Print ID # ___________

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School Name ____________________ Student Name __________________________
(Print First, Last)

2012 Edmonton Junior High Math Contest

Part A: Multiple Choice

1. D	
2. C	
3. B	
4. E	
5. B	

Part B (short answer) / Part C (short answer)

Part B (short answer)	Part C (short answer)
6. 6.4	15. $\frac{1}{2}$ or 0.5
7. 53	16. 403
8. 4	17. point B
9. 2025	18. 5
10. 31	19. 0.14
11. 375	
12. 0	
13. 12	
14. 2	

Part A: __________ × 4 + ________ × 2 = __________ Blank answers ≤ 3.
Correct blank

Part B: __________ × 5 = _________ MARKER
Correct ONLY

Part C: __________ × 7 = __________
Correct

Total: = __________

Instructions:

1. Calculator, grid paper and scrap paper are permitted. You may write on the booklet.
2. Programmable calculators and cell phones are not allowed.
3. Each correct answer in Part A is worth 4 points, each correct answer in Part B is worth 5 points, and each correct answer in Part C is worth 7 points. In Part A each blank is worth 2 points each up to a maximum of 3 blanks.
4. Each incorrect answer is worth 0 points.

5. Unanswered questions in Parts B and C are worth 0 points.
6. You have 60 minutes of writing time.
7. When done, carefully REMOVE and HAND IN only page 1.

Edmonton Junior High Math Contest 2012

Place your answers on the answer sheet provided.

Part A: Multiple Choice: Each correct answer is worth 4 points. Each unanswered question is worth 2 points to a maximum of 3 unanswered questions.

1. The product of four positive integers is 1365. Which of the following could not be the sum of any three of the integers?

A) 15
B) 21
C) 23
D) 24←
E) 25

The prime factor of 1365 are: 3, 5, 7, and 13.
3 + 5 + 7 = 15
3 + 7 + 13 = 23
3 + 5 + 13 = 21
5 + 7 + 13 = 25
It is not possible to make a sum of 24.

2. Eight cards are placed facedown. Each has one of the following numbers: 2, 3, 6, 7, 8, 9, 15, and 18. If you and your friend each turn over one of them, what is the probability, to the nearest whole %, that the sum of the pair of turned up cards is odd?

A) 47%
B) 50%
C) 57%←
D) 60%
E) 67%

There are 4 even numbers and 4 odd numbers. Odd plus odd is even. Even plus even is even. Odd plus even is odd. There are 8 out of 14 pairs that are odd plus even. Therefore, the probability of an odd sun = $\frac{8}{14}$ or 0.57. Represented as a %, 57 %.

3. Five different integers have a sum of -6. The first integer is two greater than the third. The second and fifth integers are opposite. The fourth integer is two greater than the second, and it is double the third. Which of the following must be one of the integers?

A) 7
B) 6 ←
C) 5

D) 4
E) 3

Let x = the 3^{rd} number
x + 2 = the 1^{st} number
2x = the 4^{th} number
2x – 2 = the 2^{nd} number
-2x + 2 = the 5^{th} number
x + x + 2 + 2x + 2x – 2 + -2x + 2 = -6
x = -2
Therefore, the integers are: 0, -6, -2, -4, and 6

4. A cube is 3 cm × 3 cm × 3 cm. The complete surface is painted green. It is cut into congruent cubes that are 1 cm × 1 cm × 1 cm (unit cubes). What is the ratio of unit cubes with exactly 2 green faces to unit cubes with exactly 3 green faces?

A) 5:27
B) 4:3
C) 2:3
D) 3:4
E) 3:2←

Of the 3^3 or 27 unit cubes, 12 have green on exactly 2 faces and 8 have green on exactly 3 faces. Therefore, 12:8 or 3:2 is the ratio.

5. A square picture has a side length of 28 cm, and a circular picture has a diameter of 30 cm. They both have a uniform 5 cm border enclosing the exposed area of its photo. For which picture is the ratio of the exposed area of its photo to its border area, the greatest, and what is that ratio?

A) The square picture, with a ratio of 0.70:1
B) The circular picture, with a ratio of 0.80:1 ←
C) The square picture, with a ratio of 2.07:1
D) The circular picture, with a ratio of 2.27:1
E) Both have the same ratio of 0.75:1

A square with side length of 28 cm, would have a square of 18 cm within since there is a 5 cm wide border around the inner square. The ratio would be
$18^2 : 28^2 – 18^2$
324 : 460
0.70 : 1
A circle with diameter 30 cm, would have a circle within with diameter 20 cm since there is a 5 cm wide border around the inner circle. The ratio would be
$10^2 : 15^2 – 10^2$
100 : 125
0.8 : 1
Therefore, the circular picture has the greatest ratio which is 0.8 : 1.

Part B: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 5 points.

6. Connor walked an average of 6 km/h from his home to a park. When he returned along the same route, he averaged 6 km/h. At the half way point from the park to home, he remembered he left his watch behind and ran back to the park at 9 km/h. Once he found his watch, he walked back home at 6 km/h. What was Connor's average speed, rounded to the nearest tenth of km/h, for the whole trip.

Choose the distance from home to the park to be the LCM of 6 and 9, which is 18. Therefore, It takes Robert 3 h to walk from his home to the park. At the half way point, he walked for 1.5 h and ran for 1 h, after which he walked back home for 3 h. The total time spent was 3 + 1.3 + 1 + 3 or 8.5 h. The total distance is 3(18) or 54 km. So, Robert's average speed is $\frac{54}{8.5}$ or approximately 6.4 km/h.

7. . An artist mixes a shade of paint that is $33\frac{1}{3}$% red, 15% yellow and the rest blue. He then mixes a second shade of paint using only red and blue in a 3:1 ratio. If the artist then combines 300 mL of the first shade with 120 mL of the second shade and 85 mL of blue, what percent of the resulting shade of paint will be blue? (Express your answer to the nearest 5%.)

1st shade is $100 - 33\frac{1}{3} - 15$ or $51\frac{2}{3}$ % blue
2nd shade is 25% blue
Total ml that are blue:
$300(51\frac{2}{3}\%) + 120(25\%) + 85 =$
$155 + 30 + 85 =$
270 ml of blue paint
Total number of ml of paint: 300 + 120 + 85 = 505
$\frac{270}{505}$ expressed as a % is 53.465
Rounding to the nearest whole % the answer is 53%.

8. In a collection of 48 coins, one is counterfeit, and it has a mass that is slightly less than all of the other identical coins. What is the minimum number of mass comparisons required, using a balance scale, to ensure that the counterfeit coin is discovered?

Divide the 48 coins into 3 piles of 16 coins each.
1st weighting:16 coins, on left pan, 16 coins on right pan, 16 coins on table.
Divide the 16 coins into three piles.
2nd weighing: 5 coins on left pan, 5 coins on right pan, 6 coins on table.
3rd weighing: Now, there will be 2, 2, 1 or 2, 2, 2

4^{th} weighing: The last weighing will have 1 coin on each pan.
Total of 4 weightings.

9. The square root of $\frac{1}{25}$ of a composite number is 4 less than the sixth prime number. What is the composite number?

The sixth prime number is 13, so

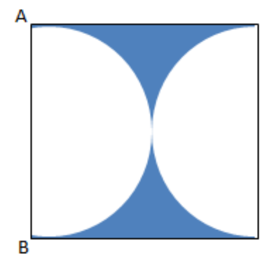
$$\sqrt{\frac{x}{25}} = 13 - 4$$

$$\sqrt{\frac{x}{25}} = 9$$

$$\frac{x}{25} = 81$$

x = 2025
The composite number is 2025.

10. What is the area of shaded region shown at the right, to the closest whole square centimetre, if the length of semicircle AB is 18.85cm?



Since the length of the semicircle is 18.85 cm, the circumference of the circle is 37.7 cm.
Divide the circumference by π:

$$\frac{37.7}{\pi} = d$$

12 = d
Therefore, the diameter of the circle is 12 cm.
To find the area of the shaded region, subtract the area of the circle with radius 6 cm from the area of the square with side of 12 cm.
$12^2 - (6^2)\ \pi =$
30.9
Rounding to the closest whole number, the answer is 31 cm.

11. A pet shop has 1500 pets that are either: cats, dogs or birds. Of this total, 55% are cats and 20% are dogs. A group of cat lovers bought cats only, until just 40% of the remaining pets were cats. How many cats were bought by the cat lovers?

Cats: 55 % of 1500 is 825.
Dogs: 20% of 1500 is 300.
Birds: 1500 – 825 – 300 = 375.
Let n be the number of cats purchased.
825 – n = 0.4(1500 – n)
n = 375
There were 375 cats purchased.

12. In the Real Number System of Mathematics, how many of the following statements are true?

- 1 is a prime number.
- $\sqrt{-25} = -5$
- $\pi = 3.14$
- $\frac{x^3}{x^0 - 1} \geq 0, x > 0$
- $-n^4 = (-n)^4$
- $\sqrt{x^{100}} = x^{10}$
- If a 3-D object is a right square prism, it must be a cube.

All of the statements are false. Therefore, the answer is 0.

13. Wong's Fortune Cookies are sold in 4 different sizes of packages. Compared to the largest size, the three smaller sizes contain, respectively: $\frac{1}{3}$ as many cookies, $\frac{1}{2}$ as many cookies, and $\frac{3}{4}$ as many cookies. If you buy one of each of the four sizes of packages, you will get 62 cookies. How many cookies are in the second-smallest package?

Let x represent the largest size
$\frac{1}{3}$x represents the smallest size
$\frac{1}{2}$x represents the second smallest size

$\frac{3}{4}$x represents the third smallest size

$x + \frac{1}{3}x + \frac{1}{2}x + \frac{3}{4}x = 62$

x = 24

Therefore, the second smallest size contains $\frac{1}{2}$(24) or 12 cookies.

14. What is the ten's digit of the smallest multiple of 13 whose unit's digit is greater than 0 but less than its hundred's digit?

The hundred's digit must be at least 2. The first such multiple of 13 is 204, but the unit's digit is greater than 2. The next one is 221, which satisfies the conditions of the problem. The ten's digit is 2.

Part C: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 7 points.

15. John wrote a computer program which changes numbers. If he enters a number *x*, the program outputs the number: $\frac{1}{1-x}$. For example, if John enters the number 3, the output is: $\frac{1}{1-3} = -\frac{1}{2}$.

Two is the first number John enters which results in an output of -1. Each output is then used as the next input. What will be the computer's 2012th output?

The following table show input values and output values for the first 6 inputs:

# of Inputs	Input value	Output value
1	2	-1
2	-1	½
3	½	2
4	2	-1
5	-1	½
6	½	2

The output values repeat for every 3 inputs. In 2010 inputs, there would be 670 repeats of the outputs of: -1, ½, and 2. Therefore, the 2011th input would yield -1, and the 2012th input would yield ½.

16. How many positive integers up to 2012 have the ones (units) digit either 1 or 8?

The numbers which have the one's digit 1 are:
1, 11, 21, 2011
If we erase the 1 at the end we obtain exactly the numbers:
0, 1, 2, …., 201.
Thus there are exactly 201 + 1 = 202 such numbers (the extra number coming from the fact that we start our counting at zero).
The numbers which have the one's digit 8 are:
8, 18, 28, …., 2008
If we erase the 8 at the end we obtain exactly the numbers: 0, 1, 2, …, 200.
Thus there are exactly 200 + 1 = 201 such numbers (the extra number coming from the fact that we start our counting at zero.
Thus in total there are 202 + 201 = 403 such numbers.

17. A level road of length 16 km links A and B. From B, the road goes uphill to C. Kelly and Kerry both ride their bikes at 8 km/h on level roads, 5 km/h uphill and 10 km/h downhill. Kelly starts from A and goes towards C, while Kerry starts from C and goes towards A. They meet at a point on the level road 4 km from B. They continue to their respective destinations and turn around immediately. Where will they meet the second time?

When Kelly and Kerry meet the first time, Kelly has traveled 12 km on level ground. Therefore, she had traveled for 12/8 or 1.5 h. The distance from B to C is unknown, but the fact that both Kelly and Kerry have traveled for 1.5 h can be used to find the distance from B to C.
1.5 = 4/8 + d/10
10 = d
The distance from B to C is 10 km.
The following chart can be used to determine when Kelly and Kerry will meet again.

	Time in h from starting points	**Distance traveled (km)**	**Location**
Kelly	1	8	Half way between A and B
Kerry	1	10	B
Kelly	2	8	B
Kerry	2	8	Half between B and A
Kelly	3	5	Half way between B and C
Kerry	3	8	A
Kelly	4	5	C
Kerry	4	8	Half way between A and B

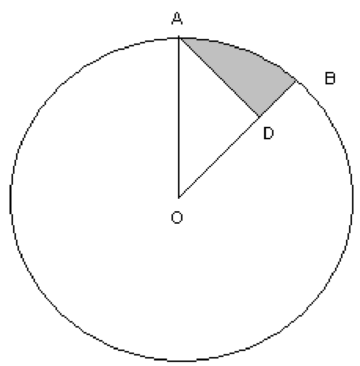
Kelly	5	10	B
Kerry	5	8	B

Therefore, Kelly and Kerry will meet again at point B.

18. Three regular polygons of equal side lengths fit perfectly around a point. If the first polygons has 5 sides and the second has 10 sides, what is the number of sides of the third polygon?

The measure of each interior angle of a regular polygon with 10 sides is:
180(n – 2)/n =
180(10 – 2)/10 =
144°
The measure of each interior angle of a regular polygon with 5 sides is:
108 °
To fit perfectly around a point, the sum of the interior angles of the three polygons must be 360°.
360 – 144 – 108 =
108°
Therefore, the third polygon is a regular pentagon, which has 5 sides.

19. In the diagram below, O is the centre of a circle of radius 1. A and B are two points on the circle. $\angle$AOB = 45°. D is a point on radius OB so that segment AD is perpendicular to OB. Find the area of the shaded region. Write the answer in decimal form, round the answer to the nearest hundredth.



To find the area of circle O use πr^2 where r = 1, therefore, the area of circle O is π.

Sector AOB represents $\frac{1}{8}$ of the circle, so the area of sector AOB is $\frac{\pi}{8}$.

Triangle AOD is an isosceles right triangle with hypotenuse 1, so the area of triangle AOD is $\frac{1}{4}$.

The area of the shaded region is found by subtracting the area of triangle AOD from the area of sector AOB.

$\frac{\pi}{8} - \frac{1}{4}$ or $\frac{\pi - 2}{8}$.

Written as a decimal, the answer is 0.14