THE SITUATION PROCESS IN PROBLEM-SOLVING: HELP OR HINDRANCE?

T.P. Atkinson
University of Alberta
Edmonton, Alberta

Among the objectives of teaching mathematics, we find two which are given considerable attention. With the risk of over-simplification, may I refer to them as

1. the content of mathematics,
2. its application to our environment.

I shall leave for you to decide what the first involves. I am concerned in this paper with the second; specifically, I propose to consider problem-solving, which is one phase of the teaching towards it.

The procedure which is referred to as the "situation process" technique of problem-solving requires basically four steps:

- Analyze the physical situation to determine the action that takes place.
- Write a mathematical sentence, using numerals for known numbers and placeholders for unknown numbers, insuring that the operations involving the numbers bear a direct relationship to the action identified in the first step.
- Do the computation necessary to determine the unknown number or numbers.
- State the answer to the question asked in the problem.

To illustrate the steps, consider a rather simple problem situation described as follows:

> Eight children were playing a game. Some more children arrived to play with them. Then there were 17 children playing together. How many more children had arrived?

1. The analysis of the situation reveals that two groups of children have been combined to produce one group. The addition of two numbers and their sum is suggested.

2. The mathematical sentence is $8 + n = 17$, because 8 is a numeral naming the number of the first group, n is a placeholder for a numeral since the number of the second group is not known and 17 is a numeral naming the number of the group resulting from the combining of the two groups.

3. To determine the number named by n, one subtracts 8 from 17. But why? The computation is justified by reasoning that when the first group (whose number is 8) is removed from the combined groups (whose number is 17), the group whose number is not known remains. Thus an auxiliary sentence $17 - 8 = n$ may be thought of, and the computation is evident.

4. The answer to the question is: "Nine more children arrived to play."

Discussion and argument among mathematically experienced individuals often develop because there are several sentences equivalent to $8 + n = 17$. (Sentences are equivalent if the replacement or replacements for n that make the sentence express a true statement are the same in both cases.) Here the sentences $17 - 8 = n$ and $n + 8 = 17$ are equivalent to $8 + n = 17$. Some teachers aver that the first of these may be used because it reflects the thinking which a child must use to justify the computation; others support the second in that the union of sets is commutative.

It is more difficult to adhere to just one point of view when one considers the comparison situation. Consider the example:

> Eight children were playing in one yard and 17 children were playing in another yard. How many more children were playing in the second yard than in the first?

The technique involved is the matching of the members of the set of 8 in one yard with a subset of 8 of the 17 in the other yard. When the subset of 8 is removed from the set of 17, the remaining subset indicates "how many more". Hence the action suggests the sentence $17 - 8 = n$.

It may be argued that one can consider recruiting a set of children to combine with the set of 8 children so that the new set has 17 children in it. The number property of the recruited set provides the answer to "how many more". Such action suggests the sentence $8 + n = 17$. I am satisfied that one of these sentences is better than the other in reflecting the action a child would follow in comparing two groups of objects which he could move about. However, the distinction between the two analyses may be more blurred here than in the first situation.

In problems involving the combining of several sets having the same number of elements each, the multiplicative principle is involved. In general terms the mathematical sentence is

> (Number of sets) times (number of elements in each set) equals (number of elements in the combined sets).
>
> Consider the example:
>
> A man bought 4 copies of the same book. He paid $32 for them. What was the price of each book?
>
> The particular sentence based upon the general one is $4 \times n = 32$.

Arguments to the effect that this is not the only permissible sentence are based upon the mathematical understandings that multiplication is commutative and that division and multiplication are related (inverse) operations. When two factors and a product are involved, knowledge of any two will lead to the third by one of the two operations.

The "situation process" treatment of the operation of division is based directly upon the action of separation of a set of objects into a number of equivalent subsets. Again, speaking in general terms, the sentence is as follows:

> (Number of elements) divided by (number of elements in each subset) equals (number of subsets).

If I wish to distribute 84 pencils to 6 children so that each child receives the same number of pencils, I think $84 \div n = 6$. In order to justify the computation, I must think of allocating one pencil at a time to each of the 6 children so that in effect I am using sets of 6 pencils. Thus, the sentence $84 \div 6 = n$ arises in one's mind; because of its more direct connection with the actual computation, it is regarded as being equally acceptable as the other sentence.

I wish to draw to your attention several more type situations, but for the sake of brevity I shall simply describe the situation and write two or more possible mathematical sentences arising from it. I would like you to decide if the sentences are equally good or if one is better than another.

Example: A confectioner had 192 candy bars to pack into boxes, each of which held 12 bars. How many boxes did he use?

$\frac{12}{1} = \frac{192}{n}$	12 bars per 1 box, 192 bars per n boxes.
$192 \div 12 = n$	Separate 192 bars into subsets of 12 bars.
$n \times 12 = 192$	A number of boxes of 12 bars each will hold 192 bars.

Example: A teacher separated her group of children into four teams for a game in the gymnasium. She had seven children on each team. How many children were there in the class?

$\frac{7}{1} = \frac{n}{4}$	7 children per 1 team, n children per 4 teams.
$n \div 7 = 4$	Separate the group of n children into subsets of 7.
$4 \times 7 = n$	4 teams of 7 children each make up the whole class.

Example: A store advertised condensed milk at 2 cans for 35 cents. How much should a shopper pay for 8 cans?

$\frac{2}{35} = \frac{8}{n}$	2 cans of milk per 35 cents. 8 cans of milk per n cents.
$\frac{8}{2} = \frac{n}{35}$	The relationship of 8 cans to 2 cans

must be the same as the relationship of n cents to 35 cents.

$n = (8 \div 2) \times 35$	$(8 \div 2)$ gives the number of sets of 2 cans. This is the number of groups of 35 cents to be paid.

<u>Example</u>: A farmer sowed 72 acres of wheat. He sowed 3 times as many acres of wheat as of oats. How many acres of oats did he sow?

$\frac{3}{1} = \frac{72}{n}$	3 acres of wheat to 1 acre of oats, 72 acres of wheat to n acres of oats.
$72 = 3 \times n$	72 is 3 times the number of acres of oats.
$72 \div n = 3$	Separate 72 acres of wheat into 3 lots of the same size, each one to match the number of acres of oats.

<u>Example</u>: In a class there were half as many boys as girls. There were 22 girls in the class. How many boys were there?

$\frac{\frac{1}{2}}{1} = \frac{n}{22}$	$\frac{1}{2}$ of a boy to 1 girl, n boys to 22 girls.
$n = \frac{1}{2} \times 22$	Think of "half" as meaning one of two equivalent subsets.
$2 \times n = 22$	2 groups of n boys each are equivalent to 22 girls.

Do the foregoing examples suggest that there are several equally valid ways of writing the mathematical interpretation of the action in a situation? I have tried not to state my point of view in this presentation so far. Now I wish to summarize my ideas, most of which are based upon the presentation, some from related experiences not specifically mentioned.

1. The "situation process" technique is a valid approach to problem-solving because it provides a method of isolating the mathematical aspects of a physical situation in a meaningful manner.

2. Because there can be different points of view about the action, there may be more than one acceptable mathematical sentence. However, the criterion of choice should be the action in the situation, not mathematical convenience.

3. The writing of sentences should be used as a testing device only when the specific skill of translation is being examined and when the action is unequivocal.

4. The principles underlying mathematical operations and the relationships between them (e.g. commutativity of addition; division as the inverse of multiplication) arise out of mathematics more than out of physical situations. By illustration: $2 \times 5 = 5 \times 2$, but 2 sets of 5 books are different from 5 sets of 2 books.

5. When computational devices are being learned, their basis upon physical situations should always be clear. After they are learned, shortcuts based upon mathematical considerations may be justified.