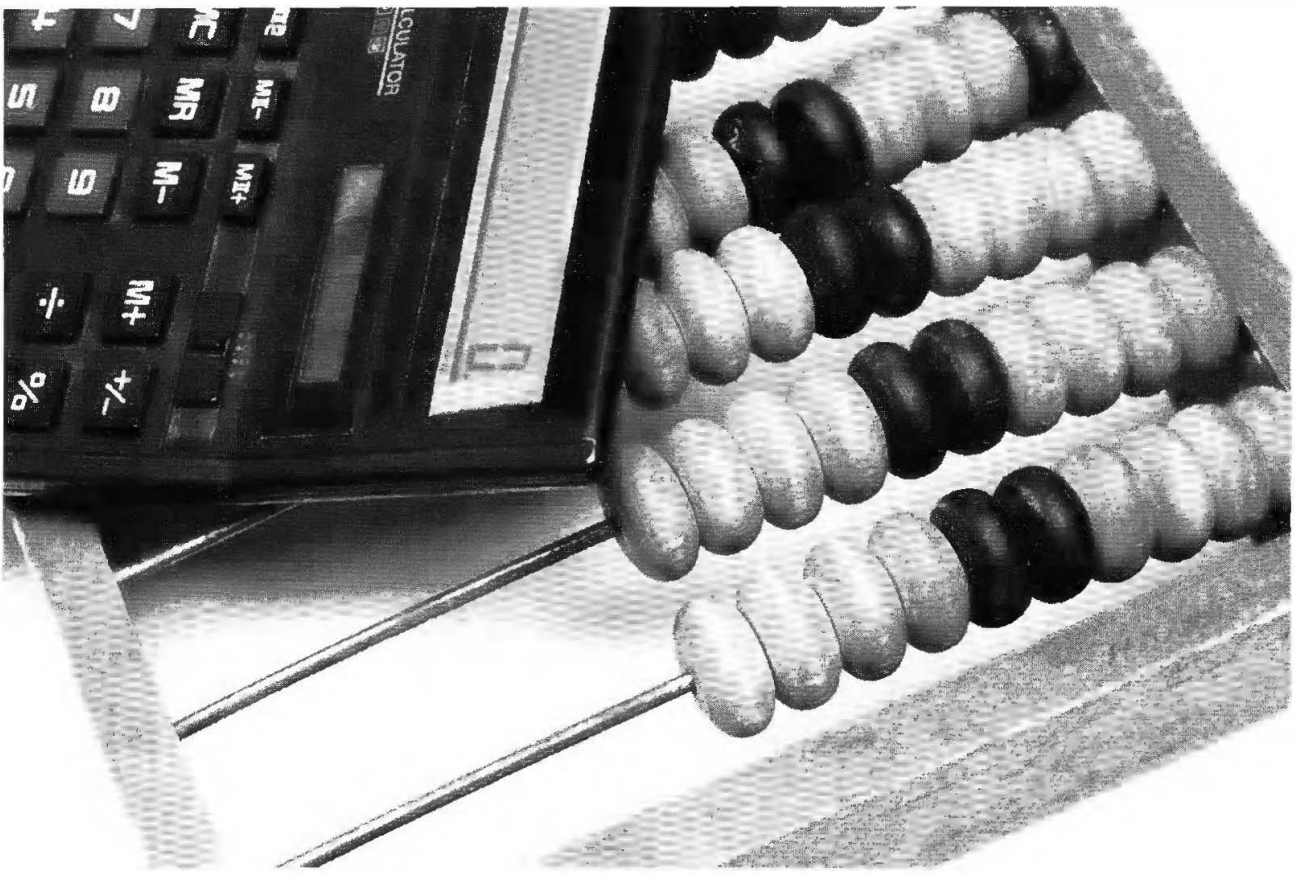
Why Use History in a Mathematics Classroom?

Glen Van Brummelen



We've all experienced the problem. Given the mass of material we are required to cover in our math classes, it seems all but impossible to find avenues for creativity in our lectures. When small time windows open up, we tend to show extra problems, or new applications, or some favourite theoretical wrinkle that we had been saving for such an occasion. Why bring in history? It takes time and effort, and displaces other subjects. What's the advantage?

Simply put, history provides a path for the entire mathematical experience. Typically, our students are asked to solve problems and prove theorems, a limited part of what mathematicians do. The full story involves motivation: what is the context within which the subject arose, and why is it so appealing that it deserves our attention? Next is research: once the problem is identified, how do we articulate lines of attack that have already been made that might be adapted to the new situation? Third is critical thinking: how do we transition from received knowledge to new situations? Finally, we have implications: how does the solution affect us, the academic community or society? Good history of mathematics synthesizes all these aspects. Bringing it into the classroom can provide for our students a much broader and deeper mathematical experience. Most crucially, history is a natural means to attain these goals: we follow real people, who struggled as our students do, and eventually (usually) triumphed. We learn best through stories, and true stories are often the best ones.

Some Examples

Motivation

All mathematical subjects arose due to some need, either from within mathematics or from outside of it. Trigonometry was invented in ancient Greece to convert geometric models of the motions of the planets into quantitative predictions. Today, it is still a significant tool for moving back and forth between geometry and numerical measurement. Now, the need for some subjects may not have been the same in the past as it is today. Logarithms, for example, were invented in the early 17th century as a calculation device for astronomers to reduce the work involved in finding products and roots of numbers. Today our computing power renders this use obsolete. Nevertheless, this historical route can provide a meaningful context for students' first exposure to the subject; the benefits of the theory are obvious, even if its original motivation is no longer active.

Research

Coming to terms with methods that have been devised to attack difficult problems is, by definition, a study in history. Examples begin as early as ancient Babylon, where the geometric practice of "completing the square" seems to have led to solutions of problems related to quadratic equations. In differential equations, the discovery of a function that is its own derivative ($f(x) = e^x$) was exploited repeatedly to solve many problems from the early 18th century onward, and it even helped lead to a unification of exponential and trigonometric functions through complex analysis.

Critical Thinking

Every mathematical community makes shared decisions about the validity and power of various competing approaches. For instance, medieval Indian mathematics valued solutions that we might describe as approximate or iterative, while ancient Greek and medieval Islamic mathematicians preferred direct arguments and calculations. One of the most difficult concepts for modern students to understand is that such commitments are also present today. In order to think creatively, one needs to make informed judgments about alternate avenues of attack; one must know what the community's rules are before one decides to bend or break them. An example of such a struggle is the work of the 12th-century Iranian astronomer al-Samaw'al, who rejected an instance of ancient Greek use of approximation to calculate trigonometric tables. Nevertheless, he still needed to produce the tables. His creative solution was to redefine the base circle to contain not 360, but 480 degrees, which bypassed the need for approximation!

Implications

It is often said that the most powerful mathematical results are those that lead to new and interesting questions or that open mathematics to new applications. Witnessing the enlargement of the social role of certain types of mathematics can be a meaningful lesson in measuring its cultural significance. For instance, in early modern Europe the unification of trigonometry with logarithms brought mathematics into the hands of surveyors, architects and navigators. This transformed mathematics from a primarily theoretical discipline into an engine that eventually helped to reshape modern culture through science and technology. Students aware of these cosmic shifts are better able to place themselves in the intellectual landscape and to act in their profession with more reflectiveness.

There is one additional aspect of mathematical work that history can support: communication. Since history encompasses entire narratives from initial conception to final product and societal impact, there is a unique opportunity here to improve students' ability to write and otherwise present ideas. Students can write essays; they can make presentations on the background and significance of subdisciplines; they can write short-answer responses to questions about the significance of and interconnections between theories. It is usually difficult to find opportunities to improve mathematics students' rhetorical skills; history provides a powerful solution.

Challenges

Although the potential benefits of history are diverse, several dangers must be avoided.

- Misunderstanding history as mere biography: Textbooks often give snapshots of mathematicians' lives and works in the page margins, mistakenly believing they have done a service to history. They have not. Many of these biographies are unrelated in any direct way to the narrative in the text, so they unintentionally reinforce the tacit misconception that the mathematics itself is ahistorical. Genuine history in the classroom should be part of the presentation of the mathematics; its benefits can only be realized with deeper integration.
- Entering history without sufficient depth: The history of mathematics is a deeply challenging

endeavour, requiring sophistication in two disciplines with very different aims and modes of thought. Unfortunately, not everything one finds in the library or online is reliable, either historically or mathematically. The mathematics teacher should consult reliable sources; looking up reviews in professional journals is an effective way to screen out low-quality content.

- Assuming that history is a universal panacea: Although history is helpful in learning many mathematical concepts, assuming that it always leads to positive results is dangerous. Choose moments where the historical context genuinely interacts with the subject and is appropriate to students' concerns and maturity levels.

Places to Start

For topics in the undergraduate curriculum, there is no better place to begin than Victor Katz's history of mathematics textbooks. For accuracy, mathematical rigour and thorough coverage, they are unsurpassed; and they provide many connections to the rest of the literature. At an elementary level, consider William Berlinghoff and Fernando Gouvea's *Math Through the Ages* (2nd edition). Finally, the MAA Notes series has published a number of volumes of historical episodes ready for classroom use, edited by Victor Katz, Amy Shell-Gellasch, Dick Jardine and others.

Many modern theories of education attempt to address the plague of passivity in our students by promoting active educational experiences, such as the Moore method and inquiry-based learning. History provides the kind of engagement these innovations attempt to foster. However, history can also enhance the traditional mathematics classroom. By considering the entire cycle of mathematical development, and by asking students not merely to perform calculations but also to reflect upon them, history makes students more powerful, more thoughtful and more significant. In short, it makes them better mathematicians.

Glen Van Brummelen is coordinator of mathematics at Quest University in Squamish, BC. He is (twice) past president of the Canadian Society for History and Philosophy of Mathematics, and is currently the MAA governor-at-large for Canadian members. He is author of The Mathematics of the Heavens and the Earth: The Early History of Trigonometry *(Princeton 2009) and* Heavenly Mathematics: The Forgotten Art of Spherical Trigonometry *(Princeton 2013), the latter based on his course at Quest.*

Reprinted with the permission of the Canadian Mathematical Society; this article was originally published in CMS Notes, *February 2015. Minor changes have been made to conform to ATA style.*

Comments from Your Executive

Like most things that are more than 2,000 years old, mathematics has a rich history that is replete with interesting characters and unlikely events. And yet, after 12 years of school mathematics very few people know any of that history. Indeed, even after four more years of mathematics at university, it is possible to know very little of its history. In the article, Professor Glen Van Brummelen gives some good reasons why the history of mathematics should be incorporated into our classes. There is also good advice on how to do it. Equally important are his words on how not to do it.

Indy Lagu is the postsecondary mathematics representative of MCATA and a full-time professor at Mount Royal University in Calgary.