Designing Action Mathematics for Low Ability Students

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INTRODUCTION

All of us probably have felt the frustrations of working with low ability groups of mathematics students. They seem frustrated, almost ready to fail, and hence often become resentful of the authority a teacher standing in front of them represents. Yet if we consider ourselves teachers, we have to believe that such students want to learn and in particular want to learn mathematics if given a real opportunity.

What is the nature of an instructional environment that provides this "real opportunity"? It will be the purpose of this paper to describe the basis for one such environment - a mathematics laboratory - to give extensive samples of materials actually used with 14- to 15-year-old students and to comment on the effects of the use of such materials.

The mathematics laboratory seems to be in vogue today. Actually, the laboratory idea can be traced back to the turn of the century and is frequently related to the methodology of progressive educationalists. Whereas the laboratory previously was used to show the social utility of already learned mathematical notions, today's mathematics laboratory is thought of as a vehicle for the actual learning of mathematical ideas.

There is considerable theoretical justification for using activity approaches as instructional vehicles in mathematics. Piaget (1967) suggests that there are two modes by which young children develop new ideas: imitation and play. Piaget (1967) and other researchers (Sutton-Smith 1968, Vance 1969) find that play also has the effect of allowing for new uses of previously learned ideas and that play seems to sponsor acts of idea generation in children. It would seem that as children enter school however, the instructional activities in mathematics seem to emphasize imitation and, in fact, deny this tremendous learning resource of purposeful play. We feel this denial is particularly acute for the low achiever in mathematics. It would seem that the bright successful student in mathematics, particularly in the secondary school, can mentally "play" with the ideas of mathematics. However, the low achiever, particularly those of lower ability, appears to have few symbolic mental images available and seems in need of some structured concrete experiences.

The word "structure" in the last sentence brings to mind ideas of Bruner (1966). Bruner suggests that students need to be faced with problems as a learning tool. Yet these problems cannot be so vaguely stated as to allow rejection by students nor so difficult as to create frustration for students. For the low ability student these two considerations are very important. The materials and instructional procedures must be designed to provide adequate structure and feedback to the student and yet allow for the "playing" with mathematical ideas, and provide motivation for student construction of mathematical ideas.

One such source of motivation arises from the work of Z. P. Dienes. Dienes (1967) suggests that the proper learning of mathematical ideas requires experience with examples of the idea which allow for both perceptual and mathematical variations of the idea. It would seem for the student of low ability that a variety of teaching techniques and settings would also prove valuable.

The above discussion provides the following principles for designing instructional settings for low ability mathematics students:

1. The situation must allow for concrete "play" with mathematical ideas.

2. The situation must pose interesting questions but these must be developed in steps which these students can handle.

3. The material used must be readable.

4. An idea must be developed in *several* concrete settings.

5. The materials and setting must provide adequate feedback to the student.

6. To provide for the above, and to give pedagogical variety, several teaching settings should be used.

The sample materials on the area of a circle which follow are part of a complete package on measurement which the authors developed on the basis of the above principles. They were used with classes of low ability students in the following manner.

A. Parts 1, 2 and 3 were used in mathematics laboratory sessions. In these sessions small groups of students (usually 2 or 3) worked with concrete materials following the direction on the activity sheets provided. These students can be assigned to groups on several bases. Although the students in our experiment did not experience reading difficulties, it is advisable to have at least one reader in a group. Usually the group functioned in such a fashion that one student performed the activity while another recorded results. It should be noticed that while questions are posed, they are in small steps. The idea, in this case the formula for area of a circle, is currently defined in each setting.

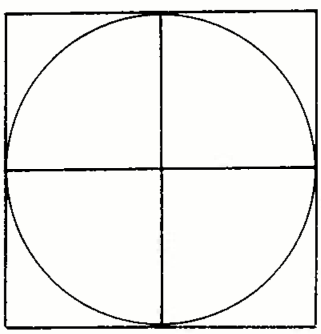
B. Prior to and during the use of part 4 of the material, class sessions were held to review the accomplishments of the labs and to obtain closure on the central ideas. Part 4 of the materials allowed for supervised practice items.

C. Part 5 (a) of the materials again was designed for use by individuals and small groups. In the case of every topic, interesting practical applications were provided. These problems appeared highly motivating and also provided a further application-oriented variation of the major mathematical topics involved.

D. Part 5 (b) is an active extension of part 5 (a). Since many low ability students seem to like problems presented in a large and/or realistic setting, part 5 (b) allowed for some simple but meaningful applied projects.

AREA OF A CIRCLE, PART 1

1. In the diagram below, a circle has been drawn inside a square that just fits around it.



2. Notice the four smaller squares that are formed by the outside square and the four radii drawn in.

(a) Is the side of each small square the same length as the radius of the circle? ______________________________

(b) What is the area of each small square if the radius of the circle is 5 units? ____________ r units? ____________

(c) Is the area of the circle less than the area of the four small squares? ____________

(d) About how many times as large as the area of one small square is the area of the circle? ____________

AREA OF A CIRCLE, PART 2

1. Take a sheet of graph paper and the four wooden discs.

(a) Place one of the discs on the paper and draw a circle by tracing around the disc with a pencil.

(b) Draw four radii in the circle and a square that just fits around the circle, as shown in Question 1, page 1. This will give four smaller squares, as before.

(c) How many units long is the radius of the circle? ____________ (Count the spaces; each space on the graph paper is 1 unit.) Round this number to the nearest whole unit and record it in the second column of the table below.

(d) Is the side of each of the four squares the same length as the radius of the circle? ____________ Therefore, the area of each square is ______ x ______ or ____________ square units. Record this number in the third column of the table.

(e) Find the approximate area of the circle by counting the number of whole squares within the circle and adding to this number an estimate for the partial squares within the circle. Round this result to the nearest whole number and record it in the fourth column of the table.

(f) Divide the number in the fourth column (A) by the number in the third column (r^2). Round this result to the nearest whole number and record it in the last column of the table.

2. Repeat the above work with the three remaining discs.

	Radius (r)	Area of Square (r^2)	Area of Circle (A)	$A \div r^2$
Disc 1				
Disc 2				
Disc 3				
Disc 4				

3. Look at the numbers in the last column of the table. Are they all the same?____________ Is the number 3 in every case? ____________________

4. Do these results suggest that the area of a circle is about 3 times the square of its radius? ________________________________

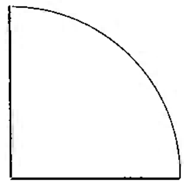
5. In actual fact, the area of a circle is π times the square of its radius. (You will recall π from the formula $C = 2\pi r$ for finding the circumference of a circle.) We can therefore find the area of a circle by multiplying the square of its radius by π. This suggests the formula $A = \pi r^2$ where A is the number of square units in the area of the circle and r is the number of units in the radius. As before, the value of π is taken as 3.14 or 22/7.

AREA OF A CIRCLE, PART 3

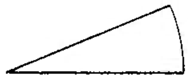
1. Take the plastic disc and a piece of paper.

(a) Using the disc, trace out a circle on the paper and cut along this circle to make a paper disc.

(b) Fold the paper disc along a diameter and then fold it again so that it looks like this:



(c) Fold it a third time and then a fourth, so that after the fourth folding it looks like this:



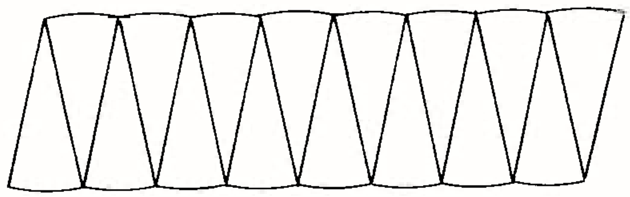
2. Unfold the paper, Sketch a picture below to show what the disc looks like with the fold lines.

3. Cut the disc along the fold lines.

   (a) How many separate pieces do you have? ____________________

   (b) What shape does each piece resemble? ____________________

4. Arrange 16 pieces as shown below.

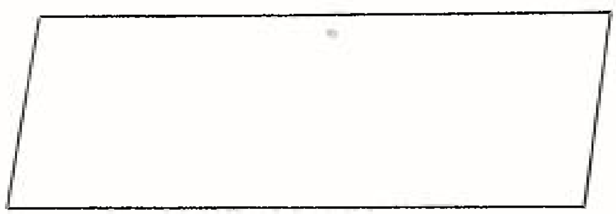


5. Does this arrangement look like a figure you know?____________Does it look like a parallelogram?____________________

6. (a) Which dimension of a circle is the height of this "parallelogram"?______ Is it the radius, r?____________

   (b) Which dimension is the length? _______________
   Is it one-half the circumference? _________________
   Since the circumference of a circle is given by $C = 2\pi r$, then one-half of the circumference is $\frac{1}{2} \times 2\pi r$ or πr. Therefore, the length of this "parallelogram" is πr units.

   (c) Show these dimensions in the diagram below.



   (d) Can you find the area of this "parallelogram"?_________Is it $\pi r \times r$ or πr^2 square units?________________

   (e) Is this also the area of the circle?______Therefore, the area of the circle is__________square units.

7. Can we therefore find the area of a circle by using the following formula?

   $A = \pi r^2$

AREA OF A CIRCLE, PART 4

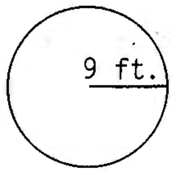
1. Use the formula $A = \pi r^2$ to find the area of each circle shown below. (Use 3.14 or 22/7 for π.) If the diameter is given, first find the radius by dividing the diameter by 2.

   Example: If a circle has a diameter of 8 in., then $r = 8 \div 2 = 4$. Therefore

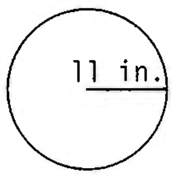
   $$A = 3.14 \times 4 \times 4$$
   $$= 50.24$$

   The area of the circle is 50.24 sq. in.

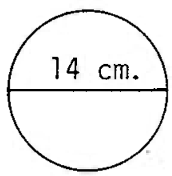
   (a)



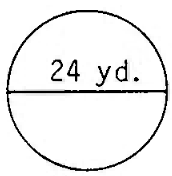
   (b)



   (c)



   (d)



2. Find the area of a circle having

   (a) radius 4.5 cm.

   (b) radius 7.7 ft.

(c) diameter 26 in.

(d) diameter 13 yd.

3. Is the area of a circle with diameter 20 cm. the same as the area of a circle with radius 10 cm.? ______________

AREA OF A CIRCLE, PART 5(a)

1. Find the area of a circle having a radius of

   (a) 13 in.

   (b) 2.5 cm.

2. Find the area of a circle having a diameter of

   (a) 7 ft.

   (b) 22 yd.

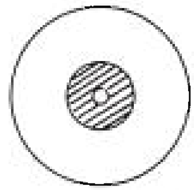
3. The free throw circle on the basketball court has a radius of 4 ft. What is the area of the circle?

4. What is the area of a circular patio, 10 ft. in diameter?

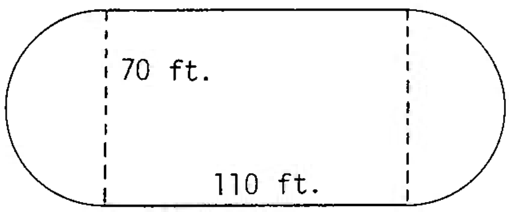
5. A rotating lawn sprinkler can water a lawn for a distance of 35 ft. in every direction. How large an area can the sprinkler cover?

6. How many square feet of cloth are needed for a circular table cover if the table is 3 ft. in diameter and the cloth hangs 6 in. all around?

7. The picture below shows a 12 inch phonograph record. The label in the center is 4 in. in diameter. The rest of the record is playing surface. What is the area of the playing surface?



8. What is the area of the running track pictured below?



9. A tinsmith cut a circular piece of metal 6 in. in diameter from a square piece 6 in. on each side. How much metal was wasted?

10. The girls in the cooking class each rolled out some dough into a rectangle that was 14 in. by 10 in. Their cookie cutter was 2 in. in diameter. (a) How many cookies was each girl able to cut out? (b) What area of dough was left for rolling out again?

11. The boys in the gardening class are planting flowers in a circular flower bed, 18 ft. in diameter. (a) How many flowers should they order if each flower requires 2 sq. ft. of garden space? (b) How much will the flowers cost at 60¢ each?

12. A circular rug, 8 ft. in diameter, is placed on a rectangular floor, 10 ft. by 12 ft. The part of the floor not covered by the rug is to be tiled. (a) How many square feet of tile will be needed? (b) About how many tiles will be needed if each tile measures 9 in. by 9 in.?

13. The pressure on the piston of a sports car engine is 70 lb. per sq. in. What is the total pressure on the piston if it has a diameter of 3.4 in.?

14. A circular wading pool, 28 ft. in diameter, is surrounded by a concrete walk 7 ft. wide. Which do you think is larger in area, the pool or the walk? ________________ Prove your answer.

15. At the center of one side of a house 30 ft. on a side, a dog is tied by a leash 40 ft. long. What is the total area over which the dog can play?

AREA ACTION, PART 5(b)

1. Find the area of the surface of the circular flower bed in the school courtyard.

2. Find the area of the base of the storage tanks at the Esso terminal.

3. Find the area of the "bullseye" and of the successive rings of a dart board.

4. Find the area of the bottom and the top of an angel-food cake pan.

SUMMARY

In studying the materials and the discussion of their use the design principles should be apparent. The materials provide students the opportunity to "play" with mathematical ideas in a non-complex feedback-rich setting. The instructional settings are varied; there are concrete materials labs, class discussions, problem lab sessions and project labs each with a particular purpose. There are two important by-products of the use of such materials and settings. The low ability student gets a rare opportunity to learn on his own. Further, the teacher assumes non-authoritarian roles as guide, instructional manager and small group tutor.

The authors were responsible for a large scale controlled experiment, testing the materials and design principles discussed herein. The results were very encouraging. As well as reacting very favorably to the pedagogical variety, students in the lab groups were significantly higher than a non-lab group in achievement, in attitude towards mathematics and in feeling that they had learned to work independently.

It is hoped that the reader will try to apply the design principles outlined above in developing instructional settings for your low ability students. Perhaps you will find your teaching frustrations turned to teaching fun.

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