Edmonton Junior High Math Contest 2018

Part A: Multiple Choice

1. In a triangle, all angles are integer values. The measure of the smallest angle is 20°. What is the measure of the largest possible angle?

   (a) 80° (b) 89° (c) 139° (d) 140° (e) 160°

2. The original price of an item is reduced by 20%, and the new price is further reduced by 20%. The final sale price is the same as a single reduction of what percentage of the original price?

   (a) 30% (b) 36% (c) 40% (d) 60% (e) 64%

3. The first three terms of a *geometric* sequence are 16, 24, 36, ... With the exception of the first term, each term thereafter is the product of the previous term and a constant.

   The first three terms of an *arithmetic* sequence are 30, 45, 60, ... With the exception of the first term, each term thereafter is the sum of the previous term and a constant.

   If both sequences are to continue, for which n does the nth term of the geometric sequence {16, 24, 36, . . .} first become larger than the nth term of the arithmetic sequence {30, 45, 60, . . .}?

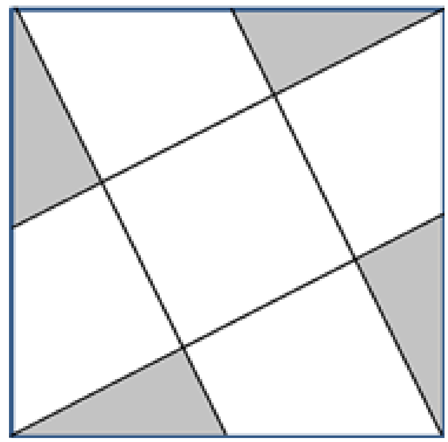
   (a) 4 (b) 5 (c) 6 (d) 7 (e) 8

4. Solve for x.

$$\frac{x}{2{,}018} + \frac{2{,}017(2{,}019)}{2{,}018} - 2{,}019 = -1$$

   (a) −2,018 (b) 2,018 (c) 0 (d) −1 (e) 1

5. Midpoints of each side are connected to one of the vertices of a square. What percentage of the square is shaded?



   (a) 20% (b) 25% (c) 30% (d) 35% (e) 15%

6. In the sequence of numbers 1, 3, 2, −1, . . . , each term after the first two is equal to the term preceding it minus the term preceding that: $t_n = t_{n-1} - t_{n-2}$. What is the sum of the first 100 terms of the sequence?

   (a) 0 (b) −1 (c) 21 (d) 16 (e) 5

7. A woman, her brother, her son and her daughter (all relations by birth) are chess players. The worst player's twin (who is one of the four players) and the best player are of opposite sex. The worst player and the best player are the same age. Who is the worst player?

   (a) woman (b) son (c) brother
   (d) daughter (e) insufficient information to determine

Solutions

1. The two remaining angles have a sum of 160°. Since the measurements of the angles are integers, the next smallest angle should be 21°. Thus, the largest angle would be 180 − 20 − 21, or 139°. The answer is **(c)**.

2. If the item is priced at \$100, a 20% reduction results in a reduced price of \$100 × 0.8 = \$80. A 20% reduction of the reduced price is \$80 × 0.8 = \$64. The twice-reduced price is 64% of the original, or 100 – 64 = 36, a single reduction of 36% of the original. The answer is **(b)**.

3. Each term of the geometric sequence is multiplied by the constant 1.5. Each term of the arithmetic sequence is added to by the constant 15. We can simply list out the next few terms of each sequence.

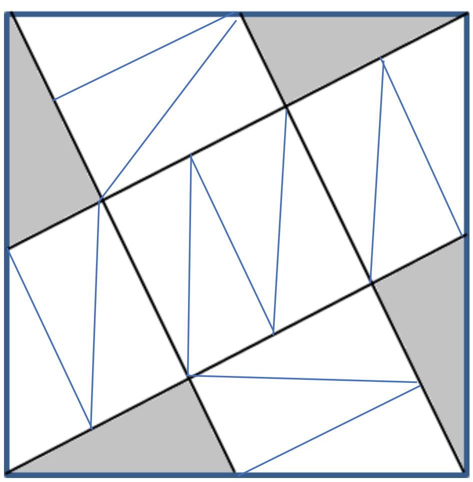
Geometric sequence	16	24	36	54	81	121.5
Arithmetic sequence	30	45	60	75	90	105

We can see that the geometric sequence is larger starting from the sixth term. The answer is **(c)**.

4. $$x = (2{,}018)^2 - (2{,}017)(2{,}019) = 1$$

The answer is **(e)**.

5. The diagram can be subdivided into congruent triangles, as follows.



The entire square is made up of 20 congruent triangles. The shaded part has four pieces out of 20. This means that the shaded part covers 20% of the square. The answer is **(a)**.

6. Writing out the sequence, we have 1, 3, 2, −1, −3, 2, 5, 3, −2, −5, −3, 2, 5, 3, −2, −5 , −3, 2, 5, 3, −2, −5 . . . . After the first four numbers in the sequence, there is a repetition of these six terms: −3, 2, 5, 3, −2, −5. The remaining 96 terms are grouped six terms at a time, and each group has a sum of 0. The sum is

$$1 + 3 + 2 + -1 + (96/6)(0) = 5.$$

The sum of the first 100 terms is 5. The answer is **(e)**.

7. If the son is the worst player, the daughter must be his twin. The best player must then be the brother. This is consistent with the given information, since the brother and the son could be the same age. The assumption that any of the other players is the worst leads to a contradiction.

If the woman is the worst player, her brother must be her twin and her daughter must be the best player. But the woman and her daughter cannot be the same age.

If the brother is the worst player, the woman must be his twin. The best player is then the son. But the woman and her son cannot be the same age, and the woman's twin, her brother, cannot be the same age as the son.

If the daughter is the worst player, the son must be the daughter's twin. The best player must then be the woman. But the woman and her daughter cannot be the same age.

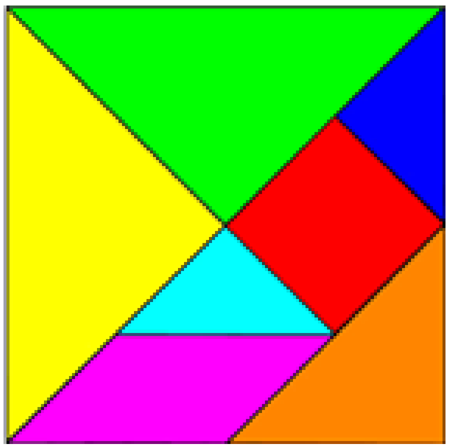
The answer is **(b)**.

Part B: Short Answer

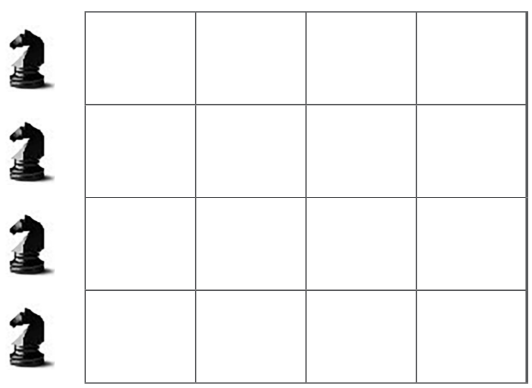
8. A large cube is assembled together with 125 smaller identical cubes. This large cube is placed on a tabletop and spray-painted on five of the faces. After drying, the large cube is disassembled, with all the smaller cubes placed in a bag. How many cubes have exactly two faces painted?

9. Juice cans are sold in a pack of six or twelve. There are 12,448 cans in the warehouse waiting to be put into different packs. If the cans are placed in any combination of six- and twelve-packs, how many cans will be left over?

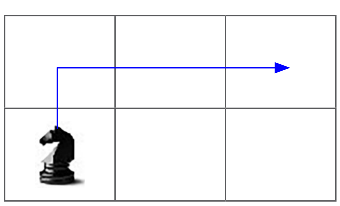
10. Below is a picture of an ancient Chinese puzzle, called a tangram. The seven pieces are arranged to form a large square. If the edge of the large square is one unit, find the perimeter of the smaller square in the picture. Leave the answer in simplest radical form.



11. Sally places four knights on a 4×4 board.

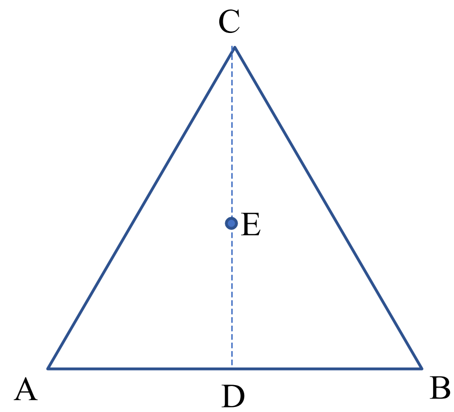


Knights can only attack a square that is 2×3 away.



If any square is attacked by at least two different knights, Sally colours the square black. What is the maximum number of black squares possible?

12. The $\triangle$ABC is an equilateral triangle with a height of $2\sqrt{3}$. E is the midpoint of the altitude CD. Find distance $\overline{AE}$. Leave the answer in radical form.



13. In $\triangle$ABC, $\measuredangle$CAB = 108° and AB = AC. The bisector of $\measuredangle$ABC meets CA at E, and the perpendicular to BE at E meets BC at D. Determine $\measuredangle$ADE.

Solutions

8. Of the twelve edges, only eight edges have cubes that have exactly two faces painted. Each edge has three such cubes, giving a total of $8 \times 3 = 24$ cubes. The answer is **24**.

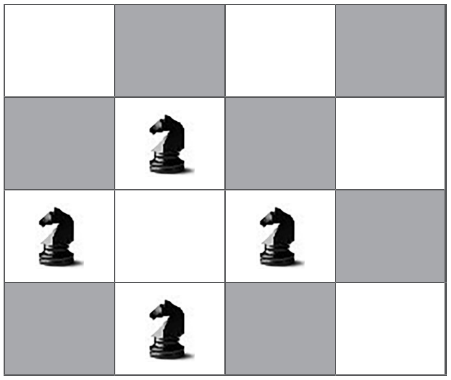
9. Let n represent the number of six-packs, and let m represent the number of twelve-packs. Then

$$6n + 12m = 12{,}448$$
$$6(n + 2m) = 12{,}448$$

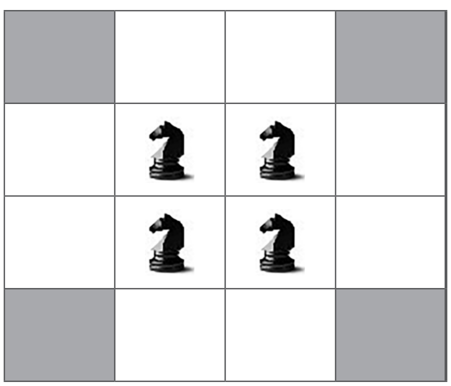
12,448 ÷ 6 = 2,074, with a remainder of 4.

Four cans would not fit into six- or twelve-packs. The answer is **four**.

10. All the triangular pieces (small, medium and large sizes) are isosceles right triangles. This means that they are all similar. The largest triangle measures $\sqrt{2}/2 \times \sqrt{2}/2 \times 1$. The smallest triangle is a reduction with scale factor of 1/2; it measures $\sqrt{2}/4 \times \sqrt{2}/4 \times 1/2$. The side of the small square is the same as one of the legs of the small isosceles triangle, $\sqrt{2}/4$. The perimeter of the small square is $4(\sqrt{2}/4) = \sqrt{2}$. The answer is $\boldsymbol{\sqrt{2}}$ **or 1.414**.

11. 

Where knights will doubly attack all the blackened squares. In total, there are seven black squares. To prove that this is the maximum, we note that any knight in one of the centre squares will attack four squares, and any knight on an edge will attack three squares. Note that the above configuration maximizes the number of black squares, given that two knights are on an edge and two knights are in the centre. We only have four knights, so the only way for Sally to colour eight squares is if all knights are attacking four squares or, in other words, if they are all in the centre.



Clearly, this does not result in eight blackened squares. The answer is **seven**.

12. The △ABC is an equilateral triangle with interior angles at 60° each. The △CBD is a 30° – 60° – 90° triangle with side lengths in the ratio of $1 - \sqrt{3} - 2$. Since $\overline{CD}$ is $2\sqrt{3}$, we must have $\overline{CB}$ = $\overline{CA}$ = $\overline{AB}$ = 4. It follows that

$$\left(\overline{AE}\right)^2 = \left(\overline{AD}\right)^2 + \left(\overline{DE}\right)^2$$
$$\overline{AE} = \sqrt{(2)^2 + \left(\sqrt{3}\right)^2} = \sqrt{7}.$$

The answer is $\sqrt{7}$ **or 2.646**.

13. Extend BA and DE to meet at F. By symmetry, BED and BEF are congruent right triangles. Now ∡EFB = ∡EDB = 90° – ∡EBC = 72° while ∡EAF = 180° – ∡CAB = 72° also. Hence, EA = EF = ED. Note that we have ∡BEA = 180° – ∡CAB – ∡ABC = 54° and ∡DEA = ∡BED + ∡BEA = 144°. It follows that ∡ADE = 1/2(180° – ∡DEA) = 18°. The answer is **18**°.

Part C: Short Answer

14. Each of the digits 0–9 is written on a card. You can select any number of cards from the group. The digits can then be rearranged in any order to form a number. What is the minimum number of cards from which a multiple of 3 with up to three digits can always be formed?

15. A class of 31 students invites some students from another school to the Valentine's Day dinner. There are 19 tables at which one or two students may sit. If each boy has exchanged valentine cards with exactly three girls and each girl with exactly two boys, how many students have attended the dinner?

16. How many different five-digit numbers have the property that if one digit is removed, reading from left to right, the number 2,018 will be obtained?

17. In the school badminton club, there are 18 boys and 18 girls, with 12 in each of Grades 7, 8 and 9. The school wants to enter as many mixed doubles teams as possible in the city championship. The partners in each pair must be from the same grade. What is the minimum number of mixed doubles pairs the school can enter?

18. Consider a non-negative number boring if it is made of only the same digits and cool if it is made of only distinct digits. Single digits (1, 4, 3 and so on) are only cool. Note that digits with repeats (such as 21,330) are neither cool nor boring. What is the smallest positive integer greater than or equal to 11 that cannot be represented as the sum of a boring and a cool number?

19. In △BAD, BD = 3, AD = 4, and AB = 5. C is the point on the extension of BD such that DC = 1. PQRS is a rectangle with P and S on BC, Q on AB, and R on AC. Determine the maximum perimeter of PQRS.

Solutions

14. If we have only two cards, the numbers on them may be 1 and 4, and none of 1, 4, 11, 14 or 41 is a multiple of 3. Hence, two cards are not enough. We claim that three cards are enough. If any of 0, 3, 6 or 9 is on one of them, we have a one-digit multiple of 3. Suppose this is not the case. If we have all of 1, 4 and 7 or all of 2, 5 and 8, we have a three-digit multiple of 3, since 1 + 4 + 7 = 12 and 2 + 5 + 8 = 15 are multiples of 3. If this is also not the case, we take one number from each triple. Their sum will be a multiple of 3, and they will form a two-digit multiple of 3. The answer is **three**.

15. The ratio of boys to girls is 2:3. Hence, the total number of students is a multiple of 5. It is greater than 31 and less than 19 × 2 = 38. Hence, it must be 35. The answer is **35**.

16. If the deleted digit is in the first place, before 2, we have exactly nine choices for adding a digit, because we cannot add 0. It may appear that we have ten choices if the deleted digit is in the second place, between 2 and 0. However, 22,018 has already been counted. So we have only nine choices. Suppose the deleted digit is in the third place, between 0 and 1. We still have only nine choices, since we cannot choose 20,018. Hence, the total number of choices is 9 × 5 = 45. The answer is **45**.

17. Suppose that all Grade 7 students are boys and all Grade 8 students are girls. Then the school can enter exactly six Grade 9 teams. Suppose the number of teams that can be entered is less than six. Then there are more than 24 students with no partners. Such students from each grade must be of the same gender. By symmetry, we may assume that all students without partners from Grade 7 or 8 are boys, and all students without partners from Grade 9 are girls. Since the total number of boys is equal to the total number of girls, we have more than 12 girls with no partners, and they are all in Grade 9. This is a contradiction since there are only 12 students in Grade 9. The answer is **six**.

18. We claim that the smallest positive integer is 110. Note that the difference between 110 and any two-digit boring number will also be a boring number, so there are no cool numbers that can sum with a boring number to obtain 110.

    We now prove that this is the minimum. All two-digit boring numbers are the sum of the given boring number and 0. We know that all two-digit boring numbers are separated 10 numbers, which correspond to adding the cool numbers 1, 2, . . . , 10. Thus, any two-digit number is the sum of a boring number and a cool number less than or equal to 10. For three-digit numbers, note that we only need to consider numbers greater than 99 + 10 = 109, of which there are none that are also less than 110. Thus, the minimum is proved.

    The answer is **110**.

19. We have

$$\frac{BD}{DA} = \frac{BP}{PQ} = \frac{3}{4}$$

and

$$\frac{CD}{DA} = \frac{CS}{SR} = \frac{1}{4}.$$

Note that PQ = SR. Hence,

$$BP + CS = PQ\left(\frac{3}{4} + \frac{1}{4}\right) = PQ.$$

It follows that the perimeter of PQRS is equal to

$$\begin{aligned} 2(PQ + PS) &= 2(BP + CS + PS) \\ &= 2BC = 2(3 + 1) = 8. \end{aligned}$$

Since the value is constant, the maximum value is also 8. The answer is **8**.