Geometry with *Three Pigs, One Wolf and Seven Magic Shapes*

Carole Kamieniecki

Learning Geometrically

According to van Hiele (1999), children go through five stages as they develop geometric concepts. At the first level, visualization, children identify shapes based on a prototypical shape; the shape is judged based on its overall appearance only, and specific attributes are not considered (Hannibal 1999; van Hiele 1999; Ryan and Williams 2007). One example of a problem that can occur at this stage is that when a square is rotated 1/8 turn, so that one vertex is pointed down, children may no longer identify it as a square; they may call it a diamond (Margerm 1999). Teachers need to question the child to help correct this type of misconception. In the previous example, questions such as

- What changed about the square when I turned it to make it a diamond? Why is it no longer a square? and
- What happens when I turn it some more (turn it another 1/8 turn so the vertex is on the bottom left)?

should allow the child to think about what really changed. One of the ways to help prevent children from developing this misconception is to expose them to a variety of geometric shapes in different forms and positions (Hannibal 1999).

The second stage of van Hiele's (1999) levels is the descriptive level, at which children judge a shape based on its properties. For example, it is a square because it has four straight sides that are the same length. At this stage, children may still base their judgements on a prototypical shape and may not recognize a shape as being of a specified type. For example, a triangle like Triangle 1 below may not be called a triangle because it does not resemble the child's prototypical triangle concept (Ryan and Williams 2007).

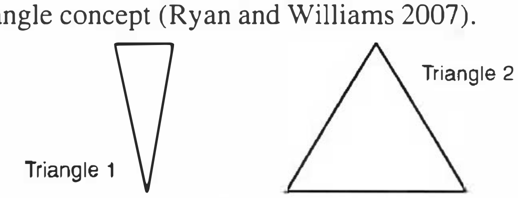


Figure 1

Triangle 1 may not match the prototypical triangle, Triangle 2, that a child pictures in his or her mind.

When students have a misconception, such as the one above, teachers once again need to provide some probing questions to help dispel the misconception. Here are some questions that would apply to the triangle problem above:

- What is different about this shape (Triangle 1)? (Children may respond that the pointy part is down and the flat part is up.)
- What is the same about the two shapes? (You want the child to indicate that both triangles have three sides and three points.)
- What happens if I turn this shape (Triangle 1) a half-turn? (Turn the triangle as you ask the question.)

These questions should help the children think about why they thought Triangle 1 was not a triangle and reconsider how they determine what attributes make a triangle.

Van Hiele (1999) indicates that mathematical "language is important for describing shapes" (p 311) at this level, so students need exposure to the language of geometry. Terms such as *face*, *edge* and *line* may hold different meanings for children, and teachers need to define and use these terms frequently so children can assimilate the new definition (Ryan and Williams 2007).

The third level, abstraction and informal deduction, is when the "properties [of shapes] are logically ordered" (van Hiele 1999, 311). Ryan and Williams (2007) indicate that "the concept [of the shape] becomes ...'concrete'... because its invariance ... under different transformations is recognized" (p 84). Teachers need to expose students to a wide variety of shapes and help students to understand what attributes of shapes are "relevant and irrelevant" (p 85) so students can proceed to this stage. Van Hiele (1999) feels that many students do not achieve this level of geometric thinking and have difficulty when encountering more advanced geometric thinking in higher grades. Ryan and Williams (2007) also indicate that many students 12 to 15 years old have not achieved this stage. It is important to provide probing questions to students in division I to dispel misconceptions so that, as they progress through division II and into secondary education, students can achieve this third level and continue on to van Hiele's (1999) fourth and fifth levels.

A Geometrical Task

1. Give each student a tangram. Explain that a tangram is a Chinese puzzle with seven geometric pieces (tans) that can be used to create different shapes. Demonstrate how the tans go together to form the square. Have students make the square (standard tangram shape) with their tans. Ask students to answer the first question on the worksheet.
2. Have students separate the tangram, examine each piece and fill in the chart on the worksheet. Ask students to demonstrate a flip and a turn with the tans.
3. Explain to students that we will read the book *Three Pigs, One Wolf, and Seven Magic Shapes* (Maccarone 1997) and students will construct the different shapes in the story with their tangram pieces. Read the story, allowing time for students to build each of the shapes in the story. Place an enlarged picture of the shape on the interactive whiteboard or overhead. Tell students to indicate if they are having difficulty making the shapes in the story. Check to see how the students approach the task of building the shape. If they are having difficulty, provide an outline of the shape for the students to fill in with their tans.
4. After the story is done, have students create their own shapes with their tans. Students should choose one of their shapes to draw on their worksheet and then colour it with pencil crayons.
5. As the students are filling out the chart, ask the following questions where applicable:
   - Why does flipping the triangle/square/parallelogram make it/not make it different? How is it different or why is it not different?
   - Why does turning the triangle/square/parallelogram make it/not make it different? How is it different or why is it not different?
6. Both as students are building the shapes in the story and afterward, ask them some of the following questions (pick a few that are appropriate for what the student is doing):
   - Why do you think that shape fits there?
   - Why doesn't that (pick the shape the child is using) fit in that spot?
   - Why did you make the square with those two triangles?
   - Why do you think the square fits in that spot?
   - What would happen if you flipped that piece? (National Council of Teachers of Mathematics [NCTM] 2012)
   - Can you use a different arrangement of pieces to make the shape? (NCTM 2012)
   - I see you rotated the square. Did rotating it change the shape? If the student says yes, ask why; if the student says no, ask why not.
7. To increase the students' mathematical vocabulary, make comments about what they are doing to the shapes as they build shapes in the story and after (NCTM, 2012).
   - I see that you are rotating that triangle to make it fit.
   - I like how you tried flipping the parallelogram to see if it fit.

Curriculum Connections

Grade 3 Mathematics

Specific Outcome 7—sort regular and irregular polygons, including

- triangles
- quadrilaterals
- pentagons
- hexagons
- octagons

according to the number of sides (Alberta Education 2007, 42)

Grade 3 Art

Purpose 2: Students will illustrate or tell a story.

- Concept A: A narrative can be retold or interpreted visually
- Concept C: Material from any subject discipline can be illustrated visually
  (Alberta Education 1985, C.7)

Grade 3 Language Arts

General Outcome 2—Students will listen, speak, read, write, view and represent to comprehend and respond personally and critically to oral, print and other media texts.

Specific Outcome 2.2—Respond to texts.

- Experience various texts—choose a variety of oral, print and other media texts for shared and independent listening, reading and viewing experiences, using texts from a variety of cultural traditions and genres, such as nonfiction, chapter books, illustrated storybooks, drum dances, fables, CDROM programs and plays.
  (Alberta Learning 2000, 31)

Engaging in the Task

I chose three children of different ages to work on my math task because I was interested to find out if there were any differences in their geometric ability; all three children are mine. The youngest child is a 7-year-old male in Grade 1; he enjoys working with numbers and is at a beginning reading level. The second child is a 9-year-old male in Grade 4; he is an average to above-average student who does well in math, science and reading when he takes his time and applies himself. The third child is a 12-year-old female in Grade 7; she is an above-average student who excels in all subjects. I worked with the two boys together for 35 to 40 minutes and I worked individually with the girl for about 30 minutes. Both interactions took place on the same day; the girl did not observe the interaction with the boys.

All three children filled in the worksheet, but I needed to assist the 7-year-old male in filling out his worksheet. The completed worksheets are attached at the end of this article. I have also included some of the interactions and pictures from the videos in the "Reflection" section.

Reflection

All three children were reluctant participants at first, because the activity entailed doing math and was expected to be like school. The thing that surprised me the most was that the older two children had not used tangrams previously in math. All three children were surprised at how hard it was to make the square out of the seven tans. I expected this, because I have played with tangrams and I still find it difficult to make the square. Once the children had become frustrated with trying to make the square, I gave them the tangram template sheet showing how the square is made with the tans. I was surprised that the 7-year-old found it easier to use the template than the 9-year-old did. If I did this activity again, I would start by giving just the square outline and have the children try to fill it in with the tans. Having the children fill in the outline first would allow them to do more exploration with flipping and rotating the tans.

By doing this expanded initial exploration with flipping and rotating the tans, the children should begin to understand how the shape changes when the tan is flipped or rotated. One of the things I could have improved during the task was getting the children to relate how they manipulated the tans to make the square to flipping and rotating th tans when filling in the chart. Relating these two activities should provide additional assistance for the children to see how a flip or rotation would change the tan.

When the children started filling in the chart, there were both things I expected and things that surprised me. I expected the young children to not be clear on what they needed to do to flip the tan. At first, the two boys were unsure about what flipping the shape involved, but when I asked a question about pancakes ("What do you do to a pancake when you cook it?") they immediately understood a flip. Some children may not have had the experience of making pancakes, so the teacher would need to find another analogy that may be relevant in the life of that child. Another word that caused problems was the word *different* in the chart. The boys told me that nothing would make the shape different because it was always the same; they stated, "All you do is move the shape around, you don't change it in any way" (N Kamieniecki and C Kamieniecki, personal communication). After I asked the boys to flip their tan I asked them what had changed; the answer I got from the youngest was "It's pointed in a different direction" (C Kamieniecki, personal communication). I indicated that this is what *different* means. Both boys worked individually for a few minutes at flipping the triangle to see if it changed anything. After they decided on how many flips made the triangle different, I asked them to show me how they came up with their answer.

Figure 2: 7-year-old flipping a triangle

The youngest child would only flip the triangle along the base, and he rationalized that the flip made the triangle different because it was pointed in a different direction. Both the 9-year-old and the 12-year-old were not limited to flipping the triangle at one point of the shape; they would perform a flip using a vertex, a base or one of the sides. One of the things that the 12-year-old girl mentioned is that flipping does not change the shape itself; it simply changes the position of the shape relative to where it was at the start. As a Grade 7 student, she has experienced working with the translocation of geometric shapes; for her, any translocation of a tan was considered as making the shape different even if the position of the vertices and faces were the same.

I was pleasantly surprised to find out that the 7-year-old did still call the square a square even when one vertex was pointed downward so that the square resembled a diamond shape. I asked him why it was still a square when it was in that position and his response was "It is still a square because the only thing that that is different is the corner is pointed down" (C Kamieniecki, personal communication). I had expected the 7-year-old to have the misconception that the square was a diamond when it was turned in this position. I believe that this child would be considered to be starting to understand the third stage of van Hiele's (1999) geometric thinking because he judges a shape based on its properties but does not limit himself to a prototypical shape.

The other thing that surprised me was that none of the three children knew what to call the parallelogram. The 12-year-old called it a quadrilateral, which it is, but did not remember the term *parallelogram*, although she did remember what a trapezoid was. When she completed her chart, the term was provided by her brother who had already completed the task. As an aside, another child who is 14 years old and in Grade 9 also did not remember the term *parallelogram*. I believe that the two older children had difficulty remembering the term because geometry is not given a lot of teaching time in elementary school, as evidenced by the number of 2-D and 3-D shape and space outcomes in relation to the total outcomes for each grade.

Having the children build the shapes along with the story was quite engaging for the two boys. The boys enjoyed the humorous relationship of the book to the original story of the Three Little Pigs. Both boys found building the different shapes in the book challenging but not frustrating. The 7-year-old boy had some slight difficulty with the sailboat shape, which used most of the tans in diagonal positions (relative to their prototypical positions). When he became frustrated with trying to build the sailboat, I asked him to stop and look at the shapes in the picture. While he was looking I asked him to pick one shape to start with; he started with the large triangle in the sail and was then able to complete the sailboat easily. The 9-year-old boy had a lot of difficulty with the candle shape, which required proper placement of the correctly sized triangle. While he was attempting to construct the triangle I asked him if rotating the triangle he had chosen would change the shape so that it would fit. After he tried rotating the triangle, he said it would not fit. I asked him to show me why he did not think it worked, and he pointed to the picture of the candle in the book and said that it stuck out and was too big.

For the last activity, the 9-year-old created his own shape, a house, and after he had constructed it and recorded it, we explored what would happen if he chose different shapes or changed the position of the shape by rotating or flipping a tan. He discovered that he could make different houses, but he could not make the same house. When the 7-year-old was doing the last activity, building his own shape with the tans, I asked him if the parallelogram would still fit if he flipped it. He answered no and when I asked him why, he said he would show me; he demonstrated what would happen if he flipped the parallelogram. The 12-year-old girl did not enjoy the story but did engage with building the shapes, as did the 14-year-old, who was just passing through.

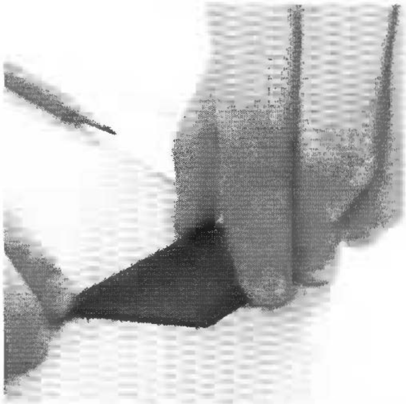


Figure 3: 9-year-old flipping the parallelogram

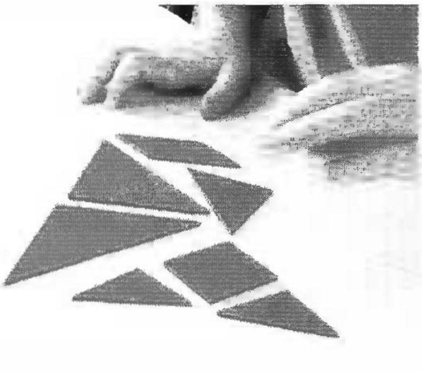


Figure 4: 7-year-old building the sailboat from the book

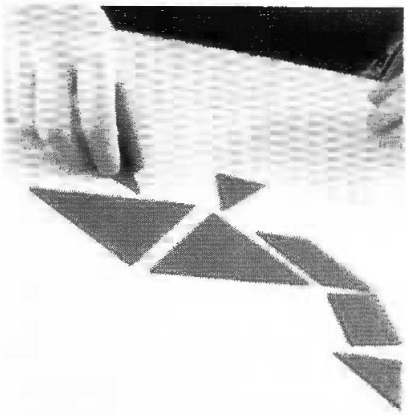


Figure 5: 12-year-old building the duck from the book

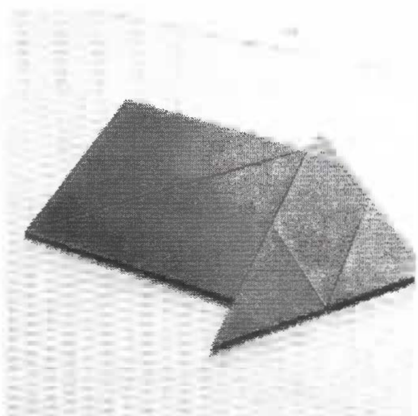


Figure 6: 9-year-old's completed shape

Assessment of Task

The two boys were very engaged with the task and even expressed their enjoyment of playing with the tangrams. The girl did not appear to be as engaged, but when I asked her what she did not like, she replied "It's math and I always have this attitude about math" (M Kamieniecki, personal communication). She was also my testing station for math last year and so has a jaded opinion of being a math guinea pig. The three children followed the task instructions and required few directions. The task connected math to language arts and art. I was able to note student thinking about the task using question prompts.

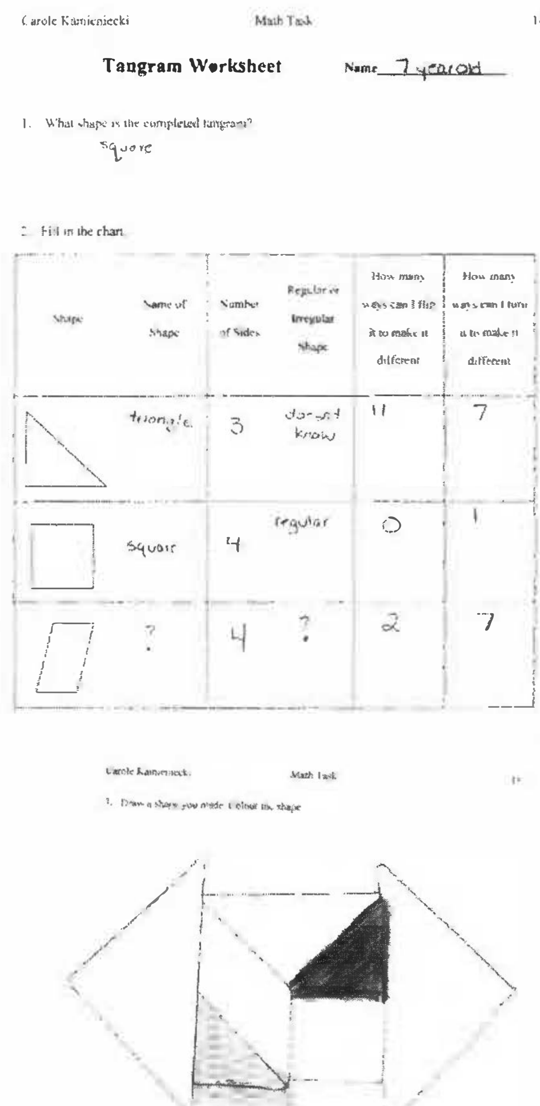
Modifications of the Task

The first modification I would make is to give students the square tangram outline to fill in if they have difficulty making the standard square tangram shape. I would also ask questions about how they are manipulating the individual tans to make them fit, such as

1. Why did you select that piece to go in that spot?
2. Why did you flip that piece to make it fit in that spot? What made it different?
3. Why did you rotate that piece to make it fit in that spot? What made it different?

I think these questions would allow the children to think about the word *different* and what it means to say that a shape is different. Asking why may make children feel uncomfortable; if it does, the questions could be rephrased:

1. How does this piece fit in that spot?
2. Show me how flipping that piece made it fit.
3. Show me how rotating that piece made it fit.



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Tangram Worksheet Name 7 year old

1. What shape is the completed tangram?

square

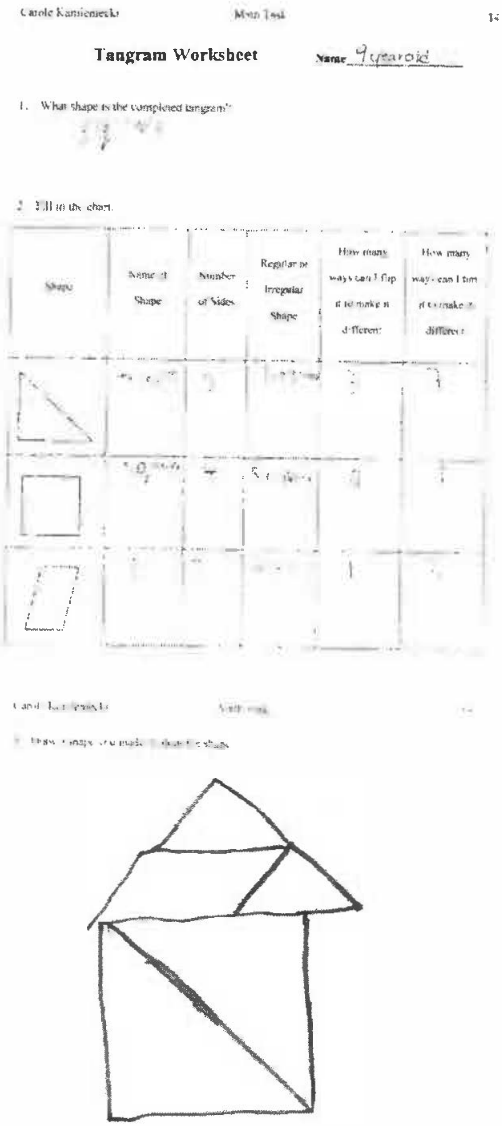
2. Fill in the chart.

Shape	Name of Shape	Number of Sides	Regular or Irregular Shape	How many ways can I flip it to make it different	How many ways can I turn it to make it different
	triangle	3	don't know	11	7
	squair	4	regular	0	1
	?	4	?	2	7

Carole Kamieniecki Math Task

3. Draw a shape you made. Colour the shape

Figure 7: 7-year-old's worksheet



Tangram Worksheet

1. What shape is the completed tangram?

2. Fill in the chart.

Shape	Name of Shape	Number of Sides	Regular or Irregular Shape	How many ways can I flip it to make it different	How many ways can I turn it to make it different

Figure 8: 9-year-old's worksheet

Tangram Worksheet

What shape is the completed tangram?

[illegible]. Fill in the chart.

Shape	Name of Shape	Number of Sides	Regular or Irregular Shape	How many ways can I flip it to make it different	How many ways can I turn it to make it different

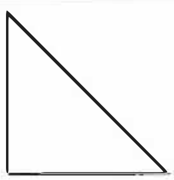
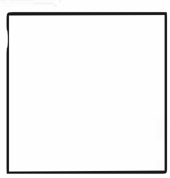
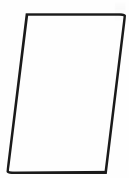
3. Draw a shape you made. Colour the shape.

Figure 9: 12-year-old's worksheet

Tangram Worksheet   Name

1. What shape is the completed tangram?

2. Fill in the chart.

Shape	Name of shape	Number of sides	Regular or irregular shape	How many ways can I flip it to make it different?	How many ways can I turn it to make it different?
					
					
					

3. Draw a shape you made. Colour the shape.

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Prior to earning her BEd, Carole Kamieniecki worked as a veterinarian and thought that she would like to focus on teaching science. However, during her math curriculum classes she developed an intense interest in math. She received her BEd in elementary education in November 2013; she is currently working as a substitute teacher and volunteer math and science tutor for junior and senior high school students. She continues to attend math-related professional development sessions to discover more ways that students can explore mathematical concepts, to better understand where students can become confused, and to develop a better understanding of how students can find the connections between mathematical concepts.