MODERN MATHEMATICS IN ELEMENTARY SCHOOL - A CRITICAL VIEW

William A. Steeves
Department of Education
Quebec, P.Q.

If I were to choose a text for my remarks, it would be an observation by Harold Wilson, Prime Minister of Great Britain: "We have had far too many words chasing too few ideas." I shall now proceed to live up to that statement, "... too many words chasing too few ideas."

Since 1959, or a little earlier, there has been a startling revolution in mathematical content of textbooks, in teaching methods, and in teacher attitude. All the mathematical thinking of many interested educators seems to have burst forth all over this continent and indeed the whole world. Public demands for better mathematical and scientific education gave an impetus to this and literally everybody has felt the impact. Publishers, authors, many teachers at the elementary, secondary, and university levels of education seized on the movement to attempt to revitalize the mathematics programs. Psychologists have restated the laws of learning and propounded new ones based on recent studies and research. The schools have been swamped with all sorts of new texts, materials, and devices, most of which have said or implied that here is something new which every school must have or it will be hopelessly behind. Who wants to be behind the times?

It should be remembered, of course, that changes now affecting mathematics are part of a wider movement enveloping the whole pattern of education.

I shall not attempt to give what I think should be a new program for the elementary school, Kindergarten to Grade VI. There are many new programs which are done much better than I could do them. Let me just make two statements.

Firstly, I do not subscribe, emphatically do not subscribe, to developing a program or flow chart of elementary school mathematics based on any one set of materials, be that set of materials blocks, rods, apples, oranges, or rabbits. Perhaps I lean a little toward rabbits because they can multiply quickly. I do believe that teachers should have available whatever materials there are and should use those which they can use best. Two years ago when I visited several Canadian cities, I observed teachers in Vancouver using rods, in Richmond - blocks, in Burnaby - various materials - charts, blocks, rods, and unifix, in Edmonton - circles on the chalkboard, and in Toronto - workbook material.

Secondly, I subscribe substantially to the program suggested by Dr. H.F. Fehr in his article "Sense and Nonsense in a Modern Mathematics Program", to be found in *The Arithmetic Teacher*, February, 1966.

Dr. Fehr says that, "If at the end of sixth grade almost all the children know the decimal system of notation; can read and write the numerals for whole and fractional numbers; can at an adult level of performance do all four computational operations on whole numbers and fractions, both in common and decimal notation; have an intuitive understanding of the rationale and structure

underlying these computations; can apply this knowledge meaningfully to the solution of problems involving measure and per cents; and know and recognize common geometrical figures and relations among them; we shall have achieved an outstanding and notable advance in elementary school mathematical education. This is something for us to aim at."

Now as to method - today the so-called "discovery method" is in vogue. If you are not using the discovery method of teaching elementary school mathematics, you are behind the times. We tend to go in circles; progress is slow. I am reminded of the story of the snake. The head probes ahead of the tail. It keeps probing this way and that, and long after the head has passed a certain point, the tail arrives. Naturally, the tail is a little out of sorts about this until it discovers that the head has come around to the same spot. I would remind you that in this action the tail, in trying to keep up with the head, has made some progress and that the whole snake has moved ahead a little. So it is with education; we never go back quite to where we were; we simply change the names and use different language.

Discovery is not new. The vast scientific and mathematical knowledge of today came to us largely through discovery. Men discovered, tested, and fought to have their discoveries accepted. The Church condemned Galileo's discoveries, but you recall the Pope wouldn't sign the condemnation document because he was infallible and was smart enough to realize that if at some future time Galileo was proved right, it wouldn't look well for the infallible Pope to be proved wrong.

Today the discovery method means letting the child discover things for himself. This is good, but the teacher must provide the environment and be the guide and leader. The guiding philosophy today is that the child should think and figure things out for himself. As Sophocles said long ago, "One must learn by doing things, for though you think you know, you have no certainty until you try."

Let us see how it works: A group of children are seated around a pile of blocks. Each child selects a certain number. The teacher asks, "How many blocks have you?" One child replies, "two". "Would you like some more blocks?" The child takes three more blocks. "How many blocks did you take this time?" The child counts and says, "three". "Good, how many blocks do you have altogether?" The child counts again and says, "five blocks". And so he discovered that a group of two blocks and a group of three blocks gave him a group of five blocks. "Can you arrange the five blocks in different groups?" He discovers that he can have a group of four blocks and one block by itself, a group with only one block, a set of one element.

Another child selects three blocks and three more blocks. He finds that he has six blocks and discovers that two groups of three blocks made six blocks; that he can rearrange them in three groups of two blocks. And so he learns the beginning of addition and multiplication and, if you like, subtraction and division. He writes what he has learned on the chalkboard.

All through his school life the child should have opportunities of learning by finding out. What is needed if democracy is to survive is a type

of mind which demands to hear both sides of every story and weighs one against the other objectively. This is the essence of democracy.

When children explore for themselves, they make discoveries which they want to communicate to their teachers and to other children, and this results in discussion. It is this changed classroom relationship which is the most important development of all.

The teacher sets the stage and provides the opportunities, the motivation, and the necessary stimulation. I recall being in a Grade III class recently. The children were doing subtraction with borrowing with three place numbers. After one child had explained and demonstrated one way on the chalkboard, four children each worked on the question in a different way and then the class discussed which way seemed best. These were children being taught to figure things out for themselves and to evaluate their work.

Textbooks today are full of examples which give the pupils an opportunity to think for themselves if the teacher will let them. One trouble is that teachers insist on reading the examples to the children and in so doing interpret the problem. Usually they are interested only in the answer and not in how the child thought in order to get the answer.

In problem solving, the child must learn to restate the problem in simple mathematical form; he must also learn that the unknown may be represented in this form by a symbol. In interpreting problems, the child must learn to state mathematically the information given in the problem. In other words, he must learn to think logically, and although we do not mention the word logic to him, we must endeavor to be logical even if the elementary school child himself is not logical.

While it is our hope that the child will learn to think and discover, it is the teacher's responsibility to see that he gains a wide variety of experiences with concrete material, semi-concrete material, and abstractions. New textbooks are full of many ideas for varying the pupil's work and giving him the opportunity and guidance he needs to help him develop his ability to think independently. Every effort has been made to help the teacher to give the child the variety he needs in his work. Experimental approaches to multiplication and division, finding area by counting squares, different methods of dividing a fraction by a fraction, "what's my rule" questions, cross number puzzles, experiments in design with geometrical figures, are but a few. Have you ever asked a Grade VI class to see in how many ways they can divide a two-inch square into eight equal parts?

Teaching the new elementary school mathematics program requires that the teacher be continuously conscious of the child's question, "Why?" This question must be answered for the child, either by the child's experimenting or by the teacher. Therefore, every effort must be made in our teaching to help the child to know why our numbers are the way they are, why we carry, why we divide whole numbers and fractions the way we do, why we multiply the number of square inches in a row by the number of rows to find area, and why we multiply the diameter of a circle by pi in order to find the circumference. These are a few of the child's "whys".

I mentioned earlier that the discovery method is very popular in educational thinking today and that its use will prove most helpful in teaching new programs in elementary mathematics. However, be assured, it is no panacea; the millenium has not arrived. It is still necessary that children learn and practise so that their responses to many facts will be automatic. They must, over the years, gain a storehouse full of information which will be ready for use instantly. While all our new methods are aimed at motivating the child, holding his interest, and giving him the satisfaction of achievement, our efforts will be largely fruitless if the child does not gain precise knowledge and the ability to use it. To accomplish this, we must give the child an opportunity to check his work or the teacher must check it so that the pupil knows that he has achieved and that he has accomplished something now. It is not good enough for the child to wait for a day, or even much longer, before he knows what he has accomplished. We can learn from industry. Some of the in-service training programs of the Northern Electric Company, for example, are so planned that the trainee knows not only whether his responses are correct or not but he knows what effect his mistakes will have on production, and he knows it immediately.

This brings us right up to the teacher. It makes little difference how much money we spend, how good our buildings are, how good our equipment is, how many TV's and radios we have; the education of our children will be as good as are our teachers. The fact of the matter is that the only really important and indispensable factor in helping children to learn is a good teacher. No devices, whatever they may be, will replace him. It is still true that a good teacher on one end of a log and a student on the other is the best learning situation.

Fehr says that in order to implement a sensible program we need two essentials: suitable textbooks and good teachers. He goes on to say, "But a good mathematics program depends not so much on a textbook or a syllabus as on the teacher and the methods of teaching. Let us make no mistake! Any program, no matter how sensible, modern, and balanced it may be, can degenerate into mere dogma in the hands of a dognatic teacher." Have you ever heard a teacher putting a class through "rod drill"? White is 1, red is 2, light-green is 3, and so on. I have!

There are certain basic qualifications which every teacher should have:

1. the basic teacher training and education offered by various teachers' colleges and universities (I have said 'training and education' because I believe that each teacher requires training in the routine of class management in its broadest sense);
2. fundamental education in the subjects taught in school and as broad a knowledge as possible in the special subjects to be taught (for the elementary teacher it may be enough to have studied only high school mathematics successfully, but I doubt it; the teacher needs a knowledge of the number systems, the theory of sets, mathematical structure and relationships and much more if he is to teach the new programs in mathematics well);
3. the teacher also needs a good knowledge of the elementary school course of study (it is not enough for him to know the work of just the grade

which he is teaching - a Grade III teacher needs to know the work of Grades I and II and that of Grades IV to VI, yes, and some high school work also).

These three elements - teacher training and education, a liberal education with a depth of knowledge in the special subject field, and knowledge of the school curriculum - are basic. In addition, each teacher must be a student. The teacher must not be content with preparing his lessons from the material in the textbook or manual; he must constantly be searching for new ideas, for variations in method, and for enrichment materials.

The teacher must teach in the classroom. It is quite impossible now to have the pupils learn new programs in mathematics by assigning work and forgetting about the children for the next 20 or 30 minutes. Teachers just must not do this any more, not even in the high school grades. Why?

Firstly, the pupils in any class, be it large or small, vary in ability and must be taught in groups or given individual attention.

Secondly, an assignment that can be done without the teacher's guidance is too easy. It is necessary for the teacher to go around to see whether or not the children are having difficulties and, if they are, to give them the necessary assistance. Recently I visited a class in which the pupils had been given a sheet of questions including about 10 subtraction questions with borrowing. The teacher allowed some children, and one in particular, to work all these 10 questions incorrectly and thus waste time when a little help at the beginning would have put the child on the right track.

Thirdly, the new programs are activistic in nature. They require discussion, probing, eliciting answers from the pupils, evaluation of answers, checking of written work, preparation and use of all sorts of illustrative materials. The teacher just cannot do this from behind the teacher's desk, that stronghold of ignorance.

Some of you may be wondering just what this has to do with the new programs in elementary school mathematics; after all, this talk was supposed to be a critical view of the new programs. I want to tell you that it has everything to do with the new programs. The new programs are almost useless without the teacher. We have in our educational setup today eminently qualified educators who can devise wonderful procedures set forth in manuals and we have these programs available for teachers, but the biggest problem of all is to improve what goes on in the classroom. Unless this is done, the new programs will be largely ineffective. The new textbooks and manuals are very complete. They set forth suggested methods, activism, or discovery, but the teacher must make himself thoroughly familiar with them so that he can guide the children in their learning.

If the teacher is to help the children to learn or think more clearly, he must be concerned with what they are thinking and how they think. Writing tests and checking answers are not enough. The teacher must talk to individual pupils and find out how they are thinking. Ask them to explain what they are doing. Now you may say, "How can a teacher with 25 or 30 wiggling boys and

girls find time to interview each one separately?" Well, fortunately, this is not necessary with all pupils but only those who are having trouble. For example, take the Grade III child mentioned earlier who experienced difficulty in subtraction with borrowing. I observed that the child was following the correct procedures except that she forgot she had borrowed from the tens place. I sent her to the chalkboard and asked her to explain each step of her work. This she did quite well and loud enough so that any pupil who wished to listen could hear. It was only necessary to ask her how many tens she had left after she borrowed to draw her attention to her error. She corrected the question, worked a couple more at the board and was on the right track. Other members of the class having the same or a similar difficulty learned from the discussion, and the child at the board showed that her thinking was, on the whole, satisfactory.

One of the most difficult problems we face in the improvement of elementary mathematics have to do with the indifferent or lukewarm teacher. The inservice education of our teachers is a vast undertaking. Fortunately, most teachers are very anxious to do a better job and they, in turn, will (so we hope) infect those whose work is less effective with a desire to improve. During the past five or six years we have in fact witnessed the most unbelievable change in the attitude of teachers of mathematics and their desire to teach more effectively. They are vitally interested, full of enthusiasm, keen about learning the new programs, and becoming increasingly effective teachers. There is also evidence that the children, too, have caught the spirit.

Research is another great need. To my knowledge, there has been no research which has established whether or not the new programs are better than the traditional programs, but the research undertaken during the past 20 years into the psychology of learning provides unchallengeable evidence that sound and lasting learning can be achieved only through active and continued participation.

There has been and is much criticism of educational research. S.L. King reported that at the 50th Anniversary Meeting of the American Educational Research Association there was now "the conscious emphasis on processes of teaching and learning rather than on their end products". Much of the research reminds me of how the hippopotamus got its name. It seems that when God was naming the animals his assistant asked him why he called this one the hippopotamus. God replied that he thought it looked as much like a hippopotamus as any animal he had ever seen.

The individual interview technique, although slow and fraught with many variables, has promise, but it must be very carefully planned. Also, we know so little about what or how children are thinking. Some of you may have read that charming collection of essays, *O Ye Jigs and Julips* by Virginia Hudson, written when she was 10 years old. In her essay *Etiquette in Church* she writes, "Etiquette is what you are doing and saying when people are looking and listening. What you are thinking, is your business. Thinking is not etiquette".

Brownell, (*The Arithmetic Teacher*, April, 1966), says that, "We tend to minimize the complexity of evaluative research." Glennon, (*The Arithmetic*

Teacher, May, 1966), quotes Cronback as suggesting that existing research on inductive teaching, the discovery method, "has not begun to give the answers needed for firm recommendation of the schools."

The researcher has great difficulty in trying to keep pace with the panacea-mongers. May I suggest to you, "Be hospitable to new ideas, but beware of panacea-mongers!"

There is great pressure on schools today to adopt all sorts of new devices. These must be evaluated, and decisions must be made as to whether or not they are suitable. Also, there is great pressure on schools to introduce new topics and to teach old topics earlier. Should we not always ask ourselves, "Why?"

It is my thesis that within the next few years you will see the present new programs in elementary school mathematics revised and revitalized. There will be many new procedures to be evaluated. There will be a greater emphasis on the learning of fundamental concepts and relationships relevant to the child's needs and environment as he grows and develops. There will be greater emphasis on how we analyze problems, the method of attack, and the selection of pertinent facts. Methods will improve and the activist approach will become the normal; future teachers, having been taught in school by this approach, will use it with greater ease and refinement. Notice that I said the 'activist approach', not method. There are many teachers, I fear, who think that the 'discovery method' means 'any' method, and this so often means 'no' method. While I believe that it is good for children to devise their own methods, I also believe that these methods should be evaluated and the best selected. Whatever method is used, it must follow sound mathematical principles and be well done.

To sum up, we have fine new programs of elementary school mathematics, but many of them emphasize pure mathematics to the exclusion of what is more practical for the average child. New programs must be examined carefully, and we must beware of new programs which claim to be the 'great answer' for which we are searching. Also, remember that no program replaces the competent and devoted teacher.

The so-called discovery method is very promising in the hands of the teacher with imagination and ability, but let us be honest with ourselves, too few teachers have the imagination and ability to use it really effectively. Also, let us remember that some things are not worth the time it takes to discover them.

There must be good inservice courses for teachers, but beyond these courses there must be small study groups in which the teachers have an opportunity to discuss their work and they themselves must study, experiment in their classrooms, and constantly and continually evaluate and reevaluate the effectiveness of their methods.

Teachers using the new programs must teach. They must plan their work very carefully and provide activities which are challenging and which take into account the individual differences of the children in their classroom. This

means also very clearly that they must not be content with merely keeping children busy.

Above all, let us remember that we are teaching children, children who, we hope, will take places in a free society. However, freedom needs framework. For the individual that framework is character; for society it is the law. In education today, there is much bewilderment and some dismay. It is our duty to teach our children that they will respect the creative restrictions which make public order.