Appendix II: High School Mathematics Competitions in Alberta

A. Recent History

Professor **Alvin Baragar**, Chair of the Alberta High School Mathematics Competition Board, has retired from the University of Alberta. His successor is his former colleague, Professor **Ted Lewis**, whose hotline for quick consultation is 492-4565. Prof **Claude Laflamme** of the University of Calgary has also joined the Board. As it turned out, Dover Publications Incorporated of New York sponsored our contest for only one year, but the Board is nevertheless grateful for the support.

At the moment, the Canadian Mathematical Society is contemplating overhauling the structure of the Canadian Mathematical Olympiad. As a result, the Alberta High School Mathematics Competition (A.H.S.M.C.) may no longer serve as a qualifier for the national contest. However, it will continue to be the flagship of mathematics competitions in the province.

The newsletter on problem-solving *Postulate* has temporarily ceased publication. In its place, the A.H.S.M.C. Board has been publishing the *Alberta High School Mathematics Competition News*, which contains various announcements from the Board as well as a selection of past contests.

In 1990, Edmonton became the first Canadian city to participate in the **International Mathematics Tournament of the Towns**. This is an inter-city competition based in Moscow, with the Central Organizing Committee under the leadership of Professor **Nikolay Konstantinov** and Professor **Nikolay Vasiliev**. In recent years, some Calgary students also participated through the Local Organizing Committee in Edmonton. Currently, Toronto is the only other Canadian city taking part, with the workload undertaken by Professor **Eugene Kantorovitch**.

The Central Organizing Committee offers no prizes except diplomas in Russian for students who achieve a certain standard. Quite a number of Edmonton area students have won multiple diplomas. They include **Calvin Li** of Archbishop MacDonald, **Jason Colwell** and **Matthew Wong** of Old Scona, **Steven Laffin** of J.H. Picard and **Byung-Kyu Chun** of Harry Ainlay. The Local Organizing Committee offers certificates as well as modest prizes in the form of mathematics books and journals.

In 1993, **Daniel van Vliet** of Salisbury Composite High School and **Matthew Wong** attended the Tournament's Problem-solving Workshop in Beloretsk, Russia. **Clemens Heuberger**, the author of one of the student projects in Appendix I, was also there and won a prize.

In 1995, Canada hosted the International Mathematical Olympiad for the first time. The Problem Selection Committee consisted largely of members of the A.H.S.M.C. Board. National team member **Byung-Kyu Chun** won a bronze medal.

B. Sample Multiple-choice Questions of the A.H.S.M.C., 1967-1983

Multiple-choice questions were first introduced into the Alberta High School Mathematics Competition in 1967. There were 25 of them in 1967-70, and 20 in 1971-83. These were to be attempted, along with a number of essay-type problems, in a single paper. Since 1983-84, the two parts of the competition are written in separate sittings. The following questions have been typeset by **Hubert Chan** of Archbishop MacDonald High School.

1967

1. When the base of a triangle is increased 10%, and the altitude to this base is decreased 10%, the change in area is

(a) 1% increase
(b) $\frac{1}{2}$ % increase
(c) 0%
(d) $\frac{1}{2}$ % decrease
(e) 1% decrease

2. If $\frac{4^x}{2^{x+y}} = 8$ and $\frac{9^{x+y}}{3^{5y}} = 243$, then xy is

(a) $\frac{12}{5}$
(b) -4
(c) 4
(d) 12
(e) 6

3. The value of $\frac{(4-\sqrt{5})(2+\sqrt{5})}{7+\sqrt{5}}$ is

(a) $\frac{8}{7} - \sqrt{5}$
(b) $\frac{4-\sqrt{5}}{11}$
(c) $\frac{8+4\sqrt{5}}{11}$
(d) $\frac{5}{\sqrt{5}-1}$
(e) $\frac{1+\sqrt{5}}{4}$

4. The graph of the equation $x^2 - 4y^2 = 0$ is

(a) a parabola
(b) a point
(c) an ellipse
(d) a pair of straight lines
(e) none of the above

5. If $x - y < x$ and $x + y < y$ then

(a) $y < x$
(b) $0 < x < y$
(c) $x < y < 0$
(d) $x < 0, y < 0$
(e) $x < 0 < y$

6. The number of values of x satisfying the equation $\frac{2x^2 - 10x}{x^2 - 5x} = x - 3$ is

(a) zero
(b) one
(c) two
(d) three
(e) more than three

7. The radius of the circle whose equation is $x^2 + y^2 - 16x - 10y + 64 = 0$ is

(a) 4
(b) 5
(c) 6
(d) 8
(e) 10

8. One root of the equation $\left(\frac{1}{1+x}\right)^3 + \left(\frac{1}{1-x}\right)^3 + \frac{1}{2} = 0$ is $i = \sqrt{-1}$. Then the number of real roots is

(a) zero
(b) one
(c) two
(d) three
(e) four

9. The sides of a triangle are 8, 13 and 15 inches. The number of square inches in its area is

(a) 52
(b) $20\sqrt{2}$
(c) 60
(d) $30\sqrt{3}$
(e) none of the above

10. If for all x we have $1 = ax^2 + (bx + c)(x + 1)$, then

(a) $c + a + 2b = 0$
(b) $a + b + 2c = 0$
(c) $b + c + 2a = 0$
(d) $ab = c^2$
(e) $bc = a^2$

11. When $x^3 + k^2x^2 - 2kx - 6 = 0$ is divided by $x + 2$, the remainder is 10. Then k must be

(a) 2
(b) -2

(c) 2 or -3
(d) 2 or -1
(e) none of the above

12. A student wrote that the product of $a + i$ and $b - i$ was $a + b + i$ where $i = \sqrt{-1}$. If this answer was correct, then the minimum value of ab is

(a) 2
(b) 1
(c) 0
(d) -1
(e) -2

13. The converse of the statement "If $a = 0$, then $ab = 0$" is

(a) If $a \neq 0$, then $ab \neq 0$.
(b) If $a \neq 0$, then $ab = 0$.
(c) If $a = 0$, then $ab \neq 0$.
(d) If $ab = 0$, then $a = 0$.
(e) If $ab = 0$, then $a = 0$ or $b = 0$.

14. Let ABC be a triangle with $\angle A < \angle C < 90° < \angle B$. Consider the external angle-bisectors at A and B, each measured from the vertex to the opposite side (extended). If each of these line segments is equal to AB, then A is

(a) 6°
(b) 9°
(c) 12°
(d) 15°
(e) none of the above

15. The sum and the product of two numbers are each equal to $s + \frac{1}{s} + 2$ where $s > 1$. Then the difference between the squares of the reciprocals of the numbers is

(a) 1
(b) 2
(c) $(\frac{s-1}{s+1})^2$
(d) $\frac{s-1}{s+1}$
(e) at least 1

16. The distance that a body falls from rest varies as the square of the time of falling. If it falls from rest at a distance of 256 feet in 4 seconds, then during the tenth second it falls a distance (in feet) of

(a) 288
(b) 304
(c) 320
(d) 336
(e) 384

17. The roots of the equation $x^3 + 3px^2 + q^2x + r^3 = 0$ are in arithmetic progression. Then we must have

(a) $pq^2 = 2p^3 + r^3$
(b) $p = 0$
(c) $q = 3pr^3$
(d) $3p + r^3 = 2q^2$
(e) $3pr = q^2$

18. In calm weather an aircraft can fly from one city to another 200 miles north of the first and back in exactly 2 hours. In a steady north wind the round trip takes 5 minutes longer. The speed of the wind (in miles per hour) is

(a) 8
(b) 16
(c) 32
(d) 35
(e) 40

19. The length of the common chord of two intersecting circles is 16 feet. If the radii are 10 feet and 17 feet, then the distance (in feet) between the centres is

(a) 27
(b) 21
(c) $\sqrt{389}$
(d) 15
(e) none of the above

20. The number of positive integers less than 500 that are divisible by neither 3 nor by 5 is

(a) 269
(b) 267
(c) 265
(d) 234
(e) 201

21. The system of equations $x + (k - 2)y = 1$ and $(k + 2)x - 3y = 1$ can be solved for x and y in terms of k, provided that

(a) $k \neq 1$
(b) $k \neq 0$
(c) $k \neq -1$

(d) $k \neq 1, k \neq -1$
(e) none of the above

22. The smoke trail of a steamship sailing due east at 30 knots is in a direction 60° west of north. It overtakes a freighter sailing east a 10 knots, whose smoke trail is in a direction 30° west of north. The wind must be blowing from the direction

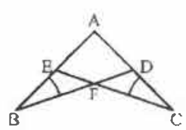
(a) 30° east of north
(b) 135° west of north
(c) due north
(d) due south
(e) 150° west of north.

23. When the last digit of a certain six-digit number N is transferred to the first position (the other digits moving one place to the right), the number is exactly one-third of N.
The sum of the six digits is

(a) 28
(b) 27
(c) 26
(d) 25
(e) 24

24. In the diagram, CA = CF and ∠B = ∠C.
Then we must have

(a) AE = EF
(b) AE = AD
(c) AD = $\frac{1}{2}$CF
(d) AE = EB
(e) AE = DF



25. The guests at a party play as follows: each player in turn names a real number (not 0 or 1), the rules being (i) no number is to be repeated; (ii) either the sum or the product of every pair of successive numbers must be 1.
The greatest positive number is

(a) 2
(b) 4
(c) 6
(d) 8
(e) limitless

1968

1. The equation $x^2 + x + 2 = 0$ has

(a) two positive roots
(b) two negative roots
(c) one positive root and one negative root
(d) no real roots
(e) none of the above

2. The equation $(a + b)x = 3$ in x will have no solution when

(a) $a = b$
(b) $a = -b$
(c) $a + b = 3$
(d) $a + b = -3$
(e) under any circumstance

3. The most general parallelogram which has equal diagonals is a

(a) rhombus
(b) square
(c) rectangle
(d) trapezium
(e) none of the above

4. What is the value of $5^{\log_5 6}$?

(a) 1
(b) 5
(c) 6
(d) $\log_6 5$
(e) none of the above

5. Which of the following constructions is impossible, using only an unmarked ruler and compass?

(a) trisecting a given angle
(b) trisecting a given line
(c) bisecting a given line
(d) bisecting a given angle
(e) none of the above

6. Given that $\log_{10} 2 = x$ and $\log_{10} 3 = y$, then $\log_{10} 15$?

(a) $1 + x + y$
(b) $1 - x - y$
(c) $1 + x - y$
(d) $1 - x + y$
(e) none of the above

7. Let S be the set of points, (x, y), in the plane satisfying both $x^2 + y^2 \leq 1$ and $x^2 + y^2 \geq r^2$. A value of r such that S is the empty set is

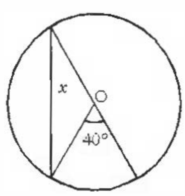
(a) 1
(b) -1
(c) $\frac{1}{2}$
(d) $-\frac{1}{2}$
(e) none of the above

8. If S, T, and V are any sets, then $[(S\cap T)\cup(S\cap V)]$ is the same set as
(a) S
(b) $T\cup V$
(c) $T\cap V$
(d) $S\cap(T\cup V)$
(e) none of the above

9. What is the greatest number of possible points of intersection of three lines in the plane, of different slopes?
(a) 2
(b) 4
(c) 6
(d) 8
(e) none of the above values

10. What is the greatest number of possible points of intersection of three circles, in the plane, of different radii?
(a) 2
(b) 4
(c) 6
(d) 8
(e) none of the above values

11. If O is the centre of the given circle, what is the value of the angle, x?



(a) 30°
(b) 40°
(c) 60°
(d) 80°
(e) none of the above

12. A metal disc has one face marked "1" and the other face marked "2". A second metal disc has one face marked "2" and the other marked "3". Assume that, when tossed, the two faces of a disc are equally likely to turn up. If both discs are tossed, what is the probability that "4" is the sum of the numbers turning up?
(a) $\frac{1}{4}$
(b) $\frac{1}{2}$
(c) $\frac{3}{4}$
(d) $\frac{1}{3}$
(e) none of these values

13. Which of the following statements about $\frac{1+\sqrt{2}}{1-\sqrt{2}}$ is true?
(a) it is irrational
(b) it is rational
(c) it is imaginary
(d) it is positive
(e) none of the above

14. What is the longest rod that can be put in a rectangular box of dimensions 6'×3'×2'?
(a) 6 ft
(b) $3\sqrt{5}$ ft
(c) $2\sqrt{10}$ ft
(d) 7 ft
(e) none of the above

15. If $i =$, then i^6 is
(a) 1
(b) -1
(c) i
(d) $-i$
(e) none of the above

16. If $i = \sqrt{-1}$, then $i\,\frac{1+i}{1-i}$ is
(a) 1
(b) -1
(c) i
(d) $-i$
(e) none of the above

17. The solution set of the inequality $x^2 - x - 2$ is the interval
(a) $-2 \le x \le 1$
(b) $-2 < x < 1$
(c) $-2 \le x < 1$
(d) $-2 < x \le 1$

(e) none of the above

18. If $a > 0$, $b > 0$, $a > b$, which of the following is false?

(a) $\frac{1}{a} > \frac{1}{b}$
(b) $a^2 > b^2$
(c) $a^3 > b^3$
(d) $\sqrt{a} > \sqrt{b}$
(e) none of the above

19. The solution set in the plane of the equation $y^2 = xy$ is

(a) a parabola
(b) a rectangular hyperbola
(c) a pair of straight lines
(d) a circle
(e) none of the above

20. The distance between the two points represented by the complex numbers $1 - 2i$ and $2i - 2$ is

(a) 5
(b) $-1 + 4i$
(c) $\sqrt{17}$
(d) $3 - 4i$
(e) none of the above

21. At the end of a party, everyone shakes hands with everyone else. Altogether there are 28 handshakes. How many people are there at the party?

(a) 8
(b) 14
(c) 20
(d) 56
(e) none of the above

22. How many odd multiples of "3" are there between 100 and 200?

(a) 15
(b) 17
(c) 33
(d) 49
(e) none

23. Let $aabb$ be a four digit number in base r, with $b = 0$. This number is divisible by

(a) r only
(b) $r + 1$ only
(c) $r - 1$ only
(d) more than one of these
(e) none of the above

24. If $\{a_1, a_2, a_3, \cdots\}$ is an infinite sequence of positive numbers with $a_k \geq 1$ for $k = 1, 2, \cdots$, and we define $U_n = a_1 \cdot a_2 \cdots a_n$, then which of the following statements about I, II, and III below is true?
I: $U_n = U_{n+1}$ II : $U_n < U_{n+1}$ III: $U_n > U_{n+1}$

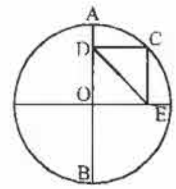
(a) I only
(b) I and II only
(c) I and III only
(d) III only
(e) none of the above

25. A rectangular floor 24' × 40' is covered by squares of sides 1' each. A chalk line is drawn from one corner to the diagonally opposite corner. How many tiles have a chalk line segment on them?

(a) 40
(b) 56
(c) 63
(d) 64
(e) none of the above

1969

1. O is the centre of the circle. CDOE is a rectangle. DE is 5 and CE is 3. The diameter of the circle is



(a) $4\sqrt{2}$
(b) 8
(c) 10
(d) $10\sqrt{5}$
(e) cannot be determined

2. A man spends $\frac{1}{3}$ of his money and loses $\frac{2}{3}$ of the remainder. He then has \$12. How much money had he at first?

(a) \$56
(b) \$27
(c) \$108
(d) \$112
(e) none of the above

3. For all real numbers a and b

(a) $a^2 + b^2 \geq 2ab$

(b) $a^2 + b^2 > 2ab$
(c) $a^2 + b^2 < 2ab$
(d) $a^2 + b^2 \leq 2ab$
(e) none of the above

4. The total number of subsets that can be formed from a set containing six elements is

(a) 4
(b) 8
(c) 16
(d) 32
(e) none of the above

5. A gambler visited three gambling houses. At the first he doubled his money, and then spent \$30, at the second he tripled his remaining money and then spent \$54, and at the third he quadrupled his remaining money and then spent \$72, and he then had \$48 left. How much money did he start with?

(a) \$29
(b) \$30
(c) \$31
(d) \$32
(e) \$33

6. Let r be the result of doubling both the base and the exponent of a^b, $b \neq 0$. If r equals the product of a^b by x^b, then x equals

(a) a
(b) $2a$
(c) $4a$
(d) 2
(e) 4

7. The symbol $|a|$ means a if a is a positive number of zero, and $-a$ if a is a negative number. For all real values of x the expression $\sqrt{x^4 + x^2}$ is equal to

(a) x^3
(b) $x^2 + x$
(c) $|x^2 + x|$
(d) $x\sqrt{1 + x^2}$
(e) $|x|\sqrt{1 + x^2}$

8. In the base ten number system, the number 526_{10} means $5 \cdot 10^2 + 2 \cdot 10 + 6$.If in the base r number system the equation $1000_r - 440_r = 340_r$ holds, then r is

(a) 2
(b) 5
(c) 7
(d) 8
(e) 12

9. While three watchmen were guarding an orchard, a thief slipped in and stole some apples. On his way out he met three watchmen one after another. To each he gave one half of the apples he had at the time and plus an additional two. Thus he managed to escape with one apple. How many apples did he steal originally?

(a) 16
(b) 22
(c) 32
(d) 76
(e) none of the above

10. If one man can dig a hole in one hour, and a second man can dig a hole in one and one-half hours, how many minutes must they work together to dig a hole?

(a) 16
(b) 36
(c) 46
(d) 56
(e) none of the above

11. If the radius of a circle is increased by one unit, the ratio of the new circumference to the new diameter is

(a) $\pi + 2$
(b) $\frac{2\pi + 1}{2}$
(c) π
(d) $\frac{2\pi - 1}{2}$
(e) $\pi - 2$

12. A square and an equilateral triangle have equal perimeters. The area of the triangle is square inches. Expressed in inches the diagonal of the square is

(a) $\frac{9}{2}$
(b) $2\sqrt{5}$

(c) $4\sqrt{2}$

(d) $\frac{9\sqrt{2}}{2}$

(e) none of the above

13. A hungry hunter came upon two shepherds. One shepherd had three loaves of bread; the other shepherd had five loaves of bread all of the same size. The loaves were divided equally among the three men and the hunter paid 8 cents for his share. How should the shepherds divide the money?

(a) 1 and 7
(b) 2 and 6
(c) 3 and 5
(d) 4 and 4
(e) none of the above

14. The average of a set of 50 numbers is 38. If two numbers of the set, namely 45 and 55 are discarded, the average of the remaining set of numbers is

(a) 38.5
(b) 37.5
(c) 37
(d) 36.5
(e) 36

15. A circle is inscribed in an equilateral triangle, and a square is inscribed in the circle. The ratio of the area of the triangle to the area of the square is

(a) $\sqrt{3}:1$
(b) $\sqrt{3}:\sqrt{2}$
(c) $3\sqrt{3}:2$
(d) $3:\sqrt{2}$
(e) $3:2\sqrt{2}$

16. Every day at noon a ship leaves New York for Lisbon and at the same instant a ship leaves Lisbon for New York. Each trip lasts exactly 8 days. How many ships from Lisbon will each ship from New York meet?

(a) 11
(b) 13
(c) 15
(d) 17
(e) none of the above

17. Points P and Q are both in the line segment AB and on the same side of its midpoint. P divides AB in the ratio 2:3, and Q divides AB in the ratio 3:4. If PQ = 2, then the length of AB is

(a) 60
(b) 70
(c) 75
(d) 80
(e) 85

18. Suppose we have two equiangular polygons P_1 and P_2 with different numbers of sides. Each angle of P_1 is x degrees and each angle of P_2 is kx degrees, where k is an integer greater than 1. The number of possibilities for the pair (x, k) is

(a) infinite
(b) finite but more than two
(c) two
(d) one
(e) zero

19. Given that the following have the same perimeters, which has the largest area?

(a) square
(b) equilateral triangle
(c) circle
(d) regular pentagon
(e) two or more of these are the same

20. The angles at A, B, C, D, E of a pentagon ABCDE are in the ratio 5:3:8:5:6. The largest of these angles has the value in degrees

(a) 90°
(b) 110°
(c) 130°
(d) 150°
(e) none of the above

21. Solve for n: $\binom{n}{8}=\binom{n}{24}$

(a) $n = 8$
(b) $n = 16$
(c) $n = 24$
(d) $n = 32$
(e) none of the above is a solution

22. The number of solutions of $2^{2x} - 3^{2y} = 55$ in which x and y are integers i s
    (a) zero
    (b) one
    (c) two
    (d) three
    (e) greater than three, but finite

23. In racing over a given distance d at uniform speed, A can beat B by 20 yards, B can beat C by 10 yards, and A can beat C by 28 yards. Then d, in yards, equals
    (a) cannot be detennined from information provided
    (b) 58
    (c) 100
    (d) 116
    (e) 120

24. If $x_{k+1} = x_k + \frac{1}{2}$ for $k = 1, 2, \cdots, n-1$ and $x_1 = 1$, find $x_1 + x_2 + \cdots + x_n$
    (a) $\frac{n+1}{2}$
    (b) $\frac{n+3}{2}$
    (c) $\frac{n^2-1}{2}$
    (d) $\frac{n^2+n}{4}$
    (e) $\frac{n^2+3n}{4}$

25. Three disks labelled 1 to 3 are put in a bag. Three other disks labelled 1 to 3 are put in a second bag. A disk is drawn from each bag and stacked in a pile on a table. This is repeated two more times. What is the probability that at least one of the stacks will contain disks with the same number?
    (a) $\frac{1}{6}$
    (b) $\frac{1}{3}$
    (c) $\frac{1}{2}$
    (d) $\frac{2}{3}$
    (e) none of the above

1970

1. The number $10a + b$, where a and b are digits, is divisible by nine if
    (a) $a + b = 7$
    (b) $a + b = 8$
    (c) $a + b = 9$
    (d) $a + b = 10$
    (e) none of the above

2. Given $a \neq b$ and if $ax+b^2 = a^2-bx$, then $x = ?$
    (a) $a + b$
    (b) $a - b$
    (c) $b - a$
    (d) $a^2 + b^2$
    (e) none of the above

3. For the function $f(x) = x^{50} - 2a^{47}x^3 + a^{50}$ the following is a factor.
    (a) $(x - a)$
    (b) $(x - a^2)$
    (c) $(x + a)$
    (d) $(x + a^2)$
    (e) none of the above are factors

4. Which of the following inequalities are true for all real positive x?
    (a) $x + \frac{1}{x} < 2$
    (b) $x + \frac{1}{x} \leq 2$
    (c) $x + \frac{1}{x} > 2$
    (d) $x + \frac{1}{x} \geq 2$
    (e) none of the above are true

5. A steamer was able to go twenty miles per hour downstream and fifteen miles per hour upstream. If on a return trip, the steamer took five hours longer coming up than going down, the total distance travelled by the steamer is
    (a) 500 miles
    (b) 600 miles
    (c) 700 miles
    (d) 800 miles
    (e) none of the above

6. Suppose that n is a positive integer. Then $\frac{n^2+(n+2)^2}{2}$ is
   (a) sometimes an integer
   (b) always a perfect square
   (c) sometimes a perfect square
   (d) never a perfect square
   (e) none of the above

7. The solution set of the inequality $x^2(x-1)^2 \le 0$ consists of
   (a) an interval
   (b) two intervals
   (c) an interval and a point
   (d) an interval and two points
   (e) none of the above

8. The expression $\sqrt[8]{x^8} + \sqrt[7]{7^7}$ is always equal to
   (a) x
   (b) $2x$
   (c) $2x^2$
   (d) 0
   (e) none of the above

9. Given the binary operation * between two integers m and n such that $m * n = m^2 + n^2$, which of the following do not hold?
   (a) commutative law
   (b) associative law
   (c) $m * n$ is an integer
   (d) $m * n \ge 0$
   (e) all of the above hold

10. Given the four quadrants labelled as follows: $\begin{array}{c|c} \text{II} & \text{I} \\ \hline \text{III} & \text{IV} \end{array}$
    the solution set of the simultaneous inequalities $x^2 + y < 0$, $x^2 + y^2 > 4$ lies entirely in quadrants
    (a) I and II
    (b) II and III
    (c) III and IV
    (d) IV and I
    (e) none of the above

11. Two cyclists race on a circular track. The first can ride around the track in six seconds, and the second in four seconds. If they start off at the same point, the second cyclist can overtake the first in
    (a) 12 seconds
    (b) 14 seconds
    (c) 16 seconds
    (d) 18 seconds
    (e) none of the above

12. An equilateral triangle is inscribed upside down in a larger equilateral triangle. The ratio of the area of the smaller to the area of the larger is
    (a) $\frac{1}{12}$
    (b) $\frac{1}{6}$
    (c) $\frac{1}{4}$
    (d) $\frac{1}{3}$
    (e) none of the above

13. If $f(n) = n^2$, where n is an integer, and $n \ne k$, then $\frac{f(f(n)) - f(f(k))}{f(n) - f(k)} =$
    (a) $n^2 + k^2$
    (b) $n^2 - k^2$
    (c) $\frac{n^2+k^2}{n^2-k^2}$
    (d) $\frac{n^2-k^2}{n^2+k^2}$
    (e) none of the above

14. Which of the following inequalities hold for all real x and y?
    (a) $\sqrt{x^2+y^2} < x + y$
    (b) $\sqrt{x^2+y^2} \le x + y$
    (c) $\sqrt{x^2+y^2} < |x| + |y|$
    (d) $\sqrt{x^2+y^2} \le |x| + |y|$
    (e) none of the above

15. The equation of the line through the origin and perpendicular to the line $y = 3x + 1$ is
    (a) $y = 3x$
    (b) $y = -3x$
    (c) $y = \frac{x}{3}$

(d) $y = -\frac{x}{3}$
(e) none of the above

16. Given that $\log_a b = c$, then $\log_{a^2} b$ = ?
(a) c^2
(b) $\frac{c}{2}$
(c) $2c$
(d) $\sqrt{c}$
(e) none of the above

17. A polynomial which passes through the points (3,0), (4,2) and (0,6) is
(a) $y = x - 6$
(b) $y = x^4 + x^3 + 3x - 5$
(c) $y = x^2 - 5x + 6$
(d) $y = x^2 - 9$
(e) none of the above

18. Let $f(x) = 3x^2 + Kx + 1$, where K is a real constant. If r and s are the roots of $f(x)$, which of the following is impossible?
(a) $r = s$
(b) $rs = 1$
(c) $r + s = 1$
(d) r and s are both positive
(e) all of the above are possible

19. If A is the area of an equilateral triangle of side length S, then the area of an equilateral triangle of side length $2S$ is
(a) $2A$
(b) $4A$
(c) A^2
(d) $2A^2$
(e) none of the above

20. The following figures all have area one square unit. Which has the smallest perimeter or circumference?
(a) circle
(b) square
(c) equilateral triangle
(d) regular pentagon
(e) regular hexagon

21. The value of K so that the polynomial will be divisible by $x - 3$ is
(a) 1
(b) -1
(c) 2
(d) -2
(e) none of the above

22. The domain of the function $f(x) = \frac{\sqrt{x+1}}{x}$ is
(a) a single point
(b) an infinite interval
(c) an infinite interval and a single point
(d) an infinite interval with a single point deleted
(e) none of the above

23. Let $a_1, a_2, a_3, \cdots$ be a sequence of numbers with the property that the sum of the first n of them is $\frac{n}{2}(n+1)$. Then
(a) $a_k = k$
(b) 1
(c) $k + 1$
(d) $k - 1$
(e) none of the above

24. If a, b, c are positive integers such that $a + b = 2c$, then $2^a \cdot 2^b$= ?
(a) 2^c
(b)
(c) 4^c
(d)
(e) none of the above

25. A man drives from Edmonton to Calgary at a speed of 30 miles per hour (MPH). At what speed must he drive from Calgary to Edmonton so that the average speed for the whole trip is 40 MPH?
(a) 45 MPH
(b) 50 MPH
(c) 55 MPH
(d) 60 MPH
(e) none of the above

1971

1. Given that x is inversely proportional to y, y is inversely proportional to z and z is inversely proportional to v, the relation between x and v is (k is a constant)

(a) $x = kv$

(b) $x = \frac{k}{v}$

(c) $x = kv^2$

(d) $x = \frac{k}{v^2}$

(e) none of the above

2. Given the function $f(x) = 2^{x-1}$, $f(-1) =$

(a) $\frac{1}{4}$

(b) 1

(c) -1

(d) $-\frac{1}{4}$

(e) none of the above

3. A triangle with sides of length 12, 13 and 5

(a) is a right triangle

(b) is an acute triangle

(c) is an obtuse triangle

(d) does not exist

(e) none of the above

4. If $p = \frac{x+1}{x}$ where $\frac{1}{100} \leq x \leq 100$, for what value of x does p have its smallest value?

(a) 101

(b) $\frac{1}{100}$

(c) $\frac{101}{100}$

(d) 100

(e) none of the above

5. For what values of M and N is the equation $M^{\log N} = N^{\log M}$ true? Both logarithms are base 10.

(a) no values of M and N

(b) all values of M and N

(c) all negative values of M and N

(d) all positive values of M and N

(e) none of the above

6. A triangle with sides of length 5, 16 and 8

(a) is a right triangle

(b) is an obtuse triangle

(c) does not exist

(d) is an acute triangle

(e) none of the above

7. At what time between four and five o'clock is the minute hand exactly two minutes ahead of the hour hand?

(a) 4:21

(b) 4:22

(c) 4:23

(d) 4:24

(e) none of the above

8. Let n be an integer. Define $f(n)$ to be the number of positive integers not exceeding n. Then $f(n) + f(-n) =$

(a) 0

(b) $-n$

(c) n

(d) $2n$

(e) none of the above

9. Let $x = 1 - 2^t$ and $y = 1 + 2^{-t}$. Which of the following is true for all t?

(a) $y = \frac{1}{x-1}$

(b) $y = (x - 2)\cdot\frac{1}{x-1}$

(c) $y = (2 - x)\cdot\frac{1}{x-1}$

(d) $y = x\cdot\frac{1}{x-1}$

(e) none of the above

10. A perfect number is a positive integer such that it is equal to the sum of all positive integers smaller than it which divide evenly into it. Which of the following is perfect?

(a) 4

(b) 6

(c) 8

(d) 10

(e) none of the above

11. Let S be the set of points defined by the inequalities $0 \leq y \leq 1$, and $0 \leq x \leq 1$ and $y \leq x + \frac{1}{2}$. The area of the region determined by S is

(a) $\frac{1}{8}$

(b) $\frac{3}{8}$

(c) $\frac{5}{8}$

(d) $\frac{7}{8}$

(e) none of the above

12. A gear of radius 1 revolves around a fixed gear of radius 2. During one complete revolution, the smaller gear will rotate

    (a) 360°
    (b) 540°
    (c) 720°
    (d) 900°
    (e) none of the above

13. For any integer n, the expression $n(n+1)(n+2)$ cannot assume the value

    (a) 0
    (b) 731
    (c) 1320
    (d) 7980
    (e) none of the above

14. The equation of the perpendicular bisector of the line segment with end points (1,5) and (-3,2) is

    (a) $8x - 6y + 29 = 0$
    (b) $8x + 6y + 29 = 0$
    (c) $8x + 6y - 29 = 0$
    (d) $8x - 6y - 29 = 0$
    (e) none of the above

15. Given the quadratic equation $ax^2 + 2bx + 3c$, the absolute value of the difference of the roots is

    (a) $\frac{\sqrt{b^2-4ac}}{2a}$
    (b) $2\cdot\frac{\sqrt{b^2-4ac}}{2a}$
    (c) $4\cdot\frac{\sqrt{b^2-4ac}}{2a}$
    (d) $\frac{\sqrt{b^2-3ac}}{2a}$
    (e) none of the above

16. A triangle ABC with $\angle A = 30°$ is inscribed in a circle. The radius of the circle is

    (a) BC
    (b) $\frac{1}{2}(AB+AC)$
    (c) $\frac{1}{2}BC$
    (d) AB
    (e) none of the above

17. If $y^2 - 1 + \log_{10}x = 0$, the values of x for which y takes on real values are

    (a) $x \leq 10$
    (b) $0 < x \leq 10$
    (c) $0 < x$
    (d) all x
    (e) no x

18. The number 123456789012345678901234567890 is not divisible by

    (a) 2
    (b) 3
    (c) 5
    (d) 9
    (e) none of the above

19. Let ABC be a triangle such that $AB = 5$ and $AC = 7$. Let AH be the altitude from A to BC. Then if $AH = 1$, $BC =$

    (a) $\sqrt{24}$
    (b) $2\sqrt{24}$
    (c) $\left(1+\sqrt{2}\right)\sqrt{24}$
    (d) $\sqrt{2}\sqrt{24}$
    (e) none of the above

20. Let x and y be positive numbers. Let $a = \frac{1}{2}(x+y)$, $b = \sqrt{xy}$, and $c = \sqrt{\frac{1}{2}\left(x^2+y^2\right)}$. Which of the following is always true?

    (a) $a \leq b \leq c$
    (b) $c \leq b \leq a$
    (c) $b \leq c \leq a$
    (d) $b \leq a \leq c$
    (e) none of the above

1972

1. A quadrilateral ABCD is inscribed in a circle of radius 5. If the centre of the circle lies on AB, and the length of AD is 7, then diagonal BD has length

    (a) 2
    (b) 3
    (c) $2\sqrt{6}$
    (d) $\sqrt{51}$
    (e) none of the above

2. A three digit number abc is chosen. The difference between bca and abc is calculated

and found to lie between 400 and 500. The number equals

(a) 400
(b) 404
(c) 429
(d) 495
(e) none of the above

3. A pentagon has angles 110°, 143°, 87°, and 52°. Its remaining angle equals

(a) 148°
(b) 110°
(c) 178°
(d) 90°
(e) none of the above

4. In 9 years, John will be $\frac{3}{5}$ as old as his father was ten years ago. In 10 years, John will be half as old as his father was one year ago.
John's age is

(a) 8
(b) 18
(c) 12
(d) 16
(e) none of the above

5. The expression i^{2073}equals

(a) i
(b) $-i$
(c) 1
(d) -1
(e) none of the above

6. Let a and b be the roots of $x^2 - 7x + 3 = 0$. Then $a^3 + b^3$ equals

(a) 91
(b) -18
(c) 324
(d) 360
(e) none of the above

7. The long hand of a clock points exactly at a minute and the short hand points exactly two minutes ahead of the long hand. The time is

(a) 3:17
(b) 4:26
(c) 7:36
(d) 11:58
(e) none of the above

8. The expression $\cos^4\theta - \sin^4\theta$ equals

(a) $2\cos^2\theta - 1$
(b) $\cos\theta$
(c) $2\sin^2\theta - 1$
(d) $\sin\theta$
(e) none of the above

9. What is the next number in the series 1, 1, 2, 6, 15?

(a) 9
(b) 31
(c) 16
(d) 28
(e) 42

10. Which number of the following divides $9^5 + 33^5 + 39^{10} - 4^{10}$?

(a) 4
(b) 7
(c) 3
(d) 2
(e) none of the above

11. ΔABC and ΔDEF have sides AB, BC, DE, and EF of length S. $\angle ABC$ is 2α and $\angle FDE$ is α. Area of ΔABC/Area of ΔDEF is

(a) $\frac{2}{3}$
(b) $\frac{1}{2}$
(c) 2
(d) 1
(e) none of the above

12. *The expression* $f(x) = \log_2\left(-\frac{1}{x^2}\right)$. Then $f(-\frac{1}{4})$ equals

(a) 0
(b) 4
(c) -4
(d) -3
(e) none of the above

13. The sum of the first twenty odd integers is

(a) 420
(b) 800

(c) 400
(d) 190
(e) none of the above

14. The expression $\left|\frac{2+5i}{3-4i}\right|$ equals

(a) 7
(b) -7
(c) $\frac{5}{\sqrt{29}}$
(d) $\left(\frac{5}{\sqrt{29}}\right)^{-1}$
(e) none of the above

15. Let p be an integer. $2x^2 + px + 3 = 0$ has real roots for

(a) all p
(b) no p
(c) $p \leq 24$
(d) $p \geq 24$
(e) none of the above

16. If $g(t) = -g(-t)$ for some function g then which of the following are true?

i) $g(t) \geq 0$ for $t \geq 0$
ii) $g(t) + g(-t) = 0$
iii) $g(0) = 0$
iv) $g(t) = t$
v) $(g(t))^2 + 2g(-t) + 1 \geq 0$

(a) all
(b) i) ii) iii)
(c) ii) iii) v)
(d) i) iv) v)
(e) none

17. The expression $i^{4421} + i^{3663}$ equals

(a) 2
(b) -2
(c) $2i$
(d) $-2i$
(d) none of the above

18. A triangle with sides of length 5, 7 and 9 is

(a) impossible
(b) obtuse
(c) acute
(d) right
(e) none of the above

19. A quadrilateral with sides 16, 38, 7, 12 is

(a) impossible
(b) convex
(c) concave
(d) a trapezoid
(e) none of the above

20. Which of the following are true if x and y are any real numbers?

(a) $x^2 + 9y^2 \leq 6xy$
(b) $x^2 + 9y^2 \geq x$
(c) $x^2 + 9y^2 \geq 1$
(d) $x^2 + 9y^2 \geq 9xy$
(e) none of the above

1973

1. A triangle ABC is inscribed in a circle of radius three. Given that BC = 2 and AC = 6, then AB equals

(a) 5
(b) 6
(c) $4\sqrt{2}$
(d) $\sqrt{2} + 2\sqrt{3}$
(e) none of the above

2. The line $-x + 3y = 9$ meets the parabola $y^2 = 4x$ in

(a) no points
(b) one point
(c) two points
(d) four points
(e) none of the above

3. Of the following numbers, select the largest.

(a) $2 + 3\sqrt{3}$
(b) $3 + 2\sqrt{2}$
(c) 4
(d) $\sqrt{39}$
(e) $2\sqrt{10}$

4. Let u and v be the roots of $x^2 - 5x + 3 = 0$. Then $u^2 + v^2$ equals

(a) 9
(b) 15
(c) 19
(d) 25

(d) 25
(e) none of the above

5. If a quadrilateral is circumscribed about a circle, then
   (a) the sum of two diagonally opposite angles is 180°
   (b) it must contain a right angle
   (c) it must have two equal sides
   (d) the sum of two opposite sides is half the perimeter of the quadrilateral
   (e) none of the above

6. If * is commutative and associative, and if a * b = c, $c * a = a$, then $b * a * c * a * b$ equals
   (a) a
   (b) b
   (c) c
   (d) $b * b$
   (e) none of the above

7. What is the last digit of 728^{4921}?
   (a) 2
   (b) 4
   (c) 6
   (d) 8
   (e) none of the above

8. What natural number is $\left\{\left[\left(\sqrt{3}\right)^{\sqrt{3}}\right]^{\sqrt{3}}\right\}^{\sqrt{3}}$?
   (a) 3
   (b) 9
   (c) 27
   (d) 1
   (e) none of the above

9. The Edmonton-Calgary Airbus can fly the 189 miles in 40 minutes on a calm day. One day with a headwind, the time was 45 minutes. What was the speed of the wind in miles per hour on the day with the headwind?
   (a) 40.5
   (b) 38.2
   (c) 31.5
   (d) 60.5
   (e) none of the above

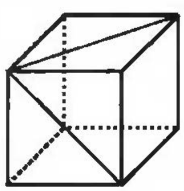
10. AB is a diameter of a circle with centre C, and D is another point on the circumference of the circle, such that ∠BCD = 72°. What is ∠BAD?
   (a) 18°
   (b) 24°
   (c) 30°
   (d) 36°
   (e) none of the above

11. A quadrilateral with sides 5, 3, 5, 7 (taken in order) must be a
   (a) parallelogram
   (b) non-isosceles trapezoid
   (c) isosceles trapezoid
   (d) rhombus
   (e) none of the above

12. A pentagon MNPQR has MN = 2, NP = 7, PQ = 4, QR = 5, RM = 1. The sum or the lengths of the diagonals MP + MQ + NQ + NR + PR cannot possibly equal
   (a) 53
   (b) 33
   (c) 18
   (d) 16
   (e) none of the above

13. 999,999,999,999 is divisible by
   (a) 23
   (b) 77
   (c) 101
   (d) 162
   (e) none of the above

14. Consider two diagonals of a regular cube, as sketched. They meet at an angle of
   (a) 90°
   (b) 60°
   (c) 45°
   (d) 72°
   (e) none of the above



15. If $f(x) = 2^x x^2$, then $f(1) =$
   (a) 0
   (b) 1
   (c) 2
   (d) 4
   (e) none of the above

16. The athletic banquet at Dudgeon High School costs \$450.00, and the committee decided that the cost would be shared equally by all those attending. 75 of those eligible to attend did not, and as a result the cost to each attendee was 50¢ higher than it would otherwise have been. How many were eligible to attend?

    (a) 200
    (b) 225
    (c) 300
    (d) 325
    (e) none of the above

17. Find the sum of the first 100 odd numbers (i.e., $1+3+5+\cdots+199$).

    (a) 10,200
    (b) 10,201
    (c) 10,001
    (d) 10,000
    (e) none of the above

18. A given quadrilateral can be inscribed in a circle if

    (a) the sum of its angle is 360°
    (b) the sum of any two opposite angles is 180°
    (c) it has at least two right angles
    (d) the sum of opposite sides is half the perimeter
    (e) none of the above is sufficient

19. Find the next number in the sequence -3, 1, 5, 9, 31, 53, 75, 97, 101, 501

    (a) 301
    (b) 700
    (c) 505
    (d) -3
    (e) none of the above

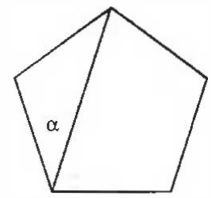
20. The altitude of a regular tetrahedron of edge length 1 is

    (a) $\frac{\sqrt{3}}{2}$
    (b) $\frac{\sqrt{6}}{3}$
    (c) $\frac{\sqrt{3}}{3}$
    (d) $\frac{2\sqrt{3}}{3}$
    (e) none of the above

1974

1. Given a regular pentagon (see sketch), the angle α is

    (a) 18°
    (b) 27°
    (c) 36°
    (d) 45°
    (e) 54°



2. Find the smallest natural number n for which $1 + 2 + 3 + \cdots + n > 5000$.

    (a) 10
    (b) 99
    (c) 100
    (d) 101
    (e) 1,000

3. Let $\frac{1}{2}\left(\sqrt{5}+\sqrt{7}\right), b = \sqrt{6}$, and $c = \sqrt[4]{35}$. Then

    (a) $a < b < c$
    (b) $b < a < c$
    (c) $c < a < b$
    (d) $c < b < a$
    (e) none of the above

4. John and Susan are both younger than 5 years old. Three times John's age equals twice the age Susan will be five years from now. Susan's age is

    (a) 0
    (b) 1
    (c) 2
    (d) 3
    (e) 4

5. Consider the curve C: $y = \sin x$, $-\infty < x < \infty$. The line $y = \frac{1}{2}$ intersects C

    (a) once
    (b) twice
    (c) never
    (d) infinitely often
    (e) five times

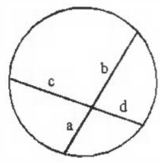
6. A right-angled cross having segments a, b, c, d (see sketch) is inscribed in a circle of radius 2. The largest possible value of a + b + c + d is

    (a) $2\sqrt{2}$
    (b) 8

(c) $4\sqrt{2}$

(d) $6\sqrt{2}$

(e) none of the above



7. The length of the segment joining any top vertex of a cube of side 1 to the midpoint of the bottom side is

(a) $\sqrt{3}$

(b) $\sqrt[3]{2}$

(c) $\sqrt{23}$

(d) $\sqrt{8}$

(e) $\frac{1}{2}\sqrt{6}$

8. The graph of the equation $x^2 + \frac{y^2}{9} = 1$ is

(a) the empty set
(b) (0,0)
(c) two straight lines
(d) an ellipse
(e) a hyperbola

9. If $0 < x + y < 3$ and $1 < x - y < 2$, then

(a) $1 < x < 5$
(b) $|x| < 1$
(c) $x > 1$
(d) $\frac{1}{2} < x < \frac{5}{2}$
(e) none of the above

10. The expression $4^{\log_4 3}$ equals

(a) 3
(b) 4
(c) 64
(d) $4\frac{1}{3}$
(e) none of the above

11. A regular polygon of 300 sides, F_1, is inscribed in a circle, as is another regular polygon, F_2 with 600 sides. The perimeters P_1, P_2 of F_1, F_2 respectively, satisfy

(a) $P_1 = P_2$
(b) $P_1 < P_2$
(c) $P_1 > P_2$
(d) $P_1 + P_2 = P_1 - P_2$
(e) none of the above

12. A right triangle with area 12 and hypotenuse 5 has a perimeter of

(a) 49
(b) 7
(c) 12
(d) 37
(e) none of the above

13. If $\tan\theta = ½$, then $\sin 2\theta =$

(a) $\frac{1}{\sqrt{2}}$

(b) $\frac{1}{\sqrt{5}}$

(c) $\frac{4}{5}$

(d) $\frac{2}{\sqrt{5}}$

(e) none of the above

14. The function achieves a minimum at the value

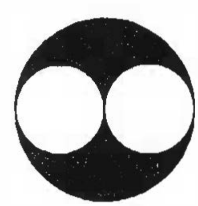
(a) $x = 0$
(b) $x = \sqrt{2}$
(c) $x = 2$
(d) $x = 2\sqrt{2}$
(e) does not attain a minimum

15. If b and c are odd integers, which of the following could be the roots of the equation $x^2 + bx + c = 0$?

(a) 5, 7
(b) 4, 7
(c) $3 + 2i$, $3 + 4i$
(d) $5+\sqrt{7}, 5-\sqrt{7}$
(e) none of the above

16. Two circles of radius 5 are inscribed in a circle of radius 10 (see sketch). If A_1 is the shaded area and A_2 the unshaded are, then

(a) $A_1 > A_2$
(b) $A_1 = A_2$
(c) $A_1 < A_2$
(d) $A_1 + A_2 = A_1 - A_2$
(e) $A_1 = \pi A_2$

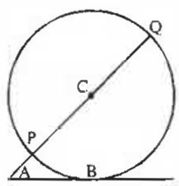


17. Suppose we have a cloth divided into 4 horizontal stripes, and suppose we wish to create different flags by coloring the stripes. If we can use the colors red, blue, white, green and yellow, how

many different flags can we make? (Adjacent stripes must be different colors.)

(a) 80
(b) 210
(c) 320
(d) 400
(e) none of the above

18. A point A is chosen outside a circle with centre C. The tangent from A meets the circle at B, while AC meets the circle at P and Q. Given $\overline{AB} = 10$ and $\overline{AP} = 2$, the radius of the circle must equal



(a) 12
(b) $12\sqrt{2}$
(c) $2\sqrt{10}$
(d) 24
(e) not enough information.

19. Given the base 4 numbers 332 and 32, find their product in base 4.

(a) 100210
(b) 31210
(c) 30211
(d) 21301
(e) none of the above

20. A parallelogram is circumscribed about a circle. It is necessarily a

(a) rectangle
(b) square
(c) parallelogram with a 60° angle
(d) rhombus
(e) none of the above

1975

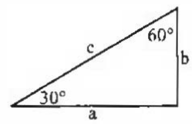
1. The sum of three consecutive positive integers is <u>always</u>

(a) odd
(b) even
(c) a perfect square
(d) divisible by 3
(e) none of the above

2. Which of the following holds true?

(a) $\log_3 2 < \log_2 3$
(b) $\log_3 2 = \log_2 3$
(c) $\log_3 2 > \log_2 3$
(d) $\log_3 2 = 1$
(e) $\log_2 3 = 1$

3. For the triangle shown, which of the following is true?



(a) a = b
(b) b = 2a
(c) c = 2a
(d) c = 2b
(e) none of the above is true

4. "The operation ° is commutative" means

(a) $x \circ 1 = 1$
(b) $x \circ x = x$
(c) $x \circ y = y \circ x$
(d) $x \circ (y \circ z) = (x \circ y) \circ z$
(e) none of the above

5. If $x = .1102$ (base 3), then x^2 is (base 3)

(a) .11021102
(b) .10102
(c) .01222111
(d) .010211
(e) none of the above

6. Given a square inscribed in a circle inscribed in an equilateral triangle, if each side of the triangle has length 6, what is the length of each side of the square?

(a) $\frac{1}{2}\sqrt{6}$
(b) $\sqrt{3}$
(c) $\sqrt{6}$
(d) $2\sqrt{3}$
(e) none of the above

7. Which is larger, the volume of a sphere of radius 1 or the volume of a right circular cone of height 1 and base radius 2?

(a) these volumes do not exist
(b) they are equal
(c) the sphere
(d) the cone
(e) none of the above are true

8. $\dfrac{a^4 + a^2b^2 + b^4}{a^2 + ab + b^2} =$

(a) $a^2 + ab + b^2$
(b) $a^2 + ab - b^2$
(c) $a^2 - ab - b^2$
(d) $a^2 - ab + b^2$
(e) none of the above

9. Five years from now Bill will be twice as old as he was two years after he was half as old as he will be in one year from now. His age is

(a) 16
(b) 13
(c) 8
(d) 41
(e) cannot be determined

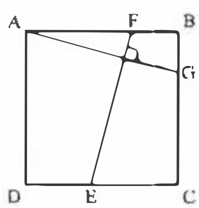
10. The number $1.131313\overline{13}$ is the same as

(a) $\frac{112}{99}$
(b) $\frac{113}{99}$
(c) $\frac{100}{99}$
(d) $\frac{1131313}{1000000}$
(e) none of the above

11. A jar contains 15 balls, of which 10 are red and 5 are black. If 3 balls are chosen at random the probability that all three will be red is

(a) 0
(b) $\frac{2}{3}$
(c) $\frac{4}{9}$
(d) $\frac{8}{27}$
(e) none of the above

12. The square ABCD has side length 1. Given that AG⊥EF and that $\overline{EF} = \sqrt{\frac{7}{16}}$, $\overline{BF} = \sqrt{\frac{1}{7}}$, then $\overline{AG}$ is

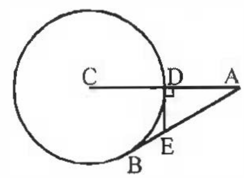


(a) $\sqrt{\frac{7}{6}}$
(b) $\sqrt{7}$
(c) $\sqrt{\frac{6}{7}}$
(d) $\sqrt{\frac{8}{7}}$
(e) none of the above

13. Which is the largest of

(a) 2^{4^3}
(b) 2^{3^4}
(c) 4^{2^3}
(d) 3^{2^4}
(e) 3^{4^2}

14. A circle of radius 3 has centre C. Let A be at a distance 5 from C and AB be a tangent to the circle. Let AC meet the circle at D and let E lie on AB with ED⊥AC. Then length $\overline{ED}$ is



(a) 1
(b) 2
(c) $\sqrt{2}$
(d) $\sqrt{3}$
(e) none of the above

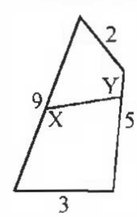
15. The system of equations $2x - 3y = 4$, $2y - 4x = 8$ has

(a) ten solutions
(b) two solutions
(c) one solution
(d) no solutions
(e) none of the above

16. Suppose that a_1 is an integer not divisible by 3 and that $a_1^2 + a_2^2 \ldots + a_n^2$ is divisible by 3, where $a_2, \ldots, a_n$ are integers. Then n is

(a) arbitrary
(b) at least 3
(c) at most 2
(d) always odd
(e) none of the above

17. According to the diagram, $\overline{XY}$ cannot equal



(a) 2
(b) 4
(c) 7
(d) 10
(e) any of these

18. Assume the earth is a perfect sphere and a wire is stretched tightly around the equator. The

wire is lengthened one metre and then expanded uniformly so as to form a somewhat larger circle. The new radius will be approximately how many metres larger than the old one?

(a) .016
(b) .032
(c) .16
(d) .32
(e) 1

19. School X has 100 students and school Y has 50 students. These schools are to be replaced by a single school Z. If the students live in the immediate vicinities of their respective schools (X or Y), where should Z be placed so as to minimize the total distance travelled by all the students?

(a) at X
(b) at Y
(c) halfway in between
(d) one third of the way from X to Y
(e) at none of these

20. An urn contains 100 balls of different colors, 40 red, 27 green, 26 blue, and 7 white. What is the smallest number of balls that must be drawn without looking to guarantee that at least 15 balls have the same color?

(a) 86
(b) 50
(c) 43
(d) 39
(e) none of the above

1976

1. What is the value of $5^{\log_5 6}$?

(a) 1
(b) 5
(c) 6
(d) $\log_6 5$
(e) none of the above

2. Let S be the set of points, (x, y), in the plane satisfying both $x^2 + y^2 \le 1$ and $x^2 + y^2 \ge r^2$. A value of r such that S is the empty set is:

(a) 1
(b) -1
(c) ½
(d) -½
(e) none of the above

3. If S, T, and V are sets, then $[(S\cap T)\cup(S\cap V)]$ is the same set as

(a) S
(b) $T\cup V$
(c) $T\cap V$
(d) $S\cap(T\cup V)$
(e) none of the above

4. A metal disc has one face marked "1" and the other face marked "2". A second metal disc has one face marked "2" and the other marked "3". Assume that, when tossed, the two faces of a disc are equally likely to turn up. If both discs are tossed, what is the probability that "4" is the sum of the numbers turning up?

(a) $\frac{1}{4}$
(b) $\frac{1}{2}$
(c) $\frac{3}{4}$
(d) $\frac{1}{3}$
(e) none of the above

5. Which of the following statements about is $\frac{1+\sqrt{2}}{1-\sqrt{2}}$ true?

(a) it is irrational
(b) it is rational
(c) it is imaginary
(d) it is positive
(e) none of the above are true

6. If $i = \sqrt{-1}$, then i^6 is

(a) 1
(b) -1
(c) i
(d) $-i$
(e) none of the above

7. The solution set of the inequality $x^2 - x - 2 < 0$ is the interval

(a) $-2 \le x \le 1$
(b) $-2 < x < 1$
(c) $-2 \le x < 1$

(d) $-2 < x \leq 1$
(e) none of the above

8. At the end of a party, everyone shakes hands with everyone else. Altogether there are 28 handshakes. How many people are there at the party?

(a) 8
(b) 14
(c) 20
(d) 56
(e) none of the above

9. Let $[x]$ denote the largest integer not exceeding x. Thus, e.g. $[2] = 2$, $[3.99] = 3$, $[-.5] = -1$. Which of the following statements are always true?

i) $[x + y] = [x] + [y]$
ii) $[2x] = 2[x]$
iii) $[-x] = -[x]$
(a) i) only
(b) ii) only
(c) iii) only
(d) all
(e) none

10. If the sum of the first n positive integers is $\frac{n(n+1)}{2}$, the sum of the first n positive odd integers is

(a) $\frac{n(n+1)}{4}$
(b) $\frac{n(2n+1)}{2}$
(c) n^2
(d) n^2-4
(e) none of the above

11. Suppose that $d=x^2-y^2$ where x and y are two odd integers. Which of the following statements are always true?

i) d is odd
ii) d is divisible by 4
iii) d is a perfect square
(a) i) only
(b) ii) only
(c) iii) only
(d) ii) and iii) only
(e) none of the above

12. If each term of the sequence $a_1, a_2, \cdots, a_n$ is either +1 or -1 then $a_1 + a_2 + \cdots + a_n$ is always

(a) 0
(b) 1
(c) $\begin{cases} \text{odd if n is odd} \\ \text{even if n is even} \end{cases}$
(d) $\begin{cases} \text{odd if n is even} \\ \text{even if n is odd} \end{cases}$
(e) none of the above

13. If x is a real number satisfying the equation $x^{x^x} = 2$ then x is equal to

(a) ∞
(b) 2
(c) $\sqrt[4]{2}$
(d) $\sqrt{2}$
(e) none of the above

14. The number of pipes of inside diameter 1 unit that will carry the same amount of water as one pipe of inside diameter 6 units of the same length is

(a) 6π
(b) 6
(c) 12
(d) 36
(e) none of the above

15. $2^{-(2k+1)} - 2^{-(2k-1)} + 2^{-2k}$ is equal to

(a) 2^{-2k}
(b) $2^{-(2k-1)}$
(c) $-2^{-(2k-1)}$
(d) 0
(e) none of the above

16. Let P be the product of any 3 consecutive odd integers. The largest integer dividing all such P is

(a) 15
(b) 6
(c) 5
(d) 3
(e) none of the above

17. If $|x-\log y|=x+\log y$ where x and $\log y$ are real, then

(a) $x=0$
(b) $y=1$
(c) $x=0$ and $y=1$
(d) $x(y-1)=0$

(e) none of the above

18. Each of a group of 50 girls is blonde or brunette and is blue- or brown-eyed. If 14 are blue-eyed blondes, 31 are brunettes and 18 are brown-eyed, the number of brown-eyed brunettes is

(a) 7
(b) 9
(c) 11
(d) 13
(e) none of the above

19. After finding the average of 35 scores, a student carelessly included the average with the 35 scores and found the average of these 36 numbers. The ratio of the second average to the true average was

(a) 1:1
(b) 35:36
(c) 36:35
(d) 2:1
(e) none of the above

20. If the line $y = mx + 1$ intercepts the ellipse $x^2 + 2y^2 = 1$ exactly once, then m^2 is equal to

(a) $\frac{1}{2}$
(b) $\frac{2}{3}$
(c) $\frac{3}{4}$
(d) $\frac{4}{5}$
(e) none of the above

1977

1. If $a > b > 0$, then

(a) $\frac{a+1}{a} > \frac{b+1}{b}$
(b) $\frac{a+1}{a} \geq \frac{b+1}{b}$
(c) $\frac{a+1}{a} \leq \frac{b+1}{b}$
(d) $\frac{a+1}{a} < \frac{b+1}{b}$
(e) none of the above are true

2. Let AB be a diameter of a circle of radius 1 and let C be a point on the circumference such that $\overline{AC}=\overline{BC}$. Then the length $\overline{AC}$ is equal to

(a) 2
(b) $\frac{1}{2}$
(c) $\sqrt{2}$
(d) $\frac{1}{\sqrt{2}}$
(e) none of the above

3. Out of 100 people, 60 report that they receive the daily news by watching television, whereas 70 read the newspaper. Of those who read the newspaper, 70% also watch television. The number not receiving any news by television or newspaper is

(a) 15
(b) 19
(c) 23
(d) 27
(e) none of the above

4. $(64^{.9})(32)^{-.08}$ equals

(a) 64
(b) 32
(c) 24
(d) 8
(e) none of the above

5. Let $f(x)$ be a non-constant polynomial with real coefficients. If $f(x)=f(x-1)$ for all x then $f(x)$

(a) has exactly one root
(b) cannot exist
(c) has exactly two roots
(d) has either no roots or an infinite number of roots
(e) satisfies none of the preceding

6. Suppose k is a real number such that $0 < k < 1$. Of the two roots of the quadratic equation $kx^2 - 3x + k = 0$,

(a) both are positive
(b) both are negative
(c) both are zero
(d) one is positive and one is negative
(e) none of the above

7. Let ℓ be a line in the real plane passing through the points (1, 1) and (3, 5). Then ℓ passes through the point $(2, y)$ where

(a) $y = 4$
(b) $y = 2$
(c) $y = 3$

(d) y = 5

(e) y is none of the above

8. ΔABC is an equilateral triangle with sides of length 1, and $DE \| CB$. If the area of ΔADE is equal to the area of the trapezoid DEBC, then the length DE equals

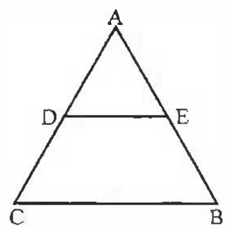
(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\frac{\sqrt{2}-1}{\sqrt{2}}$

(e) $\frac{\sqrt{3}-1}{\sqrt{3}}$



9. The inequality $(x + 1)(x - 1) \geq x^2$ is valid

(a) for all real x

(b) for no real x

(c) for all $x > 1$

(d) for all $x > 0$

(e) for none of the above

10. Suppose a bowl contains 3 red balls and 3 yellow balls. The probability that two balls drawn out without replacement will both be red is

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{3}$

(d) $\frac{1}{6}$

(e) none of these

11. $\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+1}}}$ equals

(a) 1

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{3}{5}$

(e) $\frac{5}{8}$

12. $\frac{xy-x^2}{xy-y^2} - \frac{xy}{x^2-y^2}$ can be simplified to

(a) $\frac{x^3}{y^3-yx^2}$

(b) $\frac{x^2}{y^2-x^2}$

(c) $\frac{x^2+y^2}{x^2-y^2}$

(d) $\frac{x^4+xy^3}{(x^2-y^2)(xy-y^2)}$

(e) none of the above

13. If the radius of a sphere is increased 100%, the volume is increased by

(a) 100%

(b) 200%

(c) 300%

(d) 400%

(e) none of the above

14. The expression x^4+16 equals

(a) $(x^2 + 4)(x^2 + 4)$

(b) $(x^2 + 4)(x^2 - 4)$

(c) $(x^2 - 4x + 4)(x^2 + 4x + 4)$

(d) $\left(x^2 - 2x\sqrt{2} + 4\right)\left(x^2 + 2x\sqrt{2} + 4\right)$

(e) none of the above

15. The price of a book has been reduced by 20%. To restore it to its former value, the last price must be increased by

(a) 25%

(b) 10%

(c) 15%

(d) 20%

(e) none of the above

16. OABC is a rectangle inscribed in a quadrant of a circle of radius 10. If OA=5, then AC equals

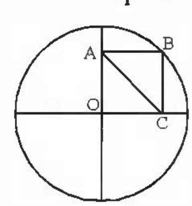
(a) $2\sqrt{5}$

(b) $\frac{1}{\sqrt{2}}$

(c) 8

(d) 12

(e) none of the above



17. The lengths of the medians of a right triangle which are drawn from the vertices of the acute angles are $\sqrt{73}$ and $2\sqrt{13}$. The length of the third median is

(a) $\sqrt{73+52}$
(b) $\sqrt{73}+2\sqrt{13}$
(c) 5
(d) 10
(e) none of the above

18. A car travels 240 miles from one town to another at an average speed of 30 miles per hour. On the return trip the average speed is 60 miles per hour. The average speed for the round trip is

(a) 35 mph
(b) 40 mph
(c) 45 mph
(d) 50 mph
(e) 55 mph

19. The expression $\log_3 6 + \log_3 3 - \log_3 2$ equals

(a) $\frac{5}{2}$
(b) 3
(c) 2
(d) 1
(e) 0

20. The slope of the line passing through the points (3, 4) and (1, 9) is

(a) $-\frac{5}{2}$
(b) $\frac{5}{2}$
(c) 5
(d) -2
(e) 6

1978

1. Which of the following inequalities is true for all positive numbers x?

(a) $x + \frac{1}{x} > 2$
(b) $x + \frac{1}{x} < 2$
(c) $x + \frac{1}{x} \geq 2$
(d) $x + \frac{1}{x} \leq 2$
(e) none of the above

2. A steamer was able to go twenty miles per hour upstream and twenty-five miles per hour downstream. On a return trip the steamer took two hours longer coming upstream than it took coming downstream. The total distance travelled by the steamer was

(a) 100 miles
(b) 200 miles
(c) 400 miles
(d) 800 miles
(e) 150 miles

3. If n is a positive integer, then $n^2 + 3n + 1$ is

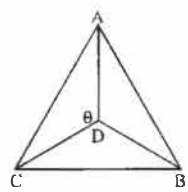
(a) always a perfect square
(b) never a perfect square
(c) sometimes a perfect square
(d) sometimes an even integer
(e) none of the above

4. The solution set of the inequality $x^2(x^2 - 1) \leq 0$ is

(a) an interval
(b) two intervals
(c) a point
(d) an interval and a point
(e) all real numbers

5. In the diagram, ΔABC is an equilateral triangle, ΔBCD is an isosceles triangle, and $\angle CDB$ is a right-angle. Then the angle θ is

(a) 45°
(b) 90°
(c) 120°
(d) 135°
(e) none of the above



6. Given the binary operation * between two positive integers, m, n such that $m*n = mn + 1$ (mn is the usual multiplication of m and n), which of the following does <u>not</u> hold

(a) commutative law
(b) associative law
(c) $n*n$ is a positive integer
(d) $m*n \geq 2$
(e) $m*n$ is odd whenever m is even

7. Label the four quadrants of the (x, y) plane as follows:

II	I
III	IV

Then the solution set of the simultaneous in equalities $x^2 - y < 0$, $x^2 + y^2 < 1$ lies entirely in quadrants

(a) I and II
(b) II and III
(c) III and IV
(d) IV and I
(e) none of the above are correct

8. If $f(n) = n^2$, where n is an integer, then $\frac{f(f(n+1)) - f(f(n-1))}{f(n+1) - f(n-1)}$ equals

(a) n^2
(b) $2n^2 + 2$
(c) $n^2 + 1$
(d) $n^4 + 1$
(e) none of the above

9. Which of the following inequalities hold for all pairs of real number x, y?

(a) $\sqrt{x^2 + y^2} \le x + y$
(b) $\sqrt{x^2 + y^2} \le x^2 + y^2$
(c) $\sqrt{x^2 + y^2} \le xy$
(d) $\sqrt{x^2 + y^2} \le |x| + |y|$
(e) none of the above

10. Two similarly proportioned boxes have their surface areas in the ratio 4:1. Their volumes are in the ratio

(a) 9:1
(b) 8:1
(c) 3:1
(d) 2:1
(e) none of the above

11. The roots of the quadratic polynomial $2x^2 + kx + 1$ are r and s. Which of the f ollowing are impossible

(a) $r = s$
(b) $r\,s = 1$
(c) $r + s = 1$
(d) $r + s = 0$
(e) all of the above

12. A hat contains three slips of paper, of which one bears the name John, one bears the name Diana and the other bears both names. If John and Diana each draw a slip, the probability that they each draw a slip with their own name is

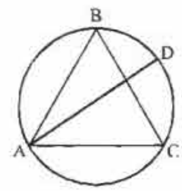
(a) $\frac{1}{9}$
(b) $\frac{1}{6}$
(c) $\frac{1}{4}$
(d) $\frac{1}{3}$
(e) none of the above

13. The value of k such that $x^6 - kx^4 + kx^2 - kx + 4k + 6$ is divisible by x-2 is

(a) 1
(b) 5
(c) 7
(d) 11
(e) there is no such value of k

14. ΔABC is an equilateral triangle inscribed in a circle of diameter 1. If $\overline{AD}$ is a diameter of the circle, then the length $\overline{BD}$ is

(a) $\frac{1}{2}$
(b) 1
(c) 2
(d) $\frac{1}{3}$
(e) $\frac{1}{\sqrt{2}}$



15. The equation of the line through the point (1, 1) that is perpendicular to the line $y=-2x-3$ is

(a) $y = \frac{2}{3}x + \frac{1}{3}$
(b) $y = \frac{1}{3}x + \frac{2}{3}$
(c) $y = 2x - 1$
(d) $y = \frac{1}{2}x + \frac{1}{2}$
(e) none of the above

16. If $\log_a b = c$, then $\log_a(b^c)$=

(a) bc
(b) 6^c
(c) c^c
(d) c^2
(e) $2c$

17. A circle and a square can never have in common exactly

(a) one point
(b) two points
(c) three points
(d) four points
(e) all of the preceding are possible

18. Let $\{a_1, a_2, a_3, \ldots\}$ be a sequence of real numbers such that the sum of the first n of them is $n^2 + n$. Then a_n is equal to

(a) n
(b) $2n - 1$
(c) $2n + 1$
(d) 1
(e) none of the above

19. The domain of the function $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$ is

(a) a single point
(b) an infinite interval
(c) a finite interval
(d) an infinite interval with a point deleted
(e) none of the above

20. A polynomial whose graph passes through the points (-1, 7), (1, 0), (2, 0) is

(a) $x + 8$
(b) $x^2 - 3x + 2$
(c) $x^2 + 9$
(d) $x^3 - x^2 + x - 1$
(e) none of the above

1979

1. A triangle has sides of lengths 1, 2 and $\sqrt{3}$. Its area is

(a) $\frac{1}{2}$
(b) $\frac{\sqrt{3}}{2}$
(c) 2
(d) $2\sqrt{3}$
(e) none of the above

2. $P(x) = 4x^4 - kx^2 + 1$ has double roots if

(a) $k = 1$
(b) $k = 2$
(c) $k = 3$
(d) $k = 4$
(e) none of the above is true

3. For what values of a, b is the equation $(a^{\log_{10} b})^{ab} = (b^{\log_{10} a})^{ba}$ true?

(a) all values of a and b
(b) no values of a and b
(c) all positive values of a and b
(d) all negative values of a and b

4. The equation of the line that is perpendicular to the line $x + 2y=3$ and passes through the point (4, 5) is

(a) $x - 2y=3$
(b) $2x - y = 3$
(c) $2x + y = 3$
(d) $-2x + y = 3$
(e) none of the above

5. Let S be the set of points defined by the inequalities $x + y \geq 1$ and $x - y \leq 1$, $y \leq$ The area of the region determined by S is

(a) $\frac{3}{8}$
(b) $\frac{1}{2}$
(c) $\frac{5}{8}$
(d) 1
(e) none of the above

6. X is a square of diagonal 1, Y is an equilateral triangle of side 1 and Z is a right angled isosceles triangle whose equal sides have length 1. Comparing the areas of these figures

(a) X is larger than both Y and Z
(b) Y is larger than both Z and X
(c) Z is larger than both X and Y
(d) X, Y, Z all have the same area
(e) none of the above

7. A positive integer is squarefree if it cannot be divided exactly by the square of an integer larger than 1. The number of positive squarefree integers less than 20 is

(a) 0
(b) 3
(c) 9
(d) 13
(e) none of the above

8. The solutions of the equation $(\sin\theta + \cos\theta)^2 = 1$ are

(a) all multiples of $\frac{\pi}{2}$

(b) all multiples of π

(c) all odd multiples of $\frac{\pi}{2}$

(d) all even multiples of π

(e) none of the above

9. The sum of the first 27 odd positive integers is

(a) 153

(b) 196

(c) 144

(d) 216

(e) none of the above

10. The expression $\left(\left(\left(\sqrt{2}\right)^{\sqrt{2}}\right)^{2}\right)^{\sqrt{2}}$ is equal to

(a) 2

(b) $\sqrt{2}$

(c) 4

(d) 8

(e) none of the above

11. If α, β are the roots of $x^2 + 7x - 5 = 0$, then $\alpha^2 + \beta^2$ is equal to

(a) 59

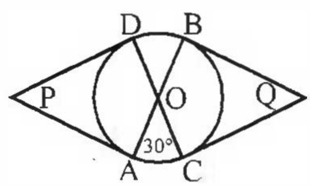
(b) 47

(c) -15

(d) 35

(e) none of the above

12. In the figure, AOB and COD are straight lines and O is the centre of the unit circle, while PD, PA, QB, QC are tangents to the circle. The distance from P to Q is



(a) 2

(b) 3

(c) 4

(d) $\frac{1}{2}$

(e) none of the above

13. For which values of x is a triangle with sides x, $x+1$, $x+2$ an acute-angled triangle?

(a) $x = 1$

(b) $x > 2$

(c) $x < 4$

(d) $x > 3$

(e) none of the above

14. Which of the following inequalities is always true for any pair of real numbers x, y?

(a) $x + y \leq xy$

(b) $(x + y)^2 \geq xy$

(c) $x + y \geq xy$

(d) $(x + y)^2 \geq x + y$

(e) none of the above is always true

15. A twelve-hour digital watch displays the hours, minutes and seconds. During one complete day it registers at least one figure 3 for a total time of

(a) 1 hour and 5 seconds

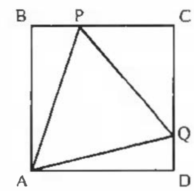
(b) 1 hour, 15 minutes and 15 seconds

(c) 2 hours and 24 minutes

(d) 3 hours

(e) none of the above

16. In the diagram, ABCD is a square of side 1 and APQ is an equilateral triangle. The length DQ is equal to



(a) $\frac{1}{2}$

(b) $\sqrt{2} - 1$

(c) $\frac{5}{8}$

(d) $2 - \sqrt{3}$

(e) none of the above

17. The product of John's and Mary's ages is five more than four times the sum of their ages. If Mary is 4 years younger than John, John's age is

(a) 13

(b) 11

(c) 9

(d) 7

(e) none of the above

18. In a poll of 1000 coffee drinkers, 40% preferred their coffee with neither cream nor sugar and 60% of the remainder preferred their coffee with cream only. After deducting both of these groups, 40% of those left preferred their coffee with sugar only. The rest preferred coffee with cream and sugar and their number was

(a) 144

(b) 216

(c) 96

(d) 172

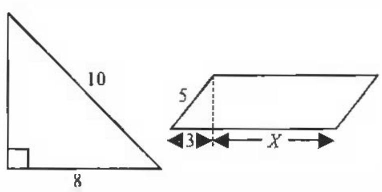
(e) none of the above

19. A sphere and a triangle <u>cannot</u> have in common exactly
    (a) 1 point
    (b) 2 points
    (c) 3 points
    (d) 4 points
    (e) all of the above are possible

20. The picture cards are removed from a pack of 52 playing cards. The number of ways of drawing 2 cards from the remaining 40 so that the sum of the numerical values is 10 is
    (a) 100
    (b) 10
    (c) 50
    (d) 70
    (e) none of the above

1980

1. There are 5 roads between the towns A and B and 4 roads between B and C. The number of different ways of driving the round trip A→B→C→B→A without using the same road more than once is
    (a) 32
    (b) 240
    (c) 400
    (d) 16
    (e) none of the above



2. In the above diagrams, the area of the triangle is two fifths the area of the parallelogram. The value of x is therefore
    (a) 30
    (b) 12
    (c) 15
    (d) 24
    (e) none of the above

3. A die is thrown repeatedly until a 6 is obtained. Assuming the die to be fair, the probability that this will happen on the third throw is
    (a) $\frac{1}{6}$
    (b) $\frac{1}{216}$
    (c) $\frac{5}{36}$
    (d) $\frac{25}{216}$
    (e) none of the above

4. Find all real values of k for which kx^2+kx+1 has no real root
    (a) $-2 < k <$
    (b) $-2 < k < 4$
    (c) $0 < k < 2$
    (d) $0 < k < 4$
    (e) none of the above

5. For which value(s) of k are the lines $9x + ky = 7$ and $kx + y = 2$ parallel?
    (a) $k = 3$
    (b) $k = \pm 3$
    (c) $k = {}^1/_3$
    (d) $k = \pm{}^1/_3$
    (e) none of the above

6. Let $x = \log_{a^2} b$. The $\log_b a^x$ equals
    (a) $\sqrt{b}$
    (b) b^2
    (c) ½
    (d) 2
    (e) none of the above

7. If $\sin 3x = 0$, then $\sin x$ equals
    (a) 0
    (b) 0 or $\pm\frac{1}{2}$
    (c) 0 or $\pm\frac{\sqrt{3}}{2}$
    (d) $0, \pm\frac{1}{2}, \pm\frac{\sqrt{3}}{2}$
    (e) none of the above

8. In the right-angled triangle ABC, the lengths of the sides AB and BC are (x^2+y^2) and $2xy$ respectively. The length of the side AC is therefore
    (a) (x^2-y^2)
    (b) $(x-y)^2$
    (c) $(x^2-y^2)^2$

(d) $(x+y)^2$

(e) none of the above

9. In a certain examination, the average mark (out of 100) of the 30 boys in a class was 60. The girls did rather better, their average being 65. If the overall average for the class was 62, the number of girls who took the examination was

(a) 20

(b) 24

(c) 18

(d) 36

(e) none of the above

10. The following is true for all real numbers x, y:

(a) $(x+y)^2 \geq (x-y)^2$

(b) $|x+y| \geq x+y$

(c) $2\sqrt{5}$

(d) $xy \geq x+y$

(e) none of the above is true for all real x, y

11. The expression $\log_{10}(144)^{144}$ equals

(a) $576\log_{10}2 + 288\log_{10}3$

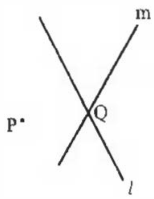
(b) $144\log_{10}2 + 144\log_{10}3$

(c) $(\log_{10}144)^{144}$

(d) $2\log_{10}144$

(e) none of the above

12. The line l has the equation $y=-2x-4$ and the line m has the equation $y=2x+4$. If P is the point (0, 0) then the equation of the line through P and Q is



(a) $y = x$

(b) $y = -2x$

(c) $2x + 3y = 1$

(d) $-y - 2x = 1$

(e) none of the above

13. The maximum possible value of $3x - 3x^2$ where x is real is

(a) $\frac{5}{4}$

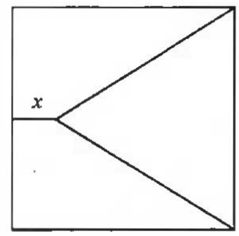
(b) 0

(c) -1

(d) $\frac{3}{4}$

(d) none of these

14. A unit square is divided into three parts of equal area as shown. Then x is



(a) $\frac{1}{3}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{4}$

(d) $\frac{1}{\sqrt{3}}$

(e) $\frac{1}{2}$

15. A unit square is divided into three parts by three lines of equal length as shown. The length of each line is

(a) $\frac{1}{4}$

(b) $\frac{5}{8}$

(c) $2-\sqrt{2}$

(d) $\frac{1}{\sqrt{2}}$

(e) $\frac{3}{5}$

16. The expression equals $\sqrt{\dfrac{\sqrt{5}-2}{\sqrt{5}+2}}$

(a) $\frac{1}{\sqrt{5}-\sqrt{2}}$

(b) $\frac{1}{\sqrt{2}+5}$

(c) $\sqrt{5}-2$

(d) $\sqrt{5}+2$

(e) $\frac{1}{5-\sqrt{2}}$

17. If $f(x) = x^2 - 5$ and $f(4+a) - f(4a) = 0$ then a is

(a) $\frac{4}{3}$ or $-\frac{4}{5}$

(b) 4 or $-\frac{4}{5}$

(c) $\frac{4}{3}$ or $\frac{4}{5}$

(d) -4 of $\frac{4}{3}$

(e) -4 or $-\frac{4}{5}$

18. Two circles and a straight line are drawn in the plane to form exactly N bounded regions. N <u>cannot</u> be

(a) 3

(b) 4
(c) 5
(d) 6
(e) all these are possible

19. A bowl contains 2 marbles of each of 4 colors. If you randomly remove 3 marbles from the bowl without replacing them, what is the probability that you have removed two of the same color?

(a) $\frac{1}{4}$
(b) $\frac{1}{7}$
(c) $\frac{80}{7}$
(d) $\frac{1}{2}$
(e) none of the above

20. If n is a positive integer so that $1+2+\cdots+n=(n+1)+(n+2)+\cdots+118+119$ then n is

(a) 60
(b) 69
(c) 76
(d) 84
(e) 89

1981

1. The circle $x^2+y^2=1$ and the ellipse $\frac{x^2}{4}+(y-2)^2=1$ intersect in how many real points?

(a) one
(b) two
(c) three
(d) four
(e) no real points

2. If $x°$ Fahrenheit equals $-x°$ Centigrade when x is

(a) 10
(b) -40
(c) $-\frac{16}{9}$
(d) $-\frac{80}{7}$
(e) none of the above

3. The values of k for which $x^2+(x+1)(k+1)=0$ has real solutions in x are

(a) $k=-1$ and $k=3$
(b) $-1 \le k \le 3$
(c) $k \ge 3$ and $k \le -1$
(d) all values of k
(e) none of the above

4. sec x cosec x is equal to

(a) sec x + cosec x
(b) tan x + cot x
(c) sin x + cos x
(d) $(\sec x + \operatorname{cosec})^{-1}$
(e) none of the above

5. The sum of the roots $3x^5 - 30x^4 - 105x^3 - 105x^2 + 72x$ is

(a) 3
(b) -30
(c) 10
(d) -35
(e) none of the above

6. For what values of k can the correct solution of $\log(3x+2)+\log(4x-1)=2\log k$ be obtained by "cancelling the log" and solving $(3x+2)+(4x-1)=2k$?

(a) for no values of k
(b) for $k=3$
(c) for $k=11$
(d) for all values of k
(e) none of the above

7. Seven Canadian coins, none of which has a value greater than 25 cents, add up to 81 cents. The number of nickels is

(a) none
(b) one
(c) two
(d) five
(e) none of the above

8. Let ABCD be a square of side 1 and P be any point. The minimum total length of line-segments PA, PB, PC and PD is

(a) 2
(b) $2\sqrt{2}$
(c) $2+\sqrt{2}$
(d) 4
(e) none of the above

9. What is the value of

$$(1+\sqrt{2}+\sqrt{3})(1+\sqrt{2}+\sqrt{3})(1-\sqrt{2}+\sqrt{3})(1-\sqrt{2}-\sqrt{3})$$

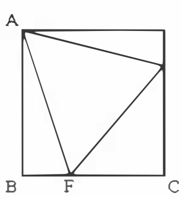
   (a) -8
   (b) 4
   (c) $2\sqrt{3}+2\sqrt{2}$
   (d) 0
   (e) none of the above

10. In how many ways can one choose 5 of the first 10 positive integers so that no two are consecutive?
   (a) 2
   (b) 4
   (c) 6
   (e) 8
   (e) none of the above

11. The only integer solution of $(3x)^{3x}.(2x)^{2x}=3^{18}.2^{9}$ is x =
   (a) 1
   (b) 2
   (c) 3
   (d) 0
   (e) there is no solution

12. In each meeting, every pair of people shook hands once. At two successive meetings, the second one having a higher attendance, 100 handshakes in all took place. The second attendance was higher than the first by
   (a) 1
   (b) 10
   (c) 11
   (d) the situation is impossible
   (e) none of the above

13. In the diagram, ABCD is a square of side 1 and AEF is an equilateral triangle. The length of AE is

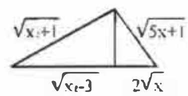


   (a) $\sqrt{2}$
   (b) $\sqrt{3}$
   (c) $\sqrt{6}-\sqrt{3}$
   (d) $\sqrt{6}-\sqrt{2}$
   (e) none of the above

14. What is the sum of all the digits appearing in the first 99 positive integers?
   (a) 900
   (b) 1800
   (c) 4950
   (d) 9900
   (e) none of the above

15. Joe tosses a fair coin three times in succession. Moe tosses three fair coins all at once. The probability that Joe gets more heads than Moe is
   (a) $\frac{1}{2}$
   (b) $\frac{3}{4}$
   (c) $\frac{5}{8}$
   (d) $\frac{7}{16}$
   (e) none of the above

16. In the figure, the value of x must be



   (a) 1
   (b) 2
   (c) 3
   (d) 4
   (e) none of the above

17. Let $f(x, y)$ be a function which is not identically zero and such that $f(x, y)=kf(y, x)$ for all x and y. The possible values of k are
   (a) 0
   (b) 1
   (c) -1
   (d) ± 1
   (e) none of the above

18. The number of positive four digit integers that give the same integer when their digits are reversed is
   (a) 81
   (b) 90
   (c) 100
   (d) 121
   (e) none of the above

19. The area of the triangle whose vertices have coordinates (0, 0), (2, 4) and (4, 2) is
   (a) 12
   (b) $6\sqrt{2}$
   (c) $2\sqrt{10}$

(d) $2\sqrt{2}+4\sqrt{5}$

(e) none of these

20. The sum:

$$\frac{1}{2x+1}+\frac{1}{(2x+1)(4x+1)}+\frac{1}{(4x+1)(6x+1)}+\ldots+\frac{1}{(2(k-1)x+1)(2kx+1)}$$

is equal to

(a) $\frac{1}{2kx+1}$

(b) $\frac{k}{2kx+1}$

(c) $\frac{1}{(2x+1)(4x+1)\ldots(2kx+1)}$

(d) $\frac{k}{(2x+1)(4x+1)\ldots(2kx+1)}$

(e) none of the above

1982

1. A plane takes off from Edmonton, flies north for 500 kilometres, then east for 500 kilometres, next south for 500 kilometres and finally west for 500 kilometres. It will land

(a) north of Edmonton

(b) east of Edmonton

(b) South of Edmonton

(d) west of Edmonton

(e) right at Edmonton

2. Merle has an assortment of pennies, nickels, dimes and quarters but is unable to change a dollar bill. The largest possible amount Merle has in coins is

(a) \$0.94

(b) \$0.99

(c) \$1.19

(d) \$1.24

(e) none of the above

3. The angle between the hour hand and the minute hand at twenty minutes past one is

(a) 60°

(b) 70°

(c) 80°

(d) 90°

(e) none of the above

4. The number of solutions (x, y, z) of the equation $x+y+z=15$ where x, y, and z are integers is $0 < x < y < z$ is

(a) 12

(b) 13

(c) 14

(d) 15

(e) none of the above

5. If a and b are the roots of $7x^2-4x+12$, then $\frac{1}{a}+\frac{1}{b}$ is equal to

(a) -3

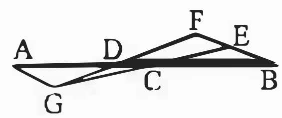
(b) $-\frac{1}{3}$

(c) $\frac{1}{3}$

(d) 3

(e) none of the above

6. In the diagram, $\overline{AC}=\overline{BC}$, $\overline{BE}=\overline{EF}$ and AG||FB. Then $\frac{\overline{BD}}{\overline{AD}}$ is



(a) less than 2

(b) equal to 2

(c) between 2 and 3

(d) equal to 3

(e) greater than 3

7. Stephen collects bugs. Some of them are spiders (8 legs each) and the rest are beetles (6 legs each). His collection consists of 8 bugs with a total of 54 legs. The number of spiders in his collection is

(a) 1

(b) 3

(c) 5

(d) 7

(e) none of the above

8. The minimum value of $x^2 + 2x - 3$ is

(a) -4

(b) -3

(c) -1

(d) 1

(e) none of the above

9. Kelly walks to school at a rate of 2 kilometres per hour. In order that the complete journey (home to school to home) is travelled at an average of 4 kilometres per hour, the rate at which the trip home must be made is

(a) 5 kilometres per hour

(b) 6 kilometres per hour

(c) 8 kilometres per hour

(d) depends on home-school distance

(e) none of the above

10. Let A, B, C, D and E be five points in space. If AB = 30, BC = 80, CD = 236, DE = 86 and EA = 40, then CE is equal to

(a) $\sqrt{236} - \sqrt{86}$

(b) $\sqrt{236^2 - 86^2}$

(c) 150

(d) 236

(e) none of the above

11. Let a, b, c and d be distinct real numbers. Let $x = \max[\min(a, b), \min(c, d)]$ and $y = \min[\max(a, c), \max(b, d)]$. Then

(a) x is always strictly greater than y

(b) x is always greater than or equal to y

(c) x is always less than or equal to y

(d) x is always strictly less than y

(e) none of the above

12. The sum of the positive divisors of 36 is

(a) 9

(b) 43

(c) 91

(d) 666

(e) none of the above

13. Peter and Trevor bought 300 grams and 500 grams of jelly beans respectively. They ate them together with Mr. Smith, each eating the same amount. Afterwards, Mr. Smith paid the boys 80¢. Peter's fair share amounted to

(a) 10¢

(b) 20¢

(c) 30¢

(d) 40¢

(e) none of the above

14. If $a = 6\sqrt{3} - 3\sqrt{13}$ and $b = 6\sqrt{10} - 15\sqrt{2}$, then a-b is

(a) at least 1

(b) positive and less than 1

(c) equal to 0

(d) negative and greater than -1

(e) at most -1

15. Five baskets contain 5, 12, 14, 22 and 29 eggs respectively. In each basket, some of the eggs are chicken eggs while the remaining ones are duck eggs. After one of the baskets is sold, the total number of chicken eggs remaining is equal to twice the total number of duck eggs remaining. The number of eggs in the basket sold is

(a) 5

(b) 12

(c) 14

(d) 22

(e) 29

16. An astronaut 2 metres tall walks once around the equator of a gigantic spherical planet. The top of his head describes a circle. The circumference of this circle is longer than the equator of the planet by

(a) less than 50 metres

(b) between 50 and 500 metres

(c) between 500 and 5000 metres

(d) between 5000 and 50000 metres

(e) greater than 50000 metres

17. The number of terms in the arithmetic series $8 + 16 + 24 + \cdots$ such that their sum first exceeds 1982 is

(a) 20

(b) 22

(c) 24

(d) 248

(e) none of the above

18. Rose and Mary play a series of three games. In each game, Rose's probability of winning is $2/3$. Rose's probability of winning at least two of the three games is

(a) less than $\frac{1}{2}$

(b) between $\frac{1}{2}$ and $\frac{2}{3}$

(c) equal to $\frac{2}{3}$

(d) between $\frac{2}{3}$ and $\frac{3}{4}$

(e) greater then $\frac{2}{3}$

19. If $\log_2 x = 2$ and $\log_{xy} z = \frac{1}{6}$, $\log_y z$ then is equal to

(a) $\frac{1}{4}$

(b) $\frac{1}{3}$
(c) 3
(d) 4
(e) none of the above

20. The expression $\frac{\cos x}{1-\sin x}$ is equal to

(a) $\sin x + \tan x$
(b) $\sin x \cdot \tan x$
(c) $\sec x + \tan x$
(d) $\sec x \cdot \tan x$
(e) none of the above

1983

1. Assume that the vertical distance between floors in a building is constant. The ratio of the vertical distance between the first and the third floors to the vertical distance between the first and the sixth floors is

(a) 2:5
(b) 1:2
(c) 2:1
(d) 5:2
(e) none of the above

2. The compound fraction $\frac{2}{3+\frac{2}{3+\frac{2}{3}}}$ is equal to

(a) $\frac{4}{9}$
(b) $\frac{22}{39}$
(c) $\frac{2}{3}$
(d) 1
(e) none of the above

3. A boy stands at the centre of a circle of radius 8 metres. A girl stands at a point 4 metres from the boy. The boy runs to some point on the circle and then to the girl. The shortest distance the boy must run in metres is

(a) 20
(b) $8+4\sqrt{5}$
(c) $8+4\sqrt{3}$
(d) $2\sqrt{68}$
(e) none of the above

4. The smallest positive integer which leaves a remainder of 2 whether it is divided by 12 or by 15 is

(a) 14
(b) 58
(c) 62
(d) 182
(e) none of the above

5. A bird in the hand is worth two in the bush. Five birds in hand and three birds in the bush together are worth \$30 more than three birds in hand and five birds in the bush together. A bird in hand is worth

(a) \$5
(b) \$7.50
(c) \$10
(d) \$15
(e) none of the above

6. In triangle ABC, $\angle A$ is at least 10° more than $\angle B$ and $\angle B$ is at least 25° more than $\angle C$. The maximum value of $\angle C$ is

(a) 35°
(b) 40°
(c) 45°
(d) 50°
(e) none of the above

7. If $f(x-1) = 3x^2 + 2x - 5$, then $f(x) =$

(a) $3x^2 + 2x - 4$
(b) $3x^2 + 2x - 6$
(c) $3x^2 + 8x$
(d) $3x^2 - 4x - 4$
(e) none of the above

8. Let $[x]$ denote the greatest integer not exceeding x. Then the set of all possible values of $[x] + [-x]$ is

(a) $\{0\}$
(b) $\{0, 1\}$
(c) $\{0, -1\}$
(d) $\{0, 1, -1\}$
(e) none of the above

9. In this problem, a, b, c are real constants. It is possible for all the points (1, 3), (2, 1) and (3, 5) to lie on a graph of the form

(a) $y = a^2x + b$

(b) $y = -a^2x + b$

(c) $y = a^2x^2 + bx + c$

(d) $y = -a^2x^2 + bx + c$

(e) none of the above

10. The sum of five numbers is 100. The sum of the first and the second number is 44, the sum of the second and third is 47, the sum of the third and the fourth is 37 and the sum of the fourth and the fifth is 35. The third number is

(a) 16

(b) 18

(c) 19

(d) 21

(e) none of the above

11. There were 16 participants in a mathematics contest. Of every two participants, at least one was right-handed. If the eventual winner was left-handed, then the number of right-handed participants was

(a) 7

(b) 8

(c) 15

(d) impossible to determine

(e) none of the above

12. If p and q are the roots of $x^2 + x + 1 = 0$, then $\frac{p}{q} + \frac{q}{p} =$

(a) -3

(b) -1

(c) 1

(d) 3

(e) none of the above

13. The smallest positive integer which can be expressed in the form $11x + 8y$ where xD and y are integers is

(a) 1

(b) 3

(c) 8

(d) 19

(e) none of the above

14. If $m = (2 + \sqrt{3})^{-1}$ and $n = (2 -)^{-1}$, then $(m + 1)^{-1} + (n+1)^{-1} =$

(a) 0

(b) 1

(c) $\sqrt{3}$

(d) 2

(e) none of the above

15. The expression $\log_{10}10 \div \log_{10}\sqrt{10}$ is equal to

(a) $\log_{10}\sqrt{10}$

(b) $\log_{10}(10 - \sqrt{10})$

(c) 2

(d) $\sqrt{10}$

(e) none of the above

16. The minimum value of $\sin^5 x + \cos^5 x$ for all $x > 0$ is

(a) 0

(b) $\frac{1}{2\sqrt{2}}$

(c) 1

(d) 2

(e) none of the above

17. A square root of the complex number $8+6i$ is

(a) $\sqrt{8} + \sqrt{6}i$

(b) $\sqrt{8} - \sqrt{6}i$

(c) $3+i$

(d) $3-i$

(e) none of the above

18. The four vertices of a regular tetrahedron consist of two opposite vertices of the top face of a unit cube together with two opposite vertices of the bottom face. The volume of this tetrahedron is

(a) $\frac{2\sqrt{2}}{3}$

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

(e) none of the above

19. Kelly rolls a fair die and scores the number of spots showing on its upper face. Kerry rolls two fair dice and scores the total number of spots showing on their upper faces. The probability that Kelly's score is higher than Kerry's score is

(a) $\frac{10}{216}$

(b) $\frac{20}{216}$

(c) $\frac{25}{144}$

(d) $\frac{1}{3}$

(e) none of the above

20. Three unit circles are drawn with centres at (l, l), (3, 1) and (5, 3). A line passing through (l, l) is such that the total area of the parts of the three circles to one side of the line is equal to the total area of the parts of the three circles to the other side of it. The slope of this line is

(a) $\frac{1}{3}$

(b) $\frac{1}{\pi}$

(c) $\frac{3}{\pi^2}$

(d) $\frac{1}{4}$

(e) none of the above

C. Sample Problems from the International Mathematics Tournament of the Towns

Students in Grades 7 to 10 take part in the Junior Tournament. Students in Grades 11 to 12 take part in the Senior Tournament. Each annual Tournament consists of a Fall Round and a Spring Round. Each Round has an O-Level Paper and an A-Level Paper. A student's score for the Tournament is the best of the four papers.

In an O-Level Paper, students are given four hours to attempt four problems. In an A-Level Paper, students are given five hours to attempt six or seven problems. Only the best three problems count for each paper. Problems may carry different weights, those in the A-Level Paper being worth more.

The questions are refreshing and unconventional, as can be seen in the following samples from the Junior Tournaments.

1. Each of 64 friends simultaneously learns one different item of news. They begin to phone one another to relate their news. Each conversation lasts exactly one hour, during which time it is possible for two friends to tell each other all of their news. What is the minimum number of hours needed in order for all the friends to know all of the news?
2. A game is played on an infinite plane. There are fifty-one pieces, one "wolf" and fifty "sheep". There are two players. The first commences by moving the wolf. Then the second player moves one of the sheep, the first player moves the wolf, the second player moves a sheep, and so on. The wolf and the sheep can move in any direction through a distance of up to 1 metre per move. Is is true that for any starting position, the wolf will be able to capture at least one sheep?
3. In a certain country, there are more than 101 towns. The capital of this country is connected by direct air routes with 100 towns, and each town, except for the capital, is connected by direct air routes with 10 towns. It is known that from any town, it is possible to travel by air to any other town, changing planes as many times as is necessary. Prove that it is possible to close down half of the air routes connected with the capital, and preserve the capability of traveling from any town to any other town within the country.
4. A pedestrian walked for 3.5 hours. In every period of one hour's duration, he walked 5 kilometres. Is it necessarily true that his average speed was 5 kilometres per hour?
5. In a ballroom dance class, 15 boys and 15 girls are lined up in parallel rows so that 15 couples are formed. It so happens that the difference in height between the boy and the girl in each couple is not more than 10 centimetres. The boys and girls are rearranged in their respective rows in descending order of height, and 15 new couples are formed, matching the tallest boy with the tallest girl. Prove that in each of the new couples, the difference in height is still not more than 10 centimetres.
6. A village consists of 9 blocks in a 3 by 3 formation, each block a square of side length 1. Each block has a paved road along each side. Starting from a comer of the village, what is the minimum distance

we must travel along paved roads, if each section of paved road must be passed along at least once, and we are to finish at the same corner?

7. Six musicians gathered at a chamber music festival. At each scheduled concert, some of these musicians played while others listened as members of the audience. What is the minimum number of such concerts that must take place in order to enable each musician to listen, as a member of the audience, to all the other musicians?

8. On the island of Camelot live 13 grey, 15 brown and 17 crimson chameleons. If two chameleons of different colors meet, they both simultaneously change to the third color. Is it possible that they will eventually all be the same color?

9. There are 68 coins, each having a different weight. Show how to find the heaviest coin and the lightest coin in 100 weighings on a balance.

10. Three grasshoppers are on a straight line. Every second, one of the grasshoppers jumps across one, but not both, of the other two grasshoppers. Prove that after 1985 seconds, the grasshoppers cannot all be in their initial positions.

11. In a tournament, each of eight football teams plays every other team once. There are no ties. Prove that at the end of the tournament, it is always possible to find four teams A, B, C and D such that A beats B, C and D, B beats C and D and C beats D.

12. There are 20 football teams in a tournament. On the first day, all the teams play one game. On the second day, all the teams play one game again. Prove that after the second day, it is possible to select 10 teams, no two of which have played each other.

13. In a game with two players, there is a rectangular chocolate bar with 60 pieces arranged in a 6 by 10 formation. It can be broken only along the lines dividing the pieces. The first player breaks the bar along one line, discarding one section. The second player then breaks the remaining section, discarding one section. The first player repeats this process with the remaining section, and so on. The game is won by the player who leaves a single piece. In a perfectly played game, which player wins?

14. A machine gives out five pennies for each nickel and five nickels for each penny. Can Peter, who starts out with one penny, use the machine several times to end up with an equal number of nickels and pennies?

15. Nine pawns form a 3 by 3 square at the lower left corner of an 8 by 8 chessboard. Any pawn may jump over another one standing next to it onto an empty square directly beyond. Jumps may be horizontal, vertical or diagonal. We want to reform the 3 by 3 square at another corner of the chessboard by means of such jumps. Can the pawns be so rearranged at the

   (a) upper left hand corner?
   (b) upper right hand corner?

16. In a game, two players alternately choose larger positive integers. At each turn, the difference between the new and the old number must be greater than zero but smaller than the old number. The original number is 2. The player who chooses the number 1987 wins the game. In a perfectly played game, which player wins?

17. There are 2000 apples, contained in several baskets. One can remove baskets as well as apples from the baskets. Prove that it is possible to leave behind an equal number of apples in each of the remaining baskets, with the total number of apples not being less than 100.

18. It is known that the proportion of people with fair hair among people with blue eyes is greater than the proportion of people with fair hair among all people. Which is greater: the proportion of people with blue eyes among people with fair hair or the proportion of people with blue eyes among all people?

19. Two players alternately move a pawn on a chessboard from one square to another, subject to the rule that the distance of each move is strictly greater than that of the previous move. Distance is measured from the centre of the starting square to the centre of the destination square. A player loses when unable to make a legal move on his turn. Who wins if both use the best strategy?

20. (a) Prove that if 300 stars are placed on the squares of a 200 by 200 board, then it is possible to remove 100 rows and 100 columns in such a way that all stars will be removed.
    (b) Prove that it is possible to place 301 stars on the squares of a 200 by 200 board in such a way that after removing any 100 rows and 100 columns, at least one star remains.