comments regarding logical reasoning and proof identified this as a weakness and as an important aspect to mathematics learning overall.

Other content areas mentioned multiple times included manipulating fractions (10 mentions), geometry (7 mentions), statistics (4 mentions) and calculus (3 mentions). Being able to work fluently with and understand fractions was noted as a weakness of students in first-year university classes after 2012. One participant commented: "I think whether or not students properly learn to manipulate fractions at the junior high level (and practise it until it is second nature) plays a bigger role in determining their success in later courses than any particular curriculum choices at the high school level." Geometry (classic plane geometry, Euclidean geometry, analytic geometry) was noted as a topic that could be more highlighted or included in the secondary mathematics curriculum, as was statistics. One participant detailed particular requests: "How to collect data for mathematical or statistical problems. How to make interpretations or conclusions from calculations based on the data. Limits of the validity of the conclusions given the limits of the data." Two responses suggested removing calculus from the secondary program; another identified limits as a common weakness.

On the positive side, one participant noted: "I think the cohorts I taught around 2010, give or take a couple of years, were the strongest I've had. The pre-Engineers were superbly prepared in every way for introductory Calculus." A second participant commented that students are "generally able to handle algebra and trigonometry with a 70 per cent or higher in Math 30-1 or Pure Math 30."

Specific Math Skills

The four skills that are mentioned repeatedly throughout the open-ended responses are student skill in the use of calculators (15 mentions), algebraic manipulation skills (15 mentions), problem-solving skills (12 mentions) and being able to connect mathematical ideas (10 mentions). Of the 15 comments regarding calculators, 3 referenced student use of calculators as a strength, for example, "The strength is using calculators for problem solving." The rest of the comments had to do with over-reliance on calculators. One respondent stated:

> A first step that doesn't require huge changes to the whole system is to take away their calculators 90% of the time. Maybe keep them only for some experiments here and there. They are not allowed them in university, so if you want to align things, start there. At the very least, they'll be able to do the basic operations and we can build from there. Also not having calculators suddenly causes the students stress and fear, and half the battle is fighting that fear.

Algebraic manipulation is a broad term that includes student skill in specific processes (factoring), algorithms (division algorithm) and symbolic computation. The comments regarding algebraic manipulation focused on both the weaknesses seen in students and the importance of this skill in mathematics. One participant specifically noted that he would "like students to arrive fully able to perform the division algorithm, manipulate fractions and factor polynomials. It would help if students could perform the division algorithm on polynomials." Another stated: "Make sure students master the standard algorithms for addition and so on. Start early with algebraic symbolic computations. Other than some basic algorithms, teach more concepts and less algorithms."

The comments regarding first-year university student problem-solving skills included analyzing a problem in order to solve it, applying learned content in new contexts, combining concepts to solve a problem and recognizing that there might be more than one way to solve a problem. One participant noted that she has "seen a significant decline in problem-solving abilities, handling of word problems." A second participant stated:

> Students are completely unwilling to experiment when faced with a problem they haven't seen before. The suggestion to "try something and see if it works" seems like a completely foreign concept… In tutorials students will insist that they are unable to proceed with solving a problem until they are given step-by-step instructions to follow.

These two comments indicate a perceived decline in student ability to solve nonroutine problems. Other comments reflect the importance of problem solving

The most frequently identified issue (5 out of 11 comments) was lack of perseverance.

to not only mathematics but student success in life in general: "Understanding how to formulate a problem and logically analyze it will be used by most of our students in every job they encounter."

The ability to make connections between concepts in mathematics was identified as a weakness in students both before and after 2012. Comments focused on the importance of being able to link ideas and see mathematics as a connected set of ideas as an important aspect of knowing and doing mathematics. For example, "Students struggle with the bigger picture of course material and typically memorize formulas/procedures" and "Students still see things as disjoint modules and are unable to see how they were connected." Knowing the connections between topics and how they work together were identified as lacking in current and former students.

Student Attitudes and Behaviours

In addition to comments regarding specific content, participants identified general skills and behaviours of students that they have noticed changing over the past few years. The most frequently identified issue (5 out of 11 comments) was lack of perseverance. One participant noticed an "inability to focus on problems for long periods. They tend to give up, rather than struggle with a complex problem." In addition, students appear to be "less willing to put in effort to learn" and "view rigorous thinking as a waste of time." Decreased willingness to put in effort may stem from having a "lack of self-confidence in the area" or from math anxiety: "Students feel anxious about how to begin to solve a math problem."

Other participants identified general attitudes and behaviours not necessarily specific to mathematics: "In general, it's not about topics and skills, but they don't have an effective approach to learning," which points to an issue that is larger than mathematics specifically. Student expectations from faculty and instructors have also changed. One participant stated:

> Present day students feel they are more "entitled" than earlier students. In a mathematics class, this becomes the expectation for the course to be a series of "type-problems" which they must learn to recognize and solve by a rote "filling in the blanks" procedure, using computations relevant to the present problem.

This is clearly antithetical to the rational thinking needed in the modern world.

This respondent sensed that students are expecting faculty or instructors to be able to categorize each type of problem that they might encounter so that the procedure to solve that problem flows from the category the problem fits within.

Three participants provided comments that focused on positive changes: "Students are very efficient and quick to arrive at an answer by closely following examples in the notes" was noted as important, but not necessarily desireable as this potentially leads students to being uncomfortable with attempting problem types that they have not seen before. One person commented that "students are more willing to work with each other on projects and are not as fearful about speaking in front of others. I think schools are doing a great job of developing the soft skill set for students." One commented that student ability in "identifying patterns" was stronger since 2012.

Discussion

There was not a noticable difference in knowledge, skills or behaviours in first-year university mathematics students that might be attributed to the changing of the curriculum in 2012. Many of the comments from university mathematics faculty and instructors indicated that they were unsure of the difference or that there was little or no difference; some were not teaching in Alberta before 2012. Questions regarding student strengths (both before and after 2012) prompted reference to weaknesses in 13 of the 38 responses; 10 of the respondents stated that there were no strengths.

Of the 16 responses to the question regarding what should be removed from the secondary mathematics curriculum, 11 identified lack of familiarity with the content of the secondary mathematics curriculum. However, 16 recommendations for what should be included are for topics that are currently in the secondary mathematics program of studies; these included translations of absolute value functions (Mathematics 20-1 Grade 11), set theory (Mathematics 30-2 Grade 12) and solving linear equations (Mathematics 10C Grade 10). These comments could be due to student claims that they had not been introduced to those topics before, the inability of students to recall specifics about those topics or inappropriate prerequisites for university courses.

Because respondents were self-selected, the current survey may indicate stronger opinions than are present in the general instructor population. Nonetheless, the responses indicate a need for more communication between secondary and university faculty about (1) what is important in mathematics teaching and learning at both levels and (2) how high school and university courses might be better aligned in terms of what is offered in Math 30-1 and Math 30-2.

Conclusion

Based on the data collected, university preferences for Mathematics 30-1 as a prerequisite for first-year mathematics courses is appropriate for calculus and linear algebra courses, but less so for reasoning and statistics courses. Based on both the numerical and anecdotal data, there does not appear to be a common understanding among university faculty and instructors about the expectations of secondary mathematics courses. Furthermore, the curriculum change in 2012 did not seem to have a noticeable impact on student strengths in first-year mathematics courses as might have been initially thought. With this in mind, continuing conversations between university faculty and instructors, secondary teachers and curriculum developers will support clearer alignment between high school and university mathematics courses. Additionally, in a time of curriculum development in Alberta, including not only those university faculty and instructors who teach primarily first-year courses but also faculty and instructors from colleges and technical institutes could help make the transition from secondary to postsecondary less difficult for students in mathematics.

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Making Sense of Money

Lisa Brooks and Janet Andreasen

It is said that money makes the world go round. While that may be debatable, what is clearer is that Grades K–12 students likely have different experiences with money than what most of us teachers had when we were growing up. This presents unique challenges to our instruction. In an effort to address these challenges, we will explore how to simplify the complex topic of money and provide meaningful ways to make connections between money concepts and other topics in mathematics. We will start by focusing our attention on coins.

The use of a structure that moves from concrete to representations and finally to abstract.

When was the last time you used coins in exchange for a purchase? We are guessing that this may not be a regular occurrence. Since the creation of checks and then credit and debit cards, not to mention Venmo, PayPal and so on, our experiences with coins, and money in general, have changed. Now think about the students we teach. Do they have experiences on a regular basis with money or, more specifically, with coins? Chances are, when students arrive in your classroom, they do so without the foundational understanding of coins and their values. So then, how and why do we teach these skills? Perhaps, more important, how do we help students make personal connections to coins and their values and use them to develop a deeper understanding of number sense in the process?

One important aspect that helps students make personal and meaningful connections to a concept is the use of a structure that moves from concrete to representations and finally to abstract (CRA) (Fyfe, McNeil and Borjas 2014). One must remember this sequence when the goal is to build understanding. Students need tangible experiences to make meaningful connections to concepts (Heibert et al 1997). Considering coins as necessary concrete items is logical; however, they do not provide the components we look for regarding concrete items. Each coin represents a value. This is different than base-10 blocks, for example. Consider the concrete manipulative we use when teaching our base-10 number system. Base-10 blocks provide a concrete model that directly shows ones, tens and hundreds. They allow students to make sense of the 10-to-1 relationship between individual units and 10 rods. Students can physically count out ones until they determine that they can trade 10 ones for a 10 rod. They can line up 10 ones to be the same length as one 10, so they can visually see the connection between the place values. This supports the students as they learn to unitize, that is, to view a 10 as a unit in and of itself as well as a set of smaller units. Later, they may draw representations of the blocks and then, finally, they are able to use a numerical representation of the blocks. The numbers would be considered abstract. Let's consider how coins are different and therefore not as concrete as we might recognize.

The one-cent coin has been taken out of circulation, but we still need to think of its value in relation to the other coins. Let's think about the next value of the coin. The nickel, while a tangible item, is not a concrete tool for understanding its value of five cents. The dime, which is worth ten cents, is actually smaller than the nickel, further confusing the matter. When students hold a nickel or a dime, it is likely that their names will not come easily, because their names don't offer any indication of five or ten. So, while actual coins or plastic coins have a place in the classroom and in instruction, they are not a helpful starting place in regard to building personal connections and meaning related to the value of coins.

Now, let's consider how we might relate money concepts to number sense. The purpose in doing so is to capitalize on what students are working toward in regard to understanding the relationships between values such as 5, 10 and 25, and to help students make connections between these concepts and the work they do with coin values.

When and How to Teach Money Concepts

It may be difficult to consider when and how to teach money concepts when they are not explicitly stated in the mathematics standards for your grade level. It may also seem necessary to hold off on teaching about coins until students have had the opportunity to develop number sense. We believe you can effectively teach both concepts at the same time. In fact, by bringing in concepts of coin values, you bring meaning and purpose to both skills. In a sense, you give students the opportunity to make more connections to model numbers in various ways. This can help them build their sense of number while also building an understanding of coins.

Starting as early as kindergarten, students can use coins for counting.

One step toward intentionality is to explore the standards indirectly related to money. Table 1 provides examples of typical mathematics standards that can be connected to work with money concepts. They are nonspecific to any particular region and are likely found in your mathematics standards. These standards can help your students to build a foundation of number sense and operations that they may then be able to apply to their work with money. Students will benefit from explicit connections being highlighted during instruction.

TABLE 1. *Typical Standards Where Money Can Be Connected*

- Say the number sequence
- Subitize
- Relate numerals to quantities
- Represent and describe quantities
- Compare quantities
- Understand addition and subtraction with whole numbers and decimals
- Understand multiplication and division with whole numbers and decimals
- Recognize and use patterns
- Solve problems with whole numbers and decimals
- Understand ratios and rates
- Understand integers and operations with integers
- Represent algebraic expressions
- Solve algebraic equations and systems of equations

Notice that initially students are benefiting from meaningful connections to build understanding. Later, they extend these connections as they make sense of problems related to money. When you teach related standards, it is appropriate to use coins even though they are not specifically stated in the standards. For example, starting as early as kindergarten, students can use coins for counting. Assuming you can find a collection of 1-cent coins, students can use them to fill in five frames and ten frames to explore one-to-one correspondence, matching and comparing. They can even explore concepts related to equivalence. Students can also explore showing values in different ways using a variety of coins. They can use frames to show and combine coins up to 20 cents. Later, they can work with multiple frames up to the value of 100 cents. Collections of nickels and dimes can be useful for providing experience in skip counting while, at the same time, reinforcing the value of the coins.

Coins and number frames provide a context that students can use as they learn the value of coins. For example, when you provide students with experiences working with a five frame and then a ten frame, they learn to use these amounts. That is, they learn to see a complete frame and recognize that amount as a unit of five or as a unit of ten as opposed to only five ones or ten ones (Fosnot et al 2001). As they develop this skill, they are making connections to the values of the coins, and they are deepening their concept of unitizing. The more intentionally we address these concepts, the more likely our students are to build understanding and to have a framework for applying flexible thinking about money concepts as well as number sense.

Money Sticks

Linking coin concepts to number sense is helpful, but students may also benefit from having a concrete model to use during instruction. In an attempt to make the coins more tangible and concrete, we can use a model called money sticks (Brooks 2017). This model may help your students make connections with what they are learning in regard to number sense. It also provides an experience that sets the stage for working with different combinations of coins. “Money Sticks are a helpful tool for bridging the gap between a tangible representation of a number and the more abstract idea of recognizing the values of coins” (p 174).

To make this model, you will need wide craft sticks, copper-coloured paint, a circular paint dobber (or anything that will transfer a small paint circle onto the craft sticks) and a collection of the coins you wish to use. How to make the money sticks: make five evenly spaced copper-coloured circles on one side of as many sticks as you would like to create. Each stick will become a concrete representation of a nickel (see Figure 1).

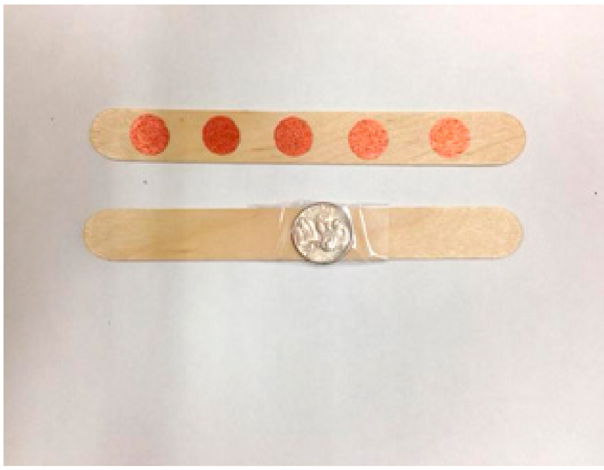

FIGURE 1. *Money stick front and back.*

What you do from that point is up to you, your imagination and your students' needs. You might start by simply having the students practise their counting skills by using one-to-one correspondence to count the dots on the stick. You can use this basic model just as you would a five frame. Students could also work on developing the skill of using one-to-one correspondence as they place a penny on each circle. They could use this model as they would use a five- or ten frame to model different numbers. Later, they can use multiple sticks to work on skip counting. You can decide to focus first on the copper dots as ones and fives, or you can talk about them as ones and pennies while also calling the stick a five and a nickel. To move from identifying the value of five to working with coins, we can replace the copper dots with pennies, and we can also secure a nickel on the back of the stick as shown in Figure 1.

Once students are accustomed to seeing the actual pennies on the stick, the copper dots will suffice for a representation of pennies. The nickel taped to the back of the stick provides the abstract representation of five, and if students forget the value of the nickel, they can flip the stick over and count each of the copper penny dots. Students may count several nickel sticks together and again, while counting, if they lose track, they can easily flip the sticks over for the needed support of counting by ones. You could also include the coin names in written form on a small piece of paper or label each side with the value of the coins. So, you might have the word form nickel and five on the back of a money stick to serve as a reminder to the word and value of the coin.

To highlight the connection of the relationships between the coins, use packing tape to secure two sticks together and include a dime on the opposite side (see Figure 2).

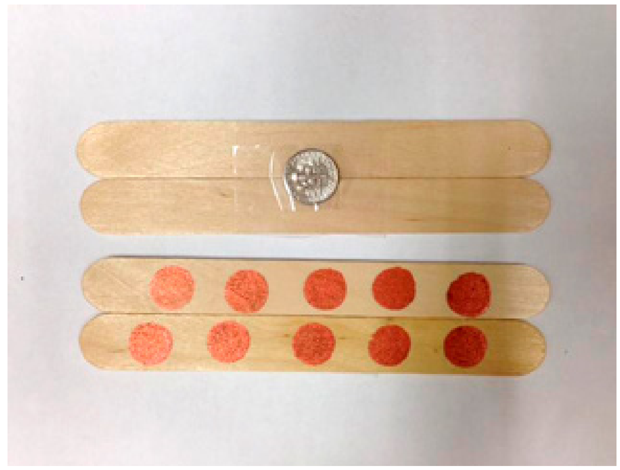

FIGURE 2. *Front and back of two money sticks taped to model a dime.*

If you create a collection of sticks, you can include values of 5s, 10s and 25s. Students could then combine sticks to show the value of a greater coin (see Figure 3).

For example, students may model 25 cents using five sticks that have been taped together and then explore other combinations to model 25 cents, such as two 10-cent sticks and one 5-cent stick (see Figure 4).

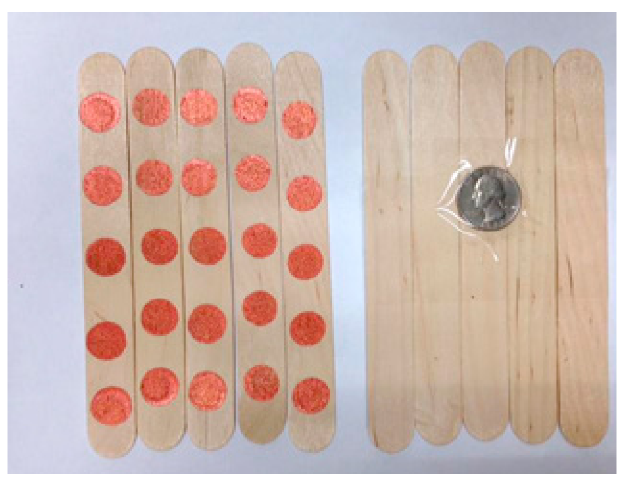

FIGURE 3. *Front and back of five money sticks taped to model a quarter.*

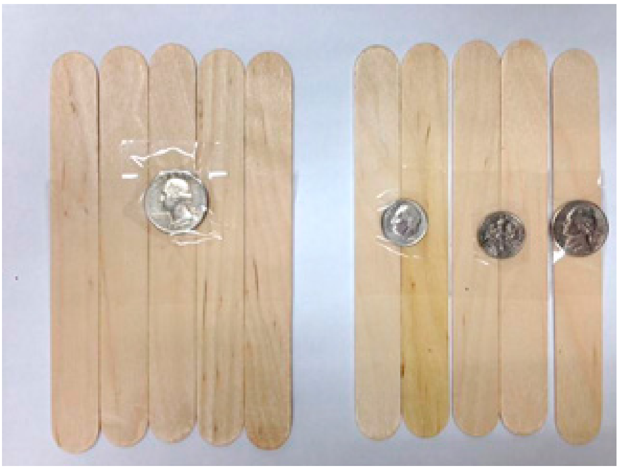

FIGURE 4. *Combinations of money sticks to model a quarter.*

Using clear packing tape creates a flexible model. Holding the model of five sticks taped to show the value of a quarter, you can fold four of the sticks toward the back and see that you have the value of one nickel facing forward with the other sticks facing to the back. This provides a way to see how the sticks work together to create the total value. Students can apply the same reasoning they use when composing and decomposing numbers to compose and decompose different amounts of money.

To engage students in problem solving, have them find different combinations for a given amount. This can be an open-ended task or you can include such limitations as the number of coins they can use. Students can create questions and share with others to solve. They can also show you how they can model a given amount with the fewest number of sticks or the greatest number of sticks. For example, they can model 50 cents with two quarter sticks or with 10 nickel sticks. You can also ask students how many different values they can make with four sticks. Remember to have students flip the money sticks so they are looking at the groupings and not the individual penny dots.

Money sticks are likely to be a helpful tool to solidify students' understanding of numbers and of the value of coins. As students progress in their understanding of both number and coin values, the sticks can be replaced with actual coins and abstract representations of numbers. The foundation they build with this model in the early years can be applied when they are required to use sophisticated strategies for thinking about money in the secondary grades. As students transition to upper elementary grades and high school, the actual coins become less critical to use, and the application of problem solving to money increases. The importance of students' understanding of the value of coins, however, does not diminish. The money sticks can continue to be used in modelling operations with whole numbers including repeated addition to find the value of multiple sets of particular coins. For example, students may be asked to determine the value of a specific number of quarters. The money sticks can be used here to repeatedly add the quarters and then to model multiplication with groups of objects. Additionally, problem-solving situations can be presented to students involving coins using the context of allowance.

> Jennifer gets an allowance of 85 cents per week. What different combinations of coins could Jennifer's parents give her each week?

Later, students begin to explore ratios and rates. An understanding of money is not a critical component when learning these concepts; however, in addition to other rich contexts, providing contextual problems situated with money can help students make sense of ratios using cents. Likewise, when they encounter situations, either within word problems or in real life, a solid grasp of money concepts will be beneficial. For example, students may be asked to compare the value of a candy bar given a size and cost.

> A candy bar that weighs 8 ounces costs 42 cents. A candy bar that weighs 12 ounces costs 65 cents. Which candy bar is the better deal?

If students are lacking understanding of coins and money, these types of situations may be difficult. Students ultimately reason with unit rates and need to understand that the most common rate in this case is cost per ounce, or cents per ounce.

As learning transitions to algebraic reasoning, the situations often will involve money and will connect to rates and repeated addition. For example, students may encounter attending a local fair where it costs \$5 to get into the fair and \$2 per ride. Students should be able to use their knowledge of money, repeated addition and rates to determine that \$2 per ride is the multiplicative factor in the equation. Additionally, using unit analysis helps students see that they need to multiply the rate (dollar per ride) by the number of rides to get the cost. Since the cost to get in (\$5) is in dollars and the total cost (C) is dollars, the cost for the rides ($2R$) should be in dollars as well. This generates an equation like $C = 2R + 5$ where C is the cost of attending the fair and R is the number of rides. Students then explore this scenario using tables, graphs and equivalent equations. This can then be connected to further algebraic ideas of systems of equations. Take the local fair situation and extend the model to two equations.

> The fair has child tickets and adult tickets. Child tickets cost $4 and adult tickets cost $6. A family bought 6 tickets and spent $36. How many child tickets and how many adult tickets did they buy?

Students' understanding of coins and money can be used to support mathematical reasoning throughout the grade levels.

As students explore this context, they use their knowledge of money and rates. They need to reason that if each child ticket costs $4, the cost of C child tickets would be 4C. Likewise, if each adult ticket cost $6, the cost of A adult tickets would be 6A. Combining these together would give the total money spent to get into the fair, in this case $36. This gives an equation of $4C + 6A = 36$. This isn't enough to solve this system, however, since there are two unknowns. The situation also indicates that the family bought 6 tickets. Here students need to recognize that A and C are the number of each type of tickets purchased, so $C + A = 6$. Once the students reason through determining the equations, they can then explore the solution using tables, graphs and algebraic methods.

Students' understanding of coins and money can be used to support mathematical reasoning throughout the grade levels. It provides meaningful context for developing number sense and an understanding of the value of the coins, using the context of money and coins in problem solving related to operations with whole numbers, reasoning about rates and ratios, and connecting algebraic contexts to situations involving money. When we focus on student understanding, we provide the means for them to make meaningful connections. It is our hope that you find these strategies helpful as you support your students in making sense of money.

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Fu-Go: Mathematical Investigations of Japan's World War II Balloon Attack in the Secondary Classroom

Josh Markle

In this paper, I recount using the Fu-Go program, an offensive launched by the Japanese Navy in World War II, as a provocation for mathematical inquiry in the classroom. Drawing on my experience as a secondary mathematics teacher in an alternative high school, I suggest some ways case studies such as this can be used to foster rich conversation and develop a disposition of expertise in the mathematics classroom.

Introduction

It is dawn in early winter 1944. At the outskirts of a small town on Japan's eastern seaboard, members of the Japanese Navy are assembled to launch one of its most cunning attacks of World War II. Lifting off Japan's coast into the cool morning light are a half-dozen balloons, each laden with incendiary devices and destined for the US mainland. It would have been a sight to behold: each balloon was approximately 10 metres in diameter with a volume of nearly 19,000 cubic feet (Mikesh 1973, 40). Appended to the bottom was a series of altimeters, each connected to a fuse designed to release sandbags along its trans-Pacific flight. If a balloon's flight went according to plan, ordnance intended to ignite wildfires and sow panic in the American populace would follow the last of the sandbags. This was Japan's Fu-Go weaponized balloon program.

Although the program, named Fu-Go after the Japanese word for balloon (*fusen*), was conceived in the late stage of World War II, Japan's experimentation with balloons dates back much earlier. Decades prior, a Japanese meteorologist, named Wasaburo Ooishi, used small weather balloons to collect data showing wind speeds in excess of 260 kilometres per hour. Perhaps most important, Ooishi established a definite pattern in the seemingly capricious winds that blew high above the island: they were seasonal, blowing far stronger in the winter months, and were relatively stable at high altitudes, between 30,000 and 35,000 feet. This was the reason for the complex apparatus appended to the balloon's undercarriage: as the hydrogen inside the balloon's paper envelope cooled and heated along its flight path, it was necessary to cut weight or vent gas in order to stay within the strong wind currents. If successful, Japanese meteorologists were able to establish a trans-Pacific flight time of 30 to 100 hours.

Looking at Fu-Go in the Mathematics Classroom

I have used Fu-Go both as primary material for an assignment in high school mathematics classes and as a lesson planning provocation in my work with preservice math teachers in their curriculum and instruction courses, all to great effect. In the former case, I taught Math 20-2 and 30-2 at an alternative high school in a small, western Canadian city. My classroom was diverse. There were students who had already graduated high school but were upgrading, who were taking this course to meet the requirements of a postsecondary program and who had had to leave classrooms in their home schools for myriad reasons, such as low achievement and poor attendance. Though diverse, the students were generally bound by one commonality: they had all experienced a lack of success in the mathematics classroom.

The "dash-2" Alberta mathematics curriculum allows for considerable depth and breadth. In addition to outcomes oriented around particular mathematics, such as quadratic functions, the Math 20-2 and 30-2 curricula require students to engage in a mathematics research project. Specifically, students must research a historical event or area of interest (Alberta Education 2008, 27). There are many opportunities to incorporate history into mathematics, in particular the history of mathematics itself. Fauvel (1991) makes a compelling case for including the history of mathematics in teaching mathematical concepts. Panagiotou (2011) describes doing so in teaching logarithms in the secondary classroom. In this case, we looked not so much to a particular historical development in mathematics, but rather what a historical event might reveal to us about our current understandings of mathematics, in particular quadratic equations.

Although it is common to tackle this kind of research project toward the end of the semester, several factors made this difficult in our context. As noted above, the students I worked with often experienced difficulty in attending regularly and frequent disruptions in their coursework. To compound this, our class was structured so that students could work at their own pace on several of the course's components. Although convenient for many students, this aspect of the course made it difficult to implement a large final project. To mitigate this, I curated a small selection of historical events and areas of interest that we could investigate as a class as we proceeded through the curriculum. The Fu-Go project was one of the most engaging and effective.

There are two texts indispensable in knowing more about Fu-Go. Mikesh (1973) is closer in nature to a technical manual and is full of schematics and maps; Coen (2014), also thorough in its detail of the program, better captures the social contexts of the American and Canadian responses to the attacks. Both are excellent resources. But perhaps the most accessible and engaging treatment of the topic is an hour-long RadioLab podcast (Abumrad and Krulwich 2015). In my work with both high school students and preservice teachers, this podcast is our primary text. In both instances, I allot time in class to listen to the podcast. Students are required to listen through headphones and are given instructions to note anything of interest to them while they listen. Instructional time is always at a premium, but I chose to allocate an hour to listening because I wanted to provide an immersive experience in which students could dwell, notice and wonder. It was also beneficial to begin discussing the material immediately afterward in class.

After listening to the RadioLab episode, we took time to organize our thoughts individually using the following prompts: What do you notice? What do you wonder? This gives students time to organize their notes, to go back to the episode and listen to parts that intrigued or confused them, and it gives me an opportunity to visit briefly with each of them. Once we felt confident to share our insights, we arranged ourselves into small groups to share briefly before having a large group conversation in which all *noticings* and *wonderings* are committed to the whiteboard. The students I have worked with have been unfailingly perceptive. Their observations range from logistic (for example, flight path, construction materials) to technical (for example, number of sandbags, mechanics of the altimeters) to emotional (for example, loss of life). They were equally insightful in their wondering. This included the questions below:

- What and where is the jet stream?
- What factors most influenced flight times?
- What is the farthest a balloon could travel?
- What is the highest a balloon will float?
- Why paper?
- How many balloons made it to the United States?
- What resources did it require to build a single balloon?

To be sure, one wondering always emerges quickly: What does this have to do with math class? And it is a legitimate question, one I believe we sufficiently answer in how we attend to the problem. The first step is deciding as a class which *noticings* and *wonderings* from our lists are mathematical in nature. Because I use this material as part of our work around quadratic functions in Math 20-2 and sinusoidal functions in Math 30-2, I am most interested in identifying those *noticings* and *wonderings* associated with the parameters in those functions, such as the general shape of the flight path, maximum and minimum heights, distance and duration.

A Provocation and Some Possible Solutions

Once we have begun to characterize the nature of our inquiry as mathematic, I offer students the following provocation: How might we describe the flight path of a balloon across the Pacific using what we know about quadratic equations? I have students begin my eschewing mathematical convention and simply drawing. Then we begin to question some of the assumptions in their drawings and how we might be able to characterize these assumptions mathematically. For example, students will often note that the balloon must undulate along its flight path and that the peaks and valleys must represent the upper and lower bounds of the jet stream, respectively. If we interpret this in the context of quadratic equations, this gives us valuable information. Consider a quadratic function in vertex form:

$$f(x) = a(x-h)^2 + k$$

In this form, we know that the vertex is given by (h, k). Assuming the upper bound of the jet stream is 40,000 feet, we can assign a value to k of 40,000.

Then we must consider the time at which a balloon reaches its maximum height (h). Students can make any assumptions they like, provided they are mathematically and physically sound. For example, if a student wants to model a balloon flight to Detroit, they need to make sure their assumptions regarding the flight's duration accurately reflect the distance travelled and the balloon's velocity, each another layer of assumption that requires research, critical thought and discussion. Generally, we first discuss these assumptions as a large group and establish some benchmarks. For example, we typically assumed the jet stream to be between 30,000 and 40,000 feet and a typical flight to the US mainland to take 72 hours (a roughly 7,500-kilometre flight at somewhere around 100 kilometres per hour). After some play with the parameters and discussion, I will ask students to provide three quadratic equations that reflect the assumptions we agree on (for example, maximum height, duration of flight)—this gives students a good opportunity to talk with each other and for me to talk

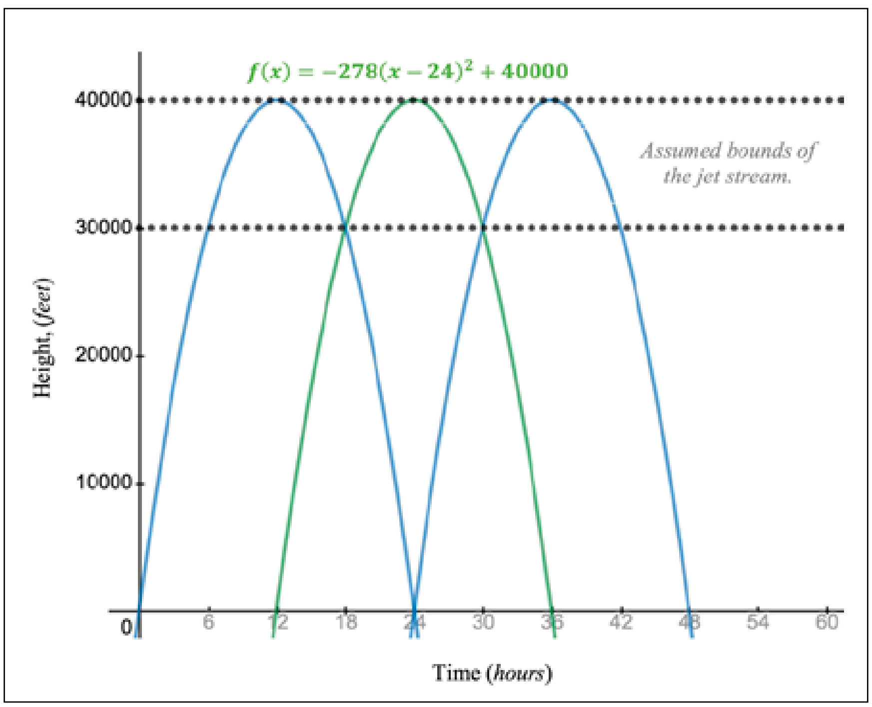

FIGURE 1. *An example of three quadratic equations characterizing a balloon's flight path across the Pacific.*

with each of them. Figure 1 show the graphs of three such quadratic equations, with the equation for the second parabola in green.

Note: A series of three quadratics show the hypothetical path of a Fu-Go balloon (the portions of the quadratic graphs between the two dotted lines). In discussing the limitations of quadratics for modelling this sort of problem, we focus on the "sharp" corners at the bottom of the low points of the travelled path.

In my formative assessment, I am most interested in understanding whether students see the connection between the function's parameters and horizontal shifts of the function's graph. Once I am confident a student has developed a sound understanding of the material, I provide each of them with a unique set of constraints they need to satisfy with a set of three quadratic equations. Constraints might include duration of flight, minimum or maximum number of drops along the flight path, minimum or maximum height, a specific location, specific velocities and so on. Students then present their work, which allows us all to gain further insight into how manipulating a quadratic's parameters affects its graph.

Mathematizing Conversation

In attempting to contextualize quadratics and parabolas, test and assignment questions often resort to situations that are accessible if prosaic, such as throwing a ball or calculating revenue. But we know that quadratics do not perfectly model either of those situations and Fu-Go is no different. The most rewarding aspect of using this material is the rich conversation it generates in class, both mathematic and otherwise. We always made sure to discuss the benefits and limitations of the assumptions we made. By the end of the investigation, students appreciate that quadratic functions are neither the tool with which the Japanese modelled the attack nor the best means to describe the flight path in hindsight. The point is not to recapitulate the development of the balloon or accurately describe its flight path, but to investigate the parameters of a particular class of functions (for example, quadratic, sinusoidal) in order to develop a more robust and intuitive understanding of their constituent elements and how each of the parts affects the whole.

Our discussions outside of the mathematics were also poignant. One interesting aspect concerns the asymmetry in Japanese and American intelligence and is drawn from the two most comprehensive accounts of Fu-Go. Mikesh (1973) notes that the notion of trans-Pacific balloon travel was a given for the Japanese. Officials there were more concerned with the parameters of flight: at what altitude should they travel? When and where should they be dispatched? (p 7). This contrasts the initial response by American authorities when they began collecting evidence of the attacks on US soil. As Coen (2014) writes, officials first theorized that the balloons were being launched from submarines just off the US coast. "There is no question about it," he quotes one US intelligence officer as saying, "[the balloon] couldn't have come from Japan" (Coen 2014, 56). One important reason for incorporating history into the mathematics classroom is to cultivate an appreciation for the reticulated nature of the development of mathematical and scientific thinking across time and cultures.

And although Fu-Go was unsuccessful from a military standpoint, it still had a profound impact. In 1945, a woman and five children were killed when they came upon unexploded ordnance beneath a balloon envelope while on a picnic in Washington state (Coen 2014, 2). According to Mikesh (1973), 17 bombs were confirmed to have landed across Alberta and nearly 40 were located in British Columbia (p 77). One balloon was found in British Columbia as recently as 2014 and there is a good possibility there are others unaccounted for (Abumrad and Krulwich 2015). We drew on many different ideas and fields in our discussions, such as ethics, geography and probability. These discussions belied the idea that mathematics is a collection of unrelated and unrelatable facts. Rather, it is enmeshed with and inextricable from lived experience—a rich source of inquiry and conversation in the math classroom.

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