ELL Students' Set-Befores and Met-Befores in Mathematics

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Recently, teachers and researchers alike have observed growing numbers of English language learning students (ELL) in American and Canadian classrooms (National Council of Teachers of Mathematics 2013). For example, Riel and Boudreau (2012) found that 15 percent of all students in Canadian classrooms do not have English as their first language. In Alberta alone, 17 per cent of all schools responding have ELL students. Of those Alberta schools, 34 per cent have at least 1 to 5 students, 39 per cent have 6 to 25 students, and 26 per cent reported more than 25 ELL students (Alberta Education 2006a, 2006b).

Not surprisingly, this demographic shift poses interesting challenges for Canadian teachers. Given that some ELL students may have received little or no formal instruction in their first language, the experience of school might be novel to them. Even for ELL students who have received prior schooling, there is the challenge of making sense of material in a language with which they are completely unfamiliar (Boaler 2008).

However, what is not obvious is that many ELL students can find a subject such as mathematics also challenging. Although mathematics is sometimes regarded as a universal language (perhaps erroneously), its structures and nuances pose a significant challenge to mathematics students—especially if they are learning mathematics in a second language. (Clark 1975; Barrow 2014). In fact, success in an English language-based mathematics classroom requires a variety of language and coding skills that go beyond merely learning mathematics (Barwell 2005, 2008; Barrow 2014).

In this paper, I aim to examine two things. First, I will look at what challenges ELL students face in terms of learning and understanding mathematics. This will be done by using some of the ideas of the respected English mathematics education researcher and theorist, David Tall, as a guide. Second, we will examine how mathematics teachers can make the task of mastering and understanding mathematics concepts and processes easier for these students.

Set-Befores and Met-Befores of English Mathematical Language

To assist in the discussion of some of the language and coding issues relating to ELL students, I will borrow some concepts and ideas from Tall's writings (Tall 2008, 2013; McGowen and Tall 2010, 2013), in particular the set-before and met-before. A *set-before* is a mental structure that humans are born with, which mature as our brains make early connections. In this category, Tall includes things like posture, identifying direction, social abilities such as gestures (eg, pointing at objects) and so on.

For math educators in particular, Tall (2008, 2013) identified the following set-befores as essential for mathematical understanding:

- The *recognition* of patterns, similarities and differences between mathematical concepts
- The *repetition* of sequences of actions until they become automatic
- The *use of language* to describe and refine the way we think about things

These three set-befores (recognition, repetition and language) form the basic skills required for learning mathematics in all of its forms. Note how the first and last in particular relate to language use. We will return to these in a moment.

In addition to the set-befores, we also need to introduce the idea of a *met-before*. For Tall, a met-before is a mental structure formed in an individual's brain based upon their previous experiences (ie, "built from experience that the individual has 'met-before'" [McGowen and Tall 2010, 169]). Though simple, the idea of a met-before can be quite helpful in dealing with mathematics, because met-befores can be supportive or problematic. A supportive met-before assists or facilitates the learning of mathematical concepts and processes; problematic met-befores, on the other hand, inhibit or make the learning of mathematics more difficult for the student (Tall 2008, 2013; McGowen and Tall 2010, 2013).

To illustrate, let us consider a phenomenon I have often seen in my junior high mathematics classes, when students first encounter the concept of multiplication of fractions. Initially, when students are first introduced to multiplication in elementary school, it becomes engrained that a small number times another small number gives a bigger number as a result (a supportive met-before). However, when students are first exposed to the multiplication of proper fractions in the junior high classroom, confusion often arises. This is because when multiplying proper fractions, the product has a much smaller value—an idea that does not appear to make sense to the students, given their previous experience (a problematic met-before).

Now, in turn, let us examine the set-befores and met-befores as they relate to ELL mathematics learning.

On the Linguistic Set-Befores and Met-Befores of ELL Students

Every student comes to class with his or her own unique experiences. But ELL students come to class with their own set-befores and met-befores that were formed prior to joining a classroom where the medium of instruction is English. This difference in background will greatly affect how the student interacts with the discourse and instruction in the mathematics classroom (Clark 1975; Cuevas 1984; Barrow 2014).

Indeed, per Tall (2013), this background, formed before entering the classroom, is important for the young student if they are to study objects correctly in a mathematical sense. Without the necessary English academic language (in an English-medium classroom), it becomes much more difficult for the students to make the steps necessary toward working in a world of conceptual embodiment, where they are able to take ideas introduced to them as they relate to the physical world and convert them into mental entities they can manipulate with their minds. Consequently, it will be very difficult for them to communicate their understandings to the teacher or to fellow students or to make sense of the materials before them.

Furthermore, in terms of language as a set-before for math instruction, it is a little more complicated, because there are two types of languages that the ELL student must master. First, there is the social language, which is the language of everyday social transactions. Luckily for the ELL student, it has been demonstrated that he or she usually has a useful working grasp of this societal language within two years (Cummins nd, 1979, 2001). However, at the same time as he or she is working to master the language necessary to function in society, the student must be also able to concurrently learn the mathematical academic language of the classroom. This academic language can take anywhere from five to seven years to master: the rich and complex vocabulary used, the unique technical jargon and symbols, the grammatical conventions unique to mathematical discourse, and specific reading techniques required to make sense of mathematical problems (Cummins nd, 1979, 2001; Collier and Thomas 1989; Slavit and Ernst-Slavit 2007; Alberta Education 2010).

At this point, it is worth keeping in mind that while the student is receiving instruction in a new language, he or she is engaged in a task of trying to compress knowledge into thinkable concepts in mathematics. In particular, the student is placed in a situation whereby he or she must decide whether to try and process the concepts in the student's native language (by first translating it) or in the new English language, or try to make sense of these ideas by using both language systems (sometimes referred to as *code switching*). This turning of the new mathematical knowledge into thinkable concepts is an important step, as noted in Tall (2013):

> Compression of knowledge enables us to think of essential ideas, without being diverted by unnecessary detail. Language facilitates this process by enabling us to name important aspects of complicated situations and talking about them to refine their meaning. This focus gives rise to a *thinkable concept*, conceived by the biological brain as a selective binding of neuronal structures, that allows us to focus our attention on it. (p 51; cf p 86)

This is particularly problematic for ELL students. Given the tug-of-war between using their original language and their new English language to categorize new ideas, encapsulate processes based on repeating actions, and define and formulate concepts for mathematical usage, it is not surprising that the research has shown that ELL students regularly create problematic met-befores as they try to make sense of the mathematics that is before them (eg, Lager 2006; Chamot et al 1992; Cuevas 1984; Bernardo and Calleja 2005).

Teachers could identify an ELL student's use and reliance on problematic met-befores by looking to see if the student is generating errors or mistakes via any one of the following pieces of evidence:

- Misusing common words or phrases in understanding word problems (Barwell 2008)—to illustrate, I have observed some of my past ELL students attempt to rephrase a given word problem in

English, but because their vocabulary is still developing, often the final question in the word problem would be read and misinterpreted by the student (for example, a question like "How many brown chickens does the farmer have *altogether*?" may lead the ELL student to try to work out the number of *all* the chickens—not just the brown chickens that the problem asked for).

- Creating and relying on faulty student-created diagrams—in the past, I have observed some of my ELL students quickly draw a figure to assist them in making sense of a question but, unfortunately, their rushed readings lead them to miss key details or words. This leads them to draw initial figures that may have the wrong dimension. For example, consider a question that asks students to find the volume of a circular swimming pool having a radius of 5 metres and a height of 2 metres, but unfortunately, in their work the students draw pictures of cylinders that have *diameters* of 5 metres. Then, after these students have found their answers, when they check their work they do not return to the original text but instead they depend solely upon their diagrams to verify their answers (see Lager 2006).
- Misinterpreting graphics—for example, a diagram of a right triangle may lead the ELL student to conclude that the base needed for an area formula is the largest side (ie, the hypotenuse), considering how it is situated on the page, when the actual base is one of the legs, even when both of the legs have provided numerical measurements (see Lowrie, Diezmann and Logan 2011).
- Not recognizing real-world constraints as they relate to word problems—for example, failing to check and notice that the answer a student provided would not be possible if they stopped and treated it as if it were a true real-world situation (Bernardo and Calleja 2005; Verschaffel, De Corte and Lasure 1994).
- Missing or neglecting semantic aspects of mathematical words, such as the difference between *divided by* and *divided into* (Lager 2006).
- Missing or misreading contextual cues that would suggest an alternative understanding of the mathematical meaning of commonly used words—for example, seeing the words *less than* in a word problem might tempt a student to leap to the conclusion that subtraction is required to find the solution when addition is actually what is called for (eg, Betne and Stanchina 2005).
- Confusing meanings for mathematical words that also have everyday meanings outside of the classroom—eg, *volume, table* or *power* (see Lager 2006; Moskovitch 2010).

These difficulties in understanding and using the language successfully in a mathematics classroom can lead to what Tall describes as an *epistemological anxiety*, or a "knowledge-based anxiety," for these ELL students (Tall 2013, 127). As Tall (2013) put it,

> Epistemological anxiety is a sign of inability to achieve the goal of relational understanding in mathematics. To relieve the frustration, the goal may switch to an instrumental understanding of being able to perform the requisite procedures ... with a level of success but a sense of underlying doubt. (p 127)

To clarify, Tall is suggesting that to avoid feeling uncomfortable when doing mathematics, students may be tempted to seek less cognitively demanding methods of understanding the mathematics before them. Thus, they will be enticed to focus on rote learning or on algorithms (ie, instrumental understanding) rather than choosing to build up the conceptual structure or schema needed to extend their knowledge beyond the task at hand (ie, relational understanding) in their work in class. In short, for an ELL student, although the rewards may be immediate and provide a quick and reliable method in a particular context, the success may be short-lived, in that the depth of the mathematical knowledge gained may not readily extend to future mathematics learning (Skemp 1976; Willingham 2009).

We now look at what could be done to help ELL students achieve success in our classrooms.

Designing Curricula and Assessing Progress of ELL Students

Looking at the above, it is clear that the goal of a good mathematics educator when working with ELL students is to promote the formation of supportive met-befores, while avoiding or preventing the formation and/or use of met-befores that could become problematic and thereby inhibit the progress of the student in understanding the mathematics being taught. For this, I offer four rules of thumb to guide teachers.

First, a mathematics teacher instructing ELL students must make efforts to ensure that problematic met-befores are avoided. This could be accomplished by ensuring that nonacademic mathematics language is avoided or minimized in problems, activities and instructions (Beliveau 2001; Lager 2006); ensuring that the language used in the classroom is suited to the level of ability of the ELL students—in particular, the teacher should ensure that the English used is more likely to be encountered by students in everyday life, and the use of passive tense should be avoided in word problems (Haag et al 2013); being cautious when the mathematical words used in the classroom are polysemic (ie, words that have two different meanings, such as *plane*, *square*, *point* and *volume*)—particularly if such words may commonly be encountered outside of the mathematics classroom (see Jarrett 1999; Dale and Cuevas 1992; Beliveau 2001); and drawing on as many resources as possible to assist the students in the formation of useful, viable and accurate mental images.

For example, realia (ie, objects from real life used in the classroom, such as using a pizza or a wheel to discuss fractions or circles), manipulatives (ie, hands-on instructional tools like fraction tiles or interlocking cubes), drawings and graphics (ie, to illustrate word problems), graphs (eg, from newspapers or magazines), gestures (eg, using a hand gesture to clarify which parts of an equation or geometric shape are being worked on) and making connections to the learners' own culture and community (eg, using a First Nations folk story to assist in the teaching of surface area or volume, or a Cree bead bracelet to discuss ratio and proportion) have all been found to be helpful in guiding ELL students (Moschkovich 2012; Nguyen and Cortes 2013; Barwell 2005; Civil and Menéndez 2010; Civil 2011; Arnason et al 2001). Further, it should be noted that providing materials in the ELL students' first language, where possible, has been deemed very helpful in promoting and furthering the students' mathematical understanding (Abedi, Hofstetter and Lord 2004; Moschkovich 2002, 2012; Barwell 2005; Clarkson 2005; Civil and Menéndez 2010; Civil 2011; Civil and Planas 2012; Nguyen and Cortes 2013).

Second, the mathematics teacher must make efforts to ensure that ELL students are guided through correct problem-solving techniques to allow them to personally filter through their met-befores and understand which certain meanings and concepts are fit to use in certain contexts. It has been acknowledged that instructing students how to tackle word problems is very helpful in guiding ELL students toward the learning of the English language and simultaneously mastering the academic language of mathematics (Cuevas 1984; Moschkovich 2012). As noted in Reyhner (1994) and Jarrett (1999), modelling and guiding ELL students through a systematic approach to problem solving is most helpful. To illustrate, let's consider this word problem:

> Allan's cat has a mass that is 2 kilograms less than Bert's cat. Together, their mass is 15 kilograms. How much do they each weigh?

When presenting such a word problem in the classroom, it has been found helpful to introduce ELL students to a comprehension technique, such as Polya's (1957) four-step problem-solving method (Al-Jamal and Miqdadi 2013). That is, initially the students must understand the problem (ie, the question denotes the sum of the masses of two cats, and they don't know the mass of either, only that one cat is two kilograms less than the other). Then the students must generate a plan to solve the problem (eg, assign the variables to the unknown weights of the cats, create the equation needed to solve, and solve for x where x is equal to the weight of Bert's cat); carry out the plan (ie, write out and solve the problem) and then check their work (eg, does the final answer make sense, based on how the question is worded? If one substitutes the found answer of 6.5 kg for Bert's cat into the planned equation, $x + (x + 2) = 15$, then does the equation still balance?).

Training the students to make sense of the text and carefully consider all the numbers, words and symbols present and their relations before attempting to solve the problem is invaluable (Adams 2003; Al-Jamal and Miqdadi 2013). It should be noted that the teaching of comprehension and problem-solving techniques to ELL students should not just focus on the keywords present. While teaching students to rely on the identification of keywords could help in some situations, such a technique could lead to the development of the formation of bad habits, create new problematic met-befores and hamper students' problem-solving skills (Carpenter, Hiebert and Moser 1983; Secada and Carey 1990). To draw on our example, imagine the potential confusion and difficulties a keyword-trained ELL student would have if he or she just focussed on the words, *less than* and *together* (ie, depending on how the words are used, *less than* can refer to a difference in value, an inequality or subtraction, while *together* can refer to an equality or a sum).

Third, when instructing ELL students, a teacher must remember that discussion and culture within the classroom can be quite important in the development of mathematical understanding. As such, the teacher should make efforts to create an environment that can encourage ELL students to attach their learning to their own experiences and participate in mathematical discussions as they learn English (Moschkovich 2012; Barwell 2008). A warm, friendly and tolerant classroom, where students will not be afraid to make mistakes as they explain a mathematical concept, can be extremely helpful for a student who is trying to learn not just mathematics but English as well (Kersaint, Thompson and Petkova 2013, 137–43).

Fourth and finally, care must be taken when performing assessments. Assessments for ELL students need to be continuous and ongoing. Ideally, the language of assessment should be in the language of instruction (Moschkovitch 2012; LaCelle-Peterson and Rivera 1994). This means that for written tests, the words used should be familiar to the ELL student; synonyms in the same word problems should be avoided; complex phrases should be reduced or simplified; the use of conditional clauses (eg, *if… then*) is limited; and active verb tenses are used (Abedi and Lord 2001; Kersaint, Thompson and Petkova 2013, 127–35). Accommodations such as audiotaping questions, the use of personalized notes for use during tests, access to word walls and glossaries, and the ability to use concrete materials such as manipulatives during tests have been shown to be effective (Kersaint, Thompson and Petkova 2013, 127–35). Further, mathematics teachers of ELL students should avail themselves of more than one type of assessment to provide a more accurate view of the students' mathematical understanding and where they may need assistance (Jarrett 1999; Buchanan and Helman 1997). These other modes of assessments could include performance assessments, project-based assessments, personal interviews and examining written responses (Kersaint, Thompson and Petkova 2013, 127–35; Moschkovich, 2010, 164). It would behoove the teacher when creating assessments to bear in mind a student's set-befores and met-befores—in particular, the concepts, knowledge, skills and applications required for the student to complete the challenge or problem presented (Jarrett 1999).

Concluding Remarks

In sum, care must be taken when working with ELL students in the mathematics classroom to avoid the raising of problematic met-befores. The mathematics teacher faced with the challenge of teaching ELL students must keep in mind the set-befores and met-befores facing his or her students. The teacher must aim to promote relational understandings in his or her students to assist them in the long term to find success, although the students may attempt to head to a weaker instructional understanding. Further, the teacher must remember to take precautionary steps to avoid the problems relating to faulty met-befores, make efforts to develop proper problem-solving techniques, promote a positive culture in his or her classroom to make the learning of mathematics welcoming and, finally, he or she must ensure that the assessments used in the classroom do not place the ELL students at a disadvantage as they try to master the challenges of two new languages: the language of English and the language of mathematics.

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