COMMENTARY

MATHEMATICS EDUCATION IN THE 1970s

A Retroperspective

David Pimm

Although I was nowhere near Alberta during the 1970s, it was during that decade that I encountered for the first time many of the names cited in the articles collected here, such as Kline and Ausubel, Bruner and Pollak—touchstone figures to whom people cleave at various times and for a variety of reasons. In this piece, you will meet some of mine.

I came to the University of Alberta full-time in 2000—staying for the entire decade as a professor of mathematics education (following Tom Kieren's retirement)—but I had visited several times and briefly taught there in the preceding decade, during which time I was frequently somewhere in Canada. But for the first half of the 1970s, I was an undergraduate and then a graduate student in mathematics (mostly in the UK), and the second half I spent in graduate school in mathematics education (in the U.S.). In his commentary on the sixties, Tom Kieren refers to Richard Skemp and his connection with Alberta. Skemp taught me as an undergraduate at Warwick University in 1974, and it was to work with him as a researcher that I

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returned in the summer of 1979. (My first task was to help him organise the third International Group for the Psychology of Mathematics Education conference, an annual meeting whose 38th incarnation will take place in Vancouver in 2014.)

However, I had come across Skemp's name before that, scrawled on desks in my high school in the mid-1960s ("Skemp is mad," "Skemp is impossible"). I recall wondering what a Skemp was, for mine was the last class who studied "old maths" at my school, we unaware travellers on its trailing edge, with the replacement "new maths" champing hard on its heels (to mix metaphors atrociously). Between 1964 and 1970, Skemp had authored a series of "modern mathematics" secondary textbooks (one of the first, if not the first, in England), entitled *Understanding Mathematics.* Hence, a possibly unintended schoolboy irony lurked in those inked and carved rejoinders. This is not simply nostalgia on my part—the schism produced by the introduction of new mathematics in (at least) Western Europe and North America during the early to mid-1960s would resound throughout the following decade (as attested to by these 10 articles, to a greater or lesser extent). In fact, I believe that its issues and preoccupations can still clearly be heard (like some ghostly echo of a Biggish Bang) in our current clashes some 50 years on.

So, though I have been an active adult participant in the field of mathematics education for some 37 years, I was also present as a consumer for 15 years before that. (My elementary school was also one of those involved in the late 1950s in experimentation with Cuisenaire rods under the watchful eye of Dr. Caleb Gattegno, who was then working at the London Institute of Education before leaving for New York City.) I start with this brief biographical sketch in order to locate myself, as the phrase goes, to assist readers in reading my comments on this engaging and interesting ten-tet of articles.

While I have some specific comments in response to particular articles, there are also general themes and concerns that spoke to me from these pages. Themes related to curriculum (and those echoes from the 1960s new math reforms and curriculum projects); the appropriate or necessary background knowledge and preparation of mathematics teachers; the nascent use of technology in mathematics teaching; and tensions in the teaching and learning of mathematics (tensions either weighed and balanced or one-sidedly insisted upon), most specifically between "fluency and practice" and "understanding" (in its various guises). Those same themes, tensions, and concerns are still with us 40 years later, possibly and on occasion in a more refined or sophisticated form. Though with regard to the reverberating clangour of the more contemporary "math wars" (see, for example, S. Wilson, 2002), as well as the issue of the appropriate mathematical preparation of teachers (especially at the elementary level), arguably the opposite is true. And as ever, like it or not, university-based mathematicians seem always to be with us, too.

Here are a couple of my touchstone mathematics and mathematics education events from the mid-1970s: the computer-assisted proof of the four-colour theorem, and the posthumous publication of Imre Lakatos's (1976) *Proofs and Refutations*. Neither of these directly relates to the teaching of school mathematics,[1] but both engage with powerful forces related to the nature of mathematical proof and the changing and changeable face of the *doing* of mathematics, one which the Bourbaki-inspired new mathematics both supported and denied at one and the same time.

Below are my specific remarks on each article, followed by a broader look at the articles as a whole.

INDIVIDUAL COMMENTARY

Nelson's piece is concerned with the mathematical preparation of Alberta teachers and "the problem of insufficient background in mathematics," reporting on a comparison with standards offered by the Committee on the Undergraduate Program in Mathematics (CUPM), a U.S. body. The opening sentence places the article within the "modern" mathematics debate, whereby practising teachers may not themselves have studied the material they were now called upon to teach. What strikes me is how this accepted CUPM framing of "the problem" seems only to have a quantitative answer—namely, the number of courses that constitute a minimal acceptable level (as if they were interchangeable and more were evidently better). However, there is no discussion of the nature of appropriate courses, nor of how any given mathematics course might relate to the teaching of school mathematics, let alone how knowing *more* mathematics (again, a quantitative framing) is supposed to help a teacher teach mathematics. Somewhat recently, Brent Davis and Elaine Simmt (2006) (both currently at Alberta universities) published a thoughtful and challenging article relating to this issue, and in 2011 Susan Oesterle (at Simon Fraser University in B.C.) completed her interview-based doctoral study of 10 tertiary teachers of elementary Mathematics for Teachers courses, providing a complex look at the interlaced tensions with which teachers of such courses are concerned.

The appropriateness of calculus within high school mathematics education, the focus of Falk's longer article, is helpful as a reminder of the specific discussion in Alberta at that time, as well as setting it within a much broader North American framework. It also signals the influence of university-based mathematicians with regard to school matters (teaching calculus seen as a college prerogative). The 'just because it can be taught, should it?' controversy (see p. 87) reminded me of a late-1970s research study in Madison, Wisconsin, where I was a doctoral student, in which grade 4 students were successfully taught operations with negative numbers, providing another instance of Bruner's much-quoted comment (see p. 87). But this teaching took much of the school year and raised the question of whether this was a good use of the students' time and mathematical attention.

Falk's piece ("Calculus Seen as an Essential Part of a Mathematical Education") also brought to mind how calculus, then as now, is the new emblem of an educated person (the way Euclidean geometry was in 19th-century England). The author's comments—concerned with how, despite there being an alternative for the second part of the Math 31 course (namely, visualisable linear algebra), it was almost never opted for—again attest to the perceived status of calculus. The question of specific curriculum status is a significant one, more recently seen in decisions about pure mathematics versus applied mathematics in this province (particularly in light of Falk's Conclusion #5). The lack of agreement about what should feature in the Grade 12 mathematics curriculum of the time makes for interesting reading, again in light of the dissension around both the recent Western and Northern Canadian Protocol (WNCP) and the Core Standards framework in the United States.[2] For a strong contemporary book on issues of teaching and one U.S. high school's mathematics curriculum (including the teaching of calculus), see Chazan, Callis, and Lehman (2007).

On a more mathematical note, Allendoerfer's requirement of "integration (via sums of series) before differentiation," which Falk cites, triggered two thoughts. First, working with integration as area rather than anti-differentiation allows for numerical approximations immediately to be made, as well as providing a clear meaning for the integral—the same way that defining π as the area of the unit circle does (as opposed to defining it as the arc length of its semi-perimeter). But it renders the fundamental theorem of calculus problematic, challenging, and unintuitive. My second thought was a memory of Dana Scott, the British mathematician, referring to analysis as a "pop-up" subject: just as you smooth down one difficulty in presentation here, another one pops up over there. There is, he claimed, no perfectly smooth presentation possible.

The third article is a reprint from Saskatchewan on a lively debate from 1971. I think the same debate, with similar voices from the same institutions, could be held today—perhaps under the aegis of the Fields Institute or the Pacific Institute for the Mathematical Sciences (PIMS). Professor Staal's acknowledgment of the benefits of the new math reminds me that there were two sources of novelty proposed: novel curriculum and novel pedagogy.[3] Historically, recollection of the latter got lost along the way in the rejection and vilification of the former. But the tension between rote learning and understanding is still with us. Some 15 years ago, I wrote a book (Pimm, 1995) in which I tried to show how the twin goals of fluency and understanding had a productive tension and interaction in mathematics education at all levels, one that was lost by opting for one over the other. Finally, the language of the debate as reported is also quite edifying: "transplant rejection," "missionary," "banana republic," "respectable mathematics." How we reveal ourselves when the blood is up!

The title of the fourth piece, "What Is CAMT?" (from July 1971), had me stumped. I had never come across mention of this organisation, presuming rather that the National Council of Teachers of Mathematics (NCTM) absorption of Canada under its *N* (there are other words one could use, I suppose: why is it not called ACTM or USACTM?) had been there right from that organisation's beginning in 1920. But, no, we learn from this piece that the Canadian Association of Mathematics Teachers (CAMT), under the auspices of the Canadian Teachers' Federation, was born in the mid-1960s and that two Canadian representatives attended the first International Congress on Mathematical Education (ICME) in 1969. The 32nd NCTM yearbook (NCTM, 1970), on the history of mathematics teaching in the United States and Canada, provides further information (pp. 430–431), referring in particular to the independence of individual provincial initiatives and reforms. Alberta was one of the earliest Canadian affiliates of the NCTM (p. 432), and MCATA co-hosted a summer meeting in Calgary in 1966 (p. 432). To date, I have been unable to ascertain at what stage Canada as a whole became affiliated with NCTM and whether that was at the same time as the demise of CAMT. It occurs to me that the subsequent national curriculum influence of NCTM in the United States (both in schools and in teacher education) through its various standards documents could perhaps not have been repeated here, due to the absence of CAMT as a national organisation.

"More to It Than You Think" is a spirited defence of the sophistication of the challenge of teaching Grade 1 students mathematics. Starting with a particular example, Loring takes on the new mathematics by analysing some of what a student needs to come to grips with in one mathematical sentence, only a small part of which is overtly mathematical. The implications of significant linguistic issues in mathematics teaching and learning is a topic I have spent my career exploring, and this short article points to many of them, including lexico-grammatical concerns (though she might have noted that *greater* signals itself as a comparative by its suffix, whereas *less*, rather than *lesser*, does not).[4] However, I was even more taken with her critique of mathematical diagrams (particularly the challenge of picturing subtraction[5]), contrasting the active operation with the static description (by means of an equation). In short, this piece is concerned with mathematics' elimination of time.

Historically, the 19th-century mathematicians Bolzano and Weierstrass were involved in what became known as "the arithmetisation of analysis" (doing away with the concept of variable moving points in calculus and defining continuity algebraically and statically). What we have here is an account of the arithmetisation of arithmetic. In summary, Loring's piece reminds us to pay attention to what is said, what is written, what is drawn, and what is meant—and, crucially, how all of these interrelate. These are new mathematics echoes that will be heard down the ages.

Van Brummelen's piece left me breathless: by its scope, by its charges, and by the continuing contemporaneity of its themes and concerns. While working on issues of new mathematics teacher induction a decade back, I spent a couple weeks in Shanghai with my then colleague Lynn Paine (see Paine, Fang, & Wilson, 2003). We learnt of the Chinese metaphor of the need for "teaching the whole fish" of mathematics. The problem with the mathematics curriculum is that it does not teach the tail of the fish (where mathematics has come from), nor its head (where mathematics is going, which is increasingly influenced, I feel, by certain sorts of electronic technology; see Rotman, 2008): it simply teaches the body (Van Brummelen's "isolated, self-sufficient body of knowledge"), in part a result of the willful amnesia of modernist mathematics itself (see Gray, 2008). Van Brummelen's question is one of *why* and not *how to* teach any particular mathematical idea.

The divorce between mathematics curricula (and their effects) and a commitment to the world and its problems is certainly alive today: authors[6] and conferences exploring working on social justice issues through mathematics;[7] the potential motivation of more human contexts, whether political or not;[8] and the recent links in the UK to requirements that *every* school subject (including mathematics) contribute to a student's moral education—to say nothing of the potential effects and costs of learning mathematics on certain learners.[9]

But the theological context of Van Brummelen's piece also reminded me of a paragraph from the Second World Conference on Islamic Education, offering a very different justification for the teaching and learning of mathematics:

> The objective is to make students implicitly able to formulate and understand abstractions and be steeped in the area of symbols. It is good training for the mind so that they may move from the concrete to the abstract, from sense experience to ideation and from matter-of-factness to symbolisation. It makes them prepare for a much better understanding of how the Universe, which appears to be concrete and matter of fact, is actually *ayatullah* signs of God—a symbol of reality. (as quoted in Pimm, 1995, p. 11)

This quotation feeds into the religio-philosophical flavour of Van Brummelen's piece. He claims that philosophy has influenced both how and what he teaches. And Thom (1973, p. 204) agrees: "In fact, whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics." But at the end, I am reminded both of Van Brummelen's assertion that "mathematics starts with situations not with theorems" and of the urban youth's likely apocryphal riposte to a mathematics teacher going on about this or that problem: "Man, I wish I had your problems!"

The next article, a reprint from Manitoba this time, nicely had me up in arms: not just with its talk of "basics," not even just with its talk of "*the* basics" (the definite article presuming an agreed-upon universality), but with its isthmus-narrow specification of what is taken to be basic (arithmetic operations on various familiar sorts of numbers). There is slippage, perhaps in the heat of the moment, from talk of "the basics" to "basic skills," and from there to the somewhat oxymoronic phrase "good understanding of the basics." Once again, we are right back in the tension between fluency and understanding (despite Biedron's one-sided and, toward the end, rather confused presentation). Philosopher Alfred North Whitehead (1925) claimed that "civilisation advances by extending the number of important operations we can perform *without* thinking about them" (p. 59; *my emphasis*). But the key *teaching* issue is how to automate successfully, in order that such procedural fluency can then become un-thought, subordinated to other things.

And this concern runs right into the next piece, the results of a survey on calculator usage and teacher beliefs. As I am writing this piece (in September 2012), I have just started teaching a master's course for secondary mathematics teachers entitled "Learning Mathematics with Computers." I started by asserting that both *calculator* and *computer* are words with a long history and that, until the Second World War, their first reference was always to a human being, the one-who-calculates or the one-who-computes (as opposed to the thing-that-calculates or the thing-that-computes).[10] This has now changed, I suspect irrevocably. Devices have been fabricated to assist both mathematical practice and its teaching/learning for the past 5,000 years, with an interesting divide between those that are used throughout the culture (such as hand-held calculators) and those that exist only in school settings (such as Dienes apparatus or algebra tiles).[11] I had my current students read this survey article, and there followed a prolonged and stimulating discussion with regard to its relevance and virtual timeliness some 30-plus years later.

By contrast, the ninth article, "Metric by 1980," had an interesting historical feel. As someone school-educated in England pre-decimalisation of the currency (which occurred in 1971), a country where the metric system currently has but a bare beachhead in the shops and, frankly, in the schools, I found this interesting reading, though the convert's enthusiasm was evident. In a nice article from 1991, Janet Ainley asks, "Is there any mathematics in measurement?"—making the point that one reason for systems of different sizes for different quantities is that the quantities involved made human sense (i.e., with regard to human scale). The SI preference for naming units primarily in relations to thousands (grams and kilograms; metres and kilometres) can lead to issues (and not just George Orwell's complaint in his *Nineteen Eighty-Four* that a litre was too much beer while a half-litre was not enough), as well as so much salient, indeed significant, information being coded into unfamiliar prefixes. Susan Pirie's (1982, 1987) work on nurses and drug dose errors (what's a power of 10 between friends?) provides a powerful instance of this challenge. The system may indeed be systematic, but this does not prevent it from causing serious difficulties. And despite having lived in Canada for the past 15 years, when I go to the grocery store and ask for 100 grams of Stong's Own ham, I simply mean "some and not too much, please"!

The final set of articles are a collection of student tasks concerned with ratio and proportion (and addressed, in places, directly to them) produced by Tom Kieren. This was the primary mathematical area that absorbed his research interests for more than 30 years. Interestingly, nothing is offered other than a set of tasks: no rationale, no sense of either where they came from or what they might be good for. At the time, I believe, there was a shortage of innovative, educational tasks of this sort, systematically foraging around a single area, building to more than the sum of their parts, quite different from standard textbook fare. (It is quite a different world we live in nowadays, drowning in material as we are.) Tom's work in this area has been both seminal and generative. It is certainly well-known and highly regarded far from Alberta, as well as within it.

SOME CLOSING COMMENTS

There is no possible way that 10 brief articles can capture a decade. Nevertheless, I would like to end this commentary by looking across the pieces a little more generally and teasing out three broad themes that emerged during the 1970s (with a little generosity with respect to temporal boundaries). Those themes are (a) technology; (b) associations, organisations, and journals; and (c) the centrality of curricular issues.

With the exception of the calculator piece, there is not a great deal of discussion about technology: educational television is mentioned, and even the calculators are described as "desk-top" more than "hand-held." No mention is made of the computer—unsurprisingly, since it was only in 1977 that the Apple II emerged, with its seemingly limitless 4KB of RAM—nor of programming as a mathematical activity. At ICME II, held in Exeter in 1972, Seymour Papert arrived with a considerable amount of bulky hardware from MIT to talk of the promise of the computer language Logo. But the hints are there of arguably the main force pulling on the head of the fish of mathematics—and mathematics education. As I mentioned earlier, it was only in 1976 that the computer-assisted proof of the four colour theorem appeared (to significant controversy). I still have an envelope from the mathematics department at the University of Illinois at Champaign-Urbana (where the provers of the theorem, Kenneth Appel and Wolfgang Haken, worked), franked in red capitals with the assertion that "Four Colors Suffice," which is the title of Robin Wilson's (2002) book on the history of the problem.

With regard to greater association and connectivity between mathematics educators of various stripes, I point obviously to the CAMT article, but also to the range of authors cited in all the articles—touchstone figures who emerged in greater numbers during this decade as mathematics education became more systematic and visible (not least in terms of journals, organisations, conferences, and doctoral programmes). It was May 1968 that saw the first publication of *Educational Studies in Mathematics*, and January 1970 when the first issue of NCTM's *Journal for Research in Mathematics Education* hit the stands. The first International Congress on Mathematical Education (which I think of as the Olympic Games, also held in the same years, apart from the first one) was held in 1969 in Lyon, France, and the first annual Psychology of Mathematics Education conference was held in 1977 (the group was created at the ICME III conference, held in Karlsruhe, Germany, in 1976).[12] In 1980, David Wheeler, based at Concordia University in Montreal, published the first issue of *For the Learning of Mathematics*, an international journal based in Canada. The three journals mentioned here provided the main face of Anglophone, academic mathematics education in Europe and North America into the 1980s.

Wheeler was also intimately involved in the creation of the Canadian Mathematics Education Study Group, a significant organisation of both university-based mathematicians and mathematics educators that meets annually to this day.[13] With regard to the goals and aims of CAMT, the closest embodiment today is perhaps the Canadian Mathematics Education Forum,[14] in the sense of the range of Canadian educators concerned with the teaching of mathematics at all levels meeting and talking together. It may just be my personal view (as it was the decade in which I moved from mathematics to mathematics education), but I see the 1970s as the time when mathematics education got itself organised, gained strength (like a tropical storm over the Gulf of Mexico?), and launched itself into and onto the world.

My third theme is the most obvious but is, nevertheless, core: the centrality of curriculum and the associated question of who gets to decide. These issues played out through the articles collected here and continue to be played out in Alberta (and elsewhere) today. Should the mathematics curriculum be broad or narrow in terms of its focus and aspirations? What should be the balance (if that is the right word) between pure and applied mathematics, and how does the affordable presence of technology that can handle "real" (that is, messy and unstructured) data influence this? New math and old math; modern math and postmodern math; the "basics" and their antipode (which I find myself unable to name succinctly).

The question of basics brings to mind a U.K. TV series started in the seventies called *Leapfrog*, which was scripted by a group of five innovative mathematics educators (including Dick Tahta) called *Leapfrogs.* The first series (polarising within the profession) was dynamic, inclusive, and broad-ranging in terms of what it considered appropriate televised mathematical experiences for upper elementary students. (One of my first pieces of educational research was to produce a 50-page report on the series to present at ICME IV, held in Berkeley, California, in 1980.) When the second series was being developed, the question of its name came up: one broad observation was that people would not know the series was about mathematics. The group decided to take the bull by the horns and reclaim ground—they renamed the series *Basic Maths.*

At bottom, any curriculum really concerns lessons,[15] and below that, the tasks that students engage with.[16] In 1961, the Russian educator Daniil Elkonin observed: "The basic unit (cell) of educational activity is the educational task.... An educational task differs fundamentally from other types of problems in that its goal and its result consist of a change in the acting subject himself, not in a change in the objects on which the subject acts" (quoted in Davydov and Markova, 1983, pp. 60–61).

So if curriculum designers thought in terms of tasks and their rationales (what changes in the "acting subject"? how? why?), as well as the efficacy of a particular task in achieving the goals inherent in the rationale (whether explicitly stated or not), a task-based mathematics curriculum (such as the partial one Kieren was offering in his pieces) could prove a delight to behold (rather than the more widely spread exhortative, assertive, or moralistic forms—the student *will*, the student *should*, the student *must*). But, realistically, I know that it is always possible to defeat the intentions of any curriculum embodied in tasks by misusing the tasks themselves, either intentionally or not, by not using them to attend to the mathematics to which they are intended to provide access.

Looking back with the hindsight of more than 35 years is always a luxury. When I agreed to write these comments, I had no idea what a personal journey it would be, nor how prophetic I would find some of the pieces (as well as the generally unchanging nature of the arguments and points of contention). I do see mathematics education generally as a pop-up subject—but, just as with analysis, that does not mean it is not worthy of engaging with. And I am grateful to have had the career that I have had, which started during a period of rapid, intricate, and continually problematic growth. I feel I have managed to live through interesting times.

NOTES

1. Sandy Dawson's (1969) doctoral dissertation at the U of A in part attempted to relate Lakatos's work (much of which had been published in a series of academic articles in a philosophy journal in the early sixties) to questions of the school teaching of mathematics.
2. See www.corestandards.org/Math.
3. At the Second International Congress on Mathematical Education (ICME), held in England in 1972, plenary speaker mathematician René Thom (1973) made similarly trenchant yet fascinating remarks about modern mathematics and the significance of Euclidean geometry (as opposed to Euclid *tout court*) as a source of rich problems and clear meaning.
4. Valerie Walkerdine (1988), in her book *The Mastery of Reason*, takes a close look at the lexical pseudo-pair *more* and *less* and its implications for a variety of subtle mathematics education issues, both inside and outside school.
5. Martin Hughes's (1986) book *Children and Number* documents grades 1 and 2 children's attempts to depict/symbolise subtraction.
6. One such author is American schoolteacher and university professor Eric Gutstein. See his book *Reading and Writing the World with Mathematics* (2005) or the reader he co-edited with Robert Peterson, *Rethinking Mathematics* (2005).
7. See, for instance, http://creatingbalanceconference.org/.
8. See, for instance, Marilyn Frankenstein's book *Relearning Mathematics* (1990).
9. See, for instance, Higginson (2006).
10. Nick Jackiw has drawn my attention to the likelihood that *computer* was first used as a job title in the 19th century for the Great Trigonometric Survey (of British India).
11. For a history of such devices used in the United States over the past two centuries, see Kidwell, Ackerberg-Hastings, and Roberts (2008), *Tools of American Mathematics Teaching, 1800–2000.*
12. See www.icmihistory.unito.it/pme.php.
13. See http://publish.edu.uwo.ca/cmesg/.
14. See http://cms.math.ca/Community/Canada/.
15. Alan Bishop talks about the need for teachers to "lessonise" the curriculum, a task that many elementary textbooks have sought to carry out for teachers, by means of the design layout known as the two-page spread.
16. There is an important and often ignored distinction between *task* and *activity*. The former is usually under the teacher's control (even if selected from a text or the Internet). The latter is the student's and occurs in response to the task provided. See Love (1989).

REFERENCES

Ainley, J. (1991). Is there any mathematics in measurement? In D. Pimm & E. Love (Eds.), *Teaching and learning school mathematics* (pp. 69–76). London, UK: Hodder and Stoughton.

Chazan, D., Callis, S., & Lehman, M. (Eds.). (2007). *Embracing reason: Egalitarian ideals and high school mathematics teaching.* New York, NY: Routledge.

Davis, B., & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics, 61,* 293–319.

Davydov, V., & Markova, A. (1983). A concept of educational activity for schoolchildren. *Soviet Psychology, 21*(2), 50–76.

Dawson, A. (1969). *The implications of the work of Popper, Polya and Lakatos for a model or mathematics instruction* (Unpublished doctoral dissertation). University of Alberta, Edmonton, AB.

Frankenstein, M. (1990). *Relearning mathematics: A different third R: Radical mathematics.* London, UK: Free Association Books.

Gray, J. (2008). *Plato's ghost: The modernist transformation of mathematics.* Princeton, NJ: Princeton University Press.

Gutstein, E. (2005). *Reading and writing the world with mathematics: Toward a pedagogy for social justice.* New York, NY: Routledge.

Gutstein, E., & Peterson, R. (Eds.) (2005). *Rethinking mathematics: Teaching social justice by the numbers.* Milwaukee, WI: Rethinking Schools.

Higginson, W. (2006). Mathematics, aesthetics and being human. In N. Sinclair, D. Pimm, & W. Higginson (Eds.), *Mathematics and the aesthetic: New approaches to an ancient affinity* (pp. 126–142). New York, NY: Springer.

Hughes, M. (1986). *Children and number: Difficulties in learning mathematics.* Oxford, U.K.: Blackwell.

Kidwell, P., Ackerberg-Hastings, A., & Roberts, D. (2008). *Tools of American mathematics teaching, 1800–2000.* Baltimore, MD: Johns Hopkins University Press.

Lakatos, I. (1976). *Proofs and refutations: The logic of mathematical discovery.* Cambridge, UK: Cambridge University Press.

Love, E. (1989). Evaluating mathematical activity. In D. Pimm (Ed.), *Mathematics, teachers and children* (pp. 240–262). Sevenoaks, UK: Hodder and Stoughton.

National Council of Teachers of Mathematics (NCTM) (1970). *A history of mathematics education in the United States and Canada* (32nd Yearbook). Washington, DC: NCTM.

Oesterle, S. (2011). *Diverse perspectives for teaching math for teachers: Living the tensions.* Unpublished doctoral dissertation, Simon Fraser University, Burnaby, BC.

Paine, L., Fang, Y., & Wilson, S. (2003). Entering a culture of teaching: Teacher induction in Shanghai. In E. Britton, L. Paine, D. Pimm, & S. Raizen (Eds.), *Comprehensive teacher induction: Systems for early career learning* (pp. 20–82). Dordrecht, NL: Kluwer Academic Publishers.

Pimm, D. (1995). *Symbols and meanings in school mathematics.* London, UK: Routledge.

Pirie, S. (1982). *Deficiencies in basic mathematical skills among nurses: Development and evaluation of methods of detection and treatment.* (Unpublished doctoral dissertation). University of Nottingham, Nottingham, UK.

Pirie, S. (1987). *Nurses and mathematics.* London, UK: Royal College of Nursing.

Rotman, B. (2008). *Becoming beside ourselves: The alphabet, ghosts, and distributed human being.* Durham, NC: Duke University Press.

Skemp, R. (1964–1970). *Understanding mathematics* (Books 1, 2, 3, 4M, and 5M). London, UK: University of London Press.

Thom, R. (1973). Modern mathematics: Does it exist? In A. Howson (Ed.), *Developments in mathematical education* (pp. 194–209). Cambridge, UK: Cambridge University Press.

Walkerdine, V. (1988). *The mastery of reason: Cognitive development and the production of rationality*. London, UK: Routledge.

Whitehead, A. (1925). *Science in the modern world*. New York, NY: Macmillan.

Wilson, R. (2002). *Four colors suffice: How the map problem was solved*. London, UK: Penguin.

Wilson, S. (2002). *California dreaming: Reforming mathematics education*. New Haven, CT: Yale University Press.