The Broken Link

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Imagine a society where toddlers are discouraged from walking. It is strongly advised to keep them in strollers outdoors and in walkers inside the house; children younger than six are rarely seen walking in the street, and even then only when assisted by a parent. Parents who do not follow their kids' every step, even in their backyard, risk attracting the neighbours' attention. There are several reasons proclaimed for this: the risk that toddlers will fall and hurt themselves while they are too young to be aware of the dangers around them; even supervised.

young children can easily be involved in car accidents in the streets. There are handicapped kids who cannot walk, run and climb as successfully as the others, which creates a feeling of inequality at an age which can be handled only with great difficulty by psychologists. It is commonly agreed that at the age of six it is allowed to start learning how to walk. Unfortunately, the precious moment is lost, the beginner walkers are too heavy for their undeveloped muscles, and most children cannot really catch up, and can only walk slowly and clumsily for the rest of their lives. There is no public concern about this issue as it is not an obstacle to driving successfully ten years later. The system has many side effects: there are countless tutoring agencies, advertising a "smooth and easy way to walking," as well as private tutors, with many desperate parents of both young and older children relying on them. For rich people, there are elite clubs where walking skills are developed earlier by licensed instructors who do the training at the appropriate age. In addition, at the most advanced level, there are free outreach activities for active runners, especially for promising athletes. However, most students are neither interested nor qualified to participate. The whole idea does not stir society since most parents grew up in a similar environment, walk with difficulty and would definitely feel uncomfortable assisting their children once they move quickly; the same situation exists with teachers at school.

The situation described above seems absurd, but it is quite appropriate when applied to mathematics, which is sometimes seen as a scarecrow of our school education and even as an obstacle for socially underprivileged students to obtain a college or university education. This is especially surprising, since the development of mathematical skills requires neither expensive tools, when compared, for example, to many sports and experimental sciences, nor an "educated" environment to be cultivated, unlike language and humanities skills. The main reason why mathe-

matics is blamed for causing depression in high school and first-year undergraduate students is probably that mathematics is not a mirror that is easily distorted. It does not allow anybody to believe what he or she desires, but tells the truth. We do not credit students for wrong solutions and answers, though there was once an article by a parent in the *Calgary Herald* (and in other sources, I believe) advocating that a partial score should be granted for totally incorrect mathematical solutions. I do not believe that first-year students are more successful in writing essays or analyzing natural science phenomena than in solving mathematical problems, but in the first case evaluation criteria may be not so strict. My colleagues in geology, for example, do not assume that students have any background, while in mathematics, we are very much dependent on the skills our students bring from high school. However, we should be careful when comparing disciplines. When I complained to my colleague who teaches history that our main problem is the poor background of students in math and that this poor background is probably not as much of an issue in the history department, she answered, "How would you teach history to students who have never heard of the first and second world wars?" (Oh, happy students! It would be interesting to know where they come from.)

One of the recommended books for junior high school is the series Math Makes Sense. There was a justified critique of this series by Dr Malgorzata Dubiel (see, for example, http://blogs.vancouversun.com/2012/04/13/math-makes-sense-not-with-these-textbooks-expert-says and https://sfu.ca/pamr/issues-experts/2012/assessing-math-teaching.html). I have just reviewed the textbooks for Grades 8 and 9 in this series; I recommend this activity to every colleague involved in teaching first-year calculus, as it is quite revealing. Your students have difficulties with adding fractions? There is no reason to be surprised: unit 3 of the Grade 8 book, titled "Operations with Fractions," deals with multiplication and division but not addition of fractions. Bringing fractions to the common denominator is not outlined; the phrase *least common multiple* is not even in the index. So, next time, in the topic of "integration with partial fractions," when students choose the product of all the denominators of algebraic fractions as a common denominator, just note that they do the best they can. By the way, concerning algebra, the only element introduced by the end of junior high school is addition of polynomials, and multiplication and division by a number and by a monomial (in all the exercises, a polynomial with integer coefficients is a result). My first impression is that the book was inspired by lazy schoolchildren who wanted material that could cause any difficulty removed from the curriculum. It does not matter that you will need these things in the future; if the addition of fractions, squaring a binomial and solving a quadratic equation can be postponed to senior high school, they probably can be avoided.

However, the lack of technical skills acquired by students in the first nine years of their school life, in elementary and junior high schools, is only a symptom. The main problem of the approach accepted in the books is that what is presented is not really mathematics (which is all about why), but some collection of recipes. In the books, I could not find any explanation for why the sum of measures of angles in a triangle is 180° (or for that matter, any other fact in geometry). The best justification that could be found in the book is to draw some triangle, measure, compute and check. Reviewing the books for Grades 8 and 9 was very educational for me: now I can understand what my students may mean by "I am good at math" and "I am bad at math" and why their image of their mathematical abilities may be so far from reality. Presumably, being good at math is viewed as being able to implement some numerical or algebraic operations according to some prescribed (and even not justified) algorithms quickly and accurately after lengthy training. It is fortunate that I was not informed earlier; personally, I struggled with long addition and subtraction in Grades 2 and 3.

There are several aspects of mathematical education: the first and most important is developing the mind in general and logical thought in particular. The second one is encouraging independent thinking and judgment: even beginners make minor discoveries, and students can challenge their instructors and parents, since reason, not authority, establishes what is true. The third part is acquiring some technical skills and learning certain methods. For some reason, only the third aspect seems to be involved in the present junior high school curriculum, and even then at a rather limited level. For example, the textbook includes exercises outlining problem-solving skills; however, practically all the problems require only one step for their solution.

The main cause of this limited representation of mathematics is the one-size-fits-all education; sometimes it may mean that the slowest students have no chance to catch up with the material, while the more advanced students have no chance to be exposed to much mathematics (which is all about reasoning, not a set of recipes). I believe while there is still a certain percentage of junior high school students in the

former category who would benefit from less intensive curriculum or curriculum spread out over the years, an even larger proportion falls into the latter category. There are free outreach activities (math nights at the University of Calgary and similar clubs at practically every Canadian university), but until a certain age it is, in fact, the parents' initiative: students cannot get to the universities on their own. And when the students grow up and get to high school it is, unfortunately, too late.

The situation can be resolved in the following way, with some foreign school systems serving as an example. First, in elementary school, when the pressure of school reports is not yet high, students should be exposed to operations with integers (mastering oral counting to 100, for example) and fractions (including addition), as well as some ideas in geometry and basic word–problem-solving skills. In the beginning of Grade 7, as a result of qualification testing, students are divided into several groups (four in the program of reference), run at the same time for several classes at the same level. Starting with senior high school, there are several streams in mathematics (A, B, C); level A includes not only enriched calculus, which is at a level between Math 31 and calculus in IB programs, but also a high level of algebra, geometry, and trigonometry. Level B roughly corresponds to the well-mastered Math 30 program, while C is a weaker level. Students can switch to a higher stream by taking exams. It is true that some of them will employ tutors for this; however, this will not be comparable to the level of private assistance that our high school and first-year undergraduate students are seeking now. Their present situation can be compared to a joint school race for all grades: first-graders have no chance; graduates have no challenge.

For our high school students to transition smoothly to universities, a solid mathematical background plays an important role. However, this link for many of our first-year science and engineering students is broken, probably at an earlier stage than senior high school.

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