Leonardo Fibonacci (c. 1175–c. 1250)

Klaus Puhlmann

Fibonacci is actually the nickname of the Italian number theorist and algebraist Leonardo da Pisa or Leonardo Pisano. Fibonacci is a short form of Filius Bonacci, meaning son of Bonacci. He is the greatest and most productive mathematician of the Middle Ages and his work is still prevalent in mathematics courses today. Fibonacci's publication of *Liber Abaci* (*Book of the Abacus*) in 1202 greatly influenced the replacement of the Roman numerals by the Hindu–Arabic system of numbers in Europe. His second version of this book was written in 1228 and still exists today. It is a comprehensive work containing almost all the arithmetic and algebraic knowledge of that day. Fibonacci, having received his early mathematical training from Muslim tutors, quickly recognized the superiority of the Hindu–Arabic decimal system, with its positional notation and zero symbol, over Roman numerals. The merits of the Hindu-Arabic decimal system were obvious to Fibonacci and he defended them in this book. While having little influence on the merchants in his native Italy, the book played an important role in the development of mathematics in western Europe in the course of the several centuries that followed. His book *Liber Abaci* contained 15 chapters dealing with the following content:

1. Reading and writing of numbers in the Hindu–Arabic system
2. Multiplication of integers
3. Addition of integers
4. Subtraction of integers
5. Division of integers
6. Multiplication of integers by fractions
7. Further work with fractions
8. Prices of goods
9. Barter
10. Partnerships
11. Allegation
12. Solutions of problems
13. Rule of False Position
14. Square and cube roots
15. Geometry and algebra, the former being devoted to problems in mensuration

Fibonacci also wrote three other works, the *Practica geometriae* (1220), the *Liber quadratorum* (1225) and the *Flos* (meaning blossom or flower)

It was not until the 19th century that the number sequence that appeared in *Liber Abaci*, was named the Fibonacci sequence by the French number theorist Edourd Lucas. The sequence of numbers 1, 1, 2, 3, 5, 8, 13, 21, ..., each of which, after the second, is the sum of the two previous ones. Or, putting it another way, these numbers are such that, after the first two, $F_n = F_{n-1} + F_{n-2}$. These numbers are called Fibonacci numbers and play an important role in mathematics and nature. The ratio of one Fibonacci number to the previous one is a convergent of the continued fraction

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\dots}}}}$$

which satisfies the equation $x = 1 + \frac{1}{x}$ and is equal to $\frac{1}{2}(\sqrt{5} + 1)$. Thus the sequence $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \ldots$ has the limit $\frac{1}{2}(\sqrt{5} + 1)$ and the ratio of successive terms approximates the golden mean. That is, $F_n / F_{n-1} = \varphi$ (Greek letter phi). Here φ is known as the golden ratio and is equal to 1.61803.... It appears in the most surprising places in nature, art and mathematics.

In nature, the number of spiral floret formations visible in many sunflowers, spiraled scales on pinecones and segments on the surface of a pineapple have been found to match Fibonacci numbers. The arrangements of leaves, buds and branches on the stalk of a plant correspond to the numbers in the Fibonacci sequence.

Fibonacci numbers, besides bearing a curious relationship to botany, also appear to exert a strange influence on art and architecture. The ratio between any two adjacent Fibonacci numbers after 3 is about 1:1.6. This is the so-called Golden Ratio, or Golden Section, which has fascinated experts for centuries because of its connection with esthetics. The ratio, more precisely expressed as 1:1.618, occurs in pentagons, circles and decagons, but most notably in the Golden Rectangle, a figure whose two sides bear the magic relationship to each other. Because it presents such a visually pleasing form, it was present in all edifices of ancient Greece and in the art of the masters. The presence of the Fibonacci numbers is also felt in the area of music. Some musicians, like Bartok, based the entire structure of their music on the golden mean and the Fibonacci sequence.

The Fibonacci numbers and sequences are truly a rich area for study because they give rise to a vast amount of substantial mathematics. Be it in the area of art, music, architecture, science, economics or engineering, physical applications and connections with various branches of mathematics abound. Students and teachers willing to explore the Fibonacci numbers and sequences will make exciting discoveries and experience wonder and amazement.

Bibliography

Kapproff, J. *Connections: The Geometric Bridge Between Art and Science.* Columbus, Ohio: McGraw-Hill, 1991.

Pickover, C. A. *Keys to Infinity.* New York: John Wiley & Sons, 1995.

Smith, D. E. *History of Mathematics. Volume 1.* New York: Dover, 1951.

Change Sides and Keep the Area the Same

One side of a rectangle is increased by 25 percent. By what percentage must the other side be reduced if the area of the rectangle is to remain the same?