The Laboratory Approach to the Teaching of Mathematics

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The mathematics laboratory has received much attention in recent years as an approach to the teaching of mathematics. This emphasis is obvious from even a casual reading of the professional journals in mathematics education. It is also exemplified in numerous publications by the Nuffield foundation and in the book *Freedom to Learn.*[1] What is the nature of this approach?

COMPONENTS OF A MATHEMATICS LABORATORY

Active Student Involvement

One of the distinguishing features of a mathematics laboratory is that pupils are actively involved in the learning process. This is in contrast to the conventional approach where children were expected to sit in five or six neat rows, passively absorbing (?) the material the teacher was presenting. In the laboratory setting, a small group of students may be cutting or folding paper in one corner of the room as they investigate some geometric concepts, another group may be out in the hallway involved with measurement while others may be rolling dice on the floor in an investigation of probability. In other words, the philosophy of the mathematics laboratory is "learning by doing".

Manipulative Aids

If pupils are to be actively involved in the learning process, they have to have something to be involved with. Pupils investigate ideas of numeration by manipulating counters, popsicle sticks, or commerically available apparatus such as Dienes' Multibase Arithmetic Blocks. An understanding of volume is achieved through pouring liquid or sand into containers of different sizes and from one container to another. The meaning of a fraction can be discovered by the manipulation of cardboard cut-outs. Similarly, other mathematical concepts are investigated through the use of appropriate manipulative or concrete aids.

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[1] Edith E. Biggs, and James R. MacLean, *Freedom to Learn.* (Don Mills, Ontario: Addison-Wesley (Canada), 1969).

Problem-Solving Through Open Search and/or Guided Discovery

A common connotation of a laboratory is a place for experimentation. Experimentation involves the search for a solution to a problem. In a mathematics laboratory, the pupil is confronted with a mathematical problem and he uses the manipulative aids to investigate various hypotheses and eventually arrives at a solution. The solution is the pupil's own solution - he has "discovered" a mathematical concept, relationship, principle, or process by himself or with the help of a partner.

The problem investigated in the laboratory may be a problem posed by the teacher, or pupils may work on mathematical problems they have encountered in their daily life. The latter is often a more meaningful problem for the pupil.

The foregoing discussion assumes that a problem is given and that the pupils search for a solution with a minimum of guidance from the teacher. This is good, but a more common approach is to pose a problem and then guide the pupils to a discovery through a series of intermediate steps. These steps are usually outlined on what is frequently called an "assignment card". Some examples of assignment cards will be given shortly. The latter approach is more structured than the open-ended approach discussed earlier. For this reason, it is the recommended approach for a teacher who is beginning to experiment with the laboratory approach to teaching mathematics.

VALUE OF A MATHEMATICS LABORATORY

Obviously, you can see problems in the laboratory approach to teaching mathematics. For example, there is the problem of organizing the class so that each pupil is engaged in meaningful activity. In addition, there are problems in assigning grades to pupils and an increased noise level which could be a problem in some schools.

Clearly, the above problems can be overcome with a little effort. But, you ask, are the benefits of using a laboratory approach worth the additional time and effort needed to overcome the problems encountered in setting it up? Why use the laboratory or activity approach?

Involves the Real World

First of all, the laboratory approach translates abstract mathematical concepts into concrete representations. This is important in the elementary school since, according to Piaget, most children in the elementary grades do not have the cognitive structure required to deal with formal abstract mathematical notions. Therefore, the best understanding should occur when pupils are allowed to discover the concepts themselves through the use of concrete aids.

Fosters Inquiry

In the laboratory setting, children learn to solve problems independently or in small groups. As students become more familiar with this approach, many of them will be able to pose their own problem and determine a method for solving it. They may even be able to devise their own materials if such are needed to solve the problem. When pupils are able to do this they have developed an attitude of inquiry.

Individualization

The laboratory approach allows pupils to progress at a rate suitable to their ability. A pupil may spend as long investigating a concept as he requires to understand it. On the other hand, children who understand a concept or process need not spend time doing repetitive exercises.

The laboratory approach allows for individualization not only in terms of rate of learning, but also with respect to the concepts covered. There probably are some core concepts which everyone needs to investigate, but the laboratory approach allows individual pupils the opportunity to explore concepts that are of interest to them.

HOW TO GET STARTED

Different teachers have used different techniques to start a mathematics laboratory in their classroom. Some have found that it works well to send one row of pupils each day to the back of the room where the teacher has set up a few activity stations. When this procedure is used, children always anxiously await their day to have lab.

Other teachers have started a mathematics laboratory by setting aside one mathematics period per week to work on projects and activities.

An important feature of the above two approaches is that they expose pupils (and teachers) to the activity approach slowly. It may be harmful to jump into the laboratory setting too deeply too quickly. Pupils need time to adjust to the new setting and to learn how to make use of their additional freedom and responsibility.

No matter what technique you use to get started, you probably want to structure the activities at first. Then, as pupils learn to accept more responsibility for learning, you can make the activities more open ended and even allow pupils to work on their own projects. Following are a number of assignment cards which have been successfully used in the elementary school. Their purpose in this article is to give you some ideas and to help get you started on an activity approach to teaching mathematics.

SAMPLE ASSIGNMENT CARDS

Measurement

The following assignment card could be used to introduce linear measurement at about a Grade II or III level. This assignment card is quite structured and does not give pupils much of an opportunity for independent exploration. The following assignment card may accomplish three objectives:

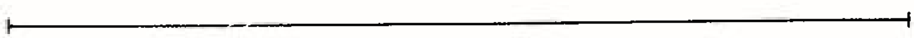
1. Improve the ability of the pupils to estimate lengths.
2. Enable pupils to discover the concept that linear measurement is the iteration of a unit, whatever the unit may be.
3. Lead pupils to the realization that standard units are needed. This objective may be accomplished by having pupils compare their answers, especially in part 3.

For parts 1 and 2 pupils will need a rod and a piece of string. A six centimeter rod (Cuisenaire) and about a 10 centimeter string work well, but the actual length of the unit is not important.

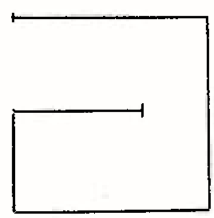
LET'S MEASURE

Use the string and stick on the table to measure.

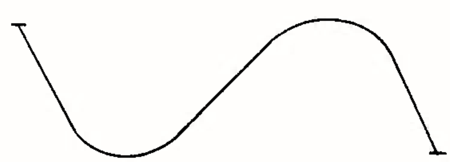
1. Look at each of these lines. First write down how many sticks or strings long you think the line is. Then measure it and write down your answer.



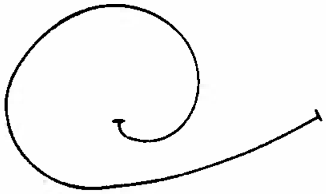
My guess: ____________________ sticks
Measured length: ______________ sticks



My guess: ____________________ sticks
Measured length: ______________ sticks



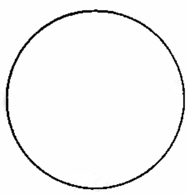
My guess: ____________________ strings
Measured length: ______________ strings



My guess: ____________________ strings
Measured length: ________________ strings

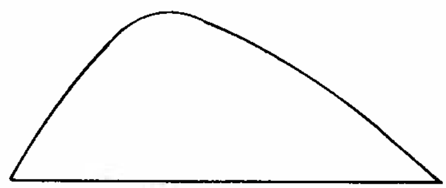
2. Decide whether the stick or the string would be the best to measure the following lines. Write down your guess and answer as before.

My guess: __________ __________
Measured length: __________ __________

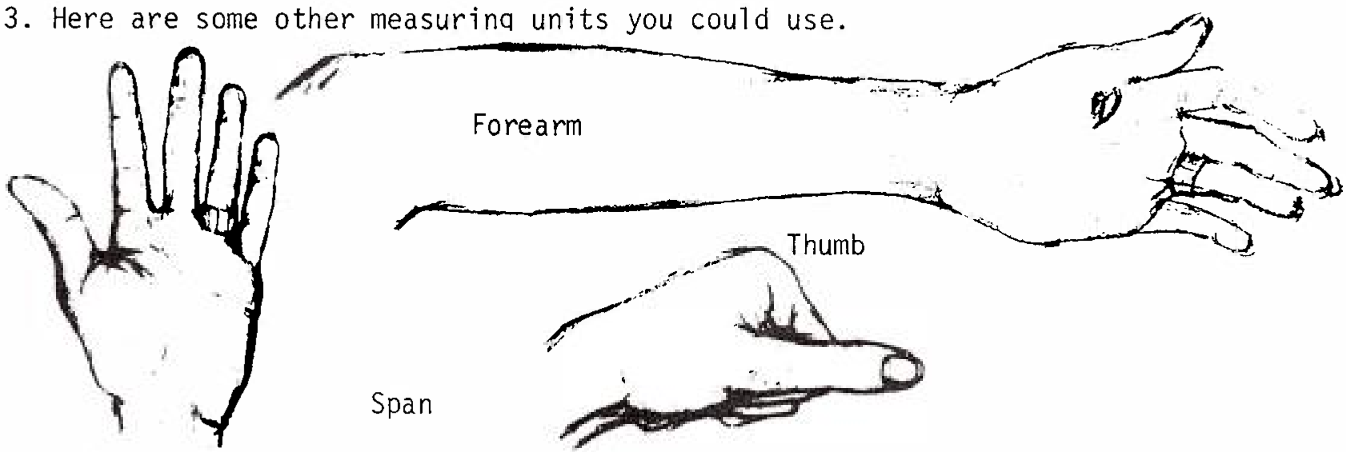


My guess: __________ __________
Measured length: __________ __________

My guess: __________ __________
Measured length: __________ __________



3. Here are some other measuring units you could use.



Use these units to measure the following. First guess and then measure.

	GUESS	MEASURE
(a) Length of the blackboard in forearms	______	______
(b) Width of this paper in thumbs	______	______
(c) Width of the door in spans	______	______
(d) Width of a book in thumbs	______	______
(e) Length of your pencil in thumbs	______	______
(f) Height of the door in forearms	______	______
(g) Width of a window in spans	______	______

Geometry

The next sample assignment card is suitable for the upper elementary grades. Notice that it is much more open-ended than the first one was.

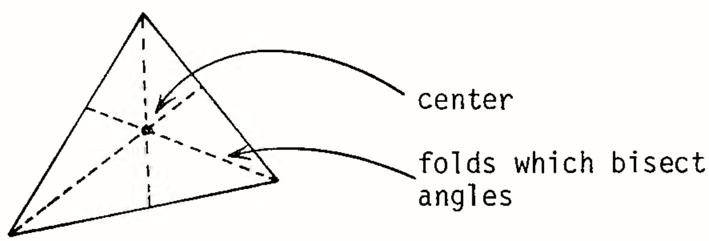
The only material the students will need for this activity is a supply of paper and a pair of scissors. This activity was taken from *MATHEX: An introduction*[2].

Finding a Center of a Triangle

1. Take a sheet of white paper. Fold it to make a triangle. Cut out the triangle.

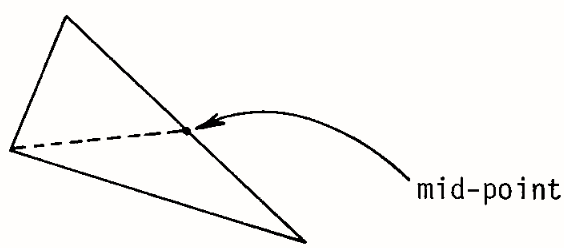
2. Use any method you like to find a center of the triangle by folding.

HINT:



3. Fold and cut out another triangle. Use a different procedure for finding a center.

HINT:



[2] L. D. Nelson and W. W. Sawyer (Editors), *MATHEX: An Introduction.* (Montreal: Encyclopedia Britannica Publications, 1970).

4. Fold and cut out a triangle whose center is the same regardless of the procedure you use to find the center.

5. Fold and cut out a third triangle. Try to find a third procedure for finding a center.

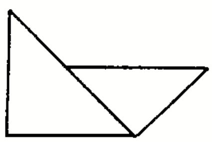
Tangram Puzzle

Puzzle

The following assignment card provides some valuable experiences with plane geometric shapes as well as being a mathematical puzzle.

1. The figure on the next page is an old mathematical puzzle called the Chinese Tangram. Trace the figure onto a piece of manila tag or cardboard and then cut out the seven pieces.

2. Experiment with the seven pieces to see how many *different* shapes you can make. For example, using two different triangles you can make the following shape:

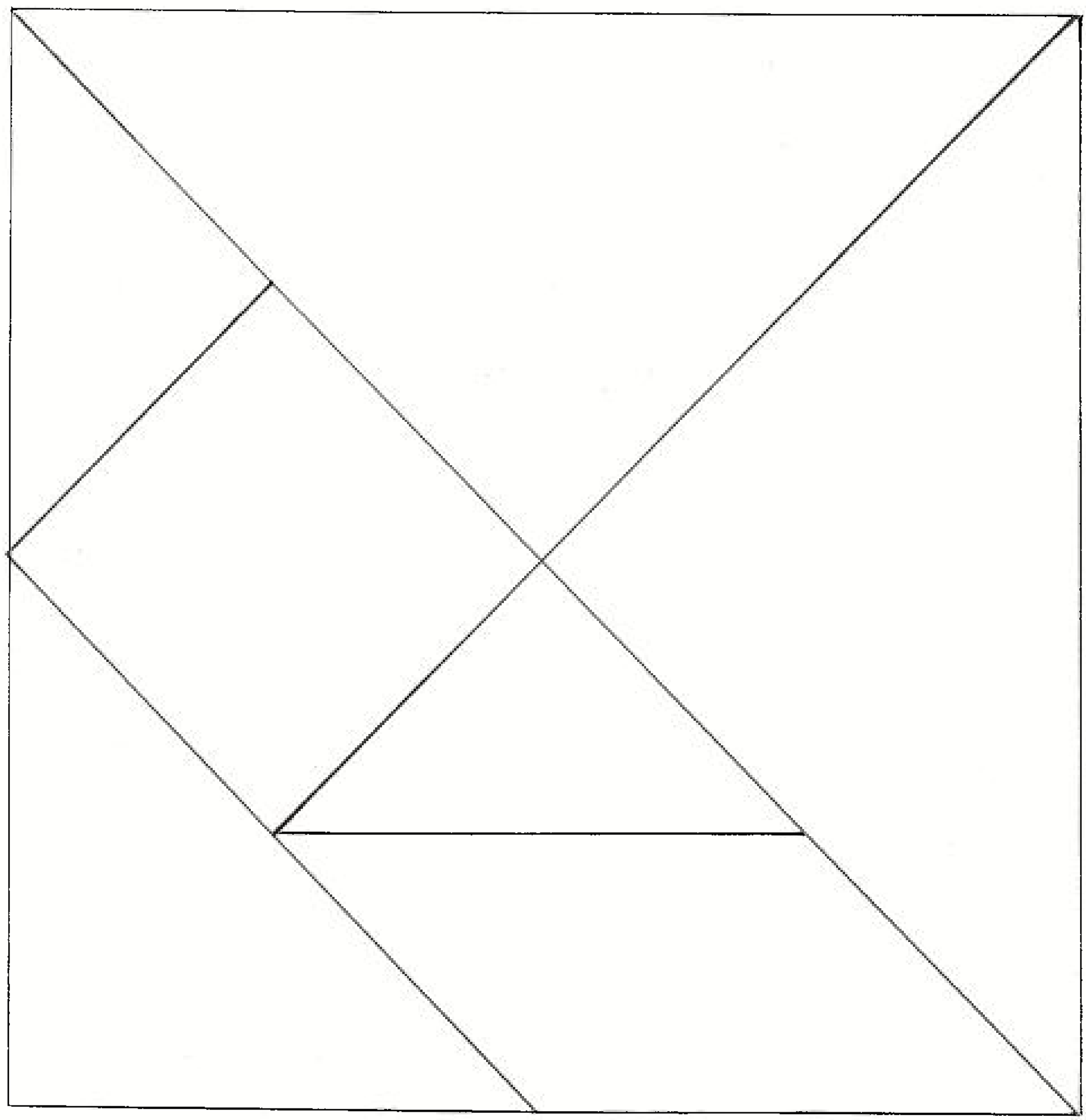
   Compare your shapes with those of your friend. There are over 175 different shapes which can be made from the seven pieces.

3. Try to put the pieces together so that they form the original square that you started with.

4. How many four-sided figures can you make using two pieces? Keep a record of your shapes by tracing them onto a sheet of paper.

5. Do the same for three pieces, four pieces, five pieces, six pieces, seven pieces.

6. Choose a partner and play the following game. Each player chooses a number of pieces from his Tangram puzzle and makes a geometric shape. Each player traces his shape onto another piece of paper and then the players exchange papers. The first person to make the other player's design from the Tangram pieces is the winner.



Chinese Tangram

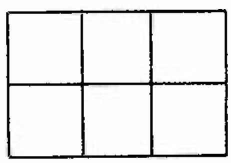
Number Theory

The last sample assignment card involves the concept of prime and composite numbers. It could very easily be expanded to include the concepts of square numbers, perfect numbers and factors of a number. The only material needed is a set of 24 squares. Squares 1" x 1" are probably convenient.

Prime and Composite Numbers

There are 24 cardboard squares in the pile in front of you. You will be asked to take some of these squares and construct rectangles or squares with them.

1. Take six squares. Make all the rectangles or squares you can. (Your figure should not have any holes in it.) For each rectangle write down the length and width as shown below.



2 x 3 (The 2 means 2 rows and the 3 means 3 columns)

Need we make a 3 x 2 rectangle? Why not?

What other rectangles did you make with 6 squares?

2. Use the squares to help you complete the following table. For eacn set of squares make and then record the size of all the rectangles or squares you can make. Two and six have been done for you. Remember 2 x 3 is the same as 3 x 2.

Number of Squares Used	Size of Rectangle or Square	Number of Different Rectangles or Squares
2	1 x 2	1
3		
4		
5		
6	1 x 6; 2 x 3	2
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		

3. For which numbers could you make only one rectangle?

   These numbers are called *prime* numbers. A prime number can be divided only by one and itself without a remainder.

4. *Composite* numbers are numbers which can be represented by more than one rectangle. They have more than 2 divisors. List all the composite numbers from 2 to 24.

5. Is 27 a prime or composite number? 29? Why?

CONCLUSION

Assignment cards such as those illustrated above can be developed for any part of the curriculum. The prescribed curriculum need not be ignored in a laboratory approach. The textbook and curriculum guide are valuable sources of ideas for activities.

It doesn't take long for children to become enthused about mathematics when they can learn by actively participating in a laboratory program. Such enthusiasm may or may not result in more significant learning but it will certainly improve children's attitude toward mathematics. Enthusiasm and active learning will result in a higher noise level. However the noise results from meaningful activity and therefore is not opposed to learning.

The laboratory approach to teaching mathematics has much to offer. Why not give it a try?