Repetition as a Means of Encouraging Tall's Met-Befores

Lixin Luo

In a classroom teaching and learning situation, it is common for individual students to respond differently to a new topic introduced by the teacher. While some students might be able to understand the new topic quickly, others might feel lost or confused. Students' different responses can be explained using David Tall's (2013) idea of *met-befores*. In this paper, I first interpret Tall's concept of met-before, and then I explore using repetition to help students to construct and activate met-befores in order to facilitate their mathematical growth.

Met-Befores

The term *met-before* is used to "describe how we interpret new situations in terms of experiences we have met before" (Tall 2013, 88). Tall defines a met-before as "a mental structure we have now as a result of experiences we have met before" (p.84). The term *met-before* refers not to a person's actual experience, but rather to the embodied influence of the person's previous conscious and unconscious experience. Met-befores are personal; two people who have learned the same topic might not have the same understanding of the topic. Met-befores can exist unconsciously and might not present themselves until a person is prompted by certain situations that make her met-befores problematic. For example, a student might not realize that she believes that "multiplication makes more" until she encounters fraction multiplication and the fact that multiplication makes less. Tall's met-befores are similar to presumptions, prejudices, attitudes or habitual ways of thinking formed through a person's former experience.

Of particular significance, met-befores affect how we interpret a new situation, thus influencing our learning. Some met-befores are supportive because they help learners to understand new experience, while some are problematic because they cause initial confusion (Tall 2013). For example, knowing $2x + 3x = 5x$ is helpful for one to understand $2x^2 + 3x^2 = 5x^2$, but understanding that addition makes a bigger number, based on one's experience with positive numbers, is problematic when one first encounters adding negative numbers. Tall sees that supportive and problematic met-befores arise naturally in mathematical learning, and the development of mathematical thinking involves a change of meaning of met-befores: some supportive met-befores might continue to be helpful in a new context while some become problematic. Thus, whether a met-before is supportive or problematic is contextualized rather than a fixed attribute. For instance, a student who has calculated the square of a real number many times would find the statement "Any real number's square is positive" easy to understand, but find the idea of $i^2 = -1$ hard to grasp.

A person can have some supportive aspects of a given concept and some problematic aspects at the same time (Tall 2013). Students who can understand a new topic quickly might have sufficient supportive met-befores or they can suppress their problematic met-befores in order to move on, while students who find the topic hard to grasp might lack supportive met-befores or have problematic met-befores that they cannot resolve.

Both supportive and problematic met-befores are important for mathematical learning, yet they are not equally valued in school curriculum (Tall 2013). Supportive met-befores are commonly valued in curriculum design through the emphasis of prerequisite knowledge and skills and in teaching practices through connecting new ideas with students' experience. Problematic met-befores, however, are rarely used in mathematics classrooms as "an integral part of learning" (Tall 2013, 89). Contradictions between the new idea and one's previous understanding are not welcome because they seem to interrupt and trouble one's learning. Tall sees curriculum's focus on supportive met-befores as a problem. He argues that problematic met-befores can have "debilitating effects in long-term learning" (p 89), and the resolution of problematic met-befores is needed for confident new learning. Therefore, Tall suggests considering ways to deliberately reveal problematic met-befores so that they can be addressed. This leads us to the use of repetition.

Using Repetition to Construct and Activate Met-Befores

Repetition is one of Tall's (2013) three fundamental mental structures that humans are born with. These structures (ie, recognition, repetition, language) take time to mature as the brains make connections in early life. Tall calls these structures *set-befores*. He argues that the development of mathematical thinking is *based on set-befores* and *built on met-befores*. The importance of repetition is somehow self-evident: without our mental ability to repeat actions to form repeatable sequences, mathematical thinking is impossible. Repetition encourages generalization and abstraction. Through repetition, one can notice patterns and compress a sequence of actions into a mental object, which becomes the object for manipulations at a higher level of abstraction. While considering ways to deliberately reveal problematic met-befores, I see the possibility of using repetition to help students construct and activate both supportive and problematic met-befores.

Supportive Met-Befores

Teachers can facilitate students' construction and activation of supportive met-befores by using examples that repeat with variation. Here is a set of examples that a mathematics teacher might write one by one on the board during a lesson on solving equations:

$$x = 0$$
$$x - \mathbf{1} = 0$$
$$\mathbf{2}x - 1 = 0$$
$$x^{\mathbf{2}} - 1 = 0$$
$$\mathbf{2}x^2 - 1 = 0$$

This set can be used at different times in a quadratic equations unit. If the students are new to solving quadratic equations, the first three linear equation examples serve as a deliberate review for students. The skills they use to solve these equations can be carried into solving the last two quadratic equations. Yet, they have to modify their skills in order to solve these quadratic equations. For example, to solve $x^2 - 1 = 0$, after students isolate the variable term, as they have done for solving $x - 1 = 0$, to obtain $x^2 = 1$, they might see x^2 somehow similar to 2 (both terms include an operation done to x), yet different (multiplying x by itself vs doubling x). Thus they have to think about a way different from dividing both sides by 2 to undo the operation in order to obtain x. The equation $x = 0$ is included as the first example because it has the form of the final stage of solving an equation.

This example illustrates a way to help students to construct and activate supportive met-befores for new learning. Each equation in the set repeats the previous one with a subtle change. Therefore, when students move from one equation to another, they have seen part of the new equation before. The new element in each equation is noted in **bold**. The students' experience with the previous equations contributes to supportive met-befores for their encounter with a new equation. These supportive met-befores facilitate students' interpretation of the new situation and enhance students' confidence as well. The repetition in this set of equations encourages generalization, and the subtle yet salient difference between equations helps to shift student attention to the change and consequently the structure of each equation. The new equation is comparable with the old ones, yet it is not a simple extension. For instance, the change from $x^2 - 1 = 0$ to $2x^2 - 1 = 0$ can be significant from a student's perspective, as many students tend to have difficulty handling a variable term with a coefficient not equal to 1. This kind of change brings in a new structure or attribute to the new equation. Thus it is possible that after working through this set of equations, students establish sufficient met-befores, which make solving equations like $2(x-1)^2 - 1 = 0$ or $2(\sin x)^2 - 1 = 0$ imaginable.

Problematic Met-Befores

Tall (2013) suggests that the teacher rationalize a problematic situation and make the contradiction between a met-before and a new situation obvious by deliberately having students recall situations during which the met-before works. For example, have students review a situation where "taking away makes less" works before being introduced to taking away negative numbers. Tall believes that this approach also facilitates new learning by enhancing students' confidence: "Giving confidence in an earlier situation may make it easier to see what is different in the new situation to address the issue in a position of confidence" (pp 88–89). From my point of view, Tall's approach is a form of repetition with variation. It starts with a review that activates and reinforces students' met-befores. Then, students encounter problems that resemble the old ones yet are significantly different, making students' met-befores problematic and demanding a breakthrough in students' thinking.

Similarly, teachers also can use repetition with variation to deliberately help students construct problematic met-befores. Here is a set of quadratic relations that can be used as an example of the method proposed.

$$y = x^2 - 1$$
$$y = 2x^2 - 2$$
$$y = -3x^2 - 3$$
$$y = \mathbf{-3}x^2 \mathbf{+1}$$
$$y = -3x^2 + 12$$
$$y = -3x^2 \mathbf{-3}$$

This set can be used in different grades for various purposes. Assume that this set is used in a Grade 10 mathematics lesson after students have learned finding zero(s) either by factoring or by completing a square. All the relations in this set repeat the ones that come before in some ways. While the first three relations are very much alike, the last three differ quite dramatically. After students have graphed the first three relations, they are likely to form an understanding that these graphs open upward, share the same x-intercepts (1 and –1), and cross the x-axis twice. They might not realize the met-befores' presence until they encounter the last three relations: these met-befores, one after another, are problematized (the fourth graph opens downward, the fifth graph's x-intercepts are 2 and –2, and the sixth graph does not touch the x-axis). This change from the similar examples (as represented by the first three) to not-quite-similar examples (the last three), as Watson and Mason (2006) argue, is important: it breaks the pattern perceived or conjectured by the learners to nudge learners into thinking mathematically. While working on the set of relations presented above, it is possible that students will begin to understand how the coefficient on the quadratic term of a quadratic relation affects the graph, or notice some commonalities of quadratic relations with two x-intercepts opposite to each other, or wonder about the common form of quadratic relations with no x-intercepts (and even whether $x^2 = -1$ is possible) after graphing the fourth, fifth and sixth relation respectively.

Repetition has potential for helping students construct and activate both supportive and problematic met-befores. This possibility is related to repetition's contribution in generalization when combined with variation. Through repeating with variation, students get a chance to generalize patterns, maintain enough supportive met-befores to be confident and perceive differences at the same time. Bateson's (2002) theory of mind asserts that mental activities are triggered by differences. Difference is needed for the mind to work. When the difference is small for a learner, her met-befores can be supportive enough for her new learning so she can progress in a smooth continuity. When the difference is big for the leaner, her met-befores can become so problematic for her new learning that a significant change in her understanding is needed for her to move on. Such interruption of the smooth continuity of a learner's cognitive development is essential because it can break the learner's equilibrium and force her into a cycle of rebuilding equilibrium. According to Piaget (in Doll 1993), it is through the recursive cycle of equilibrium–disequilibrium–equilibrium that cognitive development becomes possible. Clearly, with the help of variation, repetition has the capacity to both reinforce something old and generate something new.

Difference can be either a difference between two things or a change between a thing in time 1 and the same thing in time 2 (Bateson 2002). Thus, a looking-back activity, which invites students to revisit and reflect on the same topic later in time or from different perspectives, can enable them to perceive the difference between their met-befores and their current understanding of the concept. In this sense, repetition can be integrated into students' forward movement (ie, learning new knowledge) and their backward movement (ie, reviewing previously learned knowledge).

Conclusion

Tall's ideas of met-befores, although not entirely new, invite us to reconsider the balance of supportive aspects and problematic aspects in teaching and learning. Tall shows us that the change of met-befores from supportive to problematic is natural for the development of mathematical thinking. Thus, teachers need to consider both supportive and problematic met-befores of students. Repetition can be used to help students construct and activate met-befores, thus benefiting their mathematical growth. It is worth our attention to explore more ways to employ repetition to integrate met-befores, particularly the problematic ones, into teaching and learning mathematics.

References

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Lixin Luo has taught high school mathematics, computer science and computer engineering courses in Toronto for four years. Currently, she is working toward a PhD in the Department of Secondary Education at the University of Alberta. She is interested in doing research on high school mathematics education in both Canada and China, through the lens of complexity thinking, hermeneutics and wisdom traditions. Her dissertation topic is recursive mathematics curriculum. Lixin Luo can be reached at lixin@ualberta.ca.