Guessing and Testing - A Sequence of Fallibilistic Lessons

A.J. Dawson
Visiting Professor
Professional Development Centre
Simon Fraser University

This paper grew out of a talk entitled "Guessing and Testing" presented by Dr. Dawson at the Winnipeg Meeting of the NCTM in October, 1970.

It seems to be a characteristic of the current activity in mathematics education to focus on the active learning approach to the teaching of mathematics. Several subgroupings of methods relevant to an active learning orientation can be identified. For example, the *games approach* is designed to foster enjoyable concentrated activities by children in the classroom frequently motivated by the use of physical materials. The *discovery approach* is based on the concept that students must accept new experiences rather profoundly in order to be able to build on these discoveries in a meaningful fashion. Moreover, one must not forget the individual in the many activities being designed for the mathematics classroom. Consequently, the *individual approach* recognizes the fundamental hypothesis that every student has to do his own learning in his own unique way. As a result, it must also be recognized that individualization is a reality and not a method open to choice. Our only choice is as to how we identify this "species specific mode".

There is a danger in each of these approaches that the mathematics being taught may be lost in the methodology. As a consequence, we can see the development of the *mathematics approach*, an approach which sees mathematics as the dynamic use in the mind of relationships, and of relationships of relationships. In this view, mathematics is not an end-product to be presented to students as a finished science in a colorfully bound and illustrated book, but rather mathematics is a process, a process in which students can be involved and in which they can produce products. The *integrative approach*, on the other hand, focuses on the need for relevancy of the subject matter to every child, student and teacher involved in the study of mathematics. This approach involves an integration of mathematics with other school subjects, integration of operations, transformations, attitudes of discovery, self-respect and responsibility. One may also identify a rising concern with children as people in the classroom.

*Dedicated to E. T. Nepstad whose compassion for, interest in, and involvement with students has been a constant source of inspiration for many years.

This may be termed the *human approach*, for according to this view, no matter what goes on in classrooms, it goes on with human beings. This implies that tolerance, compassion, frustration, tiredness, hope, lack of communication and a lack of attention are attributes of every lesson - mathematics or otherwise - for these are attributes of human beings. Therefore, teachers and students need to develop attitudes of understanding, acceptance, and respect for others; they need to allow each other to utilize their own freedom; and they need to develop skills of learning how to learn. Finally, the *systems approach* realizes that human beings, mathematical concepts, educational institutions and classrooms are all very complex systems. Each student and teacher has to work within each of these systems. An alteration in any one of these systems produces changes or responses in each of the other systems. Therefore, all systems must be recognized and appreciated so that educators may advance in the direction of maximizing the efficiency of these systems in producing environments where children may learn and grow in an atmosphere which is fun, exciting, challenging and rewarding.[1]

The thrust of the above description of approaches to the teaching of mathematics is that there is no *one* best way to foster the learning of mathematics - there is not *the* approach. None of these approaches is a panacea. Consequently, arguments as to which approach is best are sterile. Instead, it is the writer's contention that the focus should be on selecting the methods and materials and on devising the classroom climate which seems most appropriate or potentially most fruitful for aiding students to learn mathematics. It should be noted at this point that the approaches described above are by no means mutually exclusive nor exhaustive.

Within any one or all of these methods, it is possible to develop various teaching strategies, strategies which are not necessarily approach oriented. The writer has recently developed strategies of teaching which seem appropriate to an active learning approach. However, these are not appropriate to just the games approach or the discovery approach or the mathematics approach. Indeed, it can be argued that the strategies to be discussed here are appropriate to many approaches.

The strategies of teaching developed by the writer spring from a philosophical foundation. A basic philosophical position was chosen and the logical implications this position has for classroom activities in mathematics were explored. The philosophical position adopted is called CRITICAL FALLIBILISM. It was developed by Karl R. Popper during the last four decades.[2] Moreover, the application of Critical Fallibilism as a foundation for describing the nature of mathematical inquiry has been explored by Lakatos.[3] The writer has extended

---

[1]The reader is directed to John Trivett's article in this publication for a further expansion and description of these approaches or aspects of an active learning approach.

[2]The development of Critical Fallibilism is documented and presented in two books by Karl R. Popper, namely, *Conjectures and Refutations*, Basic Books Inc., 1962, and *The Logic of Scientific Discovery*, Basic Books Inc., 1958.

[3]Imre Lakatos, "Proofs and Refutations", *The British Journal for the Philosophy of Science*, Vol. 14, 1963. Pp. 1 - 25, 120 - 139, 221 - 245, and 296 - 342.

Lakatos' work (and to some degree Polya's also) to the point that strategies of teaching mathematics have been developed.[4]

The purposes of the paper are to briefly explain the underlying assumptions of Critical Fallibilism, to then describe the model of instruction derived from this position, and finally to provide illustrations of some of the teaching strategies inherent to this model of instruction.

The Philosophical Position

It is the contention of those who espouse Critical Fallibilism that knowledge grows by means of conjectures and refutations. In endeavoring to solve some mathematical problem, the mathematician would conjecture a possible solution to the problem. However, simply conjecturing possible solutions is not good enough, the mathematician would then attempt to either prove or refute his conjecture.

If he is successful in refuting his conjecture, then he must seek a new conjecture guided by his new knowledge of a conjecture which was not satisfactory. On the other hand, if he proves his conjecture, it still behooves him to analyze his proof in order to identify hidden lemmas and weaknesses in his proof. In applying this philosophical position to teaching strategies, it would seem that children should be allowed the freedom to conjecture or guess hypotheses as possible solutions for their mathematical problems, to severely test these conjectures, and to prove these conjectures.

From a Fallibilistic viewpoint, then, one is able to identify three phases of mathematical inquiry: origination, testing and proving. A consideration of the possible permutations of these three phases leads to the development of a model of instruction.

A Fallibilistic Model for Instruction

If the origination phase or conjecturing phase is denoted by "O", the testing phase by "T", and the proving phase by "P", then there are six possible permutations of these phases. They are the following:

1. O - T - P
2. O - P - T
3. P - O - T
4. P - T - O
5. T - O - P
6. T - P - O

If one studies these six cycles, it becomes evident that the last three cycles are not individually possible, although they may occur as parts of longer patterms of instructional sequences. For example, if cycle one above was followed by cycle two so that the flow of cycles was the following:

(O - T - P) - (O - P - T),

[4] A. J. Sandy Dawson, "The Implications of the Work of Popper, Polya, and Lakatos for a Model of Mathematics Instruction", Unpublished Doctoral Dissertation, The University of Alberta, Edmonton, Fall 1969.

then a rearrangement of the parentheses would yield cycle six as shown below:

O - (T - P - O) - P - T.

However, even in this case, the initial testing phase is preceded by an origination phase. The point is that cycles four to six could not be the initial cycle in a longer sequence of cycles because it is necessary to have originated a conjecture before one can test it or prove it. Consequently, this leaves one with the first three cycles as being the only viable cycles. Even then, the question may be asked as to why cycle three is a viable strategy. This question will be answered below.

Cycles one and two represent two of the three strategies of teaching derivable from a Fallibilistic orientation. For ease of reference, cycle one is denoted by TP and called the testing-proving strategy. Cycle two is denoted by PT and called the proving-testing strategy. Taken together, these two cycles are called the *naive* instructional pattern because the first phase of both strategies is that of origination by a conjecturing process. Cycle three is called the *deductive* instructional pattern and is denoted by DED.

The distinguishing characteristic of this latter pattern is that origination is a deductive process or a proving process in the following sense: a conjecture could be originated in a deductive or proving fashion if one begins with a set of axioms, definitions and undefined terms and proceeds to derive consequences from these axioms. These consequences could be thought of as conjectures which were being proved, in the sense of logically derived, as they were being originated. Therefore, cycle three is a viable strategy of teaching.

From a Fallibilistic standpoint, then, origination is of two types, naive and deductive. Naive origination is characterized by a guessing procedure, a procedure which seeks to propose plausible solutions to problems. Deductive origination proves a conjecture as it is being originated.

The distinctions among these three strategies of teaching stems from the order in which particular phases are utilized. The distinctions do not arise from basic differences among the phases themselves. One exception does exist however, for as noted previously there is a difference between naive and deductive origination. Figure 1 depicts a Fallibilistic model for instruction in skeleton form, identifying the two patterns of instruction and the two strategies within the native instructional pattern.

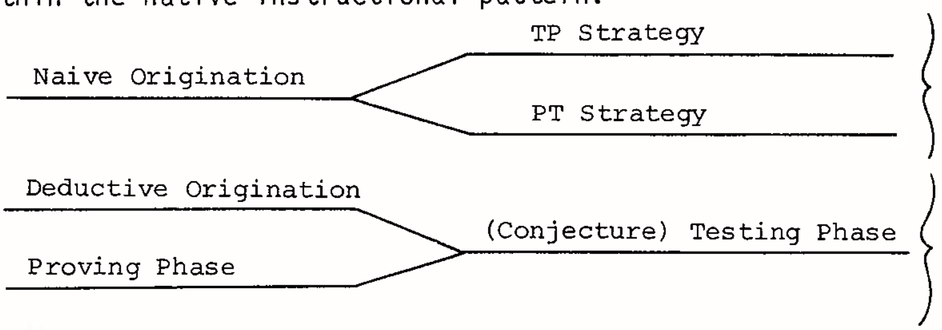


Figure 1
A Fallibilistic Model for Instruction

The next section of the paper is a record of a sequence of lessons in which the writer had students focus on the naive instructional pattern.[5]

A Sequence of Fallibilistic Lessons

A number of matters should be noted at the outset of this section of the paper. First, the word *lesson* is not meant to mean a specified block of time. Indeed, with some groups of students the sequence of lessons to be described below actually took only 45 minutes. With other groups of children, the sequence was carried out over a period of a week or more. The amount of time taken depended entirely on the group of students with whom the writer was working Generally speaking, however, the older the group of students who were involved, the shorter the period of time which was required. Nevertheless, it must be recognized that all of the mathematical topics considered in this sequence of lessons were new to the students and hence they were treated only at an introductory level.

Second, every attempt was made in all lessons to remove the teacher from the center of attention in the classroom in order that the students could interact with their peer group and the mathematics under discussion. As a result, the teacher's role becomes that of the creator of the learning environment rather than that of an authority figure in the classroom. Finally, the names of the students in the following record are not the names of students who have actually partaken of this sequence of lessons.

Lesson One: The main objective of this activity was to acquaint students with a guessing and testing strategy as a means of acquiring knowledge. As a corollary of this objective, it was desired that students would come to treat *wrong* guesses on a cognitive level rather than an affective level, the point being that even so-called wrong guesses contribute to one's knowledge. Since this lesson serves as an introduction to the guessing and testing strategy, it was desirable that the lesson be fun, exciting, challenging and rewarding. Let us join the lesson as the teacher sets the problem:

TEACHER: I would like you to try to determine the rule I am using to generate the following sequence of numbers: 4, 16, 37 . . . . Can anyone guess what the next number in the sequence would be and hence how the sequence is obtained?

DOUG: I think the next number is 1369.

SCOTT: That can't work, because 16 squared isn't 37. (Scott realizes that Doug looked at the first two numbers in the sequence, guessed the squaring hypothesis, and then squared 37 to obtain 1369.) Is the next number 49? (Scott seems to have focused on the difference between 16 and 4, namely 12, and the difference between 37 and 16, namely 21. He is guessing that perhaps these differences alternate. Incidentally, as each number is suggested, the teacher records them on the board so that there is a growing list of numbers which do not work - refuted conjectures.)

---

[5]For illustrations and explanations of the deductive instructional pattern, the reader is directed to the dissertation noted in the previous footnote, especially chapter five.

TEACHER: No, the next number is not 49. I'll tell you the next number in the sequence. It is 58. (The sequence is now 4, 16, 37, 58 . .)

SUSIE: Well, the differences now are 12, 21, and 21. Is the next number either - let's see - 70 or 79. (Susie is guessing naively that there may be a pattern of differences which is either 12, 21, 21, 12 or 21, 21, 21.)

TEACHER: No, neither of those is the next number. The next number in the sequence is 89. (General puzzlement usually follows. The sequence is now 4, 16, 37, 58, and 89. The teacher by giving additional numbers is attempting to provide a wider basis on which the students can test their guesses. As a result of the addition of this last number, Susie's conjecture seems to be refuted. In the actual classroom setting, the teacher would usually seek many more guesses before revealing new numbers in the sequence. The process has been shortened here for the purposes of writing the sequence up for this paper.)

JEFF: Does it (the pattern) have anything to do with the squaring of the numbers? (Jeff is looking for patterns not just numbers. The actual numbers serve only to *test* the pattern and the pattern is the real conjecture.)

TEACHER: Perhaps. (He's not *too* helpful.)

SUSIE: Is the next number 120 or 102? (Susie has many conjectures, but she remains focused on "differences" between numbers in the sequence. In the first case, she guesses the sequence of differences to be 12, 21, 21, 31, 31, and in the second case that it might be 12, 21, 21, 31, 13.)

TEACHER: No, neither of those is the next number.[6]

Several things should be noted in this illustration. First, the conjectures put forth by the students are naive; that is, they are guesses which are not deductively obtained, but rather which seem plausible on the basis of the sequence of numbers. Second, the students are able to test their conjectures by using their proposed "rule" to determine the next number in the sequence. They then ask the teacher for refutation or corroboration. This refutation is devoid of personal criticism of the merit of the student's guess.

The role of the teacher is to set the problem and then to inform the students if their guess as to the next number in the sequence is satisfactory or not. The teacher also acts as the recorder of these guesses so that the students may know what numbers have already been suggested. The former role of the teacher is performed without penalty or praise in order that the students may interact with the mathematical problem without attempting to conform to some preconceived behavior patterns established by the teacher.

[6]Those readers who have not already *guessed* the sequence for themselves are encouraged to send their guesses to Sandy Dawson who will refute or corroborate the reader's conjecture. His address is Professional Development Centre, Simon Fraser University, Burnaby 2, British Columbia.

Some readers may be saying something like: "Yes, that is interesting, but the mathematical problem being considered is not really of any significance." This is not the point, however. The goal of the lesson was to have the students develop attitudes which made guessing and testing an accepted classroom activity as well as to get the teacher out of the center of attention in the classroom. Consequently, the particular mathematical topic chosen is not of particular importance; the attitudes and strategies to be developed are important. This is an introductory lesson and as such it should be fun and exciting and at the same time it should present a problem which is challenging to the students.

<u>Lesson Two</u>: Once the teacher feels his students are comfortable with a guessing and testing orientation, it is possible to move to the next series of activities. Lesson two is the first of two lessons dealing with mathematical topics commonly found in a secondary school mathematics program. The relationship between lesson two and lesson three is considered in lesson four. The ideas for lessons two and three were obtained from the Madison Project materials, which are reported on in two films produced by the Project, "Guessing Functions" and "A Lesson with Second Graders".[7]

The situation the students are presented with in lesson two is as follows: three students (the *team* of experts) are asked to make up an open sentence of the form $mx + b = y$, that is, a linear equation. The remainder of the class is charged with the task of "guessing" what equation these students are using in creating a table of values for the equation. Members of the class suggest values for "x" and the team of experts using their equation determine the corresponding value of "y". In the Madison Project form of equation writing, the equation under discussion might be as follows: $(\square \times 9) - 6 = \triangle$.

A table of values is created from the suggested numbers and the team's response to these suggestions. It is usually the case that the first such rule or equation would probably be set by the teacher. However, once the students understand the situation, the teacher steps out of the situation and simply acts as the recorder of values for the table. Let us join such a class which is just beginning to discuss the open sentence given above.

TEACHER: Does the team have a rule in mind?

TEAM: Yes we do. Would someone please suggest a number?

ANITA: Try 6 please.

TEAM: 6 would give 48.

□	△
6	48

BETH: Would you apply your rule to 1 please?

TEAM: 1 yields 3 when we apply our rule.

□	△
6	48
1	3

[7]These two films are available on a rental basis from the Madison Project, 918 Irving Avenue, Syracuse, New York. The reader is also directed to two books written by Robert B. Davis, Director of the Madison Project. The two books are *Discovery in Mathematics*, Addison-Wesley Company, Don Mills, Ontario, 1964, and *Explorations in Mathematics*, Addison-Wesley Company, Don Mills, Ontario, 1966.

DAPHNE: Try 0 please.

TEAM: (After some discussion over this request, the team responds.) Yes, 0 gives us -6.

□	△
6	48
1	3
0	-6

JOHN: I think I know the equation. Is the equation you are using this one: (□ x 10) - 12 = △ ?

TEACHER: Can you test your guess, John? (The teacher has re-entered the situation now, not by saying "Yes, John is right", or "No, John is wrong", but by encouraging John to test his guess. This requires John to first determine how he might test his guess and then actually test it.)

JOHN: Well, if I put 6 in the box, I get 48. Now if I put 1 in the box I get - let's see - -12. Oh! I guess that doesn't work.

NEIL: Would you please try 2?

TEAM: 2 gives 12.

□	△
6	48
1	3
0	-6
2	12

(The rule is then applied to the numbers 3, 4 and 5 obtaining respectively the responses from the team of 21, 30, and 39 so that the table of values now appears as follows:

□	△
6	48
1	3
0	-6
2	12
3	21
4	30
5	39

Another student guesses the rule to be (□ x 7) + 2 = △ . This is quickly refuted. Finally, we witness the following dialog.)

JOHN: Look here now. The triangle numbers are increasing by 9 each time the box numbers increase by 1. When the box number is 0, the triangle number is -6. Let's see then, is the equation (□ x 9) - 6 = △ ? (This is quickly checked for each of the values already in the table and found to be satisfactory.)

JOHN: Alright then. If this is the rule then if I tell the team 8, they should tell me 66. Is that right team?

TEAM: Yes, we would tell you 66 and you do have the rule we were using.

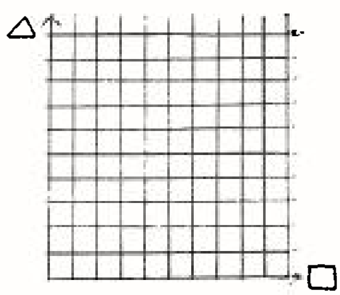
Several ingredients of this lesson must be noted. First, the teacher in responding to John's first suggestion as to what the equation was, asked John to test his guess. Obviously, the guess was made on the basis of looking at only the first pair of values in the table. What is of importance is that the teacher does not discourage John from guessing. Moreover, the guessing is on a naive level in that a solution is proposed and then tested to determine its validity. The student makes a guess, tests it, and revises it if it is refuted. In such a classroom climate, the situation is one which encourages guesses, one in which no stigma is attached to guesses which do not happen to work. There is nothing wrong with being wrong. Indeed, as Davis points out, everything one knows is to some extent wrong.[8]

Second, the means of finally arriving at the conjecture which proves to be the equation the team of experts is using begins to take on the characteristics of deductive origination in that John identifies certain patterns and bases his guess on these patterns. One can see here the beginnings of a deductive strategy of conjecture origination.

Third, we see again that the role of the teacher in this lesson is minimal once the lesson is underway. He acts as a recorder for the table of values, as one who sometimes makes a suggestion or a comment, but who does not interfere in a major way with the interaction among the students and between the students and the mathematics.

<u>Lesson Three</u>: A different guessing and testing situation is created in this lesson. On this occasion the mathematical topic is that of naming points in the usual manner in the first quadrant of a Cartesian coordinate system. The goal is to have the students identify for themselves the method or pattern which is utilized to plot points given a pair of numbers. The teacher begins the lesson by setting some limitations to the problem.

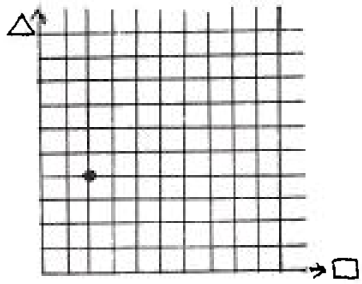
TEACHER: I have drawn a grid or set of crossed lines on the board. I would like you to try to determine how I am placing points on this grid. To do that, I would like you to tell me two positive whole numbers The first number you tell me I shall call the box number. The second number you tell me I shall call the triangle number. Would you please use numbers less than 10 for the time being. Alright, who will tell me two numbers? (The teacher would start with a grid much like the one below.)



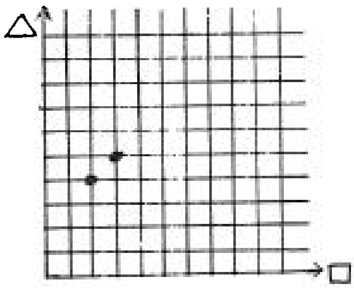
[8]Robert B. Davis, "The Madison Project's Approach to a Theory of Instruction," *Journal of Research in Science Teaching*, Vol. 2, p. 155.

COLLEEN: 2 and 4.

TEACHER: The box number is 2 and the triangle number is 4. (The teacher marks the appropriate point on the grid.)

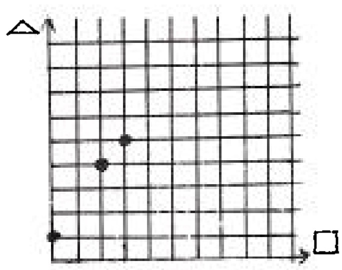


DONNA: 3 and 5. (The teacher plots the point 3, 5)

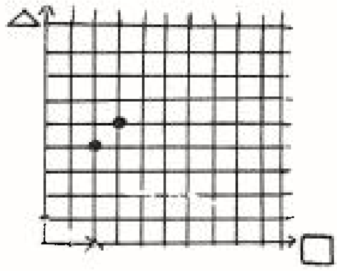


ALICE: 0 and 1.

TEACHER: That's a bit tricky, but let's see where that one goes. (The teacher plots this point after some consideration.)



TEACHER: Who thinks they know how I am placing the points on the grid? (Several students raise their hands.) Fine, will those students please come up to the board and be the team of experts. (They do so.) The team is now going to place points on the grid. The rest of you have to now tell them what two numbers would name that point. (The team place a point on the grid.)



HEATHER: That's 2 and 0.

TEAM: Yes, we agree.

TEACHER: Heather can join the team.

And so it goes. Without the teacher ever saying a word about the x-axis and the y-axis, the origin, or where to begin counting, the class is able to determine by guessing and testing how the various points are plotted and named. As a follow-up activity for this lesson, the class could be divided into two groups so that a game of tic-tac-toe (x's and o's) could be played but using a five-by-five board or a six-by-six board or whatever size board the teacher thinks appropriate. This is actually a drill activity, but it is a drill activity which in the writer's experience the students find to be fun, exciting, challenging and rewarding.

It is evident that in lesson three the teacher again removes himself from the center of the classroom situation. The students are left to guess and test relative to the mathematics under consideration. They are not guessing and testing as to what answer the teacher is wanting or seeking, a game which is too common in classrooms today.

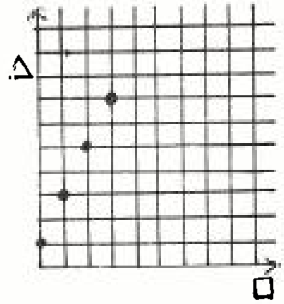
<u>Lesson Four</u>: This lesson is designed to bring together the mathematical topics the students have been investigating in the previous two lessons. We join the lesson at the point where the students have established the following table of values from an equation which a team of experts has created.

$\square$	$\triangle$
1	3
3	7
0	1
2	5

Several possible equations have been suggested and quickly refuted. The teacher makes the following suggestion:

TEACHER: Could you use the grid system we talked about recently to assist us in finding the equation the team is using?

BETH: Well, we could mark the points on the grid for each pair of numbers in the table. (This is done. It yields the following grid.)



DAPHNE: I can tell by looking at the grid what the value will be if I tell the team 4. It will be 9. Is that right?

TEAM: Yes, that is right.

JOHN: Yes, look every time we go over one on the graph, we go up two. If you look at the table of values, you see the triangle numbers increase by two every time the box number increases by one. I think I know the equation. Is it ($\square$ x 2) + 1 = Δ?

TEAM: Yes, that is the rule or equation we were using.

ANITA: Why did you guess that you add one to the ($\square$ x 2) John?

JOHN: Because I noticed last time that every equation we worked with gave the number you add or subtract when the box number is zero. (John certainly was looking for patterns and relations, and relation of relations, in order to make such an observation.)

In this fourth lesson, one can see how two mathematics topics which may seem unrelated when presented in isolation can be brought together in order to give the students greater power and range or scope for their mathematical guessing and testing techniques. By applying the graphing techniques explored in lesson three to the linear equations of lesson two, students were able to see the patterns created in the table of values in a graphical dimension.

These four lessons are but a very brief description of how Fallibilistic strategies of naive origination and testing could be introduced to students in your classroom. Some of the more fundamental issues involved in utilizing such an approach are discussed next.

Summary and Conclusions

At the outset of the paper, a brief description of a variety of current approaches to the teaching of mathematics was provided. The purpose of this discussion was two-fold. First, it was contended that none of the approaches mentioned (and even more could be added) was a panacea for our problems in teaching mathematics. The usefulness, fruitfulness and validity of any approach depends on the goals of a particular lesson, the students involved in the lesson, the teacher guiding the lesson, and the physical environment in which the lesson occurs. Second, it was argued that no one approach is inherently any better than any other approach. However, this does not mean the teacher should only utilize one method. Indeed, a teacher must use a variety of approaches in order to create an environment in the classroom which is conducive to students' learning of mathematics.

The sequence of Fallibilistic lessons and strategies of teaching must be viewed in this more global context. From a Fallibilistic viewpoint, mathematical knowledge was seen to grow as a function of conjectures and refutations. Within this overall orientation, three strategies of teaching were identified, based on the three phases of mathematical inquiry. The three phases of inquiry were those of origination, testing and proving. The naive instructional pattern was composed of two teaching strategies: the TP strategy in which naive origination was followed by a testing phase and then a proving phase; and the PT strategy where the phases of proving and subsequently testing followed the origination phase. The deductive instructional pattern (DED) was distinctive in that the origination and proving phases proceeded simultaneously giving rise to a deductively generated conjecture, a conjecture which was proven while it was being originated.

The Fallibilistic model of instruction and the strategies of teaching described were offered as being applicable to many of the approaches discussed earlier. The illustrations provided in the form of a sequence of lessons were examples of only the TP strategy of teaching. Furthermore, there were a number of fundamental characteristics of this strategy focused on in the illustrations. Among these was the desire to remove the teacher from the center of attention in the classroom in order to allow the students to interact with their peer group (the human approach) as well as to interact directly with the mathematics being studied (the mathematics approach). The teacher's role thus became one of being the creator of the learning environment. As a result, it is the teacher's job to manipulate the physical setting of the classroom, to select the mathematical topics for consideration, and with the aid of the students to develop ways of testing student conjectures independently of the teacher. It should be recognized that the decisions the teacher might make regarding one of these components would have effects on all the other components (the systems approach). The very act of removing the teacher from the focus of attention in the classroom will produce varying responses from students and will alter the learning environment in the classroom.

One of these responses, it was contended, was that students' guesses could be treated on a cognitive level rather than on an affective level. The students would cease playing the game of trying to out-guess the teacher. Instead, they could direct their energy and attention to the mathematics and to their colleagues' responses to the mathematics. For those readers who are concerned about the students being allowed to guess at answers, it must be remembered that a guess is usually called wild only if it fails; if it succeeds the guess is usually called a daring one. Furthermore, the testing situation which is created does act as a force which fosters responsible guessing rather than irresponsible guessing in the classroom.

It was suggested throughout the sequence of lessons that the tenor of the lesson should be one of fun and excitement, one which is challenging yet rewarding to the students. The creation of knowledge is a tremendously exhilarating experience whether the creator is a child or an adult. One only has to witness the joy expressed by children or the sense of accomplishment and pride exhibited by adults when they create or discover something to become convinced that learning is fun, exciting, rewarding and challenging. Why, then, cannot this also be the feeling demonstrated by students in mathematics classrooms? If the many wrong paths and false steps all of us take in learning are treated cognitively rather than affectively, if we are not branded as failures for wrong guesses, if the teacher becomes a learning facilitator rather than a knowledge transmitter, then perhaps the joy and anticipation of learning which children have when they enter school will remain with them throughout their lives.

Finally, it probably goes without saying that a teacher would not utilize a guessing and testing strategy at all times in the classroom. It is but one strategy that seeks to accomplish specific goals, but these goals are by no means all encompassing. Consequently, teachers are encouraged to have a Fallibilistic approach become part of their repertoire of teaching strategies. However, as with all such strategies, the Fallibilistic approach is not a panacea and should not be viewed as such.