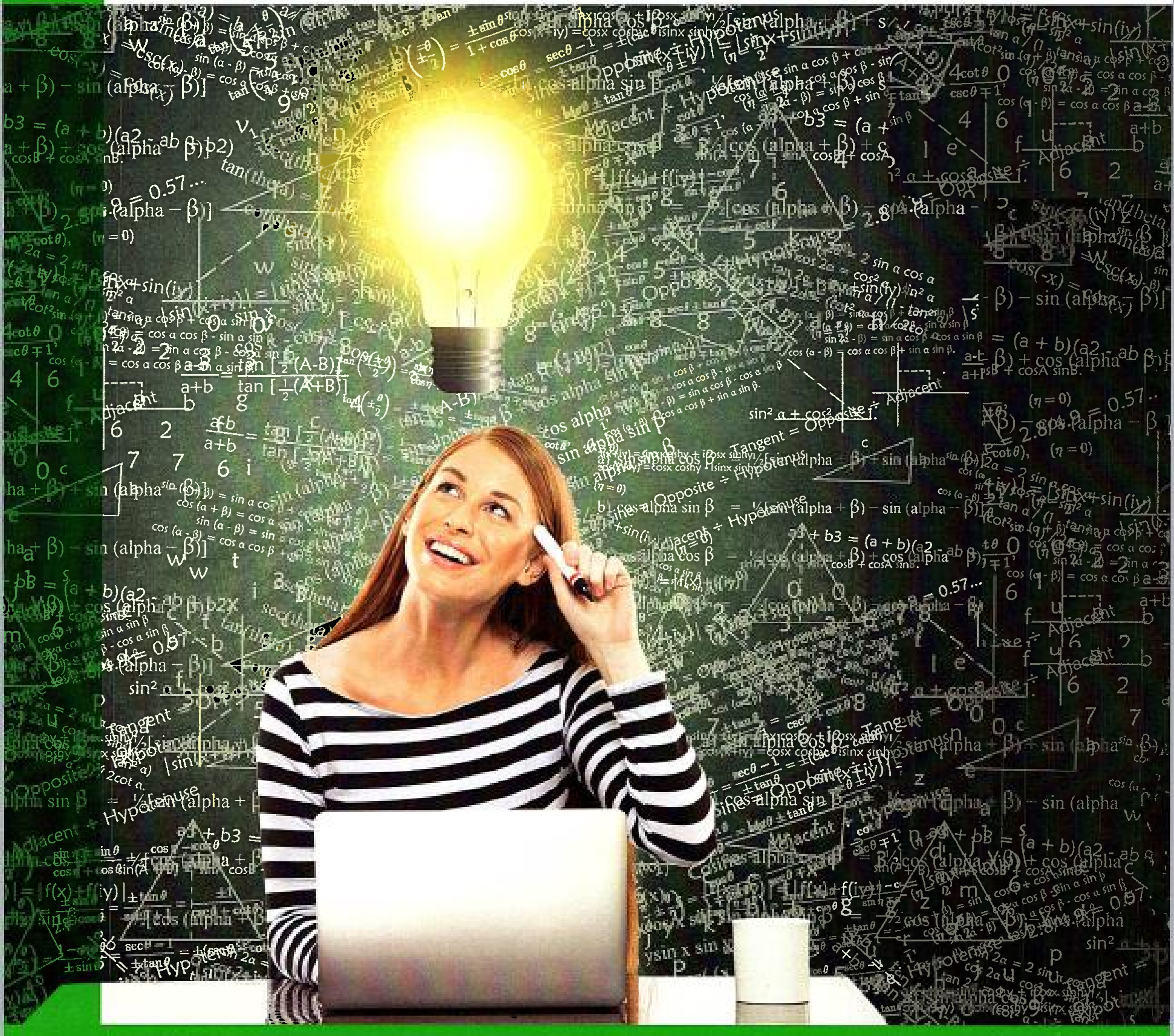
delta-k

Journal of the Mathematics Council of the Alberta Teachers' Association

Volume 55, Number 2 March 2019



Mathematical Connections

Guidelines for Manuscripts

delta-K is a professional journal for mathematics teachers in Alberta. It is published twice a year to

- promote the professional development of mathematics educators and
- stimulate thinking, explore new ideas and offer various viewpoints.

Submissions are requested that have a classroom as well as a scholarly focus. They may include

- personal explorations of significant classroom experiences;
- descriptions of innovative classroom and school practices;
- reviews or evaluations of instructional and curricular methods, programs or materials;
- discussions of trends, issues or policies;
- a specific focus on technology in the classroom; or
- a focus on the curriculum, professional and assessment standards of the NCTM.

Suggestions for Writers

1. *delta-K* is a refereed journal. Manuscripts submitted to *delta-K* should be original material. Articles currently under consideration by other journals will not be reviewed.
2. If a manuscript is accepted for publication, its author(s) will agree to transfer copyright to the Mathematics Council of the Alberta Teachers' Association for the republication, representation and distribution of the original and derivative material.
3. Peer-reviewed articles are normally 8–10 pages in length.
4. All manuscripts should be typewritten, double-spaced and properly referenced. All pages should be numbered.
5. The author's name and full address should be provided on a separate page. If an article has more than one author, the contact author must be clearly identified. Authors should avoid all other references that may reveal their identities to the reviewers.
6. All manuscripts should be submitted electronically, using Microsoft Word format.
7. Pictures or illustrations should be clearly labelled and placed where you want them to appear in the article. A caption and photo credit should accompany each photograph.
8. References and citations should be formatted consistently using *The Chicago Manual of Style's* author-date system.
9. If any student sample work is included, please provide a consent form from the student's parent/guardian allowing publication in the journal. The editor will provide this form on request.
10. Letters to the editor, description of teaching practices or reviews of curriculum materials are welcome.
11. Send manuscripts and inquiries to the editor: Lorelei Boschman, c/o Medicine Hat College, Division of Arts and Education, 299 College Drive SE, Medicine Hat, AB T1A 3Y6; e-mail lboschman@mhc.ab.ca.

MCATA Mission Statement

Providing leadership to encourage the continuing enhancement of teaching, learning and understanding mathematics.

delta-k

Volume 55, Number 2 March 2019

CONTENTS

---

 delta-K is published by the ATA for the Mathematics Council (MCATA). EDITOR: Lorelei Boschman, e-mail lboschman@mhc.ab.ca. EDITORIAL AND PRODUCTION SERVICES: Document Production staff, ATA. Opinions expressed herein are not necessarily those of MCATA or of the ATA. Address correspondence regarding this publication to the editor. *delta-K* is indexed in CBCA Education. ISSN 0319-8367

Individual copies of this journal can be ordered at the following prices: 1 to 4 copies, $7.50 each; 5 to 10 copies, $5.00 each; more than 10 copies, $3.50 each. Please add 5 per cent shipping and handling and 5 per cent GST. Please contact Distribution at Barnett House to place your order at distribution@ata.ab.ca.

From the Editor's Desk

Lorelei Boschman

Teacher mathematicians everywhere are well into working with their student mathematicians, all the while planning, organizing, pondering and creating. Setting up and implementing the mathematics program with your students is an intricate and delicately balanced endeavour, one that takes continuous effort throughout the year. This may be just the time to add something new to your mathematics lessons, program and instruction.

Consider some of the effective pieces of a mathematics lesson and pedagogy:

- Math with the teacher
- Math with others
- Math by self
- Math games
- Math manipulatives and technology
- Math writing

How many of these elements can you incorporate into a math lesson? This is a challenge that keeps math filled with energy and engagement throughout a variety of activities and teaching strategies. What are some of your math routines throughout the week and month? Can you add a new enhancement now? Have you had a conversation with other math teachers to discover some great ideas that may be just around the corner? This planning jump-starts our student mathematicians' learning; math understandings; and true, lasting growth. Consider implementing something new each month, and by the time the school year is over you will have many new additions to your mathematics lessons and program.

I would encourage you to read Jo Boaler's book *Mathematical Mindsets*—it can impact your teaching of mathematics tomorrow. How many of Jo Boaler's ideas can you implement this year in your mathematics teaching and students' learning? The cover says, "Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching." These are concepts that can make lasting and effective change. You are embarking on your own personal journey of mathematics teaching right now, so why not consider this book as part of your professional growth plan this year?

This issue of *delta-K* is diverse and ranges from global math challenges, to math and music, to stopping the cycle of math anxiety and more. What can you take away from reading this journal that will affect your own practice this year, or begin the deep thinking of creating even more mathematical experiences and growth for your students? Can you adapt these ideas to your grade and group of students?

We may have many questions as the school year unfolds, but be assured—you have the skills or have access to the skills to create and enhance your mathematics lessons for your students. As teachers, we are all truly on our own path of learning as well. Be encouraged as your year continues!

Learn Leap Fly: Interview with Two Canadian Mathematicians Taking On a Global Learning XPRIZE Challenge

Kseniya Garaschuk

This article was originally published in CMS Notes *49, no 4 (September 2017), 8–10, and is reprinted with the permission of the Canadian Mathematical Society. Minor amendments have been made in accordance with ATA style.*

"Education Notes" brings mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities and noteworthy news items. Comments, suggestions and submissions are welcome. John McLaughlin, University of New Brunswick (johngm@unb.ca); Jennifer Hyndman, University of Northern British Columbia (hyndman@unbc.ca).

While Amy Wooding was finishing her PhD at McGill in number theory, she used to take the train back and forth from Ottawa to Montreal. One day, she was listening to an interview of Peter Diamandis, a founder of the XPRIZE Foundation. He was introducing the upcoming competition for the Global Learning XPRIZE, putting out a challenge to use technology to teach basic literacy and numeracy skills to children who do not have access to teachers. The submitted proposals would be judged and the chosen finalists would then try their software in an 18-month field test with students in Africa. Now *that*, she thought, would be a really cool project to pursue. She pitched the idea of participation to her husband, Kjell, later that day; he said yes immediately. Two and a half years later, on June 21, 2017, their team, Learn Leap Fly, was named as one of the Global Learning XPRIZE's 11 semifinalists.

Now, Amy was not just an average person on the train thinking about educational technologies, and Kjell was not just an average guy saying yes to developing them. Both Amy and Kjell Wooding have mixed interdisciplinary backgrounds that are, in many ways, perfect for a project like this. After completing her BSc in math, Amy attended teachers' college before continuing to her PhD in arithmetic geometry. Kjell started as a computer engineer and then worked in industry before returning to academia to complete his math PhD. Furthermore, within their immediate families they had a plethora of skills required for this project. Among these, Kjell's family members comprised many educational specialists, while Amy's aunt runs a children's home in Kenya.

The intention of this piece is to share more of the experience of the two mathematicians, Amy Wooding (AW) and Kjell Wooding (KW), with a focus on this project. For instance, what challenges did they face and how did they address them? I sat down with them to chat about their project's past, present and future. As the interviewer, I use my initials (KG) below, with questions and prompts appearing in italics.

KG: *So you didn't have to go very far to find all the necessary skills that one might need for a project like this, but this is still a big undertaking.*

KW: I think the thing that helped us the most is that we are both fundamentally researchers. We are people who tackle hard problems for a living. So when it came to this problem, it didn't seem unusual to try to do a thing that has never been done before. But we also came to it as a blank slate. We came to it not knowing what solution we wanted to see work, just that we wanted to see a solution. Se we came to it very much with a sense of experimentation. We wanted to get in there and try things out, see what was working and what wasn't.

KG: *What was the development process like?*

KW: When we started this, there was an 18-month period before you had to submit your solution. So we took that 18-month period and we broke it up into three phases. We called our team Learn Leap Fly, and we ended up doing that as our approach to the problem. First was the Learn phase, where we sat down with educational research to find out the well-known ways to teach literacy and numeracy. Then we started identifying products available already: what software is there now, what does it do well and what does it do poorly? By the end of this phase, we had a pretty good idea about where the challenges were. For the Leap phase, we chose the set of things we wanted to try and started trying them out. We started testing other people's software to see how children responded to it. We noticed several important things right away, for example that a child's hand is sometimes too small to do things on the screen the standard way. Some of these discoveries we were making are now obvious in hindsight, but impossible to know until you do those observations and experimentations. So this was our approach throughout the project: to try things out as soon as possible and see how the children respond. The last phase, the Fly phase, was taking all of our best ideas, all the pieces that worked well, putting them together into a prototype and then flying off to Kenya to try it out with the children there, so we got to see these first interactions with technology.

KG: *This is very different, because we now assume that everybody knows how to use a tablet.*

AW: Exactly. There are some things that are assumed that are not natural. Dragging is one of them. It's a learned thing of how to interact with a mobile device, so now you need to do something designwise to try to teach children to do that.

KW: Design was a huge challenge throughout, for a number of reasons. One of them is when you are trying to appeal to children the world over. Obviously, you have to avoid using a language because they can't read yet, but you also have to avoid metaphors, because they don't always work. For example, an icon to go back to the main menu is often a picture of a house. But that doesn't mean anything to these children, since they have no context for it, so you have to find things that, whenever possible, communicate what you want to do directly. So we had to experiment to see what children would respond to, children who have never seen this technology before and wouldn't necessarily have any instruction on it.

KG: *What other things did the initial testing in Kenya reveal?*

KW: Here we have this interesting ideal of one device per child. When we did our alpha testing in Kenya, we had four tablets with us and a dozen children, so of course we would end up with multiple children per tablet. We noticed right away that we weren't just getting children in the age group we were aiming at, but we were getting everybody. At the same time.

AW: This was deliberate on our part—we wanted to test it out naturally. We didn't want to impose any constraints. We wanted to see what they would do and how they would work together, whether this is something we need to incorporate into our design, something we should focus on or not.

KW: It turned out to be very powerful. Having three to four children per tablet was actually a benefit, because it set up a social environment. Reading, for example. Some of the children knew how to read and some didn't, but they surely knew how to listen and repeat. So you got this group activity of reading together and even those children who weren't attracted to the letters wanted to participate in the social part of this. We wanted to play on this social idea. So, whenever possible, we started designing our software so it could be used not just by one child at a time, but by a group of children, all their hands on a tablet, working together.

KG: *So it has to be multitouch because there will be a lot of little hands.*

KW: Exactly, there might be a dozen hands on the screen or as many as would fit, so your activities needed to accommodate all of those hands. Designing this kind of social software is not well known, well understood or well developed, so this meant rethinking a lot of the ways we typically design our activities.

AW: When we were faced with the problem of doing personalization to groups, we came up with the idea of "digital personalities," where each tablet has its own distinct personality. It may prefer certain types of activities or stories, whereas the tablet beside it will have different preferences. The first step is to give children the chance to self-select a tablet, but then we also have things that adapt underneath it.

KG: *What about the level and progression of activities? Since you cannot rely on the same child working with the same tablet every time, how does the software accommodate that?*

AW: There are three main threads: reading, writing and numbers. Some are mixed and matched in the sense that numbers may know what level of reading you are at, so they know whether they should ask you to write the numbers out or not and so on. But

it is all driven by a set of stories and every set of activities adapts each time to reflect the story you just read. For vocabulary-building activities, it will pull words from the story. For numbers, it may draw elements of the story and ask you to count them. So the stories control the certain amount of the levelling and allow us to control user preference in the sense that the software can always connect the activities to the elements from your favourite stories. Under the hood, we have a decision engine that keeps track of the user preferences, such as the kinds of stories you like, as well as which level you are on in various activities. To add a little bit of randomness and variation, there is always a distinct aspect of the digital personality present in these decisions. There is also always some notion of curriculum, as in where you are in terms of learning progression.
KW: This is where it's convenient that we happen to be mathematicians and data scientists. We can use things like machine learning to identify the level we think the children are at and adapt the activities accordingly. But again, we always have to be aware of the fact that we never know for sure if it is the same children coming back. So those digital personalities have to be able to recognize that behaviours have changed and change the way it is acting.
AW: If the software ever feels that the user has changed in a significant way, it goes through an annealing-type process where it presents the child with a whole bunch of random options to quickly figure out their level and preferences. As behaviour stabilizes over time between different groups of users on a particular tablet, it also uses clustering to break up different types of behaviours. So if it expects to see 10 different kinds of behaviours on this tablet, it just has to figure out which one it is currently dealing with.

KG: *How does it make sure that children practise various skills, as opposed to just picking one type of activity they are already good at?*
AW: At the end of every activity, the main menu has eight choices, so a child always has a choice, but it is how we fill those choices that matters. Some of the choices will be influenced by user preferences and some by curriculum preferences. If the child only ever chooses user preference options, it will eventually be filled with all curriculum choices. So when you are trying to maximize engagement, you will present more user preference-type choices; if the child is highly engaged, the software might decide it is a good time to learn something new and swing toward more curriculum-type choices. It will oscillate back and forth to guide you in various directions.

KG: *Your software is driven by a set of stories and it pulls content from them. So if you decide to implement it in another language, you can just upload a new set of stories?*
KW: Exactly. One of the great appeals to us in this approach is that it allows us to easily engage with a new language or a new culture, say our northern communities in Canada. We would first work with that community to develop the initial set of stories, so we get some cultural and linguistic familiarity. This aspect of being able to create stories that are culturally relevant to the group is a great way to be able to tailor the content. You start with a set of stories and do the work to separate the characters, objects, story elements and everything adapts.

KG: *What is the next step for you?*
AW: This fall, we will be in kindergarten classes in Ottawa and will be introducing French content as well. Their curriculum here is 50–50 English and French timewise, which is a recent change, so the teachers are looking for more suitable resources for instruction. So we are working over the summer to get everything in French as well ready to go for the new school year in September.
KW: We were originally developing this software to be used essentially without schools and teachers, but we are also doing our testing in classes and kindergartens and after-school programs. Because it was designed for children at different levels and for children working in groups, it ended up working really nicely in classrooms, where you can set up stations with four or five devices for children to work together. It worked so well in the classroom environment that by the end of the school year we had teachers asking if they can use this in their classrooms on an ongoing basis, which was a pleasant surprise. So we figured, let's give it a shot. Starting September, we will be working with some classrooms here in Ottawa to align it better with their curriculum to introduce activities that fit with how we do kindergarten here and, again, to bring it closer to home with French and English as a start.

KG: *Last question: what has been the most exciting part of this process?*
KW: In my opinion, all of the interesting stuff we do in mathematics occurs on the boundary where it meets something else in the world. We never could have predicted when we started out that we will be using machine learning or developing Bayesian inference systems to these digital personalities. But

it's the combination, the ability to take all of these different areas together and put them into one solution that makes it really exciting.

AW: The ability to make a real difference in the world. To apply mathematics and research in a way that can change the lives of hundreds of millions of children. It's an incredible experience.

Note: Readers interested in more information on the work of Amy and Kjell Wooding may wish to visit learnleapfly.com.

Kseniya Garaschuk is an assistant professor of mathematics and statistics at University of the Fraser Valley, Abbotsford, British Columbia, and can be reached at kseniya.garaschuk@ufv.ca.

Musical Notes

Mike Pacheco, Heather Glynn Crawford-Ferre and Henry King

This article is reprinted from the "Math by the Month" column in Teaching Children Mathematics *volume 24, number 6, 2018, with the permission of the National Council of Teachers of Mathematics.* *Minor amendments have been made in accordance with ATA style.*

Mathematics provides an important way to inform and interpret music. For example, because of a mathematical breakthrough (better ways to calculate the twelfth root of 2), Johann Sebastian Bach was able to write in every key. This set of problems is intended to facilitate interdisciplinary connections and reasoning with music.

Grades 5–6

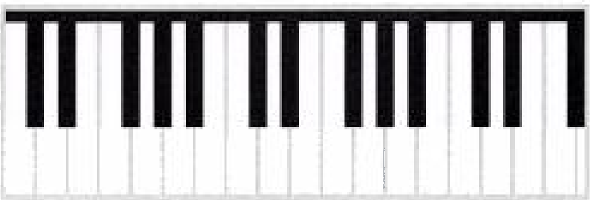
Week One

The first key on a full-size piano is the note A. The next keys are B, C, D, E, F and G, and then the pattern starts over at A. This set of notes is called an *octave*. An octave is the interval between one musical pitch and another with half or double its frequency. The first A on the piano vibrates at 27.5 Hertz, or 27.5 waves per second. Each time a note goes up an octave, the Hertz measure doubles. The highest A on a piano vibrates at 3,520 Hertz. How many different octaves can be played on a full-size piano? (Hint: How many keys are on a full-size piano?)

Week Two

A new musical artist's video has been viewed online 125,673 times since it was posted this morning. The most viewed video of all time (Psy's "Gangnam Style") has been viewed 2.79 billion times. How many more times would the new video have to be viewed to become the most viewed video? Approximately how many times larger is the number of views for the current most viewed video compared with the new video?

Week Three



Pianos have two types of keys. The white keys are traditionally the natural notes, and the black keys are traditionally the sharp and flat notes. A full-size piano has 88 keys; how many black keys and white keys does it have? What is the ratio of black keys to white keys on a piano? Look at the illustration of part of a keyboard. Piano keys have a repeating pattern of 5 black keys and 7 white keys, as the illustration shows. You can use a ratio to compare part to part. You can also compare part to whole. What is the ratio of black keys to all keys in the pattern? An electronic keyboard has 48 keys. How many black keys and white keys are on an electronic keyboard?

Week Four

Dance music is typically played at 120 beats per minute. If the bass drum plays on every third beat, how many times does the bass drum play in one minute? If you play a song at 120 beats per minute, it takes three minutes to play. If you slowed the tempo to 90 beats per minute, how long would it take to play the song? How long would the song last if you sped up the tempo to 180 beats per minute?

Grades 3–4

Week One

When composing music, a whole note can be divided into half notes, quarter notes, eighth notes and sixteenth notes. What are three different ways one can build a whole note from halves, quarters, eighths and sixteenths? How many ways can you build 3/4 of a note?

Week Two

A drum machine is an electronic device that can simulate drums or other percussion sounds for bands or disc jockeys (DJs). Drum machines are programmed to play sounds on specific beats during a song. A DJ programs the drum machine to play a bass drum sound on every third beat, a snare drum sound on every fourth beat, a clap on every fifth beat and a whistle on every seventh. On which beat will the bass and snare play together for the first time? What is the first beat that at least three instruments will play at the same time? What strategies could you use to show when sounds are lining up? Will all four sounds ever play at the same time?

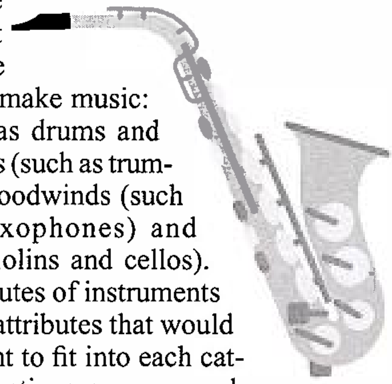
Week Three

Pop radio stations have historically shied away from playing songs that are longer than 3 minutes, 5 seconds. Some music labels started labelling songs purposely with such times as 2 minutes, 96 seconds in hopes that they would still be played on the radio. How long is that song if it is labelled using the largest number of minutes possible?

What is another way of labelling the following famous songs to mislead radio stations into playing them?

(a) Taylor Swift's "Shake It Off" is 3 minutes, 39 seconds.
(b) Meaghan Trainor's "All About That Bass" is 3 minutes, 8 seconds.
(c) The Beatles' "Hey, Jude" is 7 minutes, 11 seconds.

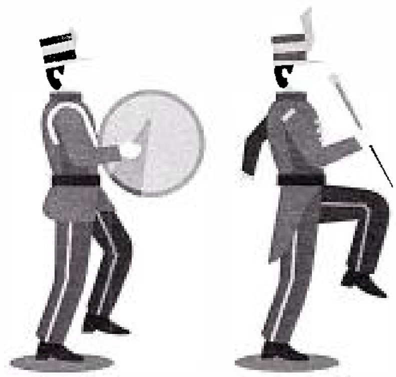
Week Four



Instruments are sorted into different categories on the basis of how they make music: percussion (such as drums and tambourines), brass (such as trumpets and tubas), woodwinds (such as oboes and saxophones) and strings (such as violins and cellos). Research the attributes of instruments and make a list of attributes that would allow an instrument to fit into each category. What connections can you make between this and categorizing shapes according to their defining attributes?

Kindergarten–Grade 2

Week One



Musicians in marching bands march in patterns. You can watch them in parades and at sporting events. You can use patterns to find out how many musicians are playing. In a drum line, the drummers march 5 in a row. How many drummers are in 2 rows? In 5 rows? Sometimes they switch and march 10 in a row. Then how many drummers are in 2 rows? In 5 rows? In 10 rows? What do you notice? What happens if you mark those numbers on a hundred grid? How many rows would there be if 100 drummers were in rows of 10?

Week Two

We can sort items on the basis of their attributes. Look at pictures of instruments (string, percussion, woodwind, brass). Play an attribute match game by matching instruments that have an attribute in common. Could instruments have more than one match? Choose an attribute and sort all of the instruments by this attribute. Look at another student's sort. Can you guess his or her rule?

Week Three

The Caldecott Honor book *Zin! Zin! Zin! A Violin* (Moss 2000) describes the 10 instruments that join to make a chamber orchestra group. List the 10 instruments. How many combinations of the 10 instruments can you make? How can you organize the keep track of your combinations of 10? How will you know when you have found all the combinations?

Week Four

Ti-Ti-Ta (eighth note, eighth note, quarter note) is a pattern you can clap and repeat. You can also play it with two instruments (bell, bell, drum). What other patterns can you create and extend? Can you model them with blocks (red, red, blue) or write them out (A, A, B)? Listen to some music. What patterns do you hear? How can you record them?

Mike Pacheco is the assistant director over assessments at the Nevada Department of Education and a graduate student in mathematics education at the University of Nevada, Reno. His professional interests include large-scale mathematics instruction improvement and educational equity.

Heather Glynn Crawford-Ferre is an educational consultant and doctoral candidate at the University of Nevada, Reno. Her professional interests include mathematics education and educational equity.

Henry King is a program specialist for the Nevada Department of Education and an accomplished musician. His professional interests include bass, guitar, ukulele and the musical education of others.

Edited by Lisa Brooks, lisa.brooks@ucf.edu, a lecturer in the College of Education and Human Performance at the University of Central Florida in Orlando; and Samantha Neff, samantha_neff@scps.k12.fl.us, a K–5 math coach at Highlands Elementary School in Winter Springs, Florida. E-mail problem collections for the editors to consider as future "Math by the Month" columns. See submission guidelines at www.nctm.org/WriteForTCM. E-mail creative solutions and adapted problems to tcm@nctm.org for potential publication, noting Readers Exchange *as the subject.*

Integrating Math and Music

Catherine Schmidt-Jones

Junior High Problem

Reading Time Signatures

Most time signatures contain two numbers. The top number tells you how many beats there are in a measure; the bottom number tells you what kind of note gets a beat.

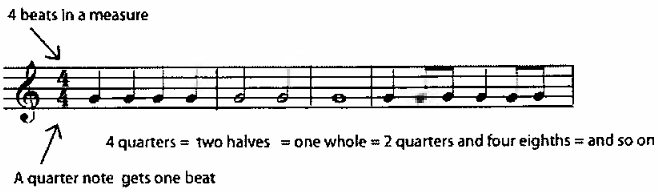


Figure 3. In "four four" time there are four beats in a measure and a quarter note gets a beat. In order to keep the meter going steadily, every measure must have a combination of notes and rests that is equivalent to four quarter notes.

You may have noticed that the time signature looks a little like a fraction in arithmetic. Filling up measures feels a little like finding equivalent fractions, too. In 4/4 time, for example, there are four beats in a measure and a quarter note gets one beat, so four quarter notes would fill up one measure. But so would any other combination of notes and rests that equals four quarters: one whole, two halves, one half plus two quarters, a half note and a half rest, and so on.

If the time signature is 3/8, any combination of notes that adds up to three eighths will fill a measure. Remember that a dot is worth an extra half of the note it follows.

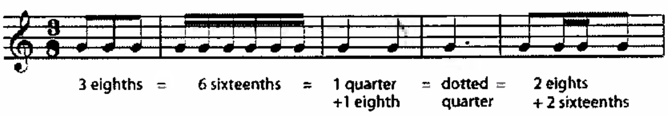


Figure 4. If the time signature is three eight, a measure may be filled with any combination of notes and rests that adds up to three eight.

Task

1. Write four measures of music in each time signature:
   a. 2/4 time
   b. 3/8 time
   c. 6/4 time
2. Share with a partner to clap each.

(From https://cnx.org/contents/aBABIQ76@16/Time-Signature)

High School Problem

The *pitch* of a note is how high or low it sounds. Musicians often find it useful to talk about how much higher or lower one note is than another. This distance between two pitches is called the *interval* between them. In Western music, the smallest interval from one note to the next closest note higher or lower is called a *half-step* or *semitone*.

Major Scales

To find the rest of the notes in a major key, start at the tonic and go up following this pattern: *whole step, whole step, half step, whole step, whole step, whole step, half step*. This will take you to the tonic one octave higher than where you began, and includes all the notes in the key in that octave.

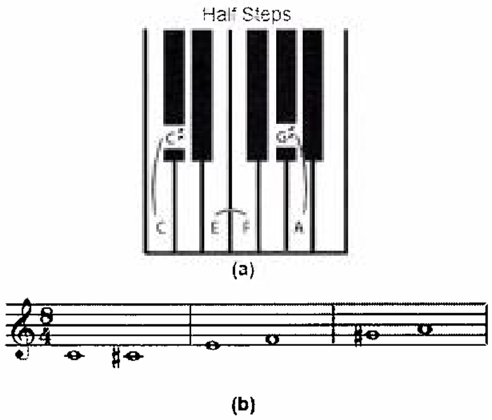


Figure 1. Three half-step intervals: between C and C sharp (or D flat); between E and F; and between G sharp (or A flat) and A.

These major scales all follow the same pattern of whole steps and half steps. They have different sets of notes because the pattern starts on different notes.

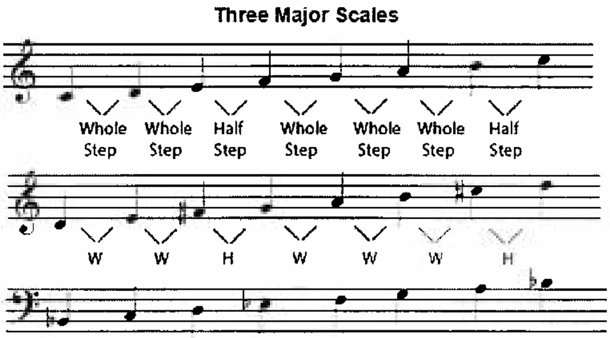


Figure 1. All major scales have the same pattern of half steps and whole steps, beginning on the note that names the scale - the tonic.

Task

1. Calculate the equal temperament frequency ratio of the tonic/first note to the following to the nearest ten thousandth.
2. Calculate the difference between the equal temperament frequency ratio and the harmonic series frequency ratio.
3. Finally, calculate the fractional equivalent to the harmonic series frequency ratio. (Please leave the frequency ratios and approximate difference columns blank for student input, and leave the last column blank for student input labelled "fraction value.")
4. Now investigate the sound of each compared to the tonic note. What do you notice about the pattern of differences and the sounds? What is the general relationship about the difference column to the sounds they make compared to the tonic note?

Comparing the Frequency Ratios for Equal Temperament and Pure Harmonic Series

Interval	Equal Temperament Frequency Ratio	Approximate Difference	Harmonic Series Frequency Ratio
Unison	$(\sqrt[12]{2})^0 \approx 1.0000$	0.0	$1.0000 \approx 1/1$
Minor Second	$(\sqrt[12]{2})^1 \approx 1.0595$	0.0314	$1.0909 \approx 12/11$
Major Second	$(\sqrt[12]{2})^2 \approx 1.1225$	0.0025	$1.1250 \approx 9/8$
Minor Third	$(\sqrt[12]{2})^3 \approx 1.1892$	0.0108	$1.2000 \approx 6/5$
Major Third	$(\sqrt[12]{2})^4 \approx 1.2599$	0.0099	$1.2500 \approx 5/4$
Perfect Fourth	$(\sqrt[12]{2})^5 \approx 1.3348$	0.0015	$1.3333 \approx 4/3$
Tritone	$(\sqrt[12]{2})^6 \approx 1.4142$	0.0142	$1.4000 \approx 7/5$
Perfect Fifth	$(\sqrt[12]{2})^7 \approx 1.4983$	0.0017	$1.5000 \approx 3/2$
Minor Sixth	$(\sqrt[12]{2})^8 \approx 1.5874$	0.0126	$1.6000 \approx 8/5$
Major Sixth	$(\sqrt[12]{2})^9 \approx 1.6818$	0.0151	$1.6667 \approx 5/3$
Minor Seventh	$(\sqrt[12]{2})^{10} \approx 1.7818$	0.0318	$1.7500 \approx 7/4$
Major Seventh	$(\sqrt[12]{2})^{11} \approx 1.8897$	0.0564	$1.8333 \approx 11/6$
Octave	$(\sqrt[12]{2})^{12} \approx 2.0000$	0.0	$2.0000 \approx 2/1$

(From https://cnx.org/contents/N6Ira9Bt@27/Tuning-Systems#s3)

Stopping the Cycle of Math Anxiety: Recommendations for Teachers and Schools

Eric Schubert

Abstract

Studied since the 1970s, math anxiety is a perpetual problem faced by schools. The cyclical nature of math anxiety poses a great challenge for educators. A negative experience with learning math is the primary cause of math anxiety, usually first occurring in elementary school. This anxiety often comes from adults who themselves have anxiety about math. Teacher and parent attitude, language and actions have an immense impact on math anxiety. Poor teaching practice, anxiety toward the subject, and negative attitudes can be passed from teachers to students, and also from parents to their children. This article examines best practices for teachers and parents to prevent and stop the cycle of math anxiety. Possible systemic solutions related to elementary math specialists are discussed through the lens of promoting growth-minded perseverance in math students. It is crucial that all adults involved in educating students be mindful of the impact of their attitudes, words, actions and decisions on students' relationship to math.

Introduction

Having taught mathematics in middle school for eight years, I have inevitably encountered students with anxiety, hatred, and fear toward math. "I can't do math," "I'm not a math person" and the popular

"MATH is an acronym: Mental Abuse To Humans" have littered my encounters with students. Math is a scary word for these students because they do not like it, or they feel they are not good at it (Beilock and Willingham 2014). Math anxiety is present in students who underperform, feel hopeless or avoid math class altogether (Anderson 2007; Ashcraft and Kirk 2001). While research on math anxiety spans over forty years, math anxiety continues to be an issue for schools today. This article discusses implications of math anxiety for teachers and investigates possible solutions by providing recommendations for teachers and educational leaders. It examines suggestions for teacher attitude, classroom environment, instructional techniques, assessment practices and parent engagement. Leadership implications include providing support for teachers and exploring potential changes to current school structure to stop the cycle of math anxiety.

Background

Math anxiety is "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems" (Richardson and Suinn 1972, 551). It occurs first in primary grades and increases through the grades, peaking in late middle school or early high school (Jackson and Leffingwell 1999; Al Mutawah 2015; Hembree 1990). Particular topics in math increase anxiety (long division, fractions and algebra), possibly due to exclusivity of math symbols, notation, language or lack of basic skills (Jackson 2008; Buxton 1981; Wu 2009; Schwartz 2002). Math anxiety is not general anxiety, low intelligence or poor math ability (Ashcraft and Kirk 2001; Morris 1981). Students may be successful in other courses but lose self-confidence and become convinced they cannot do math (Morris 1981; Tobias 1993; Dodd 1999). Math-anxious individuals do not gain competence or mastery of mathematical operations (Hembree 1990); fear or nervousness paralyzes thinking, inhibits performance (Morris 1981; Perry 2004) and prevents students from using the knowledge they do possess (Ashcraft and Kirk 2001). Math anxiety interferes with conceptual thinking and memory processes (Newstead 1998; Wilensky 1997), specifically working memory, leading to reduced accuracy of computations and decreased speed of performance (Ashcraft and Kirk 2001; Perina 2002). Students who develop math anxiety fall into a self-defeating, self-perpetuating cycle moving from negative feelings to avoidance to poor mathematical preparations to poor performance and back to negative feelings—a fatalistic attitude reinforcing convictions that they cannot do math, causing the math anxious to become math avoiders (Baroody and Costlick 1998; Preis and Biggs 2001; Morris 1981). This cycle becomes difficult to break when math-anxious individuals conclude for themselves that they cannot sustain success in any math-related situation (Preis and Biggs 2001).

Math itself does not cause anxiety; rather, it is how math has been presented or experienced (Jackson 2008; Stuart 2000). Negative experiences of failure or inadequacy often happen at school and can be influenced by teachers (Finlayson 2014; Jackson 2008; Perry 2004; Raymond 1997). Math anxiety can be passed down to students through attitudes of parents and teachers (Maloney et al 2015; Furner and Duffy 2002; Fiore 1999). Math-anxious parents may communicate their own math anxiety to children through everyday conversation, statements about math difficulty or frustrating family homework experiences (Casad, Hale and Wachs 2015; Jeynes 2007; Finlayson 2014; Maloney et al 2015). This also occurs in school: math-anxious teachers can result in math-anxious students when teachers inadvertently pass on avoidance and fear of math (Martinez 1987; Geist 2010; Wood 1998).

A Teacher's Impact

Teaching has a direct impact on math anxiety—it can take only a single teacher to create lasting math anxiety (Furner and Duffy 2002; Perry 2004). Traditional teaching methods and assessments, focus on results over understanding, and teachers who assume a direct, authoritative role may create math anxiety (Finlayson 2014; Perry 2004; Jackson and Leffingwell 1999; Harper and Daane 1998). Anxiety in struggling students is heightened when teachers give the impression that math is easy; students are told they cannot do math, and subsequently are afraid to ask questions (Finlayson 2014; Perina 2002; Perry 2004).

What teachers do in math lessons may subconsciously reflect their own thoughts and beliefs about the subject (Fernandes 1995). Students tend to internalize their teacher's attitude and anxiety (Jackson and Leffingwell 1999; Martinez 1987; Geist 2010; Wood 1998). Math-anxious elementary teachers can negatively influence instruction and "may promote the early onset of mathematics anxiety" (Hackworth 1985, 8). These teachers may create anxiety through a tendency to use more traditional and teacher-centred approaches (Hembree 1990; Gresham 2008). Elementary teachers who have math anxiety may feel uncomfortable teaching math because they do not like math or feel they are not good at it (Jackson and Leffingwell 1999; Burns 1998; Stuart 2000; Geist 2010).

Preservice elementary teachers in particular have been found to be a group with anxiety and fear of teaching math (Gresham 2008; Jackson 2008; Raymond 1997). Contributing factors to their anxiety are prior experiences as math students, influence of their teachers at schools, lack of subject knowledge and inadequate teacher preparation programs (Raymond 1997; Goulding, Rowland and Barber 2002). Fear of teaching math among some preservice and elementary teachers may reflect real or perceived knowledge deficits, in part due to university programs that may have little focus on math instruction (Levine 1993; Wingert 2014; Fennell 2006).

This research underscores the importance of great teaching and reminds us of the incredible impact that teachers have on their students. While poor or unintended practices may negatively affect or create math-anxious students, teachers have a great opportunity to make an even greater positive impact on their math students.

What Teachers Can Do

Teachers should take a holistic approach to engaging students in mathematics, considering cognitive and affective aspects of learning (Turner et al 2002; MacLeod 1992). Literature on math anxiety provides a number of key recommendations for teachers.

Create a Safe Learning Environment

"Overcoming math anxiety means that we need to examine the classroom environment and how we teach mathematics" (Finlayson 2014, 102).

Math classrooms should be communities where learners' ideas are valued, students share solution methods, mistakes are valued as sites of learning and the authority for correctness is not solely the teacher but the subject itself (Kilpatrick, Swafford and Findell 2001). Learning math is an emotionally charged practice, because students are taking risks (Schuster and Canavan Anderson 2005). Students are more willing to take risks and ask questions when they feel safe in a classroom with a supportive teacher (Finlayson 2014; Newstead 1998; Wilensky 1997; Wright 1996). Teachers are crucial, because interactions that take place in the classroom are what really matter for math learning (Kilpatrick, Swafford and Findell 2001).

A crucial pillar to creating a safe learning environment is how teachers treat mistakes in mathematics classrooms. While mistakes are important opportunities for learning and growth, students often regard mistakes as indicators of low ability (Boaler 2013). Every time students move out of their comfort zone to learn something difficult and new, neurons form new, stronger connections, and when students think about *why* something is wrong, new synaptic connections are sparked that cause the brain to grow (Boaler 2013; Dweck 2014). Teachers need to encourage risk-taking by repositioning mistakes—not as learning failures, but as valued opportunities for brain development and learning by showing them to all students to consider and recognizing their importance as sites for learning (Boaler 2013).

Success in math will come only when students learn to approach math with confidence and are confident they will not be allowed to fail (Small 2015; Shore 2005; Mighton 2003). Supportive teachers address anxiety and self-esteem to improve confidence in students (Finlayson 2014; Small 2015).

Overcome "I'm Not a Math Person" by Fostering a Growth Mindset

Dweck's (2006) research on mindsets suggests that students display either a "fixed mindset," in which they believe that one is either smart or not, or they develop a "growth mindset" and believe that intelligence and "smartness" can be learned and the brain can grow from exercise. Math, unfortunately, is typically the subject area that communicates the strongest fixed-ability messages and thinking (Boaler 2010).

Math anxiety develops out of the belief that a natural mathematical mind is needed; this myth encourages students to give up when they encounter difficulty (Furner and Duffy 2002; Mighton 2003). When faced with a mistake, math-anxious students believe they are not smart and give up. Their fixed mindset causes them to frequently avoid challenge or anything they perceive as being difficult (Dweck 2006). Math-anxious students believe that success depends on natural talent, but success actually relies on an individual's mindset. Students with a growth mindset both work and learn more effectively, display a desire for challenge, and demonstrate resilience when faced with failure (Dweck 2006).

Teachers can work to dispel the myth of "math people" by telling all students they are capable of doing math and helping them develop a growth mindset (Mighton 2003; Dweck 2006). When teachers emphasize incremental intelligence, working hard and making mistakes to learn, students are more successful (Dweck 2006). Teachers can help their students develop a growth mindset in a number of ways.

1. *Teach about the brain.* Researchers have found that when students received an intervention workshop based on developing a growth mindset, math performance improved (Dweck, Walton and Cohen 2014). Students need to be educated on the science of the brain. Brains can form new, stronger connections between neurons when faced with difficult, challenging tasks and mistakes are opportunities for learning and growth (Boaler 2013; Dweck 2014).
2. *Encourage students intentionally.* Math-anxious students need teachers to believe in them because they are unaware of their existing strengths in math (Finlayson 2014; Wright 1996). This involves teachers taking a proactive role in encouraging students to see themselves as successful, confident problem-solvers (Finlayson 2014; Furner and Berman 2003). Students need to be praised for their effort and hard work rather than for their intelligence (Dweck 1999). Praising students for their efforts or the strategies they used teaches growth mindset and fosters resilience (Dweck, Walton and Cohen 2014).
3. *Watch your language.* Be intentional about using growth mindset language and avoiding fixed mindset language. When students who face setbacks speak with worry or negative thoughts, feelings or behaviour, it is crucial to respond with constructive thoughts that promote growth mindset related to strategy, excitement of challenge, persistence and growth (Dweck, Walton and Cohen 2014) Students are on a learning journey that can be filled with hills and valleys. Using the words *yet* or *not yet* can give students greater confidence and greater persistence in future learning. The word *yet* can diffuse feelings of failure and encourage children to try again (Dweck 2014). Teachers can promote growth mindset by sending the message that students belong and have great potential (Dweck, Walton and Cohen 2014).
4. *Model a growth mindset.* Teachers must model what they expect from their students. Do not describe yourself as a *math person* or use fixed language. Treat your own mistakes as opportunities for learning, and tell your students about learning from your mistakes. When students believe that everyone's ability can grow, their achievement improves. Similarly, when teachers believe that everyone's ability can grow, student achievement improves (Boaler 2013).

Give Feedback: Recommended Assessment Practices

Math-anxious students do not accurately demonstrate their learning on assessments (Posmentier, Germain-Williams and Jaye 2013). Assessment can create anxiety due to focus on results, right answers, and specific methods instead of understanding (Perry 2004; Jackson and Leffingwell 1999; Harper and Daane 1998; Finlayson 2014). Historically, assessment focused on these products, but the *process* is equally if not more important (Finlayson 2014). Assessment should be focused not on results, but on the development and achievement of mathematical proficiency (Kilpatrick, Swafford and Findell 2001).

The assessments that teachers choose provide opportunity for messages to be communicated to students. If students work on short, closed questions with right or wrong answers, getting frequent wrong answers makes it difficult to maintain a view that achievement is possible with effort (Dweck, Walton and Cohen 2014). Similarly, engaging students in constant repetition of short, closed questions without making mistakes does not provide the same opportunities for growth and development that making mistakes in challenging work provides (Boaler 2013). By contrast, open-ended tasks provide opportunities for learning and allow students to see the possibility of higher achievement, giving them a chance to improve, in line with Black and Wiliam's (1998) research on formative assessment.

Formative assessment supports the building of confidence by encouraging and directing students to find the right answer by systematically analyzing their mistakes (Posmentier, Germain-Williams and Jaye 2013). This feedback teaches students how to convert failures into successes by providing an opportunity to correct mistakes and improve their understanding of mathematical concepts. Mistakes should not be marked with a cross but rather with a happy face and comments about the opportunity the mistake provides for learning (Boaler 2013). Assessment communicates what teachers value (Wilson and Kenney 2003), and providing second opportunities spreads the belief that students can master the concept and will be not be allowed to fail. Using a variety of assessment techniques is key. Open-ended assessments provide detail on student strategies and understanding, and long-term projects allow depth of exploration (Wilson and Kenney 2003). Use of observations and rubrics has been shown to reduce anxiety, and portfolios provide opportunity for students reflect on their work to build confidence (Wilson and Kenney 2003; Finlayson 2014). Journal writing has been shown to reduce math anxiety and increase student learning (Connor-Greene 2000; Furner and Duffy 2002; Furner and Berman 2003; Salinas 2004). Self-assessment allows students to have input into their own evaluations, which further reduces anxiety (Furner and Duffy 2002).

Assessment in any form is feedback given to students that can serve either to motivate or to discourage (Schimmer 2016). Every piece of feedback sends messages, however subtle, that shape student motivation, making students more or less tenacious learners and, consequently, more or less anxious (Dweck, Walton and Cohen 2014). As such, assessment should be designed with care.

Teach for Understanding: Instructional Strategies

Math-anxious students attribute their discouragement to a lack of interactive, creative and relevant learning experiences (Jackson 2008). Students need more than demonstration of procedures, but also experience in investigating mathematical properties, justifying solutions and analyzing problems (Kilpatrick, Swafford and Findell 2001). When teachers employ best practices for teaching math, students understand and learn math, and anxiety is reduced (Furner and Duffy 2002; Alkan 2013). Math anxiety is further reduced when math is connected to real-life situations with a series of problem-based activities (Finlayson 2014; Ernest 1989; Van De Walle 2004) and a more student-directed class with teacher facilitation (Finlayson 2014; Mutodi and Ngirande 2014; Newstead 1998; Wilensky 1997). Constructivist, collaborative, open-ended instructional design encourages a positive response from students (Finlayson 2014).

Interactions that take place within the classroom are crucial. Teachers play a key role as orchestrator of mathematical conversations and should provide opportunities for students to offer solutions, make claims, and provide explanations to their peers (Kilpatrick, Swafford and Findell 2001). Exploring multiple ways to solve problems and allowing students to choose their method empowers students to construct personal approaches and unravel misconceptions, thus building self-confidence (Finlayson 2014; Schuster and Canavan Anderson 2005). Teachers must permit students to discover why their approach does or does not work and allow them to surrender ineffective strategies to avoid creation of math anxiety and self-doubt regarding reasoning skills (Clement, Narode and Rosnick 1981). Students must learn to trust their own methods of understanding the material if teachers are to achieve the critical goals of creating understanding and confidence in their students (Clute 1984; Finlayson 2014; Small 2015).

Engage Parents

Positive parent involvement is crucial to reducing math anxiety (Alkan 2013; Furner and Berman 2003). Due to their shared position as role model, parents must be educated on the impact of their beliefs, words and actions regarding math (Casad, Hale and Wachs 2015). Parents should not talk to their children of their own dislike of math or of their own math weaknesses, because this gives the child permission to be the same way (Small 2015; Maloney et al 2015). Educating parents on the science and language of growth mindset can empower them to help develop that mindset in their children.

Home support helps to reduce math anxiety in students, and schools need to develop better tools for parents to get involved (Vukovic, Roberts and Wright 2013; Maloney et al 2015). Parents are usually willing to help with reading at home, but often assume that the school will take care of math instruction (Berkowitz et al 2015). A strategy for elementary schools may be to develop a home math program, such as encouraging parents to read math stories to help children develop better, more situated math understanding (Berkowitz et al 2015). Beyond that, schools should make efforts to educate parents as to why they should and how they can help their children develop a sense of number and shape in childhood. (Kilpatrick, Swafford and Findell 2001).

Specific parent involvement strategies remove ineffective homework help that contributes to math anxiety if parents do not understand the homework and express their lack of understanding (Maloney et al 2015). Parent strategies should include asking their child questions, teaching children to ask good questions, encouraging children to teach their parents, believing in their children and being conscious of their own behaviours related to math (Small 2015).

What Schools Can Do

Prepare and Support Teachers to Overcome Math Anxiety

Great teachers are "the key to many students' ability to learn mathematics" and possess "pedagogical content knowledge, built upon a deep understanding of how students think and develop mathematically" (Small 2012, 2). Attitude, however, is a stronger influence than skill and, consequently, addressing teachers' attitudes toward math may be more important than addressing their knowledge of math (Jackson 2008). Elective professional learning is not enough, because teachers with a reform-oriented view of math choose to engage, while others believe that increased prep time is more beneficial than subject knowledge and avoid these opportunities (Larsen 2012; Chavez, Widmer and Carroll 1982; Allen 2010). While nervousness about teaching math should lead to learning about content and asking for help, a teacher who may unconsciously hold the belief that it is acceptable to be "not good at math" may find that his or her attitude inhibits the effectiveness of addressing the lack of pedagogical content knowledge (Small 2015).

Math anxiety, therefore, must be addressed by school-level professional learning. Teachers with a lack of subject knowledge do not plan as effectively, and some elementary teachers struggle with alternative strategies, estimation and other areas due to their own misconceptions (Goulding, Rowland and Barber 2002; Good 2009). Knowing which mathematical concepts teachers struggle with can help to set priorities for future professional learning, which should include "mathematically rich activities that cause participants to acknowledge their current knowledge, recognize if this knowledge is comprised of any mathematical misconceptions, and transform their current knowledge into mathematically correct knowledge" (Good 2009, 165). The biggest challenge facing leaders of professional learning is teachers who are recovering from their own math anxiety. Therefore, it is essential that leaders treat participants of professional learning as they would a class of students. Leaders must create a climate in which teachers feel safe to express their negative feelings toward math (Chavez, Widmer and Carroll 1982). Teachers will not be successful at math until leaders build their confidence by listening to their concerns and ideas and gently moving them in the right direction (Small 2015).

Organizing Schools to Reduce Math Anxiety

A student's view of what it means to know and do math is shaped in elementary school, by generalist teachers (Fennell 2006). Few teachers, particularly in elementary schools, currently have the specialized knowledge required to effectively teach mathematics as the curriculum was envisioned; this is not a teacher's fault, because it is a significant task to obtain this knowledge—beyond what can be expected to occur in a teacher's spare time (Kilpatrick, Swafford and Findell 2001).

As generalists, elementary teachers may find it difficult to develop in-depth math knowledge and expertise in teaching mathematics (National Council of Teachers of Mathematics [NCTM] 2010). Many elementary and even middle school teachers don't view themselves as experts, yet they are tasked with teaching math with a limited understanding of the subject and a hesitance to teach for conceptual understanding they may not themselves possess, and as a result lean on traditional teaching approaches (Wingert, 2014; Lott, 2003). Their preservice background and general teaching responsibilities are not typically conducive to the continuous development of specialized knowledge for teaching math, but elementary specialists provide a solution (Wingert 2014; Fennell 2006).

Specialized Teacher Model: Elementary Content Specialists

It is unrealistic to expect elementary generalist teachers to possess deep math content knowledge when considering the advanced knowledge they must gain of how to teach reading (Wu 2009). The problem of raising the mathematical proficiency of all elementary teachers is immense, requiring considerable changes to preservice training, and could be best solved by changing the traditional model of elementary generalist teachers (Wu 2009). Math anxiety starts early, yet content specialist teachers are present only in the latter years of students' academic careers. A specialized teacher model gives one teacher, with deep content knowledge, the primary responsibility for teaching math (Fennell 2006). These teachers bring greater confidence to the classroom and use their specialized knowledge to produce a higher quality of work (Hennessy 2000; Wilson et al 2008). Arts and physical education employ subject specialist teachers at the elementary level because nonspecialists are less effective in developing skills, reverting to traditional activities they experienced as students—yet there are not specialists in elementary math, a core subject (Lott 2003). Specialists possess understanding of key mathematical concepts, and in order to teach math the right way, it is crucial to create a corps of teachers with this knowledge (Wu 2009). This allows school districts to focus professional development on a targeted group of teachers and has "economic benefits because it does not require additional teachers, just a redistribution of teaching responsibilities" (Fennell 2006). High-performing

Asian countries use elementary specialists starting in Grade 3 (Wingert 2014).

The model is not without its drawbacks, however, as studies have shown that specialist math teachers in Grades 5 and 6 have not proven to make a significant impact (McGatha 2009). Perhaps Grade 5 is too late to begin to use specialists, due to early onset of math anxiety. Conversely, it is important to consider the message the specialist model may send—that only math specialists can teach math—potentially interpreted by students as that only some adults are good at math, and it is therefore acceptable to not be good at math. While this could contribute to the cycle of math anxiety, something needs to be done to try to prevent development of math anxiety in students.

Lead Teacher Model: In-School Math Teaching Experts

A less radical change to the current structure of elementary schools is to use math specialist teachers, similar to the literacy specialist teachers already common. These on-site specialists provide continuing, comprehensive support to teachers and students in one specific school, focusing primarily on teacher development, planning lessons with teachers in the classroom and providing authentic professional development in real time (Wingert 2014). Teachers need to model the mindset they expect of their students, and this model restructures schools as places to learn, where teachers are given opportunity to learn and support to advance their craft (Stigler and Hiebert 1999).

Mathematical knowledge is a critical resource for teaching; therefore, every school needs access to math teaching expertise and should provide teachers with opportunities to develop their own knowledge about math and math education (Kilpatrick, Swafford and Findell 2001). This model gives teachers more time to effectively plan lessons, leading to improved student achievement and possibly less anxiety (McGatha 2009; Wingert 2014). Math specialists have been found to help teachers overcome insecurity, feel more confident and positive, and learn multiple ways to teach math (Blount and Singleton 2007; Wingert 2014). Short or one-off professional learning has proven to be ineffective, so by having an in-school specialist, teachers can work together to continue to learn and develop their practice (Kilpatrick, Swafford and Findell 2001). Furthermore, this ongoing, embedded professional learning can extend beyond teachers to educational assistants.

The model of supportive math specialists may be the only practical way to make change (Wingert 2014). While it may not be as time or cost efficient as the subject specialist model, it sends the message that math can be learned by all, fostering a growth mindset in teachers and students alike. When teachers believe that everybody's ability can grow (including their own), students tend to achieve at higher levels (Boaler 2013).

Conclusion

The self-defeating cycle of math anxiety is a significant problem shaped by students' experiences while learning math. Educators have a responsibility to ensure that students have positive experiences of learning math in a safe, growth-minded environment supported by sound assessment practices and instructional strategies that emphasize understanding over results. Teachers have the opportunity to model a growth mindset and positive attitudes toward math while encouraging parents to do the same by giving them specific support strategies. Educators need to research and implement programs to educate parents about math anxiety and involve parents in their child's math education. Schools need to prepare and support teachers to address math anxiety in both themselves and their students through ongoing school-based professional learning. Two possible solutions worth exploring at the school level are elementary math content specialists and lead teacher math specialists.

Leaders need to be aware of the implicit messages that they send to students regarding math anxiety, and explicitly address these in their own decision making with staff members and with parents. The best way to break the cycle of math anxiety is to prevent it from starting to begin with. All stakeholders in a student's life need to be involved and, more importantly, aware of the role they can play in reducing math anxiety by communicating a positive attitude and a growth mindset toward learning math.

References

Al Mutawah, M A. 2015. "The Influence of Mathematics Anxiety in Middle and High School Students Math Achievement." *International Education Studies* 8, no 11: 239–52.

Allen, B. 2010. "The Primary Mathematics Specialists—What Do They Think About Teaching and Learning Mathematics?" Presentation to British Congress for Mathematics Education, Manchester, April 6–9.

Alkan, V. 2013. "Reducing Mathematics Anxiety: The Ways Implemented by Teachers at Primary Schools." *International Journal of Social Science and Education* 3, no 3. Available at http://ijsse.com/sites/default/files/issues/2013/v3i3/Paper-25.pdf (accessed September 11, 2018).

Anderson, V. 2007. "An Online Survey to Assess Student Anxiety and Attitude Response to Six Different Mathematical Problems." In *Mathematics: Essential Research, Essential Practice, vol 1*, Proceedings of the 30th Annual Conference of the Mathematics Education Group of Australasia, ed J Watson and K Beswick, 93–102.

Ashcraft, M H, and E P Kirk. 2001. "The Relationships Among Working Memory, Math Anxiety, and Performance." *Journal of Experimental Psychology: General* 130, no 2: 224–237.

Baroody, A J, and R T Costlick. 1998. *Fostering Children's Mathematical Power. An Investigative Approach to K-8 Mathematics Instruction.* Mahwah, NJ: Lawrence Erlbaum.

Beilock, S L, and D T Willingham. 2014. "Math Anxiety: Can Teachers Help Students Reduce It?" *American Educator* 37, no 2: 28–32, 43. Available at https://hpl.uchicago.edu/sites/hpl.uchicago.edu/files/uploads/American%20Educator,%202014.pdf (accessed September 13, 2018).

Berkowitz, T, M Schaeffer, E A Maloney, L Peterson, C Gregor, S C Levine, and S L Beilock. 2015. "Math at Home Adds Up to Achievement in School." *Science* 350, no 6257: 196–98.

Black, P, and D Wiliam. 1998. "Assessment and Classroom Learning." *Assessment in Education: Principles, Policy and Practice* 5, no 1: 7–74.

Blount, D, and J Singleton, J. 2007. "The Role and Impact of the Mathematics Specialist from the Principals' Perspectives." *The Journal of Mathematics and Science: Collaborative Explorations* 9: 69–77.

Boaler, J. 2010. *The Elephant in the Classroom: Helping Children Learn and Love Maths.* London: Souvenir.

———. 2013. "Ability and Mathematics: The Mindset Revolution That Is Reshaping Education." *Forum* 55, no 1: 143–52.

Burns, M. 1998. *Math: Facing an American Phobia.* Sausalito, Calif: Math Solutions.

Buxton, L. 1981. *Do You Panic About Maths? Coping with Maths Anxiety.* London: Heinemann.

Casad, B J, P Hale and F L Wachs. 2015. "Parent-Child Math Anxiety and Math-Gender Stereotypes Predict Adolescents' Math Education Outcomes." *Frontiers in Psychology* 6, no 1597: 1–21.

Chavez, A, C C Widmer and C Carroll. 1982. "Math Anxiety: Elementary Teachers Speak for Themselves." *Educational Leadership* 39, no 5: 387–88.

Clement, J, R Narode and P Rosnick. 1981. "Intuitive Misconceptions in Algebra as a Source of Math Anxiety." *Focus on Learning Problems in Mathematics* 3, no 4: 36–45.

Clute, P S. 1984. "Mathematics Anxiety, Instructional Method, and Achievement in a Survey Course in College Mathematics." *Journal for Research in Mathematics Education* 15, no 1: 50–58.

Connor-Greene, P A. 2000. "Making Connections: Evaluating the Effectiveness of Journal Writing in Enhancing Student Learning." *Teaching of Psychology* 27, no 1: 44–46.

Dodd, A W. 1999. "Insights from a Math Phobic." *Math Teacher* 85, no 4: 296–98.

Dweck, C. 1999. "Caution – Praise Can Be Dangerous." *American Educator* 23, no 1: 4–9.

———. 2006. *Mindset: The New Psychology of Success.* New York: Penguin Random House.

———. 2014. "The Power of Believing That You Can Improve." TED Talk. Available at www.ted.com/talks/carol_dweck_the_power_of_believing_that_you_can_improve (accessed September 11, 2018).

Dweck, C, G Walton and G Cohen. 2014. *Academic Tenacity: Mindsets and Skills That Promote Long-Term Learning.* Seattle: Gates Foundation. Available at https://ed.stanford.edu/sites/default/files/manual/dweck-walton-cohen-2014.pdf (accessed September 11, 2018).

Ernest, P. 1989. "The Impact of Beliefs on the Teaching of Mathematics." In *Mathematics Teaching: The State of the Art*, ed P Ernest. London: Falmer, 249–54.

Fennell, F. 2006. "We Need Elementary School Mathematics Specialists Now." *NCTM News Bulletin* 43, no 4. Available at www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Skip-Fennell/We-Need-Elementary-School-Mathematics-Specialists-NOW/ (accessed September 11, 2018).

Fernandes, D. 1995. "Analyzing Four Preservice Teachers' Knowledge and Thoughts Through Their Biographical Histories." In *Proceedings of the Nineteenth International Conference for the Psychology of Mathematics Education*, vol II, Universidade Federal de Pernambuco, Recife, Brazil, 162–69.

Finlayson, M. 2014. "Addressing Math Anxiety in the Classroom." *Improving Schools* 17, no 1: 99–115.

Fiore, G. 1999. "Math-Abused Students: Are We Prepared to Teach Them?" *Math Teacher* 92, no 5: 403–06.

Furner, J M, and B T Berman. 2003. "Math Anxiety: Overcoming a Major Obstacle to Help the Improvement of Student Math Performance." *Childhood Education* 79, no 3: 170–74.

Furner, J M, and M Duffy. 2002. "Equity for All Students in the New Millennium: Disabling Math Anxiety." *Intervention in School and Clinic* 38, no 2: 67–74.

Geist, E. 2010. "The Anti-Anxiety Curriculum: Combating Math Anxiety in the Classroom." *Journal of Instructional Psychology* 37, no 1: 24–31.

Good, L. 2009. "Using Professional Development to Improve Elementary Teachers' Mathematics Teaching: An Action Research Study." EdD dissertation, University of Rochester, New York.

Goulding, M, T Rowland and T Barber. 2002. "Does It Matter? Primary Teacher Trainees' Subject Knowledge in Mathematics." *British Education Research Journal* 28, no 5: 689–704.

Gresham, G. 2008. "Mathematics Anxiety and Mathematics Teacher Efficacy in Elementary Pre-Service Teachers." *Teaching Education* 19, no 3: 171–184.

Hackworth, R D. 1985. *Math Anxiety Reduction.* Clearwater, Fla: H and H.

Harper, N W, and C J Daane. 1998. "Causes and Reduction of Math Anxiety in Preservice Elementary Teachers." *Action in Teacher Education* 19, no 4: 29–38.

Hembree, R. 1990. "The Nature, Effects, and Relief of Mathematics Anxiety." *Journal for Research in Mathematics Education* 21, no 1: 33–46.

Hennessy, S. 2000. "Overcoming the Red-Feeling: The Development of Confidence to Teach Music in Primary School Amongst Student Teachers." *British Journal of Music Education* 17, no 2: 183–96.

Jackson, C D, and R J Leffingwell. 1999. "The Role of Instructors in Creating Math Anxiety in Students from Kindergarten Through College." *Mathematics Teacher* 92, no 7: 583–86.

Jackson, E. 2008. "Mathematics Anxiety in Student Teachers." *Practitioner Research in Higher Education* 2, no 1: 36–42.

Jeynes, W H. 2007. "The Relationship Between Parental Involvement and Urban Secondary School Student Academic Achievement: A Meta-Analysis." *Urban Education* 42, no 7: 82–110.

Kilpatrick, J, J Swafford and B Findell, eds. 2001. *Adding It Up: Helping Children Learn Mathematics.* Washington, DC: National Academy Press.

Larsen, S M. 2012. "Perceptions of Elementary Mathematics Coaching." PhD dissertation, University of Toronto. Available at https://tspace.library.utoronto.ca/bitstream/1807/34777/1/Larsen_Shannon_M_201211_PhD_thesis.pdf (accessed September 12, 2018).

Levine, G. 1993. "Prior Mathematics History, Anticipated Mathematics Teaching Style, and Anxiety for Teaching Mathematics Among Pre-Service Elementary School Teachers." Paper presented at the Annual Meeting of the International Group for Psychology of Mathematics Education, North American Chapter, Pacific Grove, Calif, October 17–20.

Lott, J. 2003. "The Time Has Come for Pre-K–5 Mathematics Specialists." *NCTM News Bulletin* July/August.

Maloney, E A, G Ramirez, E A Gunderson, S C Levine, and S L Beilock. 2015. "Intergenerational Effects of Parents' Math Anxiety on Children's Math Achievement and Anxiety." *Psychological Science* 26, no 9: 1480–88.

Martinez, J G R. 1987. "Preventing Math Anxiety: A Prescription." *Intervention in School and Clinic* 23, no 2: 117–25.

McLeod, D. 1992. "Research on Affect in Mathematics Education: A Reconceptualization." In *Handbook of Research on Mathematics Teaching and Learning*, ed D A Grouws, 575–96. Available at www.peterliljedahl.com/wp-content/uploads/Affect-McLeod.pdf (accessed September 12, 2018).

McGatha, M. 2009. "Mathematics Specialists and Mathematics Coaches: What Does the Research Say?" *NCTM Research Briefs*, ed J R Quander. Reston, Va: NCTM.

Mighton, J. 2003. *The Myth of Ability: Nurturing Mathematical Talent in Every Child.* Toronto: Anansi.

Morris, J. 1981. "Math Anxiety: Teaching to Avoid It." *The Mathematics Teacher* 74, no 6: 413–17.

Mutodi, P, and H Ngirande. 2014. "Exploring Mathematics Anxiety: Mathematics Students' Experiences." *Mediterranean Journal of Social Sciences* 5, no 1: 283–94.

National Council of Teachers of Mathematics (NCTM). 2010. "The Role of Elementary Mathematics Specialists in the Teaching and Learning of Mathematics." Available at www.nctm.org/Standards-and-Positions/Position-Statements/The-Role-of-Elementary-Mathematics-Specialists-in-the-Teaching-and-Learning-of-Mathematics/ (accessed September 13, 2018).

Newstead, K. 1998. "Aspects of Children's Mathematics Anxiety." *Educational Studies in Mathematics* 36, no 1: 53–71.

Perina, K. 2002. "The Sum of All Fears: What Makes People Math Phobic?" *Psychology Today* 35, no 6: 19 ff.

Perry, A B. 2004. "Decreasing Math Anxiety in College Students." *College Student Journal* 38, no 2: 321–24.

Posmentier, A S, T L Germain-Williams and D I Jaye, D. 2013. *What Successful Math Teachers Do, Grades 6–12: 80 Research-Based Strategies for the Common Core-Aligned Classroom.* 2nd ed. Thousand Oaks, Calif: Corwin.

Preis, C. and B T Biggs. 2001. "Can Instructors Help Learners Overcome Math Anxiety?" *ATEA Journal* 28, no 4: 6–10.

Raymond, A M. 1997. "Inconsistency Between a Beginning Elementary School Teacher's Mathematics Beliefs and Teaching Practice." *Journal for Research in Mathematics Education* 28, no 5: 550–76.

Richardson, F C, and R M Suinn. 1972. "The Mathematics Anxiety Rating Scale: Psychometric Data." *Journal of Counselling Psychology* 19, no 6: 551–54.

Salinas, T. 2004. "Effects of Reflective Notebooks on Perceptions of Learning and Mathematics Anxiety." *Primus: Problems, Resources, and Issues in Mathematics Undergraduate Studies* 14, no 4: 315–27.

Schimmer, T. 2016. *Grading from the Inside Out: Bringing Accuracy to Student Assessment Through a Standards-Based Mindset.* Bloomington, Ind: Solution Tree.

Schuster, L, and N Canavan Anderson. 2005. *Good Questions for Math Teaching: Why Ask Them and What to Ask, Grades 5–8.* Sausalito, Calif: Math Solutions.

Schwartz, A E. 2000. "Axing Math Anxiety." *Education Digest* 65, no 5: 62–64.

Shore, K. 2005. "Dr. Ken Shore's Classroom Problem Solver: Math Anxiety." Available at www.educationworld.com/a_curr/shore/shore066.shtml (accessed January 18, 2016).

Small, M. 2012. *Making Math Meaningful to Canadian Students, K-8.* 2nd ed. Toronto: Nelson Education.

———. 2015. *Marian Small—Math Anxiety.* Available at https://vimeo.com/136761934 (accessed Sepember 13, 2018).

Stigler, J, and J Hiebert. 1999. *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom.* New York: Free Press.

Stuart, V B. 2000. "Math Curse or Math Anxiety?" *Teaching Children Mathematics* 6, no 5: 330–35.

Tobias, S. 1993. *Overcoming Math Anxiety.* New York: Norton.

Turner, J C, C Midgley, D K Meyer, M Gheen, E M Anderman, Y Kang and H Patrick. 2002. "The Classroom Environment and Students' Reports of Avoidance Strategies in Mathematics: A Multimethod Study." *Journal of Educational Psychology* 94, no 1: 88–106.

Van de Walle, J. 2004. *Elementary and Middle School Mathematics: Teaching Developmentally.* 5th ed. Boston, Mass: Pearson.

Vukovic, R, S Roberts and L Wright. 2013. "From Parental Involvement to Children's Mathematical Performance: The Role of Mathematics Anxiety." *Early Education and Development* 24, no 4: 446–67.

Wilensky, U. 1997. "What Is Normal Anyway? Therapy for Epistemological Anxiety." *Educational Studies in Mathematics* 33, no 2: 171–202.

Wilson, G B, R A R MacDonald, C Byrne, S Ewing and M Sheridan. 2008. "Dread and Passion: Primary and Secondary Teachers' Views on Teaching the Arts." *Curriculum Journal* 19, no 1: 37–53.

Wilson, L D, and P A Kenney. 2003. "Classroom and Large-Scale Assessment: Issues in Classroom Assessment." In *A Research Companion to Principles and Standards for School Math-*

ematics, ed W G Martin, D Schifter and J Kilpatrick, 53–67. Reston, Va: NCTM.

Wingert, P. 2014. "Are Math Specialists the Answer to Teaching Better Math?" *The Hechinger Report*, October 2. Available at http://hechingerreport.org/math-specialists-answer-teaching-better-math/ (accessed September 13, 2018).

Wood, E. 1988. "Math Anxiety and Elementary Teachers: What Does Research Tell Us?" *For the Learning of Mathematics* 8, no 1: 8–13.

Wright, C. 1996. "Learning Disabilities in Mathematics." Available at www.ldao.ca/introduction-to-ldsadhd/articles/about-education/learning-disabilities-in-mathematics/ (accessed September 13, 2018).

Wu, H. 2009. "What's Sophisticated About Elementary Mathematics? Plenty—That's Why Elementary Schools Need Math Teachers." *American Educator* 33, no 3: 4–14.

Eric Schubert is a middle school mathematics teacher and school administrator in Airdrie, Alberta. He is an MEd graduate from the University of Calgary and holds a BEd in secondary mathematics from the University of Alberta. Eric is interested in exploring ways to engage all students in meaningful mathematics and numeracy educational experiences.

Differentiating Assessment

David Martin

This article is reprinted, with the permission of the author, from a post to the blog Real Teaching Means Real Learning *on May 16, 2018. Minor amendments have been made in accordance with ATA style.*

The primary purpose of assessment is to improve student learning.
—Anne Davies, PhD

The quote above sums up why I challenged my assessment practices. In 2013, as a high school math teacher, here is the model I used:

- Teach concept
- Various quizzes during the learning journey
- Summative unit exam
- Repeat

After eight years of teaching in this model, I realized there was an issue: students who entered my class with a passion for mathematics were leaving the class beaten down, sometimes dropping out and, ultimately, not having the desire to learn more math.

Originally, I thought this was normal! When I went to high school, math classes always ended with fewer students than they started with. As a student, I remember daily expectations of having to do the odd- (or even-) numbered questions on page x, multiple worksheets, and having to prepare for weekly quizzes or tests. This was my normality. This was the machine I wanted to perpetuate when I entered teaching.

Why?

Because this worked for me. I am intrinsically motivated by mathematics, and I find prime, Fibonacci and complex numbers inherently interesting ... because they are! However, too many people have not had the chance to struggle, discover and play with these (and other) awesome mathematical ideas.

In 2013, as an educator, I saw the true problem—my assessment style was more about ranking, sorting and grading, not at all about learning. Furthermore, I was more focused on preparing students for AP or diploma exams, instead of creating an environment that allowed students to bring their passions and interests in, next to their pencils and paper. My grades were focused on what was easy to test, grade and report on, instead of what was important.

This had to change. If I was differentiating my instruction, why was I focusing on standardizing my assessment?

In 2013, I made a stand: I will only assess in a way that increases learning; if my assessment isn't increasing learning, then the assessment needs to change.

In this year, my late friend Joe Bower reminded me that "the word *assessment* comes from the Latin word *assidere*: to sit beside," an action that was rarely taken when I was assessing my students.

Here is my journey, along with the steps I took to explore what it means to provide differentiated assessment.

Manageable Outcomes

In consultation with university professors, colleagues and teachers across the province, I looked at every course outcome through the "Rock, Sand, Water" analogy: if you plan for the rocks first, then sand and then water, it will all fit; however, if you simply plan a course to cover all outcomes equally, all the outcomes will rarely fit.

During this process, I combined parts of one outcome with another, broke up some outcomes into smaller chunks and then created a list of rock, sand and water objectives:

- Rock outcomes (outcomes that pass the endurance, leverage and readiness test)—expect *all* students to master
- Sand outcomes—expect *most* students to master
- Water outcomes—expect *some* of my students to master

I then ensured that these decisions were reflected in my long-range outcomes, course outlines and daily plans. I planned my courses to ensure that the essential learning outcomes were woven throughout the entire year, while less essential outcomes were covered through the lens of a higher-leverage outcome. Of course, I still taught all the outcomes, but I decided to report on only the essential ones, regardless of how difficult it might be to do so.

Change the Tests

Having a smaller list of outcomes to report on, I decided to ensure that my summative assessments matched this philosophy. Instead of giving tests grouped by question type, I grouped questions based on outcome. Any assessment that covered more than one outcome would be given back to students with more than one grade. Each grade represented the learning of the student on a specific outcome; no longer did I average two or three outcomes into one mark and call it "Unit X Test." I then changed the categories on the online reporting program to "Outcomes" instead of "Quizzes, tests, homework, etc." Every mark, on a specific outcome, was reported in the corresponding outcome category.

Ensure That Learning Is the Focus on Every Assessment

During this time, my summative assessments were one part multiple choice, one part numerical response and one part written, simply scored by outcomes, not by question type.

I quickly realized that when my students answered a multiple-choice question wrong (or even when they guessed right), I was clueless as to how so support them from their current understanding to mastery. If I wanted learning to be the primary focus, I could not administer multiple-choice exams.

In 2014/15, I moved to an entirely written-response assessment strategy grouped by outcomes. Instead of one part multiple choice, one part numerical response and one part written, I assessed only with questions that forced students to make their understanding visible. It was during this year that I truly started to sit next to my students and provide them with written and verbal feedback that pushed their learning forward, instead of simply saying, "Here are the X questions you answered incorrectly and here are the correct answers." My feedback was focused on learning, not on the questions they answered incorrectly.

I was writing grades and comments on everything my students handed in. This was the inherent problem: I was giving both grades and comments.

Every time I handed back an assessment with a mark, I quickly noticed that students focused on their individual grade, their friend's grade and how they ranked within their peer group; most [students] completely ignored the comments. Students were not asking, "How do I understand this better?" but instead, "How do I get an A (or 90 per cent, or Excellent)?"

I had invested a lot of my time into giving useful and effective feedback; however, these comments were being overshadowed by marks. Grades were the commodity of my classroom, not learning. This had to change.

Challenge the Grading System

This is the game changer!

Simply put, I stopped writing grades, learning levels or any other ranking system on student work. Instead, I only provided feedback and asked questions that pushed learning forward. Even if a student demonstrated mastery of an outcome, I would still provide feedback or leave them with a question that pushed them beyond the scope of the outcome.

This was the most profound transformation I have ever experienced in my entire career.

Students truly became engaged in their learning, not their grade or ranking. As well, I was able to truly push my students forward when they made mistakes. When I looked at the work of my students, I simply focused on three essential questions:

- What does success on the essential learning outcome (rock outcome) look like?
- Where is the student now?
- How do we close the gap?

This ensured that the feedback I was providing to students was truly learning focused. Every written comment was also input into our online reporting system.

This meant that when a parent or a student logged into our online reporting tool, they didn't see grades but, instead, comments for any outcome. Instead of seeing "80 per cent" on an outcome, parents (and students) would see what work was needed to close the gap—without the support of a grade.

Even the conversations I was having with parents were learning focused and not grade focused. Incredible shift! At the beginning of the year, parents were apprehensive about not receiving any marks as feedback; however, when parents saw the products their children were bringing home from my courses, parents were quick to become allies of this new model of assessment.

Differentiate the Assessments

I finally started a *differentiated assessment* model. After teaching outcome X, I would have an assessment on the outcome; however, I started to use this time to also assess each student's understanding of a previous outcome. For example, students who were also being retaught ideas on outcome 2 would have questions on outcome 2 on their sheet; the ones who were working on outcome 3 would see questions on this outcome; truly, every single assessment was tiered to the individual student and what he or she had been working on in the previous weeks.

The assessments Jimmy and Jane received on this day would match only if they were working on identical material with identical errors and misconceptions, which was rarely the case. Even when designing the questions from previous outcomes, I focused the questions on the feedback the learner had received on their last assessment. For example,

- Jane might be tasked to demonstrate understanding of a specific part of a certain essential outcome, because I saw only a minor learning gap when I previously assessed her on this outcome;
- Jimmy, however, might have more questions on the same essential outcome, because when I assessed him previously I saw major learning gaps.

This is when learning became the focus of every single assessment I gave. I can honestly say that every assessment had learning as the only priority!

Looking back, I have always believed that every child can learn math to the highest levels, but only in the past three years have I taken a differentiated approach to what happens when they don't.

David Martin has a love for numbers, puzzles and everything else mathematics. After teaching high school for 10 years, and finishing his master of mathematics for teachers, he is now a division math lead. You will find him tinkering with code, counting by prime numbers or exploring the mysteries of π.

Ditch the CUBES When Solving Word Problems!

Sandi Berg

No, I'm not referring to manipulatives when I say ditch the CUBES when solving word problems. Full disclosure: in the past, I probably fell into the CUBES supporter bunch. I've learned a lot over the past few years and now focus on other ways of teaching students how to solve word problems.

First of all, some of you may be asking, "What is CUBES?" CUBES is an acronym that you may have seen posted in math classrooms throughout the years. The acronym may take a slightly different form, but they all get to the same basic idea: when solving a word problem, use CUBES

1. **C**ircle all key numbers.
2. **U**nderline the question.
3. **B**ox any key words.
4. **E**liminate unnecessary information.
5. **S**olve and check.

Rather than telling you why I no longer use this, let's just jump right in and try an example. As you look at each stage (before skipping ahead to the next image), ask yourself "What's the answer?"

You see the following word problem that makes no sense to you ...

> [redacted]

so you faithfully follow the steps ...

Step 1: Circle all key numbers.

> 89 [redacted] 12 [redacted]

What's your gut instinct for the answer?

Step 2: Underline the question:

> 89 [redacted] 12 [redacted] How many tables are needed in total?

Do you feel pretty confident about that gut instinct?

Step 3: Underline the key words:

> 89 [redacted] 12 [redacted] How many tables are needed in total?

Still feeling pretty confident?

This is where most students stop when they don't know how to solve a word problem. What answer do you think comes to mind?

Let's look at the full question. Were you right?

> 89 people have been invited to a banquet. The caterer is arranging tables. Each table can seat 12 people. How many tables are needed in total?

What operation is used in this question?

Did you know that most students who aren't sure what to do when solving a word problem will take all of the numbers they see and just add them?

Do you have a math word wall? Have you taught students to identify key words? What have they learned *in total* means?

Let's try another example:

> [redacted]

Let's start with that key word: *in total.*

[redacted] in total?

Circle all the numbers:

[redacted] four [redacted] 5 [redacted]
23 [redacted]
[redacted] 5 [redacted] in total?

What do you think the answer is?

Are you trying not to add? But if it's not adding, then what is it?

Let's look at the question:

[redacted] four [redacted] 5 [redacted]
23 [redacted] how many
grade 5 students are there in total?

Whoa ... did that 5 just get removed from the calculations? Oops! How many students might be thrown off by that?

Let's look at the full question:

If there are four grade 5 classrooms with 23 students in each class, how many grade 5 students are there in total?

What operation is used in this question? Were there any other key words that students might have identified? Would they have helped or hindered their understanding?

Let's look at another example:

[redacted]

Here are the numbers:

15 [redacted]
[redacted] 4 [redacted]

Here's the question:

15 [redacted]
[redacted] 4 [redacted]
children are there in total?

What are you going to do with the information?

15 [redacted]
[redacted] 4 [redacted]
children are there in total?

Again, let's look at the full question. Does identifying key words help here?

15 children were playing on the playground when 4 had to go home. Now have many children are there in total?

You probably guessed it ... here comes another example.

[redacted]

Let's pull those numbers out.

[redacted] 2 [redacted] 213
[redacted] 68
[redacted]

What's the question?

[redacted] 2 [redacted] 213
[redacted] 68
[redacted] How many animals do all the
zoos have in total?

What's the full set of information?

A city has 2 zoos. One zoo has 213 animals and the second zoo has 68 animals. How many animals do all the zoos have in total?

So, what's the commonality in these four questions? They all focus on the key word *in total*, which is typically taught as "addition," but every question actually addresses a different operation and some have extraneous numbers. Following CUBES (or most of the other acronyms) fails you if you don't actually make sense of the word problem.

Here's a word problem for you to try: There are 125 sheep and 5 dogs in a flock. How old is the shepherd? I'll wait while you figure it out.

Did that seem like a silly question to you? Obviously, you can't figure it out.

Robert Kaplinsky (http://robertkaplinsky.com) asks 32 students to respond to this question. How many do you think agree with you and how many do you think attempted to solve it? Once you've made your prediction, watch the video.

Were you surprised? Why did so few students make sense of this problem? How do we help students dig deeper into the word problem rather than relying on following steps like CUBES?

Brian Bushart (https://bstockus.wordpress.com) introduced me to numberless word problems (https://bstockus.wordpress.com/numberless-word-problems/) and I fell in love.

As always, let's begin with an example.

> There are some mice on the field. Some more mice come.
> 1. What do you notice?
> 2. What do you know to be true?
> 3. What do you think to be true?

Take some time to think about the three questions in relation to the two statements above them.

I have used this example in many of my workshops, and I'm always fascinated by the discourse that occurs. If the participants/students have never experienced a numberless word problem before, they might experience confusion—there are no numbers! They sometimes start by retelling me the information that's on the board. I have had to prompt them to think about what "numbers" are in the question. What words might imply numbers? They will focus on the word *some*. What does *some* mean? This is where it gets interesting. In my experience, most will say "More than one," but some will disagree and state that it means at least three. Why three? They claim that if the writer meant two, then they would have said *couple*. We often spend quite a bit of time talking about the differences between *couple*, *some* and other examples they might bring up. It's really very interesting. I'll then ask the question: If *some* means *more than one*, what do we know?

Students will have time to talk about that. They'll naturally start to think about the question, "How many mice are on the field now?" Invariably, I'll hear the answer *three*, but then they'll discuss how *more than one* plus *more than one* has to be more than three. They'll share their reasoning. I'll follow up this question with "If *some* means *more than two*, what do we know?"

After we have fully dissected this question, I will replace one word:

> There are seven mice on the field. Some more mice come.
> 1. What do you notice?
> 2. What do you know to be true?
> 3. What do you think to be true?

We'll discuss what's changed and how that changes the rest of the information we talked about. After a full discussion, I adjust the question again:

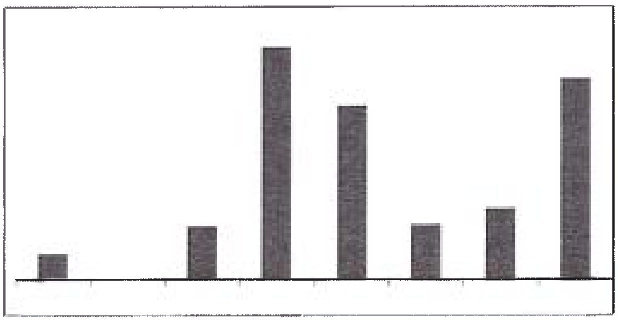
> There are seven mice on the field. Four more mice come.
> 1. What do you notice?
> 2. What do you know to be true?
> 3. What do you think the question is going to be?
> 4. Solve that question.

At this stage, students solve the question.

Questions like this are easy to create. Just pull out a question that you would give them anyway and replace the numbers with generic words like *some*, *many* and so forth (check out #numberlesswp on Twitter).

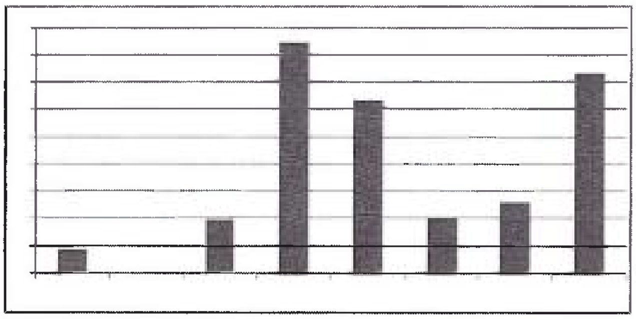
Caution! This is great at the beginning of a class, but do *not* spend an entire class talking about addition and then pull out this question. Why? If we do 40 minutes of addition questions and then have a word problem, students will just assume that it's addition. The thinking stops, and they revert to running a procedure rather than delving into the meaning behind the words.

Can you run a numberless word problem in other formats? Absolutely! Let's look at a numberless graph. I know it might be tempting to peek ahead to the upcoming graphs, but try to see what information you can figure out just by asking yourself the same three questions as in the previous section.

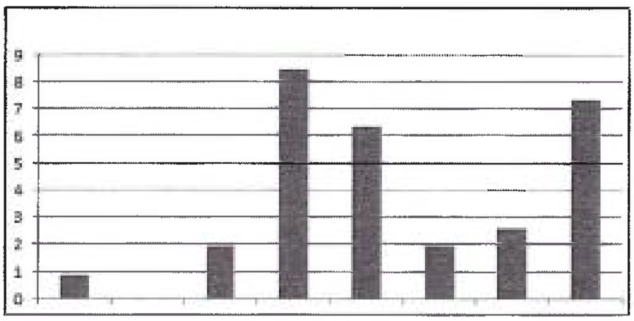


Did you see that there is one empty spot? Did you wonder if it was *zero* or if it was left out on purpose? Did you notice the third and the sixth were the same or about the same. That the fourth is the most and the first is the least? The fifth is about triple the third? And so on.

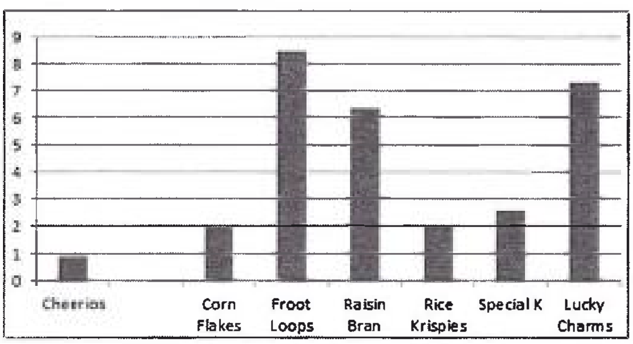
Let's add some more information. Does this help narrow down some of your statements from above?



Let's add a scale. This might help.

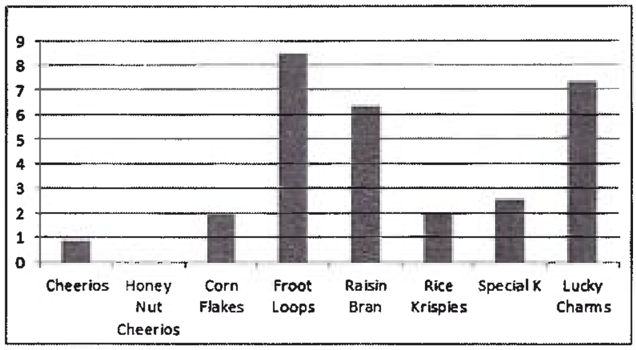


Let's add some more information.

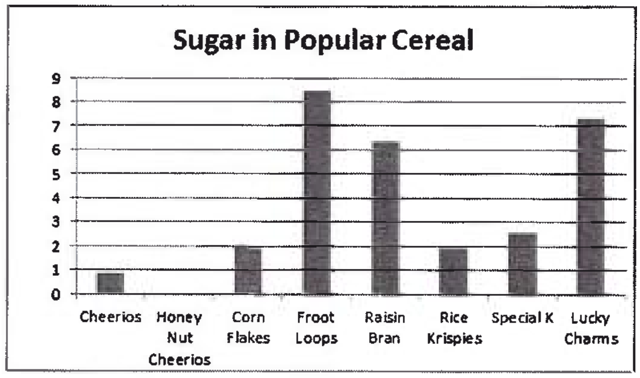


I wonder what they are comparing. Obviously cereals, but what about these cereals specifically? I wonder what the missing cereal is. Choose one.

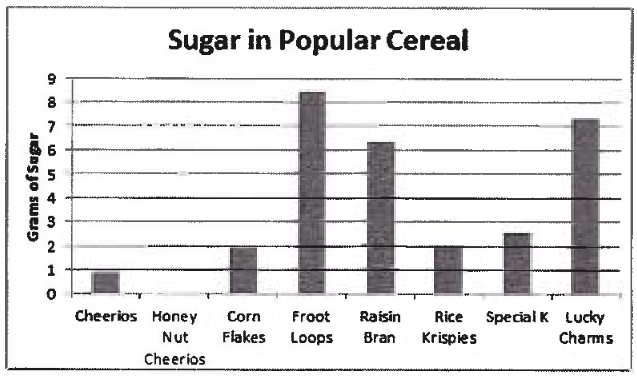
The next graph will tell you the missing cereal, but we'll come back to the one you chose in a bit.



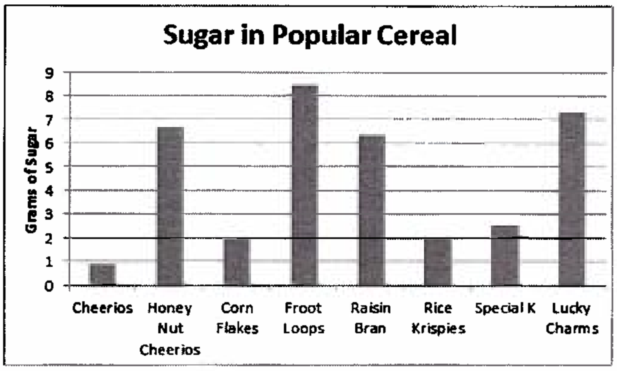
I wonder what they are actually comparing. What will help us determine this? Let's check out the title of this graph.



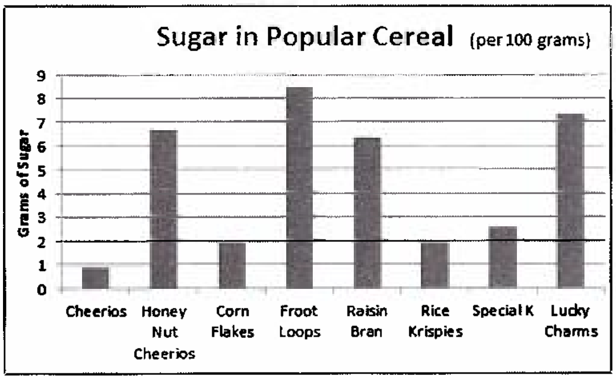
What do those numbers on the left represent?



I wonder where the Honey Nut Cheerios will fit. Make a prediction.



Have we made any assumptions? How did you visualize the amount of cereal? Did you think about it in terms of 1 cup or 0.5 cups? Did you think about it in weight? Will this make a difference?



How does this change our thoughts? Is 100g of Cheerios the same volume as 100g of Lucky Charms?

Let's return to the cereal you thought belonged in the empty spot. What would it look like if it was added to this graph? Find out!

Why is working with numberless word problems and numberless graphs a powerful activity? Once students have spent time making sense of a problem without numbers, when they get to a question they are not sure how to solve, they will slow down and think more deeply about the information provided. They won't just take the numbers, throw in an operation and "solve" the question.

Question for Reflection

How do you think your students would do in the "How old is the shepherd" problem? Try it in your class and see how they do. Are you surprised by their responses?

Activity to Try

There are many words and phrases, like *in total*, that have many mathematical meanings depending on the context of the problem. Try to create four word problems that each use that word or phrase but use a different operation, like my examples above.

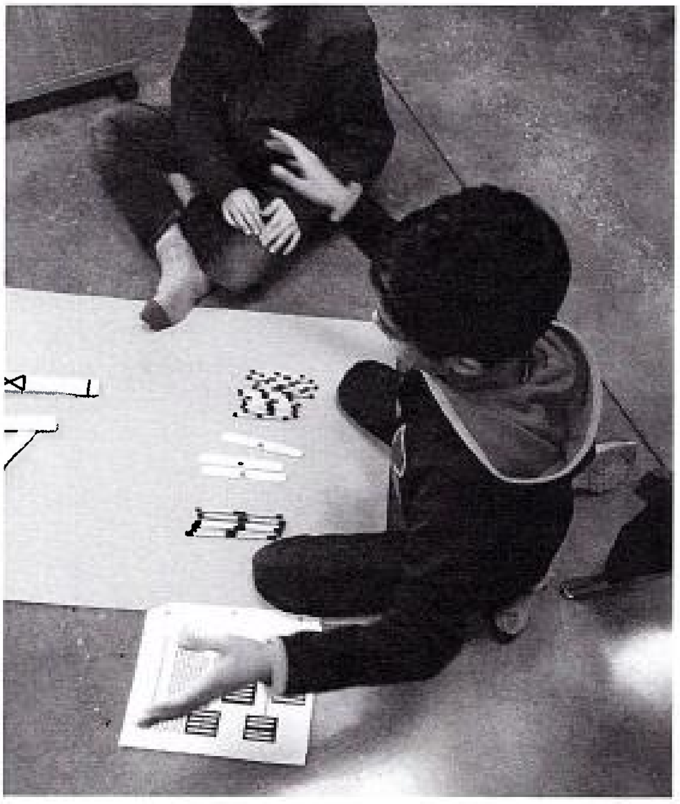
Sandi Berg is a learning services coordinator with the Learning Services Department of Chinook's Edge School Division no 73, in Alberta. She earned her bachelor of education from the University of Manitoba and a master of education in educational technology from the University of Southern Queensland. She has taught every grade from K–12 and almost every subject at least twice.

Infusing a First Nations, Métis and Inuit Perspective when Delivering Math Outcomes

Terry Lakey

Editor's note: this article was previously published in the winter 2018 issue of First Nations, Métis and Inuit Education Newsletter.

Red Deer Public Schools has approximately 900 First Nations, Métis and Inuit students. It is important that these students see themselves reflected positively in the classroom and in the curriculum. Some teachers do this by emphasizing Indigenous culture as it comes up in their curriculum delivery; some have bulletin board displays; and some encourage students to connect to their culture and share with their peers as much as they can when they engage in classroom research.



Having students see themselves in many parts of the curriculum is a way to boost their engagement and build a sense of belonging. This can instill a sense of pride in Indigenous students, encourage more dialogue and serve as a catalyst for teachable moments for the rest of the class.

The model used in First Nations, Métis and Inuit Learning Services at Red Deer Public Schools takes several approaches. The goal is to provide curriculum service delivery and support to cover as many grades as possible, making as many connections as possible. It is a work in progress.

Collaboration with teachers and administrators is essential. School personnel can approach First Nations, Métis and Inuit Learning Services with a question, a possible connection that they see or an opportunity for infusion. When an idea is brought forward by a district teacher, it is developed, refined and delivered to students. Then it is kept in a cache to be shared with others who might be interested. In the other approach, First Nations, Métis and Inuit Learning Services staff create curriculum connections, focusing on areas where infusion is possible, and then approach teachers and ask them to pilot the ideas.

The ideas for infusion are diverse and far-reaching. When a math outcome or concept is being taught, it often fits into other subject areas. Making cross-curricular connections is an excellent way to teach math, social studies, art, physical education and English language arts—all at the same time. It is also an excellent way to implement technology outcomes.

A great advantage to using Indigenous games in the classroom to teach math outcomes is that it usually involves movement. Students appreciate activity and movement whenever possible in the

classroom, and they can explore concepts through being active. This is an excellent method for keeping kinesthetic learners engaged longer. Students participate in the movement activity that leads to the learning and then reinforce their learning through paperwork, technology or class discussion. Through playing various games, students can gather data through physical activities. No textbook required! The teacher can then centre a lesson on the information gathered. For example, Google Sheets can be used to sort, analyze, organize and visualize the data. One- and two-step word problems can be created. *Mode*, *median*, *mean* and *range* can be understood. Measurement, time, greater than and less than, and base 10 blocks can all be covered. The list is endless when it comes to classroom application.

Teaching Indigenous games has a strong connection to the district's numeracy goal. However, it also capitalizes on an opportunity to expand the requirement of applying foundational knowledge about First Nations, Métis and Inuit in Alberta's revised Teaching Quality Standard (TQS). This foundational knowledge and the revised TQS are on the horizon for teachers in September 2019.

Using Indigenous games is a practical way to broaden the classroom experiences of both the teacher and the students when teaching math outcomes.

Terry Lakey is currently working with First Nations, Métis and Inuit Learning Services in Red Deer Public Schools. Prior to joining Red Deer Public Schools, in 1997, he taught with Northlands School Division. Since joining Red Deer Public Schools, he has taken on several roles. As well as being a classroom teacher for years, Terry has been involved with the development of First Nations, Métis and Inuit curriculum resources for many years. He uses the resources for his own classroom and enjoys sharing his ideas with others. He has a passion for making classroom learning engaging and hands on, and for accommodating a wide variety of learning styles.

MET Legacy Series: Theoni Pappas

Richard Seitz

NCTM's Mathematics Education Trust (MET) relies on contributions from council members and other individuals, affiliates and corporations. Gifts to the Theoni Pappas Fund support the Connecting Mathematics to Other Subject Areas Grants (9–12), which help teachers of Grades 9–12 to develop classroom materials or lessons that connect mathematics to other disciplines or careers.

Overview

This set of notes comes from e-mails, conversations and an interview with Theoni Pappas in Orlando, California. Early in 2016, we had the opportunity to visit about what it is like to write about mathematics. Pappas has written many books about mathematics, and she is known for her ability to take topics, present them with visual appeal and make the mathematics truly come alive.

The Mathematician and Mathematics Teacher as a Writer

As a young graduate in mathematics, Theoni Pappas made a decision that resulted in the start of her writing journey as an author who specializes in bringing alive the fun side of mathematics. She remarked, "When I graduated from UC Berkeley in 1966, I decided to become a teacher, so I entered the STEP education program at Stanford rather than pursue postgraduate work in numerical analysis. I know if I had not become a teacher, I would never have written math books. Instead, I would have become immersed in a specialized field of math." Since that time in 1966, we have seen a prolific collection of books that aspiring authors in the realm of mathematics writing would do well to read. Below are questions from the author of this article, followed by Theoni Pappas's responses in her own words.

Richard Seitz: It seems unusual for a calculus teacher to take the career path of becoming an author of mathematics books (including ones for younger readers). What prompted you to take on this challenge?

Theoni Pappas: I am not a calculus teacher, but rather a teacher who loves teaching and has a passion for mathematics. I like teaching any math subject, from arithmetic to calculus, and enjoy working with students in all levels of math. While in college, I learned how much I liked teaching while helping friends cope with their required math courses. From my first day of teaching mathematics, I realized I was teaching a subject that was probably the least popular in the school's curriculum and knew barriers had to be brought down if I was to be successful. I found that students usually entered the class with apprehension and fears about math. In my quest to break down math phobia, I sought out math concepts that would spark curiosity. I wanted students to ask, "How's that possible?" "How can that be?" "That can't be math." Over the years, the inevitable question all math teachers hear: "When and where am I going to use this math?" or "What does that have to do with my life?" became less and less frequent. Breaking down barriers and working with students to make them feel confident in math class is one of the hardest and most important things for a teacher to accomplish without compromising the subject matter. This was my challenge, even as it is the challenge of all teachers.

I knew what I was doing was exciting, and I wanted to share it not only with my classes but with a wider audience. One of the first books I wrote and designed was far from the typical math book of that time. Its title alone was unique: *The Joy of Mathematics.* Few lay people ever linked joy with mathematics. Rarely had a math author ever dared to "lower" the status of mathematics to the layperson's level.

Seitz: How would you say your writing differed from the traditional writing in math books at the time?

Pappas: There was an elitist attitude among math people that math books had to be written in a prescribed way, or they were not math. I purposely designed *The Joy of Mathematics* to be opened at random, copying how one discovers articles in a newspaper. The reader did not have to know what went on in the previous pages. Each topic was short and self-contained, with lots of visuals including photos and graphics. I wanted to make it as user friendly and eye opening as possible. I wanted to introduce the world of mathematics in a familiar yet different way. I knew I was entering unchartered areas, and the project might have just been criticized and/or ignored. But, I knew it was important if inroads were to be made in overcoming math phobia and the threatening feelings mathematics can conjure up. This explains why so few people could or wanted to wrap their minds around what they perceived as its abstract, cold and alienating ideas.

Seitz: Where did your writing and energy take you next?

Pappas: Next, I introduced *The Mathematical Calendars,* an active approach to mathematics in small doses. It featured short math articles each month and a problem a day for each date that ranged from arithmetic to calculus. I tried to build success into the problems, and even though the hitch that the solution was staring you in the face, the challenge remained before you. Could you figure out how to solve it? How soon would you realize the problems' answers were the dates? The calendar was extraordinarily popular, not only as a math calendar but in general; it was a tremendous success. In fact, the Smithsonian has said it was their bestselling calendar through the years. Its success made me realize that Yes! people were thirsty for more math. It all depended on how it was presented. Hence, I also introduced *The Children's Mathematics Calendars* and *The Mathematical Engagement Calendars*. From here, I and one of my colleagues formed a company called Math Products Plus, deciding to present mathematics on things people were used to using in their everyday lives. Hence, the math T-shirts were born, along with math mugs, math post-its, bumper stickers, buttons, posters etc. In essence, we helped math enter the mainstream! It was all cutting edge! It was exciting.

Seitz: I also understand that your writing took a turn into poetry and stories. Can you tell us about that?

Pappas: I feel there is no area in which math cannot be presented. For example, when I first heard a poem from the book *Joyful Noise: Poems for Two Voices,* I felt that this could be a new venue for mathematical ideas. I immediately sat down and began writing *Math Talk: Mathematical Ideas for Poems in Two Voices.* It found its audience, and it has become a mainstay in classrooms at all grade levels. Whenever and wherever I get an exciting idea on how to introduce math, I waste no time moving on it.

My work for young readers evolved as I was asked to help the children of friends and relatives with their math. I really like creating stories in which math ideas are the main characters. In addition, my cat and his mischievous behaviour were influential in introducing Penrose the math cat as the hero of many stories for children. Penrose made it a point of intruding on my math work, so eventually Penrose became a math protagonist in youth books. His books, *The Adventures of Penrose the Mathematical Cat*, *The Further Adventures of Penrose* and *Puzzles from Penrose,* have been extremely popular. Penrose even has his own fan club! I then began writing a number of math stories geared for an adult audience. Readers have told me that they forget they are reading about math as they get caught up in the stories in *Numbers and Other Math Ideas Come Alive.*

Today my math books have been translated into many languages and are attacking math phobia on a global level. My mathematical journey was not easy, but it has been fascinating, fun and exciting. Trying to introduce the beauty of math to more and more people is a never-ending journey. I am happy to see that my approach and book designs are no longer the exception but are being imitated by many writers. I am both flattered and encouraged that the beauty that mathematics holds is shared by more and more people. I am happy to have made a contribution.

Seitz: I understand you enjoy the outdoors, along with a wide range of other pursuits—painting, music (especially the flamenco guitar) and gardening. Has your love of mathematics been influential in your other activities?

Pappas: I can't say math has directly influenced activities I pursue, but I have an insatiable curiosity and love of learning. The best part of learning is the joy of discovery, of experiencing that aha! moment. I am always on the lookout for things related to mathematics, whether I find it in one of my interests or simply walking down the street. Nowadays it's wonderful to see math in our lives in so many places. It's in TV ads, on billboards, on public transportation and, especially, in the movies. It's almost a cliché, but I firmly believe math is everywhere. One doesn't have to look too deeply to find its connection to the foods we eat, the music we hear, the art we see. I never made a conscious decision to go for a hike and see what math ideas I might run into. But I know they're there, and I always keep an open mind. The same is true about my other areas of interests/activities. Everywhere I look, I see a math connection. Sometimes something is right in front of our noses, yet we don't see it. I like to write about the math I see, whether it's in mundane and familiar territory or in the complex and abstract. It's all exciting, but people are more likely to explore and read about mathematics in familiar realms.

I have always loved music and have enjoyed trying to play various instruments, from the violin as a child to trying any instrument I could lay my hands on. What better nonthreatening area to introduce mathematics than in the music and sounds that touch our lives? I loved researching the math of music and introducing my students and readers to what I had the joy of discovering. I found the music in math and math in music. For me, one of the most intriguing math/music links I learned was about music and string theory: how the vibrations of multidimensional infinitesimally small strings are believed to describe elements.

The same motivation was true for the many art/math topics I wrote about. Art, like music, is an area in which most people are comfortable and willing to explore. Which of us did not delve into art as children with finger painting and crayons? We never suspected to find math there, but it is, and that's what makes it a tantalizing area to write about. Granted, many artists may not create their works using math, but somehow it can be found in their works. Regardless, whether an artist intentionally uses mathematics in their art, such as Leonardo di Vinci or Sol de Witt, or does not give it a thought, I strongly feel it inherently exists in art. I personally chose the media of watercolour because of how water wreaks chaos with paint and art emerges.

I love to garden. Gardening is kind of a Zen experience. My mind often wanders as I'm digging or weeding, and it was during one of these times that I wrote the adult story about a gardener who is oblivious of the mathematical ideas growing in her yard. [The mathematically annotated garden appears in the book *The Magic of Mathematics*.]

For me, mathematics is more than a set of tools—it's a world unto itself, and I love to introduce people to its many areas. My main goal in my math books is that I want people to realize you don't have to be a math enthusiast to be excited by math ideas. All my activities have opened up areas in which to examine math and to communicate how I feel about it. I hope my work invites people to join me on mathematical journeys.

Seitz: Your writings include a wide variety of topics and problems across a spectrum of mathematics. What advice would you give a young teacher on how to acquire and develop a rich repertoire of mathematical ideas?

Pappas: I would encourage every teacher to follow his or her instincts and passions. If they follow what they enjoy learning, their enthusiasm will shine through to their students.

I am convinced you can find mathematical connections to any topic that interests you.

- Read or listen to a newspaper story, an article, a book, and find how it leads in numerous mathematical directions.
- Take notes and compile articles you might like to pursue when you have time.
- Randomly name 10 things you see or do during a typical day and see how many of these have something mathematically linked to them.
- Keep records and notes of things you think will work for you and your classes.
- Don't be afraid or intimidated to follow in a nontraditional approach or direction.

I would have never written the books and produced math products if I was concerned about what would be considered acceptable or conforming. Keep current on evolving math ideas. It was not long ago that most people had no idea what a fractal was or what the butterfly effect was all about. Work in conjunction with students. Let students be your scouts. They will love it and discover the art of exploring and researching. Challenge yourself and your students to look beyond your comfort zones. Delve into areas you may have never heard of or lack background in.

When I started teaching math, I had no idea that my passion for math would lead me to where I am today. I have been fortunate to be able to pursue the dream of helping to change the image of mathematics from one that was all too often considered cold, sterile and elitist to one that is exciting, fascinating and connected to so many things in our world. Now more than ever, math is achieving a new status. The digital age and so many other mathematical and scientific discoveries have helped elevate the importance of the role of mathematics in our lives. I have tried to write books that point to where mathematics is, has been and where it's going to lead us. Math is in the mundane, the complex, and the real and abstract worlds. I encourage you to seek it everywhere.

Seitz: What are some of your favourite mathematical topics?

Pappas: I'm just passionate about mathematics and discovering its many facets and quirks.

At the top of my list are non-Euclidean geometries. I especially like to explore the histories of their evolutions. For example, how Riemann introduced the ideas of elliptical/spherical geometry in his 1854 lecture. The story of how trying to see whether Euclid's parallel (fifth) postulate is actually a postulate and how this quest led to uncovering non-Euclidean geometries. Among the many stories studied, I found the one about how Saccheri's doubt in his own work delayed the discovery of hyperbolic geometry by nearly a century poignant.

Another area I like to explore is the role problems played in the development of new math ideas—how unsolved problems of mathematics challenge and lead mathematicians on many journeys and to new math ideas. I like learning about their successes and failures and periodically looking into the progress of various problem lists, such as the Millennium Prize problems or David Hilbert's list of 23 problems.

I am intrigued by types of numbers that have evolved and continue to evolve over the ages. For example, the Feigenbaum constant and its connection to chaos theory. How imaginary numbers and quaternions emerged and now influence digital graphics. The Cantor and the transfinite numbers is another great story in the history of mathematics. The topics seem endless. Many of these topics offered fertile ground for my book *Mathematical Scandals*.

My first book was *The Joy of Mathematics*. Most subsequent ones [*More Joy of Mathematics; Math Stuff; Math Snippets; Mathematical Footprints*] are designed in the same way I discover mathematics. They feature short topics to introduce the reader to ideas in the world of mathematics. There is no order to how they are presented. I just want people to be able to open the book at random and be introduced to a topic independent of what preceded it and discover something that might spark their curiosity further so they may explore it in depth. I think not needing to read these books linearly added to their popularity and made them more approachable.

Seitz: Can you recommend some topics that middle school teachers or high school teachers should introduce to their students?

Pappas: I really feel that most topics in my books and calendars can be used by middle or high school teachers. I like to choose "exotic"-sounding topics in order to catch the students' attention. I like to select topics in which all students in the class can be successful. Their past math background is not important, and most topics will probably be new to all the students in the class. Here's a short list of what I have enjoyed teaching for grade levels from Grade 4 through Grade 12:

- Fractals explored through paper folding
- Spherical non-Euclidean geometry
- Multiple dimensions and the hypercube
- Tessellations and art and design
- Time-travelling through the evolution of number systems
- Discovering the cycloid
- Discovering what the Möbius strip is all about
- Discovering mathematical ideas through paper folding, from geometric properties to the golden rectangle
- Exploring paradoxes
- Topology
- Magic squares
- What do you see? An introduction to optical illusions
- What makes some problems' solutions impossible?
- What is mathematics?
- How are the Fibonacci numbers, the golden rectangle and the equiangular spiral connected?

I am delighted that many of these topics have begun to enter today's classroom curriculum.

Seitz: What advice would you give to someone who wanted to follow your lead in becoming a mathematics author?

Pappas: Enjoy and care about what you do. Find out what ideas, areas, fields really excite you, and pursue them. Whatever you choose to write about, immerse yourself in the topic. Try to find a new way to present your ideas. It's important to find your own niche.

Theoni Pappas Biography

Theoni Pappas is committed to giving mathematics greater exposure and making it more approachable. Pappas encourages mathematics teachers to share and develop new teaching ideas, methods and approaches. Her gift to the Mathematics Education Trust (MET) helps teachers in Grades 9–12 develop mathematics enrichment materials and lessons complementing a teaching unit implemented in the classroom.

Currently, Pappas is a mathematics educator and consultant. She received her BA from the University of California at Berkeley and her MA from Stanford University. She became a member of the National Council of Teachers of Mathematics (NCTM) in 1967, when she began teaching high school mathematics. Over the years, she has taught basic math, prealgebra, algebra, trigonometry, geometry, precalculus and calculus.

Through her studies and research, Pappas has developed products that address mathematical ideas and she has written numerous books, for both the general public and educational audiences. Her books include *More Joy of Mathematics; The Joy of Mathematics; Mathematics Appreciation; Math Talk; Greek Cooking for Everyone; Fractals, Googols, and Other Mathematical Tales; Mathematical Footprints; The Magic of Mathematics; Math-A-Day; The Music of Reason; Mathematical Scandals; The Adventures of Penrose, the Mathematical Cat;* and *Math for Kids and Other People Too!*

Well, This Changes Everything, Or It Should

Chris Reed

This article has been adapted for print from an online blog post written for a university master's class and is reprinted with the permission of the author. Minor amendments have been made in accordance with ATA style. To access the original post, go to https://exploringspaceswithchris.wordpress.com/2017/11/25/well-this-changes-everything-or-it-should/.

This week in ECI 831, we were tasked with finding a tool or an app that we had not regularly used for education and reviewing it. The next day, I was talking with an amazing math teacher at my school and he said, "Check out this app, Chris. You take a photo of your math question and it solves it." I was impressed, and then he said, "Just wait. You cannot even get around this by telling the students to show their work, because this app shows it step by step. I am going to have to change how I do assignments." The bell rang and we both went to class. I knew immediately that I was going to have to review this app.

The App

The app in question is Photomath, available for free download for both Android and iPhone. To download it, go to the Google Play store or the App store for Apple.

Does It Work as Claimed?

The first thing I wanted to do was test whether the app worked as it claimed. I strongly recommend that you take a moment to watch the demonstration video that the developer produced (https://vimeo.com/147764920). The video shows a student using the calculator to verify that the work that she has done on paper is correct. It also shows her scrolling through the step-by-step process to check to see if what she did step by step matches. So now we know what to expect.

I tested this by printing off some math worksheets for multistep equations. I also went to the Photomath website (https://photomath.net/en/), where they have a list of equation examples that you can print. I also wanted to test it on handwritten samples to see how good it was at picking up printing. Here is how it did on each of these hurdles.

The Examples from Its Own Website

- Addition and subtraction—It had no problems at all and showed really detailed step-by-step instructions.
- Multiplication and division—Again, it had no problems, and that includes testing it with fractions. The step-by-step instructions also could be expanded to include more detail if needed.
- Complex arithmetic operations—It worked perfectly again.
- Factorization and algebraic fractions—It worked through complex problems without issue.

- Linear equations with restrictions—Again, no problem, and it graphed the solutions as well.
- Systems of linear equations—Here, it did a couple of things: it solved the equation using multiple methods, and it gave you options about which method you wanted to look at. It also, of course, graphed the solution.
- Integrals and derivatives—No issues. Also, the step-by-step illustration was really helpful to me, since it has been a few years and my calculus is rusty.

So yeah, it had no problems with the questions they provide as an example, but that makes sense. It better have no problems with their own stuff.

A Third-Party Worksheet

- I ran it through a selection of worksheets from a program that I had purchased years ago. It did not matter that the questions were laid out in a different format, that the variables used were different letters other than x, y, a, b, and c, or that the font was different. It did just fine.

Handwritten

- I did not do an exhaustive test of the app's handwriting detection. I just wanted to see if reasonably neat handwriting would be recognized. It did pretty well. Some things that I noticed were that you need to remember the little things like putting the degrees sign on your trigonometric functions. It liked *sin* (30°) but treated *sin* (30) as a different question. Also, it handled both forms of 4 (with the closed and open top), it handled curved 9 and straight 9, and it handled normal 7 and crossed (drafting) 7. It thought a crossed (drafting) zero was a theta, which to be honest looks exactly like a theta and more often than not *is* a theta.

For any mistake that the photo part made, there was a manual equation editor that you could use to fix it, and then it would solve the new fixed equation.

Verdict

Photomath works as stated. While there are some math questions that it could not handle yet, like graphing systems of inequalities, the app is continually being revised, and I would be very surprised if within a couple of years it couldn't do every kind of math equation that any high school student would ever face (minus the word problems, maybe).

Teacher Reaction

I wanted to know if my math colleagues thought this was a good or a bad thing. Also, I was curious how the existence of this app was affecting the math class that my colleague teaches. So on Friday, during lunch, I tracked him down and asked him about it. The first thing he asked was if I had shown [it to] any students. I hadn't. That calmed him. He said that he knows that students are going to come across this app likely sooner rather than later so he is trying to figure out what he is going to do. He already has his gradebook set up and is halfway through the semester. There is work that he expects to be done at home, and for at least this semester, some of that work counts for marks. He said that he already suspects that there are some students that might be using it because their homework is so much better done than their in class work, but he does not think it is widespread yet. He thinks this since so many students are still doing poor quality homework (a good thing in this case?).

When he said all of this, I became curious about how *he* had heard about the app. He said that he was searching for math apps to create practice quizzes and focused study help for students that were struggling. He uses a lot of the kinds of programs that we have talked about in class, things like Kahoot, an interactive quiz website (https://kahoot.it/), Socrative, a learning management system (www.socrative.com/), and more, so I am not surprised that he found this app before many others have.

He also said that while it is causing him to rethink how homework works, this was more of a wake-up call than an all-out crisis. He says you already have no guarantee that the student is the one doing the homework, and that spotting plagiarism in math is next to impossible, whereas in other subjects you at least have a chance of spotting it. The homework portion of his mark setup is relatively low, with in-class work and tests making up 80 per cent of the students' mark at this time. He was already leaning toward removing any mark for work done at home, but now he knows that this is the last semester he will ever give marks for homework.

So Now What?

If you are a math teacher, this kind of uncomfortable truce with technology is nothing new. Many of us grew up with teachers telling us to put away the calculators and to learn things by rote. Most of the current generation is growing up with a calculator that they literally carry everywhere with them. There is a lot of debate/soul searching happening about math curriculums in Canada and the world right now. The *National Post* recently ran an editorial about discovery math and why people want to get rid of it in Canada (Csanady 2016). It is not just Canada asking this question, either. The

BBC wrote an article that asks if every country should use the Shanghai method (Low 2017).

So what is the solution? If you asked this question in a coffee shop, everyone would have an opinion, and many of them would focus on the idea of back to basics, or doing more practice. This was certainly the opinion of the ministry of education in Saskatchewan in the October 2017 throne speech (Yard 2017). You would even hear a lot of the phrase "Back in my day, we ..." The problem is that we are not back in anyone's day. The tools available to students now make earlier approaches impractical and unreliable, especially since students are unlikely to avoid the use of these new tools. So how do we work with the tools available?

I know that Photomath is too new to have any studies done on it, but I remembered that Wolfram Alpha was a computational search engine that could do similar things, and I remembered that the creator of it felt that math education needed to change. So I went looking for what he had to say because I knew that he would be thinking about the issue. Was he ever! In fact, he gave a TED talk about this way back in 2010 (www.ted.com/talks/conrad_wolfram_teaching_kids_real_math_with_computers?utm_campaign=tedspread--a&utm_medium=referral&utm_source=tedcomshare).

There is a lot to process in this talk, and I strongly recommend that you watch it. His presentation boils down to the idea that we are emphasizing the wrong part of math. He sees math as a four-step process: first, posing the right questions, then converting the real-world problem into a math problem, doing the computation and, finally, converting the math answer back to a real-world application. He argues that we are spending 80 per cent of our time doing step three, which he points out is the only step that computers can do perfectly. He argues that we should be teaching students to do steps one, two and four better. In fact, he did more than just give a talk about this—he started an organization, Computer-Based Maths (http://computerbasedmath.org/), which looks at how to teach math better using computers. Teaching computer-based math was an option in 2010, when he gave this talk; in 2018, it is essential.

I recommend that you watch the TED talk if you teach anything like math or science. I also suggest that you download the Photomath app, because you are going to want to start to understand how students could use this as a positive thing. I know that I am already thinking that next semester's physics class is going to look a lot different from last spring's physics class because of the existence of this kind of stuff. I am only one person in a whole school, though. I need to make sure that what I do is not going to cause huge headaches for others. So, in the next month, I need to talk with all the math and science people in my school and figure out "Now what?" I think that Conrad Wolfram might just have a possible path forward for us.

Let me know how this affects you. I foresee good things potentially, but it will require us to change. Continuing as we are is definitely not the correct option.

References

Csanady, A. 2016. "Canada Has One of the Best Education Systems in the World, So Why Can't We Stop Debating Discovery Math?" *National Post*, Dec 7: npn. Available at http://nationalpost.com/news/canada/canada-has-one-of-the-best-education-systems-in-the-world-so-why-cant-we-stop-debating-discovery-math (accessed October 2, 2018).

Low, H. 2017. "Should All Countries Use the Shanghai Maths Method?" BBC News, January 20. Available at www.bbc.com/news/magazine-38568538 (accessed October 2, 2018).

Yard, B. 2017. "Throne Speech Suggests Math Overhaul for Saskatchewan." CBC News Saskatoon, October 27. Available at www.cbc.ca/news/canada/saskatoon/low-math-scores-throne-speech-1.4375213 (accessed October 2, 2018).

Chris Reed is a high school physics and science teacher at Regina Christian School, an associate school of the Regina Public School system. While teaching, he is pursuing a master of education in curriculum and instruction at the University of Regina, with the goal of someday being able to write a computer science curriculum designed around robotics. He is also a husband and father and, like every good Saskatchewan resident, a fan of the Saskatchewan Roughriders.

The Alberta Teachers' Association

Consent for Collection, Use and Disclosure of Personal Information

Name: ____ (*Please print*)

☐ I am giving consent for myself.

☐ I am giving consent for my child or ward.

Name: ____ (*Please print*)

By signing below, I am consenting to The Alberta Teachers' Association collecting, using and disclosing personal information identifying me or my child or ward (identified above) in print and/ or online publications and on websites available to the public, including social media. By way of example, personal information may include, but is not limited to, name, photographs, audio/video recordings, artwork, writings or quotations.

I understand that copies of digital publications may come to be housed on servers outside Canada.

I understand that I may vary or withdraw this consent at any time. I understand that the Association's privacy officer is available to answer any questions I may have regarding the collection, use and disclosure of these audio-visual records. The privacy officer can be reached at 780-447-9429.

Signed: ____

Print name: ______________________ Today's date: ______________________

For more information on the ATA's privacy policy, visit www.teachers.ab.ca.

Publishing Under the Personal Information Protection Act (PIPA)

The Alberta Teachers' Association (ATA) requires consent to publish personal information about an individual. Personal information is defined as anything that identifies an individual in the context of the collection: for example, a photograph and/or captions, an audio or video file, artwork.

Some schools obtain blanket consent under FOIP, the *Freedom of Information and Protection of Privacy Act*. However, PIPA and FOIP are ***not*** interchangeable. They fulfill different legislative goals. PIPA is the private sector act that governs the Association's collection, use and disclosure of personal information.

If you can use the image or information to identify a person in context (for example, a specific school, or a specific event), then it's personal information and you need consent to collect, use or disclose (publish) it.

Minors cannot provide consent and must have a parent or guardian sign a consent form. Consent forms must be provided to the Document Production editorial staff at Barnett House together with the personal information to be published.

Refer all questions regarding the ATA's collection, use and disclosure of personal information to the ATA privacy officer.

Notify the ATA privacy officer immediately of ***any*** incident that involves the loss of or unauthorized use or disclosure of personal information, by calling Barnett House at 780-447-9400 or 1-800-232-7208.

Maggie Shane, the ATA's privacy officer, is your resource for privacy compliance support.

780-447-9429 (direct)
780-699-9311 (cell, available any time)

MCATA Contacts

President
Alicia Burdess
aliciaburdess@gpcsd.ca

Journal Editor
Lorelei Boschman
lboschman@mhc.ab.ca

ATA Staff Advisor
Lisa Everitt
lisa.everitt@ata.ab.ca

Complete contact information for the MCATA executive is available on the council's website at www.mathteachers.ab.ca.

ISSN 0319-8367
Barnett House
11010 142 Street NW
Edmonton AB T5N 2R1