General Guidelines for Creating Mathematical Experiences

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INTRODUCTION

The original title of this paper, "Do-It-Yourself Mathematics," has a double meaning. In the first place, it refers to mathematics the student can discover and create for himself. In the second place, it is concerned with the mathematical experiences that the teacher can create for this purpose. There are many such published experiences, but most of these involve two problems. The first is that the search for and implementation of the published experiences may sometimes be as difficult as creating one's own experiences. The second problem is that of finding suitable experiences. In particular, most published experiences are concerned with skills in the basic operations, or measurement and geometry concepts appropriate for the elementary school, and few cover the other kinds of topics of the regular junior high school mathematics programs.

The intent of this paper is to indicate a few general and widely applicable guidelines for creating mathematical experiences on any desired topic. These guidelines are at present in an embryonic stage, as they have been gleaned from recent analysis of personally created mathematical experiences for junior high school students. In other words, there are no claims of finality. The guidelines are open to considerable modification. They are merely a beginning.

How does the teacher devise these experiences? With great difficulty. There is more truth than humor in this remark. In this kind of activity an element of art is invariably involved. This art will often manifest itself as the experience of the teacher grows. Hopefully, the guidelines will help the teacher get off the ground.

GENERAL PROCEDURES

Assuming the teacher has decided that he would like to create a mathematics laboratory experience for a given topic, he should first list the concepts and skills involved. These should be closely studied in order to determine behavioral objectives and to generate necessary subordinate concepts and skills. The next step is to list general and specific applications. This is the point at which the first real difficulty arises. For many topics, appropriate specific applications are not easy to find. Although it is desirable to have these, the teacher should not waste too much time in their search because there is a feasible alternative, the contrived experience. The main discussion in this paper will be concerned with the contrived experience. Let us assume that the teacher has chosen this kind of experience. The next section will deal with general guidelines for creating contrived experiences. A useful summary of the general procedures appears in the form of a check list as below:

1. Choose topic.

2. List concepts and skills in hierarchical form.

3. List subordinate concepts and skills in hierarchical form.

4. Determine general and specific applications.

5. Decide between application or contrived experience.

GUIDELINES FOR CONTRIVED GAMES

At the outset, a decision between investigative or game experiences must be made. Although either or both types can be used, it is generally more appropriate to use investigative activities for applications and game activities for contrived experiences. Examples of both these types will be given in later sections.

The first guideline is to *ensure that constructive experiences precede analytic ones*. Constructive experiences are synthetic as opposed to analytic. The student constructs or builds the concepts for himself in an informal way. Keep in mind that this is only a general guideline. If the experience is designed for post-conceptual practice in skills, this point will not apply, provided the teacher is sure that the concepts involved are intuitively understood. Furthermore, if the teacher is certain that the maturity or richness of the experiential background of the student is adequate, he may disregard this point. But if in doubt, apply it.

Aim at simplicity. It is all too easy to involve difficult concepts other than those for which the experience was devised. This is particularly true for game activities which easily can involve difficult procedures or rules. A danger is that the teacher may abandon devising laboratory experiences merely because he has made them more difficult than other methods he might utilize. One way to further the goal of simplicity is to *aim at only one game* or investigation for each unit may be considered a coherent idea. Later, in the light of experience, the teacher may opt to combine several of these.

Consider the possible experiences on an abstract-concrete dimension. Some students may be ready for the abstract at the outset, whereas others may need to work with manipulative experiences at first and graduate to abstract experiences later.

Divulge only the procedures for playing the game, not the concepts to be learned. For each move in a game there should be several *alternatives*. Application of the concepts to be learned should ensure making the best move. *The instructions should be minimal* and allow for student choice and modification.

A checklist of general guidelines for contrived games appears below. These guidelines also apply to application games, and by substitution of the word "investigation" for "game", one can equally well apply them to application or contrived investigations. In addition, for investigations one should attempt to make the outcome important for the students.

1. Constructive precedes analytic.

2. Aim at simplicity.

3. Aim at only one game for each idea.

4. Devise several concrete and abstract games.

5. Divulge only procedures, not concepts to be learned.

6. Provide moves or strategies which constitute alternatives for which application of the concepts to be learned ensure the best strategy.

7. Provide minimal instructions and allow for choice and modification.

SAMPLE CONTRIVED GAME EXPERIENCE

TOPIC: Factorization (*Seeing Through Mathematics, Book One,* Van Engen, et al., ch. 70)

CONCEPTS: Factor, prime, prime factor, unique factorization, g.c.f.

SKILLS: Recognition and production of above.

SUBORDINATE CONCEPTS: Product, quotient, divisibility, powers, factor, property of the number "1".

APPLICATIONS: General - dividing and subdividing situations.
Specific - many for factors, but none feasible for other concepts.

DECISIONS: 1. Contrived experiences.

2. Game activities

3. Assume knowledge of subordinate concepts sound.

Game 1 - Factors and Primes

Procedure

One player takes a pile of chips (or counters). Each player takes these in turn and makes his move. This procedure continues until no further moves are possible. The last player to make a move picks a new quantity. The game ends after a pre-arranged quantity of different piles has been used.

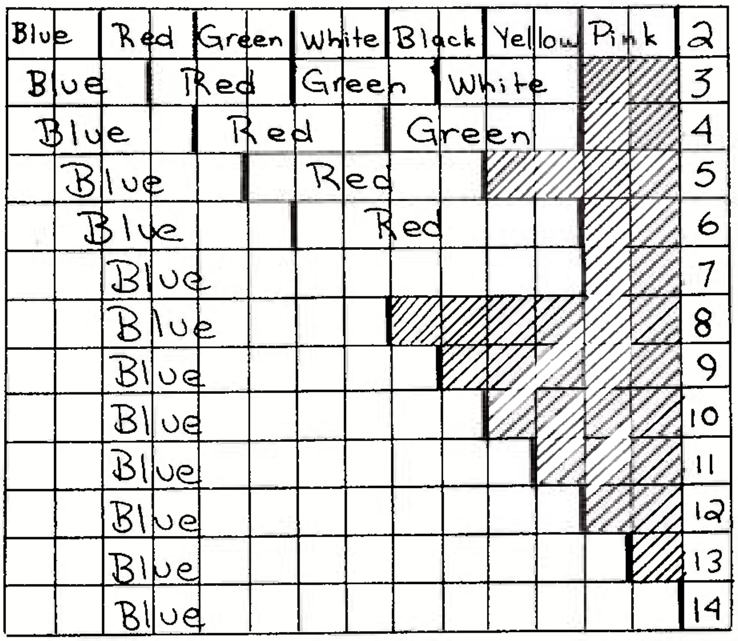
Moves

Place the chips in one row on squares of one color so that there are at least two on each square and none left over. Each succeeding move is up or down the board, but never to a row already used.

Score

One point for each successful move. Player with most points at the end of the game wins.

Game Board



Game 2 - Prime Factors and Primes

Same as game 1 except scoring and move changes as follows:

1. If a player can make a move (as previously described) of the chips on one square to a lower numbered row, he takes another move and the previous player loses one point.

2. If a player can make a move (as previously described), of all the chips in a row to a lower numbered row not already used for that pile, he takes another move and scores two points.

3. These new moves should be only demonstrated, not made.

Game 3 - Prime Factorization

The player may use as many squares in a row as he desires, but he must place the same number of chips (at least two) in each square, and this number must be equal to the row number. The other rules are the same as for game 2 except that each player can take one chip from each square in a row for a type 1 demonstration move.

Game 4 - Common Factors

Use two piles of different quantities, and two boards. Same rules as game 1 except that each player must make the same moves on both boards to score points.

Game 5 - Greatest Common Factor

Use the two board procedure of Game 4. Only moves of all chips from a row to a higher numbered row are allowed. A player scores one point for each of these moves unless it is the last move for a particular pile, in which case the score is two.

SAMPLE APPLICATION INVESTIGATION

TOPIC: Compound Conditions for More Complex Problems (*Seeing Through Mathematics*, ch. 39)

CONCEPT: To solve conditions like $b + g = 11 \wedge b - 3 = g$

Application Investigation - To Solve the Condition Mentioned

The students are asked to undertake the following activity:

From their class of 15 boys and 9 girls (the numbers in the condition can be slightly varied to fit the situation of a particular class) the students are asked to compose two teams for a proposed ball game (if possible the game should actually be played at a later date). There must be two umpires, the same number of girls on each team, and three more boys than girls on each team.

In accord with the abstract-concrete guideline, the students could be allowed to set up the condition and solve it, or write the names on paper, or each name of class members on separate sheets of paper, or, for an even more concrete experience, physically attempt to select boys and girls for the teams.

The investigation can be converted into a game by using the teams proposed by the first student who arrives at a correct solution, and allowing choice of position on team in order of completion of correct solutions.

THE TEACHER'S ROLE

Finally, a few words concerning the teacher's role. The amount of intervention by the teacher will vary. It will likely be considerable, but it should take place in a special way. The teacher has already intervened by devising the experience. Further intervention should take only the following forms:

1. Clarification of procedures, but not concepts to be learned.

2. Evaluation and diagnosis of student progress.

3. Modification of the experience, direction to other experiences, or encouragement of the student to modify the experience himself.

Arbitration disputes arising from games should be handled tactfully, most likely in a mode in which the teacher has achieved previous success.

Finally, the ultimate aim of the teacher should be that the student react with nothing else but the mathematics of the situation.