COMMENTARY

MATHEMATICS EDUCATION FOR THE TWENTY-FIRST CENTURY

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INTRODUCTION

Turn of the millennium; twenty-first century society; twenty-first century skills; twenty-first century learning; twenty-first century classroom; technology revolution; globalization; terrorism; world economy; environmental issues: these are some of the ideas that one might associate with the first decade of the new millennium, the 2000s, or the aughts. How does mathematics education fit into this picture? The ten articles in this decade of the aughts directly or indirectly offer particular ways to address this. Thus, the approach I take in this commentary is not to comment on each article in an isolated way, but to use them to paint a picture of mathematics education for the twenty-first century. In particular, I address the following themes that emerged for me from the papers: numeracy in the twenty-first century, cognitive science learning and teaching, mathematical processes, connecting mathematics to our world, and practice-based professional development.

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NUMERACY IN THE TWENTY-FIRST CENTURY

Numeracy, also described as quantitative literacy or mathematical literacy, simply stated refers to an individual's ability to understand and use mathematics effectively at school, at work, and in everyday life. Werner Liedtke's article draws our attention to the nature and importance of numeracy to students and society. As he points out, a highly numerate population is critical in ensuring the nation's ongoing prosperity, productivity, and workforce participation. Individuals who are numerate are better prepared to participate and engage in a world that is increasingly focused upon creativity and innovation and focuses upon knowledge creation and sharing. Thus, as Liedtke suggests, it is important for parents and teachers to actively engage children from an early age in appropriate activities to develop the foundation necessary to become numerate.

Given the direct relationship between numeracy and society, Liedtke's article opens the door to consider what constitutes numeracy in a twenty-first century context. Goos, Dole, and Geiger (2011) argued that a description of numeracy for this new period needs to better acknowledge the rapidly evolving nature of knowledge, work, and technology. They developed a model (Figure P5.1) to represent the multifaceted nature of numeracy in the twenty-first century.

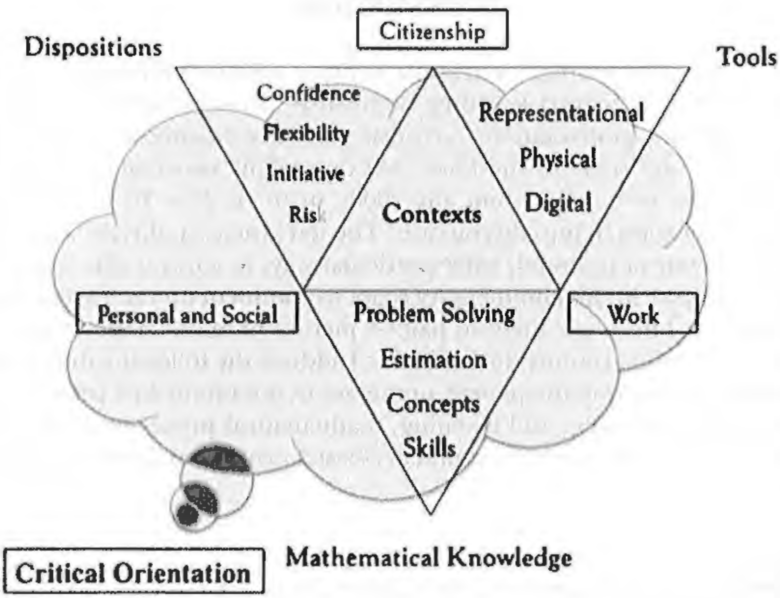

Figure P5.1 A model for numeracy in the 21st century (Goos et al., 2011, p. 33).

Based on this model and the authors' explanation of it (Goos et al., 2011, p. 34), a numerate person requires mathematical knowledge that includes not only concepts and skills, but also higher-order thinking such as problem-solving strategies and the ability to make sensible estimations. A numerate person has positive dispositions—a willingness and confidence to engage with tasks, independently and in collaboration with others, and apply their mathematical knowledge flexibly and adaptively. Being numerate involves using tools that may be representational (e.g., symbol, diagrams), physical (e.g., models, measuring instruments), and digital (e.g., computers, calculators). One needs to be numerate in a range of contexts including personal contexts, work-related contexts, citizenship-related contexts, and different curriculum contexts. The model is grounded in a critical orientation to numeracy—for example, it recognizes that a numerate person critically discusses and considers alternative solutions that give different results and have different real life consequences, evaluates the reasonableness of results, and is aware of appropriate and inappropriate uses of mathematical thinking to analyse situations and draw conclusions.

This model, then, suggests a possible scope of mathematics education for the twenty-first century. It reflects the multidimensional orientation of school mathematics that is more relevant to this new millennium. Each of the 10 articles addresses some aspect of this model which will be highlighted throughout this commentary. Thus they all contribute to our understanding of numeracy and mathematics education in this decade and beyond. They raise issues about curriculum, teaching, and learning as practised in our mathematics classrooms and offer suggestions of what we need to consider and should do to address these issues in order to transform how we engage and prepare our students for a changing world. These suggestions, as those about numeracy represented in the model (Figure P5.1), require ways of thinking about mathematics, the mathematics curriculum, and mathematics teaching and learning that are different from what are practised in traditional mathematics classrooms. This makes teachers' learning and professional development, also addressed in a couple of the articles, a critical component in realizing the change to the twenty-first century context.

The numeracy model and the articles also imply a shift or extension in the rationale for learning mathematics that includes a critical thinking component. Lynn McGarvey's article is explicitly related to this shift as she examines, "Why do we teach math to children?" She raises concern about the lack of, or inadequate depiction of, an explicit rationale in our past and present provincial mathematics curricula. As she points out, the rationale for learning mathematics when given or implied in the curriculum consists of: for the purposes of future progress of education; for its utility; for later mathematics learning; for future careers; for individual growth; and, as a way of trying to understand, interpret, and describe our world.

While these are relevant, by themselves they are inadequate in not considering, as McGarvey puts it, "How does math shape our thinking? How has it defined our culture? What has math allowed us to create? How has it constrained or perhaps disallowed possibilities for knowing, seeing, doing, and being." These are important considerations to make mathematics education more relevant to prepare numerate citizens for the twenty-first century world. They can be supported by what we now know from cognitive science and the importance of critical mathematics education and cultural relevant mathematics to a numerate society. These are addressed in the articles and the discussion in the remainder of this commentary, which highlights these and other ways in which mathematics education in this decade of the 2000s should have been evolving to meet the demands of a rapidly changing world.

COGNITIVE SCIENCE APPROACH TO LEARNING AND TEACHING

Brain-based research is offering us new ways of understanding learning. Brent Davis's article highlights some of these ways based on developments in cognitive science. Davis provides a view of mathematical intelligence that is about coming up with solutions to real problems, with answers that go beyond routine responses and that enable the person to go further than he or she could before taking on the problem. He highlights the relationship to understanding mathematical intelligence and teaching to support its enhancement. For example, he points out an important relationship between practice and intelligence; that is, the latter could be improved through the former. However, this practice must be contextualized and be meaningful and purposeful to the learner. Some of the other insightful notions related to learning in the article include: "Every lived experience entails a physical transformation of one's brain." "Every experience you have contributes to the ongoing restructuring of the brain." "For a learner to develop mathematical intelligence and robust mathematical understandings, she or he has to be aware of how mathematical concepts can be interpreted in different ways." "Intelligence is greatly enabled by a facility with contemporary tools." The article also suggests that experiences that force learners to think outside the box will enhance or extend intelligence and points out the importance of working with others and others' ideas in the production of mathematical knowledge.

In the twenty-first century classroom, all students could benefit from these ideas from a cognitive-science perspective of learning; that is, learning of mathematics can be enhanced by meaningful and purposeful practice, challenging mathematical tasks, multiple interpretations and representations of mathematical concepts, technological tools, and collaboration with others. This perspective also suggests how important teachers are to brain development. For example, if every experience impacts brain structure, then the type of experiences teachers offer students in their classrooms can enhance or limit brain development, in general, and mathematical development, in particular. Teachers who engage students in "worthwhile mathematical tasks" (National Council of Teachers of Mathematics [NCTM], 1991) and mathematical processes (discussed in the next section) and provide opportunities for creativity and critical thinking are more likely to contribute in a positive way to enhancing students' learning and mathematical intelligence and their preparation to be productive citizens in the twenty-first century context.

Many teachers continue to depend on a textbook to script their lessons. However, while textbooks can be useful references, using them as scripts can be problematic. For example, as Ann Kajander and Miroslav Lovric point out in their article, they saw evidence of secondary and postsecondary textbook formats that might have been intended to simplify the learning process for students. They note consequences of this—for example, incomplete understanding, misconceptions, inaccuracies. In general, such simplification, whether in the textbook or resulting from teachers choosing to do so to save students from being challenged, is in conflict witb a cognitive science perspective of learning and denies students the experience to enhance their brain structures.

MATHEMATICAL PROCESSES

In the late 1980s the field of mathematics education began to emphasize the importance of mathematical processes in the teacbing and learning of matbematics (e.g., NCTM, 1989). In this decade of the 2000s, these processes were reinforced by professional standards (e.g., NCTM, 2000) and included as integral aspects of K–12 mathematics curricula. Table P5.1 shows the processes of NCTM (2000) and tbe current Alberta curriculum (Western and Northern Canadian Protocol) and Ontario curriculum. These mathematical processes are not only essential to learning mathematics witb understanding and enhancing mathematical intelligence from a cognitive science perspective but are also significant in preparing students for a twenty-first century society. Through them, teachers can create classrooms to support tbe development of students who are critical and creative thinkers, prohlem solvers, risk takers, and collaborators—that is, students with skills considered to be necessary for a twenty-first century global economy and society. These processes allow students to create

TABLE P5.1 Mathematical Processes in School Curricula

NCTM	Alberta	Ontario
Problem solving	Problem solving	Problem solving
Reasoning and proof	Reasoning	Reasoning and proving
Representation		Representing
Connections	Connections	Connecting
Communication	Communication	Communicating
	Mental mathematics and Estimation	
	Technology	
	Visualization	
		Reflecting
		Selecting tools and computational strategies

knowledge that connects them with their world in ways that are personally meaningful and relevant.

As with the case of numeracy, these processes require different interpretations and teaching approaches than those in a traditional classroom. Jennifer Holm's article provides an example of this in her "perception of problem solving in the elementary curriculum." She highlights the nature of problem solving, students' role, and the teacher's role as they should be interpreted. This view of problem solving as way of thinking, learning, and teaching not only allows students to develop understanding of genuine problem solving but also allows them to be autonomous learners, develop mathematical thinking, and engage in most of the other processes. Thus, problem solving, when appropriately interpreted, by itself could significantly transform mathematics classrooms into twenty-first century oriented places that promote creative thinking; exploration, discovery, and knowledge creation; risk taking; and collaboration, sharing of ideas, and opportunities to argue and defend solutions and strategies.

In general, teaching through inquiry, another key idea in this decade, is recognized as important to support students' learning of mathematics with understanding and their development of the mathematical processes and mathematical inquiry and collaborative skills. Inquiry-based learning allows students to actively engage in in-depth exploration of mathematics concepts and to be actively engaged in the construction of mathematical knowledge with deep understanding; to make connections between prior, existing, and new knowledge and experience; to work and learn collaboratively; and to take responsibility for their own learning. Students are given the opportunity to pose questions, make conjectures, and direct their own investigations and find their own answers. So there are many similarities between inquiry and genuine problem solving as ways of learning and teaching. The article by Olive Chapman, Krista Letts, and Lynda MacLellan illustrates how teachers can transform their teaching to an inquiry-oriented perspective. This article is discussed later as part of teacher learning.

This section on processes will be incomplete without a mention of technology. While technology is not synonymous with twenty-first century learning, it is an integral aspect of it, and can play a central role in transforming classrooms to support the mathematical processes and inquiry-based learning. However, while it provides a tool for exploration/inquiry, in a twenty-first century classroom it should also be about connecting students with their world and communicating with others, enabling them to learn from others and to share their own ideas. So, while it is not dealt with explicitly in the articles, as a tool it is relevant to all of them.

CONNECTING MATHEMATICS TO OUR WORLD

The importance of connecting mathematics to the world is embodied in the process of connections. As stated by NCTM (2000), instructional programs should enable students to "recognize and apply mathematics in contexts outside of mathematics" (p. 64). However, this could be interpreted in ways that are inhibiting or liberating in terms of students' learning and relationship to the world depending on what and how contexts are used. In a twenty-first century perspective, such contexts should humanize mathematics by connecting it to students' day-to-day lives, to broader society, and to issues of equity. Three of the articles offer ways to address this.

Florence Glanfield's article draws attention to the situation that across Canada there has been an attempt to humanize mathematics in courses such as applied and consumer mathematics. These courses are generally for students who are not likely to take postsecondary programs that require mathematics as a prerequisite. While such courses allow these students to develop basic numeracy, there is a perceived issue with how "pure" and "applied" are used to label school mathematics curricula. For example, such labels could suggest that a lower level of mathematics is needed for applications and that "applied" mathematics is not important for students who are in "pure" mathematics route. This could denigrate applications to practice exercises for mathematics concepts, such as traditional word problems at the end of a unit. The bottom line is that all students should have opportunities to engage in "pure" and "applied" mathematics in appropriate and meaningful ways to facilitate their learning and connect them to the world.

David Wagner's article offers a different way of thinking of such connections for all students by considering the relationship between mathematics and society in the context of teaching for peace and living in an unpredictable world. He offers two views of peace, stating that one can be equated with focusing on product and one on process, which is analogous to two extreme views of mathematics teaching. While some balance is necessary between the two, giving the edge to process seems to be more relevant to a twenty-first century context. As Wagner suggests, the process view of peace involves "moulding ourselves to the world in order to find ourselves in it." A similar claim can be made for a process view of mathematics and the connections it allows students to have with the world. Wagner notes the importance of relevant mathematics tasks to achieve such connections, that is, tasks that have potential for building awareness and help us to see our place in the world. He offers two ways of doing this in relation to peace: (1) through word problems that express an interest in a peaceful world, and (2) through ethnomathematics (includes mathematical-rich cultural artifacts from other cultures) and critical mathematics (e.g., use tasks structured to make a variety of approaches possible and a diversity of answers correct; engage students in critical thinking/dialogue). He concludes that, in general, mathematical tasks/activities that encourage thoughtful action can format the world for peace.

Wagner's suggestions regarding word problems and critical mathematics can be generalized for other concerns of society—for example, environmental, economical, and equity issues—which have implications for world peace and a numerate society. Of particular importance in a globalized world is how we acknowledge culture. Wagner mentions ethnomathematics, which illustrates the culturally influenced uses of mathematics through its applications. However, in general, there should be a focus on culturally relevant mathematics and culturally relevant instruction that employs various methods of infusing culture into mathematics instruction. Leonard and Guha (2002) explain that "culturally relevant teaching embeds student culture into the curriculum to maintain that culture" (p. 114). They further state that "including aspects of students' culture into mathematics problems is one way to avoid the cultural deficit model and help students and teachers value the culture of the community" (p. 114). For example, Aboriginal cultures are seldom recognized for their uses of mathematical processes, yet it is through an examination of such applications in daily living practices that our understanding of our Aboriginal peoples can be enhanced.

The article by Gladys Sterenberg, Liz Barrett, Narcisse Blood, Florence Glanfield, Lisa Lunney Borden, Theresa McDonnell, Cynthia Nicol, and Harley Weston draws attention to culturally relevant mathematics from a Canadian Aboriginal perspective. It provides us with meaningful examples of activities that can be used to support culturally responsive mathematics education. It illustrates "learning mathematics from place"—that is, exploring mathematics concepts through activities at actual cultural sites and through the creation of models in the classroom. The focus is on making real connections by exploring actual places, such as through field trips to aboriginal sites. The article also highlights the importance of relationships within the aboriginal culture and impact on mathematics education, in particular, relationship of understanding and awareness of relationship with self, students, content, and relationship within and outside mathematics. In general, the article indicates how indigenous knowledge can offer a basis for teaching and learning mathematics by both Aboriginal and non-Ahoriginal students and teachers. For example, it highlights the roles language (particularly in understanding the link between indigenous knowledge and matbematics), place, community, and culture play in Aboriginal learning of mathematics that are also relevant to non-Aboriginal students' learning. The characterization of indigenous epistemologies as being experiential, storied, relational, contextual, and holistic has implications for how we view mathematics, mathematical tools, and mathematics pedagogy and is a view consistent with twenty-first century numeracy and cognitive science perspectives of learning.

PRACTICE-BASED PROFESSIONAL DEVELOPMENT

For teachers to be able to implement and sustain a twenty-first century mathematics classroom, they should understand mathematics and learning in ways discussed for students. Thus teacher education programs and professional development approaches also need to reflect the demands of the twenty-first century and implications of cognitive science research for learning. The 2000s have seen significant changes to mathematics teacher education as proposed by the research field, such as a shift in focus to the nature of mathematics knowledge for teaching and practice-based, inquiry-oriented approaches to teacber development. I address the latter here based on two of the papers.

Practice-based models of teacher learning are hased on the idea that professional development needs to be strongly linked to the professional practices of the teachers involved. The general view is tbat learning activities for the teachers should be purposefully connected to the curriculum they are teaching, student learning or work in their classrooms, content situated in an environment that models effective teaching, pedagogy of their classrooms, and a collaborative environment. Approaches that provide opportunities for teachers to explore important mathematical and pedagogical ideas that relate to their practice include teachers studying episodes of their practice by creating and analyzing narratives of their teaching; analyzing cases of sample teaching or problem situations; analyzing self-created videos of their teaching or researcher-created videos; creating, teaching, and analyzing experimental lessons during lesson study; and practicing "noticing" (Mason, 2002).

Mason (2002) defines noticing as "a collection of practices both for living in, and hence learning from, [classroom] experience and for informing future practice" (p. 29). This focus on classroom experience is a practice-based approach to teacher learning. Julie Long's article provides an example of this way of using noticing as a basis of understanding her practice. Her focus was on writing accounts-of and -for experience from her teaching. While this might seem to be a simple process, from a noticing perspective, as reflected in Long's article, it could be challenging. Noticing requires more than simply writing a description of a situation in one's teaching. For example, and more importantly, it requires one to learn something that he or she did not know before, or, as Long puts it, "draw a lesson out of the experience." Long provides an example of this based on her reflection of accounts from her teaching that contributed to her understanding of how discussion, challenge, and emotion were important to students' learning. In general, Mason's noticing includes teachers engaging in self-reflection as professional development to understand their practice with the goal of developing sensitivities to their students and to be awake to possibilities to transform their teaching. Chapman and Heater (2010) provide an example of a Calgary high school mathematics teacher's self-development in inquiry-based teaching in which noticing played an important role.

The article by Olive Chapman, Krista Letts, and Lynda MacLellan provides an example of a practice-based, school-based professional development approach to help teachers develop understanding of, and facilitate change of practice to, inquiry-based teaching. This approach shows how teachers can engage in lesson study to learn *though* and *for* inquiry that is relevant to a twenty-first century classroom. The approach was effective in helping the participants to create meaningful shifts in their thinking and teaching and to develop useful knowledge of mathematics teaching. One important feature of this approach was allowing teachers to have autonomy of it and their learning, thus making the professional development about them—that is, their perceived needs, their thinking, and their teaching. This allowed them to contextualize and personalize the inquiry process so they all could make sense of it from their perspectives and in a similar way that supported their learning collectively and individually. As a result, they got to develop their own ways of understanding key ideas being investigated through their own sense making. In addition, their choice of "communication" as the focus of their lesson study emerged as an ideal anchor for their inquiry of inquiry-based teaching. It allowed them to consider inquiry-based teaching in relation to the triad relationship among content, teacher, and students through communication. This relationship is of importance in the implementation of inquiry-based teaching. In general, this lesson-study approach offers teachers experiences that are in harmony with a twenty-first century perspective of learning.

CONCLUSION

The ten articles in this section of the aughts present a vision of mathematics education that is relevant to the twenty-first century context. They address learning for both students and teachers with implications for curriculum and teaching. They illustrate the relevance of *delta-K* to our provincial mathematics education community as a vehicle to inspire changes in our classrooms that will produce numerate twenty-first century citizens to ensure our province's ongoing prosperity, productivity, and workforce participation.

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