WHAT - ANOTHER REVOLUTION?

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In this article, Dr. Hess is referring to the American (United States) high schools, where, in many localities, no mathematics is required for high school graduation. We have not yet reached this point anywhere in Canada.

At the time when I was trying to choose a title for this paper, I had just read an article in the *Mathematics Teacher* on "The Second Revolution in Mathematics". The most appropriate question then seemed to be "What - Another Revolution?" I was rather sure that I knew how to answer this. As time passed and I did more reading and studying, I became more uncertain of my answer. Many articles have appeared in newspapers and magazines which give some light on different aspects of the question.

As a starting point, I turned to the dictionary for help. I looked up the meaning of the word "revolution". As usual, Webster came through in good style. Here is what I found:

(a) Revolution is a progressive motion of a body around a center or axis such that any line of the body remains throughout parallel to its initial position to which it returns on completing the circuit. Revolution is often combined with rotation.

(b) Revolution is a total or radical change - as a revolution in thought. In political science it is a fundamental change in political organization or in a government or constitution; the overthrow or renunciation of one government and the substitution of another by the governed.

Thus, my title allowed me to take a text and depart from it. As the occasion demands, I will use either of the definitions. In the first part of my paper I will employ the idea of a revolution as progressive motion of a body round a center and the return of that body to its initial position.

In a chapter entitled "What Shall We Teach in Geometry", which appeared in the *Fifth Yearbook of N.C.T.M.*, the lead paragraph's heading was "Revolution in Mathematics". The writer discussed the pressure from many sources to cut down on the time and material devoted to arithmetic. It was felt that much of the work with common fractions could be omitted since one used common fractions very little in everyday life. The same kind of attack was directed at secondary mathematics, particularly algebra and solid geometry.

The attacks continued. Certain concepts or topics were found to be too difficult at a certain grade level. Therefore, they were delayed to the next grade. In fact, it was thought by some that one could delay much of arithmetic to the seventh and eighth grades. In algebra, topics such as l.c.m., h.c.f., and certain kinds of factoring were dropped and inequalities treated lightly.

Beginning in the 1950's with the advent of "new" or "modern" mathematics, it has been found that youngsters can learn much more mathematics than was ever believed possible. Concepts and topics have been moved down from grade to grade. It has been found that youngsters in the primary grades can do ruler and compass constructions, that youngsters in the intermediate grades can do algebra. Inequalities have been given added emphasis. The l.c.m. and h.c.f. is again in good standing. At the high school level, statistics and calculus are being taught. Do you see that we now have a complete revolution?

Algebra has long been one of the stumbling blocks for freshmen in high school. General mathematics has been used as a way to alleviate this trouble. A next step was to relax the requirements of mathematics for graduation from high school. This received great impetus from the "Eight Year Study" which was launched in 1932. The study was designed to see whether the nation's colleges could relax their admission requirements and accept students from high schools who did not have the traditional pre-college course of study. This meant that mathematics, as well as many other subjects, was optional. The results of the study showed that no single set of academic studies insured success in college. This led many secondary schools to either abandon requirements of so many units of mathematics for graduation from high school or to materially lessen the number of units required. In June, 1952, Benjamin Fine reported in the *New York Times* that the 30 schools who had participated in the study were essentially right back where they stood in 1932. In 1956, the State Board of Education in Montana once again made two units of mathematics a requirement for graduation from high school. Nationwide, schools - elementary and secondary - are requiring mathematics. Yes, another revolution.

In the *Fifteenth Yearbook of N.C.T.M.*, published in 1940, Appendix III is entitled "Terms, Symbols and Abbreviations in Elementary Mathematics." In this, certain terms are listed. Among these terms are three: equivalent, ray, half-line. The Commission recommended that the word "equivalent" be abandoned because "this is an unnecessary substitute for the more precise expressions 'equal in area' and 'equal in volume', or (where no confusion is likely to arise) for the single word 'equal'." Concerning the words "ray" and "half-line", the Commission wished to be non-committal. Today we talk about equivalent sets which are not necessarily equal. The terms "ray" and "half-line" are used extensively. This should be a constant reminder to each of us that we do not know what youngsters will need 25 years from now. When some expert decides that a certain concept or topic is of no use and should be eliminated, we should be reminded of the amount of time that transpired between George Boole's "Laws of Thought", written about the middle of last century, and the application of Boolean Algebra to electrical circuits by Shannon during the second quarter of the present century.

Another place where the wheel seems to have failed to turn is what people often call field work in mathematics or practical applications. Some writers include this under mathematics laboratories. In this paper I wish to extend the usual notion to include mathematics library. Modern works in psychology by Piaget and others point out the value of physical apparatus in the mathematics classroom. Writers in England have recommended physical experiences for over a quarter of a century, yet the movement does not seem widespread even in that country. In the United States, textbook writers and contributors to current periodicals have devoted much effort to this phase of mathematics. With the present emphasis in the United States on education of culturally deprived children, the need for physical objects for manipulation is being stressed: various teaching aids, models and devices are needed. Classrooms are still being designed without thought for small group arrangements. Unfortunately, many mathematics teachers must share their classroom with other teachers. This hardly lends itself to the development of a mathematics classroom. There is no revolution here by either definition of the word revolution.

During the 1930's and early 40's one finds several forces at work shaping the place of mathematics in secondary schools. Since there was little else to do, more and more children attended school. This heterogeneous group presented many problems. There was much emphasis on "child-centered" schools and the core curriculum. The feeling was that a class of children should assist the teacher in selecting what went into a given course. Concepts and activities included in a course were scrutinized for their social applications. Some writers based the topics included in their books on the criterion of social application. Mathematics was considered as a tool subject. Structure seemed to be an unheard-of word.

During World War II the schools were informed that inductees knew no mathematics. Any discerning teacher of mathematics could have given his information at a much earlier time. Schools were exhorted to put in crash programs. Suddenly mathematics became important.

Following World War II and the attendant explosion of mathematics, mathematicians, businessmen, cartoonists and others turned attention to the question of mathematics. Writers found that youngsters loved mathematics if it were the right kind and if it were carefully and precisely written. Structure became the key word - understanding was most important. In some experimental programs, social applications were given slight consideration or were entirely omitted. In a NCTM publication, *An Analysis of New Mathematics Programs*, one of the things considered was "How much emphasis should be placed on social applications of mathematics?" The committee found that some programs used social applications to develop a clear concept of mathematics, while other programs omitted them because they tended to cloud the clarity of a subject's structure. Apparently more attention is presently being given to social applications. We are still on the merry-go-round.

It seems generally agreed that man's need for mathematics led him to create mathematics. When a need arises today, mathematics is created to meet the need. Earlier mathematical systems were models of physical phenomena. There was a time when leading mathematicians like Klein, Hilbert, Poincare, Weyl, and Von Newman made notable contributions to the field of physics as well as to mathematics. Some of the experimental school programs have given little consideration to such applications. In fact, some new programs in mathematics tend to separate mathematics and science to an even greater degree. There are those writers who feel that the application of mathematics may, in the near future, lead the field in restoring the contact between theory and practice. Even much more than ever before applications depend on the support of even the most abstract theories. One can find little evidence of any revolution at this point.

For some time, drill has been a word with a shady reputation. Earlier in this century a certain psychology of learning made it seem desirable to accentuate drill. It was felt that enough drill would fix the path from the stimulus to the response so that a youngster would never forget a given concept or fact. Whether a youngster understood the whys and wherefores of a concept or whether the concept or fact was of any use seemed to be unimportant.

Suddenly in the 1950's it became evident that if a pupil understood what was being done, particularly if he discovered it himself, old-fashioned drill was not needed. In fact, according to an early bit of reporter fantasy, drill was no longer needed. There were some who did not want to break entirely with tradition; so drill was included, not in the main stream, but before a child could have his candy he had to take his sulphur and molasses. Others maintained that if the right kind of problems were used, enough drill would result. All in all, drill became a word of questionable character. Within the past year we find that there are those who say that we should not throw the baby out with the bath. The fact that some things are old is no reason to discard them. There exists a sinking feeling among some that we may have a generation of youngsters who can associate and commute but who find it difficult to actually solve a problem because of weakness in fundamentals. The wheel may have revolved too far. However, Heaven forbid that we ever return to the starting point on the drill question.

In Montana there has been a great emphasis on reading for many years. You will find a section on reading at every teacher's convention as well as summer conferences at the various colleges. Administrators have been very much aware of the reading problem in the schools. More power to them! For it was - and still is - a serious problem. If progress is made for reading in general, it cannot but help reading as it pertains to mathematics. It is my personal opinion that the emphasis on speed reading has been very detrimental to reading in mathematics as well as science. Reading 1,600 words a minute may be a worthy ideal in some fields, but it certainly is not for mathematics.

The Arithmetic Teacher, Vol. 59-157, February, 1966, contains an article by Call and Wiggin on "Reading and Mathematics". I will quote some very pertinent remarks from this article:

1. There is some merit in teaching special reading skills for the solution of mathematical problems.

2. Even very good readers, as measured by the Cooperative Reading Test, have difficulty in the interpretation of the kind of reading found in word problems.

3. Part of the difficulty which teachers encounter in the teaching of mathematics is that they are not equipped to teach reading.

4. Part of the difficulty which teachers encounter in the teaching of mathematics comes from a special kind of reading disability which does not appear on standard testing instruments.

5. If, by teaching reading instead of mathematics, we can get better

results, it seems reasonable to infer that the competent mathematics teacher might get considerably better results if he were trained to teach reading of the kind encountered in mathematics problems.

Some of the modern programs place considerable emphasis on reading. It appears to me that pupils who have trouble reading old mathematics will have as much trouble reading modern mathematics. As far as I can ascertain, this is one portion of the mathematical world that has been untouched by revolution in any sense of the word. It is what one might call an unexplored wilderness.

Once a boy who was herding sheep became lonesome. One day it occurred to him to call "wolf, wolf". Upon hearing this call, his neighbors rushed to his aid only to find that he had perpetrated a fraud. Later, a wolf did actually appear, but when he called "wolf, wolf" no one came to his rescue. It seems that our use of the word revolution leaves us in about the same position. As I commented earlier, the heading of the first paragraph in Chapter II of the Fifth Yearbook (1930) of NCTM is "Revolution in Mathematics". There is little in the paragraph which leads one to believe that the upheaval was drastic. The title of a 1961 NCTM publication is *The Revolution in School Mathematics*. In a recent article in the *American Mathematical Monthly*, there is a statement "on the wake of the revolution represented by the Bourbaki attempt". When I was preparing this paper I noticed a book with a 1963 copyright in the library entitled *Stillborn Revolution - The Communists Bid for Power in Germany 1921-3*. More recently there has been talk of "A Second Revolution" in mathematics. One writer talks of three revolutions in American education. It is my contention that overuse of the word has made it too commonplace to be really effective.

One of the often mentioned characteristics of modern mathematics is the unambiguity and preciseness of the language. None of us would care to quarrel with such worthy aims. On occasion it seems that the aim may be overworked. For example, let us consider the following: "Given the equation $F(x,y) = 4x + 3y = 12$, determine the set $A = \{(x,y) \; x \epsilon N, y \epsilon N, F(x,y) = 12$, where N is the set of all integers$\}$." I believe that we will all agree that this is logically clear. Now let us compare this with: "Determine the integral roots of the equation $F(x,y) = 4x + 3y = 12$." While it is desirable and necessary to introduce and use symbols in some mathematical situations, it is neither desirable nor necessary to introduce and use symbols where there is a simpler, more straight forward way of writing the same idea. Modern mathematics has introduced an open season for inventing and using new symbols. Each time the urge arises to invent a new symbol, the question should be asked "Is this symbol necessary?" Sometimes, in an effort to make a definition of a term precise and unambigious, it becomes cumbersome and wordy. One reads the definition and then wonders how to use it. There are times when it is better if the wheel does not turn.

I am sure that all of us eagerly peruse any report of a study of comparison of achievement in modern mathematics versus traditional mathematics. I must confess that one of the earlier reports contains a result which is disconcerting. It was found that students taught by teachers who had no previous experience in that particular experimental program did better than students taught by teachers with such experience. This is indeed a revolution in every sense of the word! I will let you persue this particular point a little further in your own thinking. Remember, you experienced teachers are now expendable!

There is another point in these reports that has always intrigued me. In these reports the results are usually varied. It may be that achievement in the experimental program is not statistically different from achievement in the control program. One program may show gains higher than the other. In any case, one is likely to find the statement "they are getting something in the experimental program that we cannot measure". In all fairness, it is true that the tests used, particularly in the early studies, were devised for traditional mathematics. Might one not apply the above statement equally well to traditional mathematics?

Now let me turn my attention to the second meaning of the word revolution. This meaning is a total or radical change, or, as in political science, a fundamental change in government and substitution of another by the governed. In the usual political revolution today, this means that some people are hurt.

Most of us have read *Goals For School Mathematics*.[1] Some consider this as the opening salvo in the second revolution in mathematics. A recent article stated as a heading that modern mathematics fears it is getting old. There is evidence that the end is not in sight.

It is probably safe to say that the first revolution in mathematics was concerned about college-capable students. It is true that many experimental programs want to be included in the programs concerned with students who are not college-bound. Some persons have asked just how widespread this first revolution is. The question has also been raised as to how well acquainted writers of experimental programs are with the results of research in the psychology of learning. Materials are written to be logically sound, but are they psychologically sound? A return to an old question! Again, what is to be done with the lower one third of the school population as far as mathematics is concerned? With these unanswered questions, I will direct my attention to some other aspects of the revolution.

I mentioned earlier that in most political revolutions people get hurt. In the present mathematical revolution people were hurt. One group of people who have been hurt are the mathematics teachers. In the days of drill - ask no questions, do the problems as I tell you - the teacher soon achieved a mastery of the situation. With the textbook in one hand and the answer book in an open drawer, the teacher soon achieved confidence in his or her ability. Certainly, anyone could teach arithmetic. If one had had a course in algebra, this qualified him to teach the subject.

With the revolution, teachers were called on to teach new material in a way new to them. When one is unacquainted with something, there is always fear, suspicion and lack of confidence. To add to this, one must lead youngsters to discover concepts and facts. Incidentally, the so-called discovery method is far from new. Socrates, by questioning, tried to lead people to truth.

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[1]Cambridge Conference on School Mathematics, *Goals for School Mathematics* (Boston: Houghton-Mifflin Company, 1963).

I trust that all of you know what happened to Socrates. Back to the mainstream. Above all things, encourage youngsters to ask questions! "Now see if you can do the problem another way!" "Make generalizations; what guesses can you make?" "But my answer book shows only one way - and it does not provide answers to all questions that youngsters can ask." No wonder teachers were upset and driven almost to nervous prostration. Institutes, inservice courses, and reading have become the order of the day. A perusal of the Cambridge Report should leave everyone with the feeling that there is no end to this revolution as far as teachers are concerned. The textbook and the answer book is not enough. Here is a score for the scoreboard of the revolution!

The question of homework has been troublesome for many years. There was a time when homework in prodigious quantities was assigned students. The students reacted to this sort of thing in various ways. Some students blithely ignored the assigned work. Other students did the work faithfully. The next day they achieved a degree of popularity with a certain group of students who did not do the homework but wished to make the teacher believe that they had. Still another group called on Dad or Mother for assistance. It was a matter of paternal pride that Dad knew how to do the problems. However, some parents resented the fact that they had to do work which, in their opinion, teachers should have done. Studies were made on achievement in mathematics with and without homework. Since there seemed to be some question about its good effect, this helped usher in an era of no homework in mathematics in many schools.

With the advent of the new mathematics and urgency of more youngsters to study mathematics, the problem again came to the front but in a different guise. Again, homework became an acceptable state of affairs in mathematics. But something had happened. Dad could not do Johnnie's homework. Yes, he might get the answer - "but that is not the way we do it." Now it was the parents' turn to be hurt. They were frustrated because they saw no reason for the change. For their children to be able to do mathematics that they did not understand was adding insult to injury. Some school systems, in order to forestall some of this trouble as well as criticism of the new mathematics, did not let the youngsters take papers home. After all, if one adds 2 and 2 it is 4 and not 10. Why this nonsense? What use is this in business? One sure way to whet curiosity is to keep things a secret. In recent years numerous books and booklets have been written for parents explaining the new mathematics. Many parents have attended evening classes.

All of this should make it quite clear that parents are interested in schools and in what their youngsters are being taught. Informed parents are the best supporters of education that one can get. Another score for the mathematics revolution!

One innovation that has been the result of the mathematics revolution is the way in which textbooks, teaching aids, and materials are prepared. Before M.R., some person or two persons, occasionally three persons wrote a book. Each person concerned was usually considered an authority in mathematics or the teaching of mathematics. Due attention to the various committee recommendations was included - at least in the preface.

Various approaches have been used in the preparation of the new programs.

In some programs the writing was done by a team consisting of lay persons, teachers at level the material was being prepared, university mathematics professors, educators, psychologists, consultants, etc. In other programs a group of teachers were given training in the subject area. After this, they prepared materials to be taught. In other cases it was done by a team at the university level. Occasionally, the program was the result of the leadership of a single person. Coming out of such varied background, one would expect the programs to differ in many points.

The approaches described above indicate that a great number of persons are acquainted with a given program. Any such program is not the work of one person but the pooled efforts of the team. As a by-product of this cooperation, persons at the university level are welcomed in the elementary and secondary schools. Persons from the elementary and secondary schools are often called on to teach a course or so, particularly in the summer sessions. Speaking as one who has participated in a writing team, I believe that this is a third score for the mathematics revolution. This is a change from one form of government to another - with no one being hurt. Furthermore, this method stresses the need for the training of teachers in the new materials and methods.

A survey reported in *Fortune* Magazine a number of years ago showed mathematics as one of the most disliked subjects in school. My experience at the university level with persons going into elementary education - and who will be teaching arithmetic - bears out this finding. Somewhere back in the past many of these persons had acquired a dislike and fear of mathematics. The unfortunate fact is that these persons are inclined to continue to dislike and fear mathematics and even carry over this attitude to anything or anyone connected with mathematics. My experience over the past few years has led me to believe that the modern mathematics can be so taught as to dissipate the dislike and fear already present. This experience has convinced me that we can repair some of the damage done at an earlier time and dissipate the fear of many persons. Do not assume that we win them all - neither has any of us been nailed to the cross. Score four for modern mathematics - many persons who formerly feared and disliked mathematics gain an understanding with new mathematics.

One problem of every teacher of old mathematics as well as new mathematics was and is the slow learner. It is quite evident to anyone who has observed a group of persons that some learn faster than others. In school with the "pages to be covered" per day this means that the slower learner gets farther and farther behind. Eventually, this person turns to something else which is less frustrating. At an earlier point in this paper I suggested that the experimental programs in modern mathematics were generally aimed at the college-bound student - we hope this group includes the faster learners. Nevertheless, the question kept arising as to how slow learners would fare with modern mathematics. Some reports exuded a mild amount of optimism, but generally the results were summed up somewhat as follows: "The slow learners do no worse with modern mathematics than they do with traditional mathematics." Of course, if one is at the bottom of a given heap, then there is actually only one way to go.

I would like to dwell on the topic of the slow learner just a little longer. There are writers who contend that when properly presented, the contents of modern mathematics can be understood and enjoyed by many of the slow learners. Allowances need to be made in terms of the individual student's rate of progress. I am sure that reading is a decided factor in success. In my own mind, I am convinced that there are some students with a blind spot for certain colors. Some recent work with slow learners indicates that the number of persons who cannot learn mathematics if properly taught is much smaller than is generally assumed. It takes time for ideas to grow. It is true that NCTM and other groups have been giving some attention to the development of materials for the slow learner.

Let us remember that a slow learner will eventually be a voting citizen. If we continue to neglect this group of students, we can hardly expect royal treatment from them when they sit on school boards and in other seats of authority. The least we should try to accomplish is a friendly attitude on their part toward mathematics. Mark up a goose egg on the scoreboard for modern mathematics and its help to the slow learner.