delta-k

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Guidelines for Manuscripts

delta-K is a professional journal for mathematics teachers in Alberta. It is published twice a year to
- promote the professional development of mathematics educators, and
- stimulate thinking, explore new ideas and offer various viewpoints.

Submissions are requested that have a classroom as well as a scholarly focus. They may include
- personal explorations of significant classroom experiences;
- descriptions of innovative classroom and school practices;
- reviews or evaluations of instructional and curricular methods, programs or materials;
- discussions of trends, issues or policies;
- a specific focus on technology in the classroom; or
- a focus on the curriculum, professional and assessment standards of the NCTM.

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6. Pictures or illustrations should be clearly labelled and placed where you want them to appear in the article. A caption and photo credit should accompany each photograph.
7. References should be formatted consistently using *The Chicago Manual of Style*'s author-date system or the American Psychological Association (APA) style manual.
8. If any student sample work is included, please provide a release letter from the student's parent/guardian allowing publication in the journal.
9. Articles are normally 8–10 pages in length.
10. Letters to the editor or reviews of curriculum materials are welcome.
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MCATA Mission Statement

Providing leadership to encourage the continuing enhancement of teaching, learning and understanding mathematics.

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From the Editor's Desk

Gladys Sterenberg

As I write this, I am preparing to begin the new school year. At Mount Royal University, our BEd program is rolling out its third year, which involves practicum experiences in Calgary and area schools. Today, I heard news of schools that remain closed because of flood damage incurred in June. These are challenging times for many school boards, parents, teachers and children, who are experiencing a high level of uncertainty while trying to restore routine in uprooted contexts. By the time you read this, our community will have coped with this upheaval in school access, but right now it seems overwhelming.

The situation in Alberta this summer has reminded me of how important the mathematical processes are for our communities. In particular, I have witnessed neighbours and friends responding in compassionate ways to those who have been severely impacted by the floods. Problem solving and making connections have been at the forefront of these responses.

The US National Council of Teachers of Mathematics (NCTM) states,

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving . . .
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics.[1]

This issue of *delta-K* provides a glimpse into how this might look in a classroom. As professionals, we are constantly engaged in problem-solving processes when we reason about what students know mathematically. Marlow Ediger reminds us of the importance of observation in this process, and Chelsey Bonnett and Jennifer Hyndman share their experiences of observing students in their own classrooms. Srinivasa Swaminathan presents research around problem solving, and a problem to be used in the classroom is provided by Gregory Akulov, along with a student response from Dennis Situ. Other problems are presented in the math contests, and Karl Dilcher prompts us to consider how we might use these in our classrooms. Connections between literature, art and math are presented by Nico Rowinsky and Roberta La Haye. Included also is a content piece, by Peter Liljedahl and Minnie Liu, on how numeracy is connected to various contexts in the mathematics curriculum.

Editing this issue of *delta-K* has been an experience of connections across the teaching profession. Mathematics Council (MCATA) members have forwarded articles they have read in various publications, editors of other math education journals have sent articles they think should be reprinted, education course instructors have encouraged their students to submit copies of stellar assignments to be crafted into articles, and various people involved in math contests across our province have sent problems and solutions from past contests.

I hope that the spirit of making connections with other math teachers continues and that you find inspiration for making connections and problem solving in your own classroom. Enjoy.

Note

1. See www.nctm.org/standards/content.aspx?id=322.

Meet Your MCATA Executive

Back row (l–r): Gladys Sterenberg, Rod Lowry, Donna Chanasyk, Daryl Chichak, Mark Mercer, Carmen Wasylynuik, Carol Henderson, Lisa Everitt

Front row (l–r): Kris Reid, Tancy Lazar, Robert Wong, Marj Farris, Debbie Duvall, Indy Lagu, John Scammell

Missing: Karen Bouwman, David Martin, Alicia Burdess, Olive Chapman

Teacher Observation to Evaluate Mathematics Achievement

Marlow Ediger

There are a plethora of assessment techniques that can be used in evaluating student achievement in mathematics. Each method has its pros and cons; however, some methods do a more comprehensive job than others.

Too frequently, mandated tests are the focus in the educational literature on evaluation. Mandated tests are given once a year and then in selected grade levels. Also, feedback from these tests does not provide information on specific errors made by learners (to be used for remedial purposes). The most frequently used assessment procedure should be teacher observation. Teacher observation can be used continuously in the classroom, and the teacher can immediately diagnose and remedy difficulties faced by students.

How might teacher observation of students help in mathematics teaching and learning?

Assessing Mathematical Progress

Teachers of mathematics must have a good knowledge of the subject matter, as well as of teaching methodology, to do quality work in observing learner progress. These matters need to be uppermost in the teacher's mind when observing. The teacher should consider the following questions about student behaviour:

- Is the student on task and engaged in learning?
- Does the learner show interest in mathematics (rather than being bored)?
- What specifically does the student not understand in an ongoing activity?
- How might this student best understand how to remedy the deficiency?
- What do individual learners need as background information in order to attach meaning to the ensuing learning experience?
- Does the student's learning style favour individual or cooperative endeavours?
- Does the student reflect on past mathematical experiences?

Assignments in mathematics should make provisions for individual differences. Students have different ability and interest levels, and they need to make sequential progress. The teacher may notice that a sequence is not working when students fail to make continuous progress. Assignments should be clear and relevant, and adequate prerequisite information must precede each new process being emphasized. There is a zone of proximal development (Vygotsky 1986) for each student; thus, a student may have a current achievement level in adding negative numbers, for example, and the ensuing learnings require multiplication of negative numbers. With small steps and meaningful experiences, the teacher can help the learner bridge the gap.

Vygotsky (1986) stresses the importance of students mediating experiences through language, such as discussions in large or small groups. This might, too, involve peer-mediated discussion groups. Teacher observation of student participation in discussions should include the following considerations (Ediger and Rao 2001):

- Meaningful mathematical learnings are being developed.
- All are participating, but no one is dominating the activity.
- Ideas are circulating among the participants.
- Enthusiasm for learning is in evidence.
- Ideas are being expressed with clarity.
- In-depth discussions are being stressed.
- Optimal achievement is a focal point for each student.

What might a mathematics teacher observe specifically about "meaningful mathematical learnings"?

What is accomplished must make sense to the student. Thus, if a student is unable to come up with the correct answer to a set of three two-place numerals with carrying, what might be some possibilities for error? The teacher needs to evaluate if the learner understands the concept of addition. The student may even need to use markers to show the sum of two addends. A place value chart with ones and tens columns might well help the learner to attach meaning to adding two- and then three-digit numerals. If meaning is lacking for the student, then it is very difficult to proceed to more complex learnings, such as regrouping from the ones to the tens column. Understanding place value is very important here. Problems might even arise in terms of writing numerals legibly for ease of comprehension. Once student understanding is in evidence, the use of technology (such as handheld calculators/computers) can truly make subject-matter learnings interesting and challenging (Ediger 2006a).

Keeping Anecdotal Records and Using Student Portfolios

Teacher observations may and should be recorded. Unless a careful system of record-keeping is involved, observations can be forgotten or modified. Each record should contain vital data and be written clearly. The observer can then review patterns of student behaviour in mathematics. By recording specific errors, the teacher can diagnose and remediate students' difficulties in the sequence. For example, if a student has problems with reducing fractions to lowest terms, he or she may not be able to understand factoring. Or in dividing fractions, the learner may not attach meaning to the process of inverting the divisor and then emphasizing the operation of multiplication.

Portfolios can be an excellent way for students to demonstrate their progress over time, in ongoing lessons and units of study. The contents (chosen by the student with teacher guidance) should be a representative sampling of the learner's completed work in mathematics. The time period can be a semester or the entire school year. A student's portfolio can contain the following items, among others (Ediger 2006b):

- Student solutions to problems from the textbook
- Completed worksheets
- Student drawings of geometrical figures
- Graphs, charts and tables of data from ongoing lessons and units of study
- Printout of the student's test results
- Student self-evaluation of his or her progress, using criteria agreed upon by the student and the teacher

The portfolios should be viewed and discussed in parent–teacher conferences. Coming up with agreed-upon ways of helping a student achieve should be a goal of the conference. The home and the school need to work together for the good of the learner. Independent evaluators may also assess portfolio contents for the purposes of noting student progress and ensuring teacher accountability.

In assessing the portfolio, the following questions should be considered:

- How might the teacher guide the learner in attaining as optimally as possible?
- Which objectives need to be stressed specifically?
- What kinds of learning opportunities will help the student achieve these objectives?
- What should be done to help the student reflect on his or her progress and monitor himself or herself adequately?
- How can the student be motivated more thoroughly to develop an inward desire to learn?
- How might the student become more conscientious about careful proofreading?

Portfolios provide feedback to the teacher on how to help students overcome selected problems and continuously progress in mathematics. Decisions may then be made about large group, small group and individual student endeavours. The teacher must use the feedback wisely in order to provide for individual differences among learners (see National Council of Teachers of Mathematics 1989).

Improving the Classroom Environment

The classroom environment is highly significant in improving mathematical achievement. Through observation, the teacher can determine what environmental factors are hindering student achievement and progress. For example, when students are distracted from attending to a lesson, their sequential learnings are disrupted and they lose out on specific and major ideas. On-task behaviour is very important.

Sometimes students are rude about points presented in a discussion. This hinders the free flow of ideas. Rules for discussions should be set up, such as all students should participate but no one should dominate, interrupting others should be avoided, respect for others and their ideas should be demonstrated, and active participation is important.

Steen (2007, 12) writes the following:

Experience shows that many students fail to master important mathematical topics. What's missing from traditional instruction is sufficient emphasis on three important ingredients: communication, connections, and contexts.

Colleges expect students to communicate effectively with people from different backgrounds and with different expertise and to synthesize skills from multiple areas. Employers expect the same things. They emphasize that formal knowledge is not, by itself, sufficient to deal with today's challenges. Instead of looking primarily for technical skills, today's business leaders talk more about teamwork and adaptability. Interviewers examine candidates' ability to synthesize information, make sound assumptions, capitalize on ambiguity, and explain their reasoning. They seek graduates who can interpret data as well as calculate with it and who can communicate effectively about quantitative topics.

To meet these demands of college and work, K–12 students need extensive practice expressing verbally the quantitative meanings of both problems and solutions. They need to be able to write fluently in complete sentences and coherent paragraphs; to explain the meaning of data, tables, graphs, and formulas; and to express the relationships among the different representations.

In Closing

The demands of today's workplace require increased proficiency in mathematics, which means that mathematics achievement in the elementary, middle school and high school years is essential. Mathematics teachers need to help each student achieve optimally, and strategies must be developed to guide learner progress.

The basics should be taught in problem-solving experiences. However, with some students, essential content may be taught more systematically. The psychology of learning must be stressed in teaching and learning situations. This includes making learnings interesting and meaningful, as well as purposeful. The learning style of the individual student should also be considered. Thus, students should learn in cooperative settings, as well as individually. The student must be guided to make connections between what is acquired in the school setting and what is needed in society, in order to establish relevance.

All these strategies can be facilitated and assessed through teacher observation.

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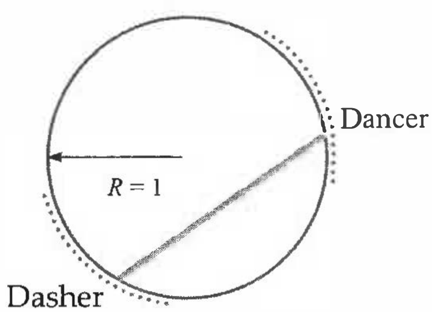
Marlow Ediger graduated from Emporia State University, in Kansas, with baccalaureate and master's degrees, and from the University of Denver with a doctorate degree. He was a public school teacher, school administrator and private school teacher on the West Bank of the Jordan River. After 30 years with Truman State University, in Missouri, he retired as professor emeritus of education. He continues to write for educational publications.

A Northern Lights Circle Problem

Gregory V Akulov

When December is incredibly frosty, the famous Dasher and Dancer preface their holiday flight to the Land of Living Skies with a spectacular warm-up. They canter gracefully at a constant rate along the Northern Lights Circle of a radius of 1 km and can be seen by everyone in the Land of the Midnight Sun. Some inhabitants even state that the curious bear Arctic, an awesome navigator from the Coffee Club Island, periodically records the exact straight-line distance between the reindeer. If the map drawn one morning is as shown below, find d. Give an exact answer.



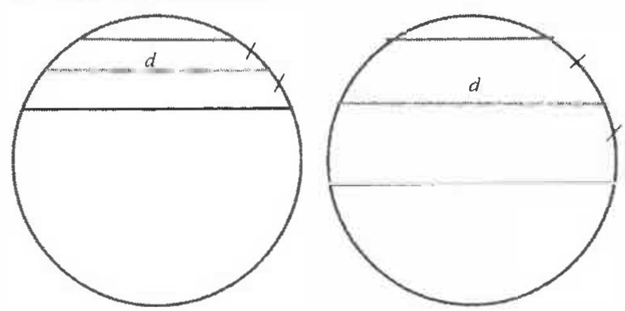
Time	Distance
AM	km
6:50	1.2
7:05	d = ?
7:20	1.6

Author's Solution

A_1, A_2 and A_3 and B_1, B_2 and B_3 are the positions of the reindeer at 6:50 AM, 7:05 AM and 7:20 AM, respectively. Since they are moving at a constant rate, it is easy to see that

$$A_1A_2 = A_2A_3 \text{ and } B_1B_2 = B_2B_3. \qquad (1)$$

Therefore, if the 1.2 km A_1B_1, d km A_2B_2 and 1.6 km A_3B_3 chords are redrawn so that $A_1B_1 \parallel A_2B_2 \parallel A_3B_3$, then (1) will remain true.



Both possible cases can be solved using an arc midpoint computation (see http://mathcentral.uregina.ca/RR/database/RR.09.10/akulov2.html):

$$d = \sqrt{(1+0.6)(1+0.8)} \pm \sqrt{(1-0.6)(1-0.8)},$$

which gives two answers $d = \sqrt{2}$ or $d = (7\sqrt{2})/5$ km.

For a student solution to this problem, see the Student Corner on page 8.

Gregory V Akulov teaches mathematics and physics at Luther College High School, in Regina, Saskatchewan. He has a PhD in mathematics (with a specialization in probability theory) from Kyiv National Taras Shevchenko University. His research interests also include theory of functions, foundations of geometry and mathematics curriculum development.

A Solution to "A Northern Lights Circle Problem"

Dennis Situ

The following is a Grade 9 student's solution to Gregory Akulov's "A Northern Lights Circle Problem" on page 7.

Editor's note: When he considers one possible case below, Dennis rounds sin (36.87) = 0.6 and sin (53.13) = 0.8. Actually, sin (36.87) > 0.6 and sin (53.13) < 0.8.

First of all, we must note that the distances between the two reindeer are not directly correlated with time; rather, it is the angle formed by the radii attached to both reindeer. The reindeer can be thought of as hands on a clock, which move at a constant rate: each minute, the minute hand moves 6° and the hour hand moves 0.5°. However, the straight-line distance between them does not change at a constant pace. Therefore, we must find the angles before we move any further.

At 6:50, we have a diagram as shown (Figure 1), with OA drawn as the perpendicular bisector of the line connecting Dasher (D^I) and Dancer (D^{II}). Because of the RHS (right hypotenuse side) property, the two triangles shown are congruent. This means that OA is also the angle bisector of D^IOD^{II}. We also have a right-angled triangle with two sides known. Therefore, we can use trigonometry to find the angles. Label angle D^IOA as α.

We have $\sin \alpha = 0.6$, so $\alpha = 36.87°$, which means that angle $D^IOD^{II} = 73.74°$.

We now examine the case at 7:20, where we draw a similar diagram (Figure 2), but with OB as the new perpendicular bisector. Angle D^IOB is to be labelled β.

We have $\sin \beta = 0.8$, so $\beta = 53.13°$. Now, $D^IOD^{II} = 106.26°$.

In order to find the distance d at 7:05, we must find the average angle, as 7:05 is the average time between 6:50 and 7:30. The average angle is given by

$$\frac{73.74 + 106.26}{2} = 90°.$$

At 7:05, the radii of the reindeer form a right angle. The diagram is given below (Figure 3).

Although we could use trigonometry to find the length d, the easiest way is to simply use Pythagoras' theorem to find that $d = \sqrt{1^2 + 1^2} = \sqrt{2}$.

At 7:05, the distance d is $\sqrt{2}$ km.

Figure 1

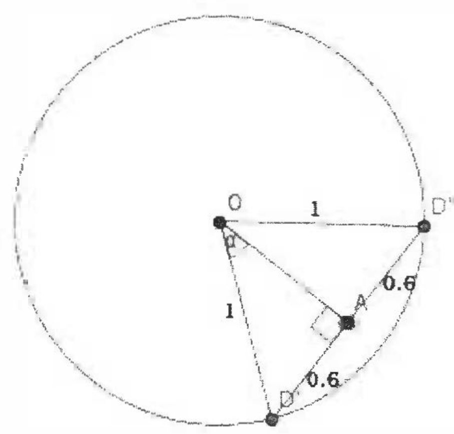


Figure 2

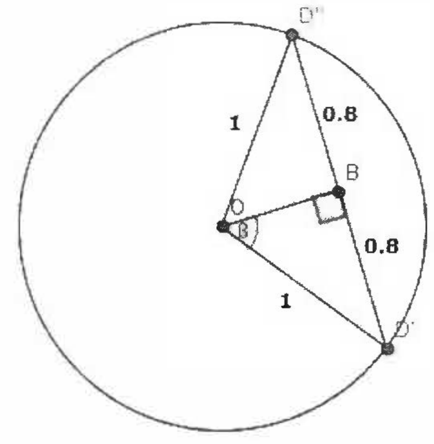
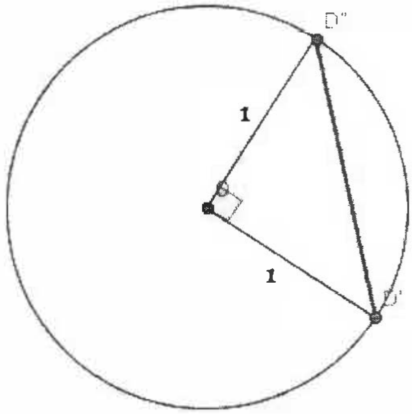


Figure 3



Dennis Situ is a Grade 9 student at Vernon Barford Junior High School, in Edmonton.

Numeracy

Peter Liljedahl and Minnie Liu

Over the last 10 years, numeracy—or mathematical literacy, as it is often called—has become more and more prominent, showing up in curriculum documents and special government initiatives around the world and in western Canada. In our local context, numeracy (or mathematical literacy) is featured in the Alberta program of studies in the front matter of every math curriculum document from kindergarten to Grade 12:

> Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences. (Alberta Education 2007, 1)

> Students are curious, active learners with individual interests, abilities, needs and career goals. They come to school with varying knowledge, life experiences, expectations and backgrounds. A key component in developing mathematical literacy in students is making connections to these backgrounds, experiences, goals and aspirations. (Alberta Education 2008, 1)

> Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. (Alberta Education 2008, 2)

> Students will develop the following mathematic competencies in the context of solving everyday problems. Students will . . . apply mathematical literacy to everyday situations. (Alberta Education 2006a, 4; 2006b, 4)

So, what is this thing called numeracy? Clearly, it is related to, but somehow different from, mathematics. To answer this question, we need to first understand where the numeracy movement is coming from.

Numeracy Movement

Around the world it has long been recognized that students are completing their compulsory education without the mathematical skills to cope with what life and work will demand of them. In reaction to this, we may want to increase the amount of mathematics being taught, to lengthen the period of compulsory mathematics, to increase or deepen the mathematics content in our curriculum, or to raise the standards in mathematics. On deeper reflection, however, it becomes evident that many of our best students—those taking mathematics beyond the compulsory level and achieving the highest marks—are just as ill-equipped to put their mathematics education to use in life and work.

This realization led to the rise of the numeracy movement—a movement designed to foster the skills the world is thirsting for in its graduates. The movement is driven by the principle that what is lacking is not more mathematics, or deeper mathematics, or greater fluency with mathematics but, rather, a greater flexibility with mathematics—a flexibility to use the mathematics we know to tackle the ever-changing and shifting landscape of life.

> Efforts to intensify attention to the traditional mathematics curriculum do not necessarily lead to increased competency with quantitative data and numbers. While perhaps surprising to many in the public, this conclusion follows from a simple recognition—that is, unlike mathematics, numeracy does not so much lead upwards in an ascending pursuit of abstraction as it moves outward toward an ever richer engagement with life's diverse contexts and situations. (Orrill 2001, xviii)

Numeracy is not mathematics. It is something different. Instead of diving deeper into the formal and abstract world of mathematics, learning more mathematics and becoming more fluent with mathematics, numeracy fosters the understanding and application of our mathematical knowledge in a quantitative sense. Unlike the field of mathematics, which continues to expand, the mathematics needed by a numerate individual is relatively finite. That is, numeracy isn't about being able to flexibly use all of mathematics to deal with "life's diverse contexts and situations" (Orrill 2001, xviii); rather, it's about being able to flexibly draw on that subset of mathematics that is most useful in dealing with these diverse contexts and situations.

Numeracy as a Toolkit

Numeracy can be seen as a toolkit of mathematical skills:

> [Numeracy] empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are skills required to thrive in the modern world. (Quantitative Literacy Design Team 2001, 2)

> [A numerate person is able to use] mathematics to make decisions and solve problems in everyday life. For individuals who have acquired this habit, mathematics is not something done only in mathematics class but a powerful tool for living, as useful and ingrained as reading and speaking. (Quantitative Literacy Design Team 2001, 8)

What the skills are that populate this toolkit is debatable . . . and contextual. In mathematics, the toolkit would contain *all* mathematics learned. In numeracy, however, the toolkit contains only those skills that are mastered and that are useful across a wide variety of contexts. So, while a formula for an arithmetic sequence may be the most efficient way to solve a problem, multiplication (or repeated addition) may be the more accessible way to solve it. That is, the formula for arithmetic series is like a specialty tool that, for most people, lies forgotten in a bottom drawer somewhere. Multiplication, on the other hand, is the well-worn, well-used, familiar tool that is easily found near the top of the toolkit. It may not be as elegant or as impressive as the formula for arithmetic series, but it is comfortable, reliable and easily accessible.

This is not to say that we should have a small toolkit. We want to continue to expand our toolkit, to add new tools to our repertoire of familiar mathematical skills that we can use to deal with our ever-expanding set of experiences. But the acquisition of a new tool should come out of necessity and be grounded in the specificity of our experiences. And it should be immediately and repeatedly useful to us. Otherwise it runs the risk of getting lost in the bottom of a drawer somewhere.

Numeracy as a Disposition

But having a toolkit full of well-worn and familiar tools is not enough. A numerate person must also be willing to use the tools to resolve the situation at hand. As such, numeracy is also a disposition—a willingness to engage with the day's problems through the use of mathematical tools.

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments, and to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen. (Organisation for Economic Co-operation and Development 1999)

> [Numeracy is] an aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work. (International Life Skills Survey 2000)

A handyman is not handy because he has tools; he is handy because he is willing to get his hands dirty using them. Likewise, a numerate individual has to be willing to engage in the work—willing to, when the situation calls for it, pull out his tools and use them.

Numeracy as Stepping Up

Thus, a numerate person is someone who is both willing and able to get the job done. This person knows the tools in his or her toolkit, has the confidence that he or she can get the job done, and is willing to engage in the problems that he or she encounters in work and life.

This has implications for what it is we expect from our students in the context of numeracy. Is the student who uses multiplication to solve a real-life problem more numerate than the one who uses repeated addition? Both students are exhibiting all the qualities of a numerate person implied in the sections above. They are both willing to get the job done. They are both using tools comfortable and familiar to them. The main difference between them is the efficiency of their strategies, but not necessarily their choice of strategy, for the second student may not have multiplication as a tool readily available to choose from. In mathematics we are concerned very much with the choice of strategy, as evolving and abstracting strategy is what moves the mathematics curriculum forward. In numeracy, however, we are much more concerned with stepping up and getting the job done with whatever tools are available to us.

> Numeracy is getting the job done with the tools you have. (Liljedahl 2010)

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The Pros and Cons of Contests

Karl Dilcher

Mathematical contests and competitions have long been among the main components of the Canadian Mathematical Society's educational initiatives. Indeed, the second entry under Education on the CMS website (http://cms.math.ca) is about competitions, and the introduction to those extensive and well-organized pages states that "competitions are an important part of learning mathematics and a fun activity for students of all ages." It then goes on to describe the CMS's support for competitions. The society has a Mathematical Competitions Committee, and at least four more of the 14 standing committees are partly or indirectly involved with competitions.

Over the years I have also been involved with competitions, both within the CMS and at Dalhousie. As a member and then chair of the Endowment Grants Committee, I played a role in the awarding of grants to several excellent proposals involving local, regional and national competitions. In my own department, before I became chair, I was involved for many years in organizing training sessions for the Putnam and the Science Atlantic (then called APICS) competitions. I supervised the Putnams and organized travel and accommodation to the APICS competitions. I even designed a (now discontinued) problem-solving course featuring competition-type problems, aimed at preparing students for various contests.

Before I go on, let me state my unambiguous support for problem solving as a mathematical activity. It was perhaps not such a coincidence that my colleague and office neighbour Swami (S Swaminathan) independently chose a topic quite similar to what I was going to write about.[1] We both have similar mathematical tastes, and our approaches to mathematics are largely problem-based. Along with many other mathematicians, we take delight in beautiful problems, and usually even more delight in our efforts to solve them. For us mathematicians the word *problem* has a positive connotation, which is certainly not the case in everyday nonmathematical usage of the word. In fact, it is difficult to convince a nonmathematician (or, to be fair, a nonscientist) that a problem can actually be beautiful.

It is also true that "mathematics is not a spectator sport" (the title of a book by George M Phillips [2005]). But is it a sport, in the competitive sense? You learn by doing; this is the premise of Phillips's book, and this is what we tell our students as we give them homework and practice problems. But do we learn better and faster by doing mathematics fast and under time pressure?

This ambiguity was also the topic of a brief article, "Pros and Cons of Math Competitions," by Richard Rusczyk,[2] founder of the very interesting and engaging web resource Art of Problem Solving. Referring to competitions for middle school and high school students in the United States, Rusczyk writes,

> The most immediate value of these math contests is obvious—they pique students' interest in mathematics and encourage them to value intellectual pursuits. Kids love games, and many will turn just about any activity into a contest, or in other words, something to get good at. Math contests thus inspire them to become good at mathematics just like sports encourage physical fitness. Eventually, students put aside the games. By then, hopefully an interest in the underlying activity has developed.

These are indeed very strong and convincing arguments in favour of math competitions. But Rusczyk goes on to caution that there are some pitfalls. In particular, he warns against what he calls "curricular contests" and contests that greatly emphasize speed or memorization. Contests need to be well designed, he argues, and should help students develop the ability to think about and solve complex problems. Rusczyk mentions two further pitfalls, namely extending children beyond their abilities, with the danger of the experience going from humbling and challenging to humiliating and discouraging. Finally, he cautions against burnout, with the danger of students not just turning against competitions, but against math in general.

I'm giving so much space to Rusczyk's article because it puts into words my own ambiguous feelings about math competitions, both as someone involved in them as an educator and minor administrator, and as a participant in a different era (the early '70s) and a different country. I myself was always attracted to mathematics because it was, and remains, one of the least competitive endeavours around. I could (and still can) be slow, very slow, and get away with it. Anything competitive has always turned me off, and I instinctively stayed away from "hot topics."

Partly for this reason, I must dispute one argument that Rusczyk brings forward in favour of math competitions: "For better or worse, much of life is competition, be it for jobs or resources or whatever." No, it doesn't have to be that way. Collaboration is always better in all spheres of life and society. So, by all means, let's build on children's love of games and competitions. But let's be mindful of the pitfalls and dangers of instilling too much of a sense of competition in children.

What does this mean for the CMS and the wider mathematical community? In spite of my words of caution, I believe we are doing all right; many of the competitions are collaborative, and there are Math Camps, Math Circles, Math Leagues, and other less competitive and more collaborative initiatives. So, in most parts of the country, there are programs for the slow kids, as well as for the fiercely competitive, and everyone in between. In any case, I hope that most will be able to say, as Terence Tao (2006) did at the beginning of his *Solving Mathematical Problems: A Personal Perspective*, "But I just like mathematics because it's fun."

Notes

1. See "Problem Solving," by Srinivasa Swaminathan, on page 15.

2. www.artofproblemsolving.com/Resources/articles.php?page=pc_competitions&

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Problem Solving

Srinivasa Swaminathan

The teaching of undergraduate and graduate courses in mathematics involves routine exposition of standard topics illustrated by solved problems from the texts. Weekly assignments are generally based on exercises from textbooks. Generally, mathematics is studied not for its own sake, but because the ultimate object is merely to pass an examination or to acquire the minimum knowledge necessary for dealing with some other subject of study. In such a situation, how much of problem-solving ability is acquired by students is doubtful; just propose a problem outside the normal curriculum—one would find that most students are unable to solve it. However, there are gifted students in almost every class. Problem-solving sessions are held to train such students so that they can compete in the annual Putnam and similar exams; they learn to apply previously acquired knowledge to new and unfamiliar situations.

Problems can be classified under different headings: mechanical or drill problems; those that require understanding of the concepts; those that require problem-solving skills or original thinking; those that require research or library work; and, finally, those that are group projects, requiring group participation.

The importance of problem solving in the learning process and also in the growth and development of mathematics has been recognized and emphasized by many prominent authors (for example, George Pólya's [1945] *How to Solve It*). New branches of mathematics have arisen from the search for solutions of challenging problems. Noteworthy examples are the successful attack on the brachistochrone problem by the Bernouilli brothers and the role played by their solution in the evolution of the calculus of variations. Mathematical theory of probability arose from the investigations by Pacioli, Cardan, Tartaglia, Pascal and Fermat. Topology and graph theory had their origin in Euler's analysis of a problem about crossing bridges. The fact that in some fields (algebraic) the resolution into prime factors is not unique as it is in common arithmetic led Dedekind to restore this highly desirable uniqueness by the invention of *ideals,* an important concept in algebraic geometry.

Many mathematical journals contain problem sections inviting readers to submit solutions. From these solutions, the editors select what they consider to be the "best" solution, which they publish along with other interesting solutions, if any. Solutions of difficult and challenging problems may lead to interesting further investigations of the devices employed. Selecting proposals poses a more challenging task to the editors than the selection of solutions; the editors seek to have a diversity of high-quality proposals in geometry, analysis, number theory, etc, rising above the level of unimaginative textbook exercises. Elegant proposals attract a wide range of would-be solvers. The criteria for elegance can be summarized in the ABCDs of elegance as follows: *A* for *accuracy, B* for *brevity, C* for *clarity* and *D* for *display* of insight, ingenuity, originality and generalization, if possible.

Periodically, collections of proposed and solved problems from well-known journals are published. Thus, *The Otto Dunkel Memorial Problem Book* (Eves and Starke 1957) was published by the Mathematical Association of America on the occasion of the 50th anniversary of the *American Mathematical Monthly,* which contains a popular section on problems. The most recent such collection is *A Mathematical Orchard* (Krusemeyer, Gilbert and Larson 2012), from the Mathematical Association of America, which contains 208 challenging, original problems with carefully worked, detailed solutions. One can spend hours browsing through this book, thinking about and trying to solve problems before looking at the solutions. As I was thinking about problem 62 of the book—which is to find the fifth digit (the ten thousands digit) from the end of the number 5 raised to the power of 5, which is raised to the power of 5, . . . up to five times!—the idea for writing this editorial occurred to me! [Answer: 0.]

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Understanding Studying and Studying Understanding

Jennifer Hyndman

Every mathematician recognizes that adding fractions of polynomials is the same as adding integer fractions—one first factors each fraction as much as possible, finds a least common denominator, rewrites the fractions with that denominator, and then adds the numerators. What is actually happening in our understanding of this? We are able to see the pattern for adding and are able to move this pattern from a simple situation to a more complex situation.

My first-year calculus students make mistakes in adding fractions of polynomials. When I show them a pattern of adding fractions of integers that mimics their incorrect polynomial addition rule, their body language response is one of total understanding of the integer situation.

What is it that they actually understand about addition of fractions of integers? They can find a prime factorization of an integer. They can find common factors of two integers. They can find a least common multiple of two integers. They can multiply integers. They can add integers. However, I doubt that the students could articulate this list of actions as part of their understanding of how to add fractions. What I am not sure of is whether this negatively influences their ability to add fractions of integers or whether this affects their ability to transfer their understanding to adding fractions of polynomials.

I continually observe how people collect knowledge and compare it to how my students learn mathematics. Gary, a member of my family, has over a thousand CDs and listens to music 18 hours a day, yet he cannot always identify the time signature of a song. He will happily ask me what the time signature is. With a little thought, I can tell him. He knows the history of every artist on every CD while I might not be able to name the band. Which one of us has a better understanding of music? Which student has a better understanding of mathematics? The student who can describe the process of adding fractions or the student who can do the process? What about the student who can both add fractions and describe the process? Most of us would agree that this latter student has the best understanding of the three students. What about the music listener who can identify the artist and the musical structure? Does that person have a better understanding of music than Gary or I? Or just a different understanding? Does this person enjoy music more or less because they instinctively hear (and cannot ignore) the underlying structure of every piece of music they hear?

As I believe (at least I think I do) that more knowledge increases understanding and enjoyment, I have been giving one-on-one sessions and running study skills workshops on how to learn mathematics for several years. The workshops were developed and initially run with my colleague, Vivian Fayowski, the coordinator of the University of Northern British Columbia's (UNBC) Academic Success Centre. For one year, they were also part of a UNBC Early Alert research project with Dan Ryan, Kerry Reimer and Peter MacMillan. The focus of these workshops has been, in essence, to explore how to organize mathematical information and how to internalize and articulate mathematics.

The most common response to the question of how a student studies is that they "do" problems. Initially, they are unable to articulate what they mean by "doing" a problem. Several minutes of prompting eventually yields words like *write, read, copy, draw, type, speak, hear, listen* and *rewrite.* Is their inability to describe their own actions relevant to their difficulties in learning mathematics? I think so. However, I also believe one needs to be able to articulate what one is doing and then internalize it so it is nonverbal.

I am part of my own observations of learning. As a student in dance classes, I am continually being challenged by learning new styles of dance and new choreography. Not long ago I suffered the misfortune of not being able to figure out exactly where I was supposed to be while on stage in a group number. This was unusual for me and, to prevent it from happening again, I thought long and hard about what had happened. I certainly knew the choreography thoroughly as I had been talking the group through the steps to help us practise. This talking turned out to be the problem. I had "learned" the choreography as if it included speaking. When I walked on stage and had to smile instead of talking, I was literally lost.

On the verbal, visual and kinesthetic scales of learning I am highly kinesthetic, very visual and almost nonverbal. Speaking the steps had interfered with my own ability to reproduce the steps without speaking. What are our students actually learning when they study mathematics or when they study any subject? What do we actually test for in a midterm or an exam? Are our students self-aware enough to realize which study methods work for them and which don't? Do they even know more than one study method?

While thinking about this article, I asked my family how they studied in university and how they learned to study. The actions they described all fit under my umbrella of things to do to study. What was more interesting were their comments on how they learned to study. David C's first reaction was that he had no memory of ever receiving specific instruction, and then he said he might have had some in high school English. David H's comment was that it was like learning to be a parent; you just do it. David H's son thinks his college course on time management is a waste of time as he is learning nothing new. When I work with students who are failing courses, they frequently and proudly admit they spent very little time on the courses and think that they will fix their grades by "spending more time studying." However, when asked what they will do in this additional time, they say, "Do problems," which brings us back to the earlier-mentioned inability of students to describe what this means.

As instructors, what should we or can we do to assist students to be more self-aware in their studying (without giving time-management courses that are a waste of time)? The lucky students, like the Davids and me, figured out effective study techniques that fit in the time we had available. Other students do less well than they are capable of. I think we should be teaching study skills as part of the ongoing education of our students.

Here are some of the things I would do if I were to teach the perfect course as part of the perfect university degree. The course objectives provided to the students would have components of both mathematical content and study skills. The lectures would have study techniques embedded in the content development. The content to be examined would include techniques for studying the material. Here is an illustration of the second idea: discuss the definition of continuity and then discuss techniques for memorizing a definition, such as writing it out several times, reading it out loud, reading it silently, and reciting it from memory with your eyes closed versus with your eyes open. An exam question could be as simple as "List three techniques for memorizing a definition" or as self-reflective as "What study technique works best for you when you try to memorize the definition of *continuity?*"

Of course, at least in my opinion, learning is an activity that spirals. One initially learns a very rudimentary approximation of a concept and then rethinks and refines the approximation until, with focused attention, one understands the same thing as others do. Where does studying study skills fit in this spiral? It cannot be too early, but it must be early enough to be useful. When I work with students, I often come to the conclusion that they have to be ready to hear what I have to say about study skills (or any subject) before they can actually take in the knowledge.

Returning to learning how to add fractions of polynomials, the spiral of knowledge for this starts with the spiral for adding fractions of integers, layers on the language of polynomials, and then repeats the original spiral another time. How could we help our students learn to add fractions? Test questions like "Explain the steps in adding the following fractions of integers (polynomials)" would be preceded by homework questions like "Build mind maps for integers and for polynomials that illustrate the concepts of adding two fractions. Discuss the similarities and differences." The intrinsic patterns that mathematicians see can be brought into the light for our students.

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I've Got Problems: Chapter 1 of *Sally Strange: And How She Learned to Stop Worrying and Love Grade 7 Math*

Nico Rowinsky

tuesday september 15th

If I was given the choice between going to math class or going to the orthodontist for a tightening, I'd probably choose the orthodontist. But I'm only 11 and I don't get to make those choices.

Yesterday, I had the painful tightening. Today, I'm here. Math class.

I move through the room towards my seat and say hi to Chin as I squeeze by his chair. Before I get a chance to sit, the bell goes, and the familiar voice of Niles comes on the PA: "Please stand for the national anthem." I plop my bag down in the little area between my desk and Arial's. The noise of everyone getting up from their chairs carries on into the first few bars.

O Canada!

Our home and native land!

While some decide to stand quietly, others are still kinda moving and continue their morning chat in whispers. I look over to Lindsay across the room and we make weird faces for a moment until Evan interrupts our friendly game by walking in late.

With glowing hearts we see thee rise.

I watch Lindsay's weird face turn into a cute smile, followed by a tiny, flirty wave pointed towards Evan. He smiles back but continues his march. He passes my desk and gives me a nod. My heart skips a beat. Or does it beat twice as fast? I'm not sure. I can't think for a moment. It's not even 9:05 and I'm already needing some help.

God keep our land glorious and free!

Oh, I'll be fine. Evan and I are close friends. We've known each other since Grade 1. We like to joke around and tell people we're cousins, even though, I don't know, this year—something's different.

O Canada, we stand on guard for thee.

I try not to stare at him too obviously,

Ooooo Caaaa-na-da.

as he turns into his aisle,

we stand on guaaaard

and timed perfectly,

foooor

reaches for his chair,

theeeeeeeeeeee

and sits.

Seemingly on *his* cue we all sit down.

"Good morning, Winona! Today is Tuesday, September 15th—a Day 6 on our cycle—and these are your morning announcements." Niles. Where does he get the energy to be so cheery in the morning?

The announcements continue as I look down and read the graffiti in my desk to see if there is anything new since Friday. I read over Gavin's name for the millionth time. He left me a few messages last week. One of them was simply *Good game!* No *Hello*. No *Sally*. Just *Good game!* He'd watched me play and wanted me to know. Both Evan and Gavin are trying out for the volleyball team, just like we are, and they're showing their support for the girls.

I pick at my teeth—the elastics are annoying and everything feels tight in the morning—before I reach for my pencil case. Today, we're starting something new because yesterday was the last of the so-called review.

After the announcements end, Mr Rowe slowly walks from his desk to the front of the room, faces the class and. with way more drama than needed, holds up two pieces of paper. He tries to make math fun. He tries.

I'm half listening, not ready to fully commit my attention to my overly excited math teacher. It's too early, my mouth feels too tight, and two pieces of paper aren't going to do it for me.

Our room door is open and I see Niles in the hall, walking past. He pauses for a split second and looks to see if I'm okay before he continues to his Grade 8 homeroom class. He's like that. After the announcements, he checks in on me, every day. I'm not sure why, but maybe it's because it's still September. Maybe it's because I'm in Grade 7 and he's in Grade 8. Or, maybe it's because this is his second year at Winona Drive Senior School, and it's my first. But most likely, it's just what big brothers do—check on their little sisters.

Back to the action. I missed something. I turn to Chin. "What do we have to do?"

Chin is this tall, friendly giant in our class. I would say fat, but that seems rude. He's just big, I guess. He's not only friendly, he seems to pay attention just a little more than I do, so he's always there when I have one of my "zone-out" moments.

"Pay attention." He tries to sound upset. "We have to make a cylinder out of this piece of paper."

I grab the sheet from Chin and wrap one side onto the other, making a tube. "Ta-daaaa!" I throw my hands up and announce to my group, "I'm a math genius!"

"Sally, do you want to share with the class?"

Shoot. My hands went up just for show; now I'm booked. I'm totally not a math genius.

"Umm, ya."

I feel like I'm getting smaller. I hate being on the spot. Reason number 24 to hate math.

"I . . ." shrinking

"folded it like this . . ." shrinking

"to make the thingy . . ." shrinking

"like you said . . ." Mr Rowe looks at me in silence. Shrunk!

And then says, "Good. Perfect." And rolls up one of his sheets, just like mine, tapes it together and places it on the front ledge.

What? I think to myself.

"What?" Arial says, half laughing at me.

"Anyone come up with a different solution?" he asks.

I'm in shock. My short tube sits proudly on the ledge, looking a little fat (not to be rude). Lindsay shoots up her hand and responds with her own solution. Her butt almost leaves her seat as she shows off her answer. Her tube is the same as mine, just the longer ends coming together.

"Very good, Lindsay," he says as he turns to showcase Lindsay's solution beside mine.

"So, the question is . . ."

Here we go. I knew it couldn't have been that easy. *Here comes the question only the math teacher actually cares about.*

Mr Rowe raises his hand slowly as he asks, "Which cylinder would hold the most water?" His hand clearly indicates we're not supposed to yell this one out.

The usuals raise their hands with confidence (how do they know this already?), followed by a few stragglers. Then Evan calls out, "They're the same!"

Our math teacher looks directly at him with no sign of emotion. Keeping his hand up, he slowly walks over to Evan.

"Someone with their hand up, please," and he calls on Gloria while whispering something to Evan.

"The taller one holds more," comes a shy answer from Gloria, sounding more like a question.

"Why?" the math teacher's favourite response to any unsuspecting student.

"Because"—but she is not the type to just say *because*—"because it's bigger, taller, so it holds more."

"Good." He leaves Evan and now moves to the back of the class. Most of us turn to follow him except for Evan, who now might be regretting walking in late *and* blurting out his answer.

"Anyone agree with Gloria?" More than half the hands go up.

"Anyone disagree?" No hands.

Wait. One hand. It's Evan, back from his momentary mental detention.

"Evan." He calls on him as if to say, "Thank you for putting up your hand this time."

"Uhh, I think they're both the same."

Mr Rowe nods his head, satisfied that he has our attention. "Good." He walks back to the front of the class.

Good? What kind of answer is "good"? That doesn't answer anything. Which tube holds more? I didn't care before, but now I want to know. *The taller tube must hold more, right?* Gloria agrees. More than half the hands in the class agree. I wait a sec to see what Mr Rowe is about to say.

Standing in front of the board, he begins again. "Good. Now here's your challenge for today."

Challenge? What the . . .? What happened to the tubes?

Before he can continue, it's Arial who asks (on behalf of most of the class), "So, which cylinder holds more, Mr Rowe?"

"Oh. Right. Ummm, I don't know yet. We'll have to figure that out. Should we have a quick discussion before our challenge?"

So there's a discussion all right, but it doesn't give us the answer, and neither does our teacher, just some more questions.

Oh, Mr Rowe. I guess he sets it up this way. It's a week into school and although I haven't figured out any of the math yet, I think I'm beginning to figure him out a little.

This time it's a "challenge," but it's always a different word with teachers. *Challenge, task, questions, problems.* Problems, really. *I have a problem for you. Work on these problems. Did you finish your math problems?* It all sounds so negative. I clearly have a problem with the word *problem.*

Homework:

Write an explanation as to why one tube holds more than the other tube.

Sally Strange 7-1

In class you said to find how much a tube holds:

Multiply the size of the circle by the height:
circle × height

You also said that circle size is done in Grade 8, so you gave us the size of both circles:

Short tube	Long tube
circle = 62 cm^2	circle = 37 cm^2

Then I measured the height:

height = 22 cm	height = 28 cm

circle × height

Short = 62 × 22 = 1,364   Long = 37 × 28 = 1,036

The short tube holds more. Because the number is bigger.

I did most of the math myself (with a calculator), but Niles helped me figure out what to do. He also said that I haven't explained *why.* I hate "explain why" in math. I would like Mr Rowe to explain why we need to explain in math. I wrote, "Because the number is bigger," which is right. Don't ask me to explain why!

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Born in Uruguay, Nico Rowinsky grew up in Mississauga, Ontario, and studied mathematics at the University of Toronto. His first novel, Sally Strange: And How She Learned to Stop Worrying and Love Grade 7 Math, *which began as a writing assignment for a student, is a real yet sensitive look at relationships in Grade 7. The novel is available through Leanpub at https://leanpub.com/Sally_Strange_Grade_7. Nico is a middle school math teacher and lives in Toronto with his wife, also a teacher. Follow Nico on his blog (http://ynaughtmath.blogspot.ca) and Twitter (@NicoRowinsky).*

The Exploration of Patterns

Chelsey Bonnett

During my studies to become a teacher, I became interested in how children think about patterns. I designed a series of tasks that would help early learning students demonstrate the following outcomes from Alberta's K–9 mathematics program of studies (Alberta Education 2007, 53):

- Distinguish between repeating patterns and nonrepeating sequences in a given set by identifying the part that repeats.
- Copy a given repeating pattern, e.g., actions, sound, colour, size, shape, orientation, and describe the pattern.
- Extend a variety of given repeating patterns by two more repetitions.
- Create a repeating pattern, using manipulatives, musical instruments or actions, and describe the pattern.
- Identify and describe a repeating pattern in the classroom, school and outdoors; e.g., in a familiar song, in a nursery rhyme.

The goal was to have students recognize how patterns allow them to make predictions and justify their reasoning when solving routine and nonroutine problems.

I chose to work with Dave,[1] a five-year-old who attended kindergarten at a public elementary school in Slave Lake. This boy was rather bright, tended to catch on quickly, was already showing a great interest in the area of science, and was enthusiastic when approaching new tasks.

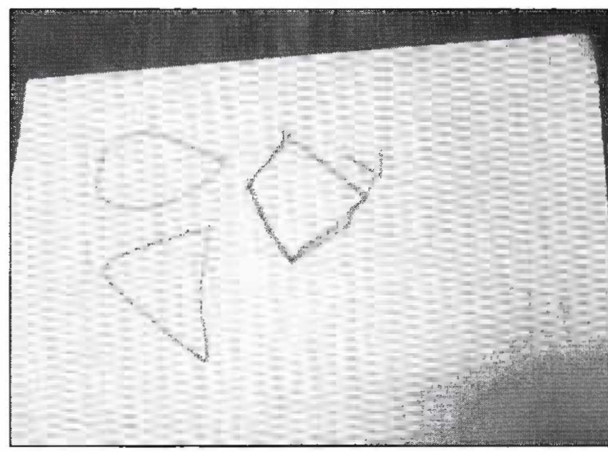
We worked together for approximately 30 minutes, going through the tasks I had planned. He was able to follow my instructions without much elaboration; he took his time thinking through what I had asked of him before responding; and, as he worked, he talked through his thinking, which is a helpful strategy he had developed for himself but which also helped me understand and follow his thought process. Dave was confident in creating and extending patterns with the use of colours, but he had great difficulty applying the same concept to shapes and number patterns.

What follows are my observations and reflective notes as I learned more about Dave's understanding of patterns.

The Tasks

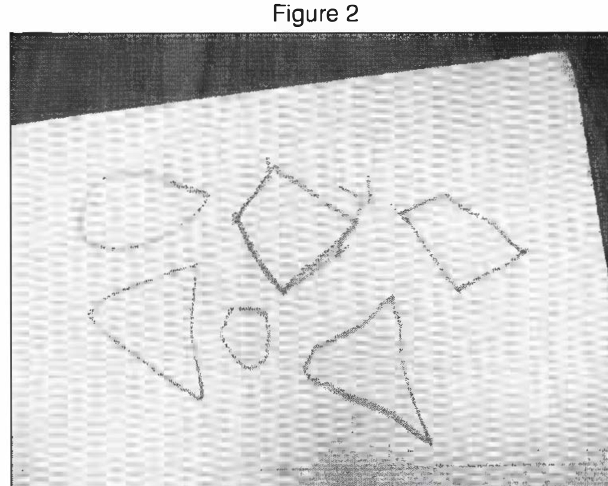
I asked Dave to draw three different shapes. He first drew a triangle and a circle. As he drew the shapes, he said their names out loud. He then said "diamond" and began drawing one, but then he paused and asked for help. I helped him finish drawing the diamond. See Figure 1.

Figure 1



I then asked Dave to repeat the pattern of those three shapes. He drew them again, although not in a particular sequence or size. See Figure 2.

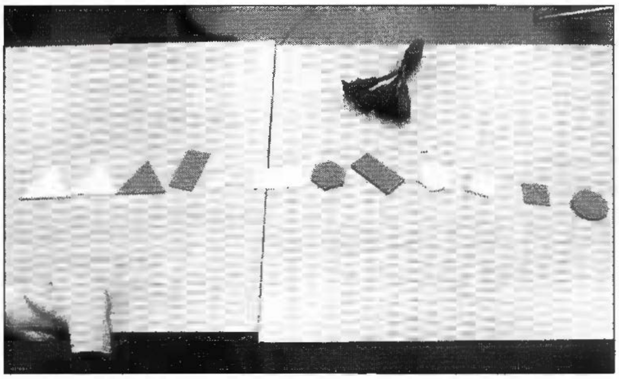
Figure 2



I asked Dave to use the manipulatives (various shapes, in various colours and sizes) to make a pattern. He asked if he should use colours or shapes, and

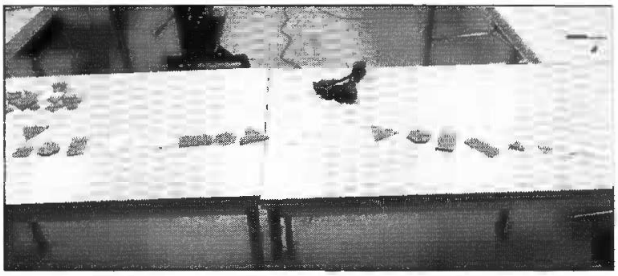
I allowed him to make the choice. He then created the pattern in Figure 3, focusing only on colours (yellow and blue). Dave successfully made the pattern three times.

Figure 3



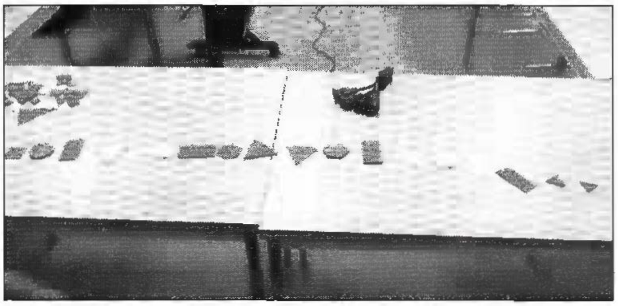
To extend this task, I asked Dave if he could make a repeating pattern using three variables. Figure 4 shows the pattern he created: blue, yellow, red, yellow, blue, red, yellow. I asked him if everything was in order, and he began going through each set of three, saying the colours. When he said "yellow" the second time, he stopped and went through the first three colours again before correcting himself and saying it should be blue, yellow, red, blue (rather than blue, yellow, red, yellow).

Figure 4



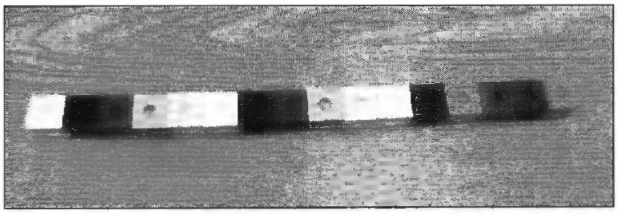
My response to him was, "Can you change this to repeat the pattern in sequence?" Figure 5 shows his solution. He rearranged the manipulatives to demonstrate a repeating pattern using three different colours (blue, yellow, red).

Figure 5



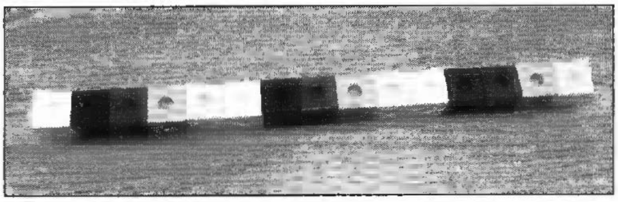
Changing manipulatives, Dave attempted to create another repeating pattern (see Figure 6). When I asked him why the end looked different from the beginning, he paused, thinking. After a moment, he responded, "I don't know. I just changed the pattern. Now it's not the same."

Figure 6



Dave pulled the blocks off the end and tried again. He said, "I just look at the beginning and know what is next." As he did this portion, he said each colour aloud and ended up with the pattern in Figure 7.

Figure 7



I then showed the repeating pattern in Figure 8 (blue, blue, orange, brown, red, blue, blue, orange, red, brown) to Dave, and asked him to continue it. He first went through, saying each colour, and he quickly recognized the mistake I had included in the pattern. I was pleased and surprised by this.

Figure 8

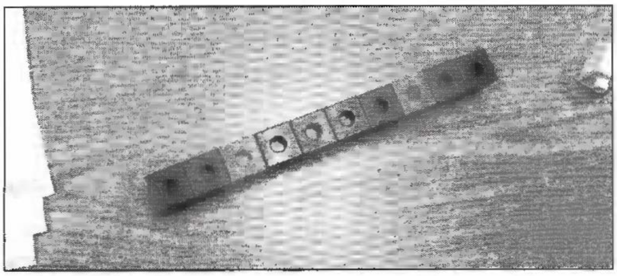


Figure 9 shows the correction Dave made to the repeating pattern (switching the red and brown blocks at the end).

Figure 9

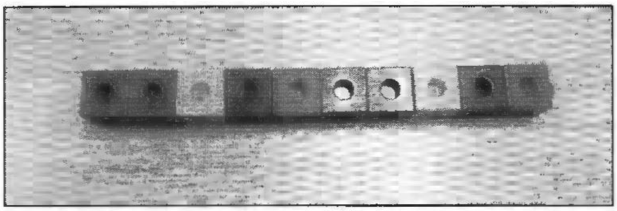


Figure 10 shows Dave checking his work after correcting and expanding the pattern I presented him with.

Figure 10

My Reflections

My work with Dave prompted me to consider whether the ability to form patterns and to develop understanding of patterns on the basis of colour or number is influenced by age or by learning style—or perhaps by both.

When I asked Dave to identify three different shapes, I was surprised that he included a diamond (although he did need assistance drawing it the first time). This demonstrated to me that he had a fairly good grasp of various shapes, and it makes me curious about how many other children would select diamond as a shape without any prompting.

Yet, when the activity transitioned into using shapes of various colours and sizes, Dave continued to work with colour-based patterns. This made it apparent that colours were much easier for him and were within his identifying comfort zone, and that he would need to spend more time transitioning his knowledge of colour patterns to be applied to shape patterns. This could also be attributed in part to his being more comfortable with drawing the patterns than with using manipulatives, but after one pattern with manipulatives, he began to grasp how he could use and manipulate them. When Dave was first presented with the manipulatives, he asked if I wanted a shape or a colour pattern. I let him make this choice as I felt it would indicate where he was more comfortable beginning a pattern lesson.

I was surprised with the strategies Dave came prepared with. As problem-solving strategies, he used talking aloud and crossing off on his fingers as he called out the colours. These strategies helped him to correct his own mistakes, to correct the mistake I had left for him to find (without letting him know it was there) and to extend the pattern. He seemed to be visual, as he could identify the patterns on the wall and the one on his sweater, but he had difficulty with shapes, numbers and physical/oral patterns. If I were to work with Dave again, I could use the talk-aloud and the physical motions he used to problem solve as a way of modelling patterns using shapes and gradually moving his thinking about patterns to include symbols, shapes, numbers, and oral and physical patterns. Working with another child his age would be beneficial for Dave, as they could communicate how they see and problem solve patterns in their environment.

A modification I would make to this lesson would be to exclude an oral or a physical pattern. I underestimated the amount of time it would take to perform this task with Dave. For Dave to develop a good grasp of patterns, I felt it necessary to have him draw a repeating pattern once, and then build it using two types of manipulatives, working with each set of manipulatives more than once. Developing these skills was crucial in order for him to move on to completing a pattern that had already been started. Because Dave spent time focusing on patterns with colours, I may need additional sessions to work toward developing patterns with shapes and numbers and oral/physical patterns. I could work with a student of this young age only so long before valuable learning stopped happening. Recognizing this, I pulled back and decided to include these tasks in separate lessons.

Dave focused on colours but lacked the ability at this point to transfer his knowledge to shapes, numbers, and oral/physical patterns. Although he had little experience with addition and subtraction, when I presented (orally and in writing) the pattern of 1, 2, 1, 2, 1, 2, he could not recognize the pattern, only that it was "wrong," and he gave me an answer of 3. Curious, I prompted him to explain this to me. In this area he could not communicate his understanding as clearly as he had with colour patterns and simply responded, "1 and 2 is always 3." While this does not indicate any understanding of number patterns, it does show that Dave has great potential to understand number operations and relationships. I don't yet know how I can use this to develop a connection to patterns. I have, however, recognized a teachable moment that I let pass by. As we wrapped up the task, Dave noticed a large calendar drawn on the whiteboard in the room. He made a connection to this and even wanted me to help him write an important event for him on the calendar, which I did. Not until later did it occur to me that this was an opportunity to make a connection to patterns, using the calendar as a medium. This showed me an important strategy I can use to move Dave's learning forward.

Note

1. Name has been changed.

Reference

Alberta Education. 2007. *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. Edmonton, Alta: Alberta Education. Also available at http://education.alberta.ca/media/645598/kto9math_ind.pdf (accessed September 25, 2013).

Chelsey Bonnett is a recent graduate of the Aboriginal Teacher Education Program, which allowed her to earn a BEd through the University of Alberta while working and remaining in her home community of Slave Lake, Alberta. She has had many teachers in her life, not all in the field of education, who have inspired her to become a teacher. She loves learning for the sake of learning and working with children, and is thrilled to be embarking on her next journey in life as an educator.

Alberta High School Mathematics Competition 2012/13

The Alberta High School Mathematics Competition is a two-part competition taking place in November and February of each school year. Book prizes are awarded for Part I, and cash prizes and scholarships for Part II. Presented here are the problems and solutions from the 2012/13 competition.

Part I

November 21, 2012

1. Each day Mr Sod visited pubs A, B, C and D, in that order, always spending \$35, \$12, \$40 and \$27 at the respective places. His total expenditure at the pubs, from the beginning of the month up to a certain moment that month, was \$1,061. Which pub would he be visiting next?

   (a) A (b) B (c) C (d) D (e) impossible total

2. Meeny, Miny and Moe were playing tennis. From the second game on, the one who sat out the preceding game would replace the loser of that game. At the end, Meeny played 17 games and Miny played 35 games. How many games did Moe play?

   (a) 18 (b) 26 (c) 36 (d) 52 (e) not uniquely determined

3. A circle of diameter r is drawn inside a circle of diameter R. For which of the following pairs (r, R) is the area of the smaller circle closest to half the area of the larger circle?

   (a) (1, 3) (b) (2, 4) (c) (3, 5) (d) (4, 6) (e) (5, 7)

4. A quadratic polynomial $f(x)$ satisfies $f(0) = 1$, $f(1) = 0$ and $f(2) = 3$. What is the value of $f(3)$?

   (a) -3 (b) 1 (c) 2 (d) 10 (e) none of these

5. ABCD is a square. E and F are points on the segment BC such that BE = EF = FC = 4 cm. The segments AF and DE intersect at G. What, in cm^2, is the area of triangle EFG?

   (a) 6 (b) $4\sqrt{3}$ (c) 8 (d) 12 (e) none of these

6. For how many integers $n \geq 2$ is the sum of the first n positive integers a prime number?

   (a) 0 (b) 1 (c) 2 d) 3 (e) more than 3

7. In a test, Karla solved four-fifths of the problems and Klaus solved 35 problems. Half of the problems were solved by both of them. The number of problems solved by neither was a positive one-digit number. What was this number?

   (a) 1 or 2 (b) 3 or 4 (c) 5 or 6 (d) 7 or 8 (e) 9

8. What is the largest possible integer a such that exactly three of the following statements are true: $a < 1$, $a > 2$, $a < 3$, $a > 4$ and $a < 5$?

   (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

9. A rectangle with integer length and width in cm has area 70 cm^2. Which of the following, in cm, cannot be the length of the perimeter of the rectangle?

   (a) 34 (b) 38 (c) 74 (d) 98 (e) 142

10. The positive integer n is such that between $n^2 + 1$ and $2n^2$ there are exactly five different perfect squares. How many such n can we find?

    (a) 0 (b) 1 (c) 2 (d) 3 (e) more than 3

11. ABCD is a rectangle such that AD – AB = 15 cm. PQRS is a square inside ABCD whose sides are parallel to those of the rectangle, with P closest to A and Q closest to B. The total area of APSD and BQRC is 363 cm^2, while the total area of APQB and CRSD is 1,113 cm^2. What, in cm^2, is the area of PQRS?

    (a) 900 (b) 1,600 (c) 2,500 (d) 3,600 (e) not uniquely determined

12. Weifeng writes down 28 consecutive numbers. If both the smallest and the largest numbers are perfect squares, what is the smallest number she writes down?

    (a) 9 (b) 36 (c) 100 (d) 169 (e) not uniquely determined

13. If the positive numbers a and b satisfy

    $$\frac{1}{a^2+4b+4} + \frac{1}{b^2+4a+4} = \frac{1}{8},$$

    what is the maximum value of $a + b$?

    (a) 3/2 (b) 2 (c) 5/2 (d) 4 (e) none of these

14. The incircle of triangle ABC is tangent to AB and AC at F and E, respectively. If BC = 1, $\angle A = 90°$ and $\angle B \neq \angle C$, what is the distance from the midpoint of BC to EF?

(a) $\sqrt{2}/4$ (b) $\sqrt{2}/2$ (c) $(3\sqrt{2})/4$ (d) $\sqrt{2}$
(e) not uniquely determined

15. At the beginning of the year, there were more robots than androids. On the first day of each month, each robot made seven androids and each android made seven robots. The next day, each old android would pick a fight with a new android, and they would destroy each other. At the end of the year, there were 46,875 million robots and 15,625 million androids. What was the difference between the numbers of robots and androids at the beginning of the year?

(a) less than 10 (b) at least 10 but less than 100 (c) at least 100 but less than 1,000 (d) at least 1,000 but less than 10,000 (e) at least 10,000

16. Let m and n be positive integers such that 11 divides $m + 13n$ and 13 divides $m + 11n$. What is the minimum value of $m + n$?

(a) 24 (b) 26 (c) 28 (d) 30 (e) 34

Solutions

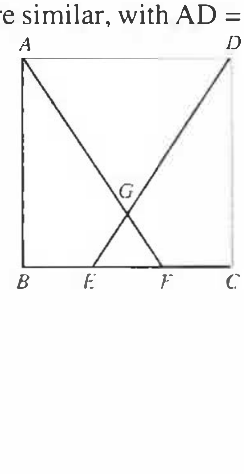
1. Note that 35 + 12 + 40 + 27 = 114 and 1,061 = 9 × 114 + 35. Thus, Mr Sod had spent \$35 on the 10th day of that month at pub A. The answer is **(b)**.

2. Since Meeny played 17 games, Miny and Moe played each other at most 17 + 1 = 18 times, and each could play at most 18 + 17 = 35 games. As Miny played 35 games, Moe did not play Meeny but played Miny 18 times. The answer is **(a)**.

3. We want $(r/R)^2$ to be close to 1/2. We have $(1/3)^2 < (2/4)^2 < (3/5)^2 < (4/6)^2 = 4/9$ and $(5/7)^2 = 25/49$. Since $25/49 - 1/2 = 1/98 < 1/18 = 1/2 - 4/9$, the answer is **(e)**.

4. Let $f(x) = ax^2 + bx + c$. Then $1 = f(0) = c$, $0 = f(1) = a + b + c$ and $3 = f(2) = 4a + 2b + c$. We have $c = 1$, $a + b = -1$ and $2a + b = 1$. Hence, $a = 2$ and $b = -3$, so that $f(3) = 10$. The answer is **(d)**.

5. Triangles GAD and GFE are similar, with AD = 3EF. Hence, the vertical height of triangle EFG is 1/3 of the vertical height of triangle ADG. Hence, it is equal to (1/4)AB = 3 cm so that the area of triangle EFG is 1/2 × 3 × 4 = 6 cm². The answer is **(a)**.



6. The sum of the first n positive integers is $[n(n + 1)]/2$. Suppose n is even. Then we must have either $n/2 = 1$ or $n + 1 = 1$. Both lead to $n = 2$. Suppose n is odd. Then we must have either $(n + 1)/2 = 1$ or $n = 1$. However, $n = 1$ is not allowed by the hypothesis. The answer is **(b)**.

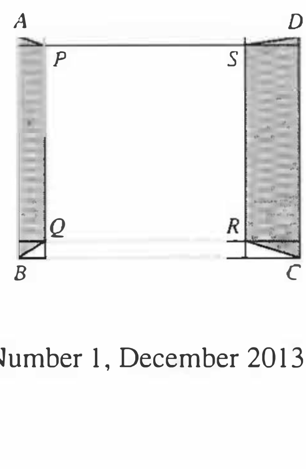
7. The fraction of problems solved only by Karla was 4/5 − 1/2 = 3/10 so that the total number of problems was a multiple of 10. The fraction of problems solved by Klaus was at most 1 − 3/10 = 7/10. Thus, the total number of problems was at least 50. If it was 50, then 10 problems were solved by Klaus alone, and as Karla solved 4/5 × 50 = 40 problems, the number of problems solved by neither was 0. The total number of problems could only be as large as 70, since 35 problems would be solved by both. In this case, the number of problems solved by neither was 1/5 × 70 = 14. It follows that the total number of problems must be 60, of which 30 were solved by both, 5 by Klaus alone, 3/10 × 60 = 18 problems by Karla alone, and 60 − 30 − 5 − 18 = 7 by neither of them. The answer is **(d)**.

8. Note that $a > 2$ and $a < 3$ cannot both be true as there are no integers between 2 and 3. Similarly, $a > 4$ and $a < 5$ cannot both be true. Since exactly three of the statements are true, $a < 1$ must be true. Hence, the largest possible value is $a = 0$, and for this value, the three statements $a < 1$, $a < 3$ and $a < 5$ are true and the two statements $a > 2$ and $a > 4$ are false. The answer is **(a)**.

9. We have 70 = 1 × 70 = 2 × 35 = 5 × 14 = 7 × 10. Thus, there are four possible shapes of the rectangle, with respective perimeters 142 cm, 74 cm, 38 cm and 34 cm. The answer is **(d)**.

10. Solving $(n + 5)^2 < 2n^2 < (n + 6)^2$ yields $50 < (n - 5)^2$ and $(n - 6)^2 < 72$. Thus, $8 \leq n - 5$ and $n - 6 < 9$ or $13 \leq n \leq 14$. The answer is **(c)**.

11. We shade the regions APQB and CRSD, while leaving the regions APSD and BQRC unshaded. Extend the sides of PQRS to the perimeter of ABCD, creating four rectangles at the corners, each of which consists of two congruent triangles, one shaded and one unshaded. The difference between the total area of the unshaded regions (not counting PQRS) and the total area of the shaded regions is 1,113 − 363 = 750 cm². The difference in the lengths of AD and AB is 15 cm.



Hence, the side length of PQRS is $750 \div 15 = 50$ cm, and the area of PQRS is 2,500 cm^2. The answer is (**c**).

12. Let the smallest and the largest numbers Weifeng writes down be n^2 and m^2 respectively. Since they are the ends of a block of 28 consecutive numbers, $(m + n)(m - n) = m^2 - n^2 = 27$. We may have $m + n = 27$ and $m - n = 1$, whereby $m = 14$ and $n = 13$. We may have $m + n = 9$ and $m - n = 3$, whereby $m = 6$ and $n = 3$. Thus, the smallest number Weifeng writes down may be $3^2 = 9$ or $13^2 = 169$. The answer is (**e**).

13. We have

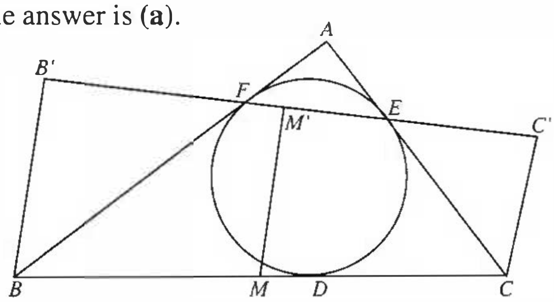
$$\frac{1}{8} = \frac{1}{a^2+4b+4} + \frac{1}{b^2+4a+4}$$
$$= \frac{1}{(a-2)^2+4a+4b} + \frac{1}{(b-2)+4b+4a}$$
$$\le \frac{1}{4(a+b)} + \frac{1}{4(a+b)}$$
$$= \frac{1}{2(a+b)}.$$

Hence, $a + b \le 4$. This maximum value is attained if and only if $a = b = 2$. The answer is (**d**).

14. Let M be the midpoint of BC and D the point where the circle is tangent to BC. Let B', C' and M' be the respective projections of B, C and M on EF. Now AEF is a right isosceles triangle. Hence, so are BB'F and CC'E. Hence, BB' = BF/$\sqrt{2}$ and CC' = CE/$\sqrt{2}$ so that

$$\text{MM}' = \frac{1}{2}(\text{BB}' + \text{CC}')$$
$$= \frac{1}{2\sqrt{2}}(\text{BF} + \text{CE})$$
$$= \frac{1}{2\sqrt{2}}(\text{BD} + \text{CD})$$
$$= \frac{\text{BC}}{2\sqrt{2}}$$
$$= \frac{\sqrt{2}}{4}.$$

The answer is (**a**).



15. Let the numbers of robots and androids be r and a respectively. After one month, these numbers became $r + 7a$ and $7r - a$. After another month, they became $(r + 7a) + 7(7r - a) = 50r$ and $7(r + 7a) - (7r - a) = 50a$. Hence, after a two-month period, the number of robots became 50 times the original number, and the same goes for the number of androids. There being six two-month periods in a year, the initial number of robots was $46{,}875{,}000{,}000 \div 50^6 = 3$, and the initial number of androids was $15{,}625{,}000{,}000 \div 50^6 = 1$. The answer is (**a**).

16. Since 13 divides $6(m + 11n) = (6m + n) + 13(5n)$, 13 divides $6m + n$. Since 11 divides $6(m + 13n) = (6m + n) + 11(7n)$, 11 also divides $6m + n$. Hence, $11 \times 13 = 143$ divides $6m + n$, so that $6m + n = 143k$ for some integer k. Since $6(m + n) = 143k + 5n = 6(24k + n) - (k + n)$, 6 divides $k + n$ so that $k + n \ge 6$. Now $6(m + n) = 143k + 5n = 138k + 5(k + n) \ge 138 + 30 = 168$. Consequently, $m + n \ge 28$, and this is attained if $m = 23$ and $n = 5$. The answer is (**c**).

Part II

February 6, 2013

1. Determine all pairs of positive integers (a, b) with $a \le b$ such that

$$\left(a + \frac{6}{b}\right)\left(b + \frac{6}{a}\right) = 25.$$

2. A set S of positive integers is called *perfect* if any two integers in S have no common divisors greater than 1. Candy wants to build a perfect set of numbers between 1 and 20 inclusive, in such a way that her set contains as many numbers as possible.
   (a) How many elements will her set have?
   (b) How many different such sets can she build?
3. Randy plots a point A. Then he starts drawing some rays starting at A, so that all the angles he gets are integral multiples of 10°. What is the largest number of rays he can draw so that all the angles at A between the rays are unequal, including all angles between nonadjacent rays?
4. In a convex pentagon of perimeter 10, each diagonal is parallel to one of the sides. Find the sum of the lengths of its diagonals.
5. Find all integers $r > s > t$ and all quadratic polynomials of the form $f(x) = x^2 + bx + c$ such that b and c are integers, $r + t = 2s$, $f(r) = 1$, $f(s) = b$ and $f(t) = c$.

Solutions

1. The given equation may be rewritten as $ab + 36/ab + 12 = 25$. Therefore,

$$(ab)^2 - 13ab + 36 = (ab - 4)(ab - 9) = 0.$$

Hence, $ab = 4$ or $ab = 9$. Note that a and b are positive integers with $a \leq b$. If $ab = 4$, we have $(a, b) = (1, 4)$ or $(2, 2)$. If $ab = 9$, we have $(a, b) = (1, 9)$ or $(3,3)$. It is easy to verify that all four are indeed solutions.

2. (a) Candy's perfect set may be {1, 2, 3, 5, 7, 11, 13, 17, 19}. We claim that this number is the highest possible. Now a maximal perfect set must contain the element 1, as otherwise we can add 1 and obtain a larger perfect set. Also, a maximal perfect set cannot contain an element that is divisible by two distinct primes, as otherwise we can replace that element by the two primes and obtain a larger perfect set. Hence, each element other than 1 is a positive power of a prime. Moreover, distinct elements are powers of distinct primes. Since there are only eight primes less than 20—namely, 2, 3, 5, 7, 11, 13, 17 and 19—our claim is justified.

   (b) Every maximal perfect set Candy can build must have the form

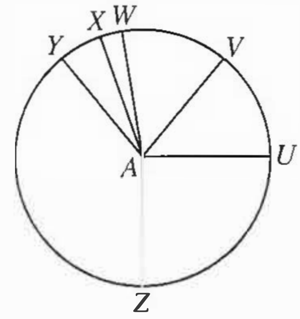
$$S = \{1, 2^{i_2}, 3^{i_3}, 5^{i_5}, 7^{i_7}, 11^{i_{11}}, 13^{i_{13}}, 17^{i_{17}}, 19^{i_{19}}\},$$

   where each exponent is a positive integer. Since $5^2 > 20$, the exponent for all primes greater than or equal to 5 must be 1. Since $2^4 \leq 20 \leq 2^5$ and $3^2 \leq 20 \leq 3^3$, the exponent for 2 must be 1, 2, 3 or 4, and the exponent for 3 must be 1 or 2. This yields eight different maximal perfect sets.

3. Let $n \geq 2$ be the number of rays drawn by Randy. Then there are

$$\binom{n}{2} = \frac{n(n-1)}{2}$$

pairs of rays. Each pair determines two angles adding up to 360°. Hence, the total number of angles between two of the n rays is exactly $n(n - 1)$. The measure of such an angle is clearly less than 360°. Since it is supposed to be an integral multiple of 10°, there are at most 35 values for the measures of these angles. Since they are distinct, $n(n - 1) \leq 35$. Now, $6 \times 5 = 30 < 35 < 42 = 7 \times 6$. Hence, $n \leq 6$. It is possible for Randy to draw six rays, determining 30 distinct angles. In the diagram below, ∠UAV = 60°, ∠VAW = 40°, ∠WAX = 10°, ∠XAY = 20°, ∠YAZ = 140° and ∠ZAU = 90°.



We now verify that the 30 angles between two rays are distinct. We have ∠WAY = 30°, ∠VAX = 50°, ∠VAY = 70°, ∠UAW = 100°, ∠UAX = 110°, ∠UAY = 130°, ∠ZAV = 150°, ∠XAZ = 160° and ∠WAZ = 170°. These are nine different angles distinct from the six between adjacent rays. All have measures less than 180°. Corresponding to these 15 angles, we have 15 other angles greater than 180°, yielding a total of 30 distinct angles.

4. Let L be the point of intersection of EC and DB. Let M be the point on the extension of AB such that MC is parallel to AE. Then ABLE and AMCE are parallelograms. Note that triangles DLC and EAB are similar, as are triangles AMC and ELD. It follows that

$$\frac{EC}{AB} = \frac{EL+LC}{AB} = 1 + \frac{LC}{AB} = 1 + \frac{DL}{EA} = 1 + \frac{DL}{CM} = 1 + \frac{AB}{EC}.$$

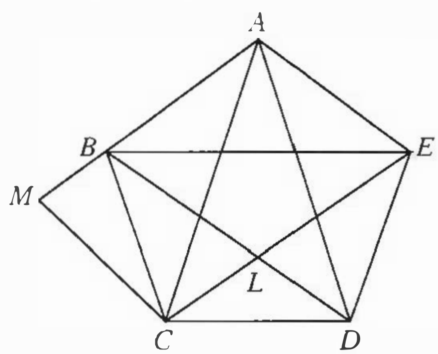
Let $x = EC/AB$. Then, $x = 1 + 1/x$ so that $x^2 - x - 1 = 0$. Hence,

$$x = \frac{1+\sqrt{5}}{2}.$$

Similarly, we have

$$\frac{DB}{AE} = \frac{AC}{ED} = \frac{AD}{BC} = \frac{EB}{DC} = \frac{1+\sqrt{5}}{2},$$

so that $EC + DB + AC + AD + EB = 5(1 + \sqrt{5})$.



Remark: The regular pentagon is used in the illustrative diagram. Many students may get the correct answer by treating only this special case, essentially proving that $\cos 36° = (1 + \sqrt{5})/4$.

5. The conditions are

$$r^2 + br + c = 1, \quad (1)$$
$$s^2 + bs + c = b, \quad (2)$$
$$t^2 + bt + c = c, \quad (3)$$
$$r + t = 2s. \quad (4)$$

From (3), $t(t + b) = 0$ so that either $t = 0$ or $t = -b$. We consider these two cases separately.

Case 1: $t = 0$. From (4), we have $r = 2s$. Substituting into (1), we have $4s^2 + 2bs + c = 1$. Subtracting (2) from this, we have $3s^2 + bs = 1 - b$, which may be rewritten as $(s + 1)(3s - 3 + b) = -2$. Hence, 2 is divisible by $s + 1$, so that $s = -3, -2, 0$ or 1. However, since $s > t = 0$, we may only have $s = 1$. It follows that $b = -1$. Hence, $f(x) = x^2 - x - 1$, with $r = 2$, $s = 1$ and $t = 0$.

Case 2: $t = -b$. From (4), we have $r = 2s + b$. Substituting into (1), we have $4s^2 + 6sb + 2b^2 + c = 1$. Subtracting (2) from this, we have $3s^2 + 5sb + 2b^2 = 1 - b$, which may be rewritten as $(3s + 2b + 3)(s + b - 1) = -2$. Hence, -2 is divisible by $s + b - 1$. From $r > s > t = -b$, we have $s + b > 0$. Hence, $s + b - 1 > -1$ so that $s + b - 1 = 1$ or 2. If $s + b - 1 = 1$, we have $3s + 2b + 3 = -2$ so that $s = -9$ and $b = 11$. Hence, $f(x) = x^2 + 11x + 30$ with $r = -7$, $s = -9$ and $t = -11$. If $s + b - 1 = 2$, we have $3s + 2b + 3 = -1$ so that $s = -10$ and $b = 13$. Hence, $f(x) = x^2 + 13x + 43$, with $r = -7$, $s = -10$ and $t = -13$.

Edmonton Junior High Math Contest 2013

Part A: Multiple Choice

1. If a stack of five dimes has a height of 6 mm, then what would be the value, in dollars, of a 1.5 m high stack of dimes?

   (a) \$1.25 (b) \$12.50 (c) \$125.00 (d) \$125.50 (e) \$1,250.00

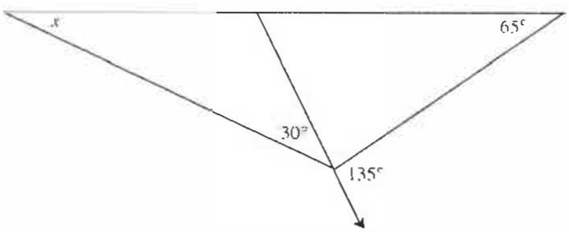
2. There are about 7.06 billion people in the world, and there are about 35 million people in Canada. What percentage of the world population is in Canada?

   (a) 0.005% (b) 0.05% (c) 0.5% (d) 5.0% (e) 5.5%

3. A large soup pot is in the shape of a right circular cylinder, and it has no lid. When filled to the top, it can hold 9.42 L of soup. The height of the pot is 30 cm. Approximately how many square centimetres of metal are needed to make the pot? Round the answer to the nearest whole square centimetre. (1 L = 1,000 cm^3, use $\pi = 3.14$ for all your calculations)

   (a) 2,198 (b) 2,218 (c) 2,838 (d) 3,010 (e) 3,140

4. Without a protractor, determine the number of degrees for x. Note: The diagram is *not* drawn to scale.



   (a) 30° (b) 40° (c) 45° (d) 60° (e) 65°

5. Robert wanted to buy Mandy a gold bracelet while it was on sale for \$160 off the regular price. He planned to pay it off with two equal monthly payments of \$340. Instead, it went on sale for only \$75 off the regular price, and he paid for it with five equal monthly payments. How much was each of his monthly payments? (Assume that there is no interest or GST.)

   (a) \$89 (b) \$136 (c) \$151 (d) \$153 (e) \$168

Solutions

1. Five dimes have a height of 6 mm. Therefore,

$$5d = 6$$
$$d = 1.2.$$

Therefore, one dime has a height of 1.2 mm.

$$1.5 \text{ m} = 1{,}500 \text{ mm}$$
$$1{,}500 \div 1.2 = 1{,}250 \text{ dimes}$$
$$1{,}250 \times 0.1 = 125$$

The value of 1,250 dimes is \$125.00. The answer is **(c)**.

2.
$$35{,}000{,}000 \div 7{,}060{,}000{,}000 = 0.00495$$
$$0.00495 \approx 0.5\%$$

The answer is **(c)**.

3.
$$9.42 \text{ L} = 9{,}420 \text{ cm}^3$$
$$9{,}420 = \pi r^2 h$$
$$9{,}420 \div (30\pi) = r^2$$
$$r = 10 \text{ cm}$$

Find the surface area of the bottom and the lateral side.

$$SA = \pi r^2 + 2\pi rh$$
$$SA = \pi(10)^2 + 2\pi(10)(30)$$
$$SA = 100\pi + 600\pi = 700\pi = 2{,}198 \text{ cm}^2$$

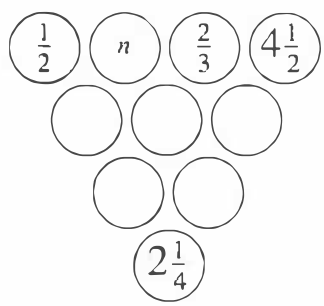
The answer is **(a)**.

4. The angle marked as 135° forms a supplementary pair with a 45° angle. The missing angle is 180 − (65 + 45 + 30), or 40°. The answer is **(b)**.

5. The regular price of the bracelet is 160 + 2(340), or \$840. The sale price with the \$75 discount is 840 − 75, or \$765. The monthly payment is \$765 ÷ 5 = \$153. The answer is **(d)**.

Part B: Short Answer

6. The number in each circle is the product of the two numbers above it. What is the value of n?



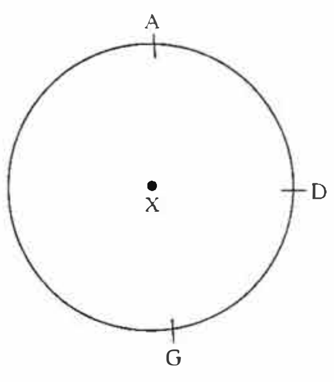
7. The sum of eight consecutive odd integers is −32. By how much does the median exceed the minimum number?

8. What fraction of the numbers from 1 to 100, inclusive, is prime? Express your answer in lowest terms.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

9. The three dimensions in centimetres (length, width and height) of a right rectangular prism are all natural numbers. The volume of the prism is 770 cm^3. What is the least possible sum that the three numbers can have?

10. Twelve points are equally spaced on a circle with centre X. Points are labelled sequentially clockwise around the circle using the letters A–L. To the nearest degree, and without the use of a protractor, calculate the measure of ∠AFX.

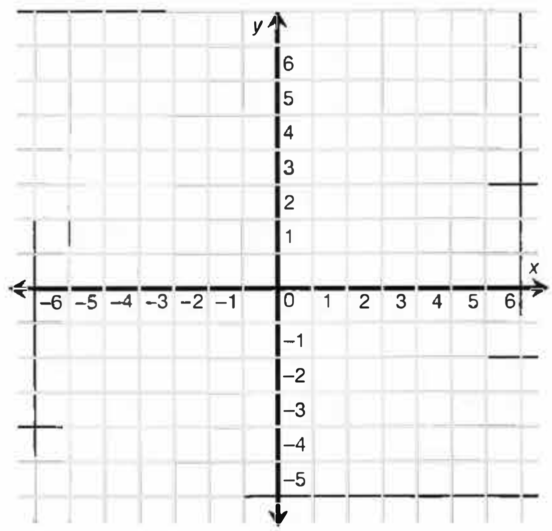


11. Kylee has a set of five cards numbered 1–5. Kassidy has a set of 10 cards numbered 1–10. If they each pick one card from their deck at random, what is the probability that the product of the two chosen numbers will be odd? Write your answer as a percentage.

12. A three-digit number has the following properties. The hundreds digit is a composite number, the tens digit is a prime number, and the units digit is greater than 2 but less than or equal to 6. How many such three-digit numbers are there in total?

13. Svitlana takes 1½ h to cycle to her friend's house if she averages 340 m/min. How many minutes should it take her to make the same trip if she travels at an average speed of 54 km/h in her car? Express the answer rounded to the nearest whole number of minutes.

14. Points A (−5, 5), B (5, 3) and C (−3, −3) are vertices of a triangle. The perimeter of ΔABC is between which two whole numbers?



Solutions

6. Start with the first two numbers in the first row.

$$\frac{1}{2}n = \frac{n}{2}$$

The value of the left circle in the second row is *n*/2.

Next, find the product of *n* and 2/3.

$$n \times \frac{2}{3} = \frac{2n}{3}$$

The value of the middle circle in the second row is $2n/3$.

Find the product of the last two numbers in the first row.

$$\frac{2}{3} \times \frac{9}{2} = 3$$

The value of the last circle in the second row is 3.

Find the product of $n/2$ and $2n/3$.

$$\frac{n}{2} \times \frac{2n}{3} = \frac{n^2}{3}$$

The value of the left circle in the third row is $n^2/3$.

Find the product of $2n/3$ and 3.

$$\frac{2n}{3} \times 3 = 2n$$

The value of the right circle in the third row is $2n$.

Find the product of $n^2/3$ and $2n$.

$$\frac{n^2}{3} \times 2n = \frac{2n^3}{3}$$

$$\frac{2n^3}{3} = \frac{9}{4}$$

Solve for n.

$$n = \frac{3}{2}$$

The answer is **3/2**, or **1.5**.

7. Let x = the first odd number, $x + 2$ = the second odd number, $x + 4$ = the third odd number, $x + 6$ = the fourth odd number, $x + 8$ = the fifth odd number, $x + 10$ = the sixth odd number, $x + 12$ = the seventh odd number and $x + 14$ = the eighth odd number.

$$8x + 56 = -32$$
$$x = -11$$

The eight consecutive odd numbers are −11, −9, −7, −5, −3, −1, 1 and 3. The median is (−5 + −3) ÷ 2, or −4. The median exceeds the minimum value of −11 by −4 − (−11) = 7. The answer is **7**.

Alternative solution: Let the eight consecutive odd numbers be $n - 8$, $n - 6$, $n - 4$, $n - 2$, n, $n + 2$, $n + 4$ and $n + 6$. The median is $(n + n - 2)/2 = n - 1$. The difference is $(n - 1) - (n - 8) = 7$.

8. There are 25 prime numbers between 1 and 100: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

$$\frac{25}{100} = \frac{1}{4}$$

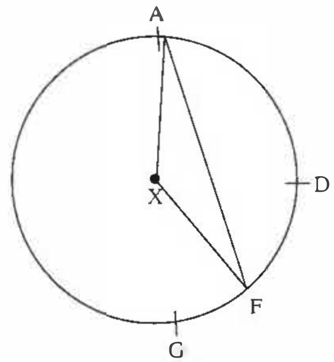
The answer is **1/4**.

9. The following table lists some of the possible dimensions and the sum of the dimensions.

Length	Width	Height	Sum
1	77	10	88
2	5	77	84
7	11	10	28
5	7	22	34
5	14	11	30
2	11	35	48
2	7	55	64

The least sum is 28. The answer is **28**.

10.



Since there are 12 points spaced equally on the circle, all 12 arcs are equal. Each arc has a central angle of 360° ÷ 12, or 30°. ∠AXF subtends five of these arcs and has a measure of 30° × 5, or 150°. ΔAXF is an isosceles triangle; therefore, ∠AFX = (180° − 150°)/2 = **15°**.

11. The sample space consists of 50 ordered pairs. Fifteen of these—(1, 1), (1, 3), (1, 5), (1, 7), (1, 9), (3, 1), (3, 3), (3, 5), (3, 7), (3, 9), (5, 1), (5, 3), (5, 5), (5, 7), (5, 9)—have an odd product. The probability is 15 ÷ 50, or **30%**.

12. The hundreds digit could be 4, 6, 8 or 9. The tens digit could be 2, 3, 5 or 7. The ones digit could be 3, 4, 5 or 6. There are 4 × 4 × 4, or 64, possible three-digit numbers. The answer is **64**.

13. Find the distance travelled.

$$d = rt$$
$$d = (340 \text{ m/min})(90 \text{ min})$$
$$d = 30{,}600 \text{ m, or } 30.6 \text{ km}$$

Find the time for the rate of 54 km/h.

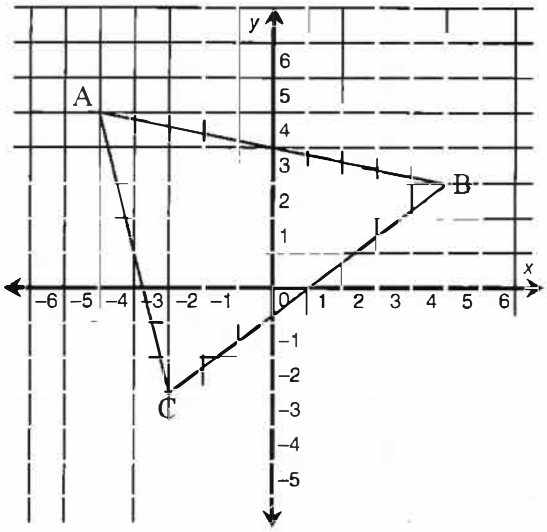
$$54 \text{ km/h} = 0.9 \text{ km/min}$$

$$t = \frac{d}{r}$$

$$t = \frac{30.6 \text{ km}}{0.9 \text{ km/min}}$$

$$t = 34 \text{ min}$$

The answer is **34**.

14. 

The distance between AC is $\sqrt{8^2 + 2^2} \approx 8.246$. The distance between AB is $\sqrt{10^2 + 2^2} \approx 10.198$. The distance between BC is $\sqrt{6^2 + 8^2} = 10$. The perimeter $\approx 8.246 + 10.198 + 10 \approx 28.444$, which is between 28 and 29.

Part C: Short Answer

15. The digits A, B, C, D, E, F, G, H and I, not necessarily all different digits, are arranged in a three-by-three configuration. The first two rows, ABC and DEF, are three-digit prime numbers. The third row, GHI, and the first column, ADG, are three-digit cubes. The last two columns, BEH and CFI, are three-digit squares. What is the value of digit E?

16. In triangle ABC, AB = 25 and CA = 24. E is a point on CA and F is a point on AB such that EF cuts ABC into two regions of equal areas. If CE = 4, what is the length of BF?

17. How many numbers between 100 and 1,000,000 have all digits the same and are divisible by 3?

18. What is the largest number whose digits are all different and which is *not* divisible by 9?

19. There exist two prime numbers, p and q, such that $2p + 3q = 99$. The sum of p and q is also the product of two other prime numbers: m and n. Find m and n.

Solutions

15. This is the configuration:

A B C
D E F
G H I

There are five three-digit cubes: 125, 216, 343, 512 and 729. ADG and GHI are cubes. If ADG is 125, then GHI is 512. If ADG is 512, then GHI is 216. No other combinations will work. A three-digit square number cannot end in 2, so eliminate ADG: 125 and GHI: 512. Therefore, the value for ADG is 512 and the value for GHI is 216.

5 B C
1 E F
2 1 6

There are four three-digit square numbers that end in 1: 121, 361, 441 and 841. There are four three-digit square numbers that end in 6: 196, 256, 576 and 676. If 5BC is a prime number, it cannot end in 2, 5 or 6. This gives CFI = 196.

5 B 1
1 E 9
2 1 6

If BE1 = 121, then 1E9 = 129. However, 129 is not prime. If BE1 = 361, then 5B1 = 531. However, 531 is not prime, either. That leaves 441 or 841 for BE1. Both give E = 4. Therefore, the value of digit E is **4**. Also note that 841 won't work, as this would give 5B1 = 581, which is not prime.

16. Connect CF. Let x represent the area of triangle CEF. Using CE and EA as the bases, the two triangles CEF and AFE have the same height. Since the area of triangle CEF = x, then the area of triangle AFE = $5x$. The area of triangle BFC = $5x - x = 4x$. The area of triangle ACF = $x + 5x = 6x$. Let m = length of BF and $(25 - m)$ = length of AF. Using FB and FA as the bases, the two triangles FCB and FAC have the same height, H.

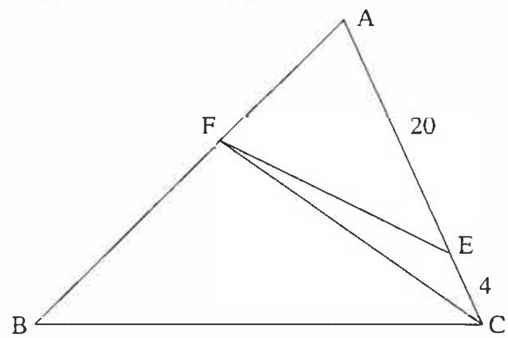
$$\text{Area of } \Delta FCB = \frac{m(H)}{2} = 4x$$

$$\text{Area of } \Delta FAC = \frac{(25 - m)(H)}{2} = 6x$$

Rewriting both equations in terms of H, we have

$$\frac{8x}{m} = \frac{12x}{25 - m}.$$

Solve for m, we have $m = 10$. The answer is **10**.



17. The number can have three, four, five or six digits.

If the number has three digits, it has the form *aaa*, with $1 \leq a \leq 9$. The sum of the digits is $3a$, which is always divisible by 3. There are nine three-digit numbers that satisfy this condition.

If the number has four digits, it has the form *aaaa*, with $1 \leq a \leq 9$. The sum of the digits is $4a$, which is divisible by 3 only when a is 3, 6 or 9. There are three four-digit numbers that satisfy this condition.

If the number has five digits, it has the form *aaaaa*, with $1 \leq a \leq 9$. The sum of the digits is $5a$, which is divisible by 3 only when a is 3, 6 or 9. There are three five-digit numbers that satisfy this condition.

If the number has six digits, it has the form *aaaaaa*, with $1 \leq a \leq 9$. The sum of the digits is $6a$, which is always divisible by 3. There are nine six-digit numbers that satisfy this condition.

In total, there are $9 + 3 + 3 + 9 = 24$ such numbers.

The answer is **24**.

18. Since the number has distinct digits, it has at most 10 digits. If the number has 10 digits, then its digits must be exactly 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 in some order. But then the sum of the digits is 45, and the number is divisible by 9. Thus, the number cannot have 10 digits.

If the number has nine digits, then one of the 10 digits must be missing. The sum of the digits then is 45 – (the missing digit). In order for this number not to be divisible by 9, the missing digit can be anything except 0 or 9.

Since we are looking for the largest nine-digit number, the missing digit must be as small as possible. Therefore, it must be 1.

This shows that our number has exactly the digits 0, 2, 3, 4, 5, 6, 7, 8 and 9. Since the largest number is wanted, the digits must be decreasing. Therefore, the number is **987,654,320.**

19.
$$2p + 3q = 99$$
$$2p = 99 - 3q$$
$$2p = 3(33 - q)$$

The p must be divisible by 3. Since p is prime, $p = 3$.

Substitute $p = 3$ into the equation:

$$2(3) + 3q = 99.$$

Solving for q, $q = 31$.

The sum of p and q is 34, which can be factored only two ways: $1 \times 34 = 34$ and $2 \times 17 = 34$.

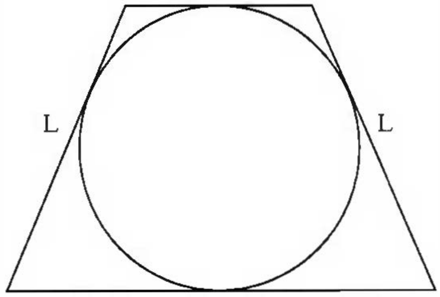
The numbers 1 and 34 are not prime, but the numbers 2 and 17 are prime. Therefore, m and n have the values of **2 and 17**.

Calgary Junior High School Mathematics Contest 2013

The Calgary Junior High School Mathematics Contest takes place every spring. The 90-minute exam is primarily for Grade 9 students; however, all junior high students in Calgary and surrounding districts are eligible. Participants write the exam in their own schools. School and individual prizes include trophies, medals, a cash award to the student achieving the highest mark, and the opportunity for the top students (and their teacher sponsors) to attend a banquet at the University of Calgary. The 37th annual contest took place on April 24, 2013.

Part A: Short Answer

1. From the set {1, 2, 3, 4, 5, 6, 7, 8, 9} all odd numbers are removed. How many numbers are remaining?
2. A bag contains red, blue and green marbles. Two-thirds of the marbles are *not* red, and three-quarters of the marbles are *not* blue. What fraction of the marbles are *not* green? Express your fraction in lowest terms.
3. Ajooni walked 9 km at 4 km/h, and then biked for 4 hours at 9 km/h. What was her average speed (in km/h) for the entire trip?
4. Notice that the digits of 2013 are four consecutive integers (because 0, 1, 2 and 3 are consecutive integers). What was the last year (before 2013) whose digits were four consecutive integers?
5. A circle is inscribed in an isosceles trapezoid, as shown, with parallel edges of lengths 8 cm and 18 cm and sloping edges of length L cm each. What is L?



6. Mary has a large box of candies. If she gives a third of her candies to her mom, then a third of the remaining candies to her dad, and finally a third of what's left to her little sister, there will only be 16 candies in the box. How many candies are in the box at the beginning?
7. I have half a litre of solution, which is 40% acid and the rest water. If I mix it with 2 L of solution that is only 10% acid, what is the percentage of acid in the mixture?
8. A two-digit positive integer is said to be *doubly divisible* if its two digits are different and nonzero, and it is exactly divisible by each of its two digits. For example, 12 is doubly divisible since it is divisible by 1 and 2, whereas 99 is not doubly divisible since its digits are equal, and 90 is not doubly divisible because it contains a zero. What is the largest doubly divisible positive integer?
9. What is the remainder when 2^{2013} is divided by 7?

Answers

1. 4
2. 7/12
3. 36/5 = 7.2
4. 1432
5. 13
6. 54
7. 16
8. 48
9. 1

Part B: Long Answer

1. You currently have $100 and two magic wands, A and B. Wand A increases the amount of money you have by 30%, and wand B adds $50 to the amount of money you have. You may use each wand exactly once, one after the other. In which order should you use the wands to maximize the amount of money you have? How much money would you have?

2. Put one of the integers 1, 2, . . ., 13 into each of the boxes, so that 12 of these numbers are used once (and one number is not used at all), and so that all four equations are true. Be sure to explain how you found your answers.

$\square + \square = \square$

$\square - \square = \square$

$\square \times \square = \square$

$\square \div \square = \square$

3. On planet X, an X-monkey has two legs and one head, while an X-hypercow has three legs and four heads. Robert has a herd of X-monkeys and X-hypercows on his farm, with a total of 87 legs and 86 heads in his herd. How many animals of each kind does Robert have?

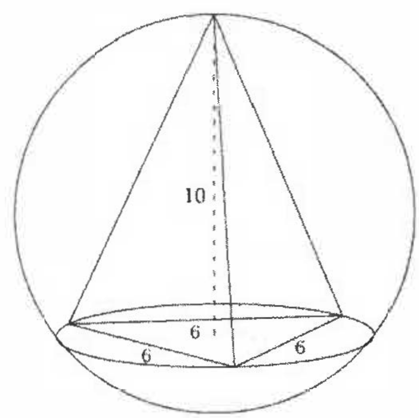
4. A pie is cut into a equal parts. Then one of these parts is cut into b smaller equal parts. Finally, one of the smaller parts is cut into c smallest equal parts. One of the original parts, together with a smaller part and a smallest part, makes up exactly three-fifths of the pie. What are a, b and c (assuming a, b and c are integers greater than 1)?

5. In a hockey tournament, five teams participated and each team played against each other team exactly once. A team received 2 points for a win, 1 point for a tie and 0 points for a loss. At the end of the tournament, the results showed that no two teams received the same total points, and the order of the teams (from highest point total to lowest point total) was A, B, C, D, E. Team B was the only team that did not lose any games, and team E was the only team that did not win any games. How many points did each team receive, and what was the result of each game?

	Total Points
A	
B	
C	
D	
E	

	Winner (or Tie)
A vs B	
A vs C	
A vs D	
A vs E	
B vs C	
B vs D	
B vs E	
C vs D	
C vs E	
D vs E	

6. The three edges of the base of a triangular pyramid (tetrahedron) each have length 6 units, and the height of the pyramid is 10. The other three (sloping) edges are equal in length. A sphere passes through all four corners of the pyramid. What is the radius of the sphere?



Solutions

1. If you first use wand A, the \$100 becomes \$130, and then applying wand B produces \$130 + \$50 = \$180. However, if you first use wand B, you obtain \$150, which (after using wand A) becomes \$150 × 1.3 = \$195. So the maximum amount is \$195, obtained by using wand B first, then wand A.

2. One answer is shown here. The fourth equation (A ÷ B = C) is the same as B × C = A, which is the same form as the third equation. Neither of these equations can use the number 1 (or

6	+	7	=	13
9	−	8	=	1
3	×	4	=	12
10	÷	5	=	2

else there would be a repeated number), so in one of these two equations the smallest number must be 2, and in the other the smallest number must be 3. If the smallest number is 3, the only possibility is $3 \times 4 = 12$. This leaves $2 \times 5 = 10$ as the only possibility for the other equation (since we cannot repeat the numbers 3 and 4). So the last two equations must use the six numbers 3, 4, 12, 2, 5 and 10, and there are various ways this can happen. For example, we could have used $5 \times 2 = 10$ and $12 \div 4 = 3$ instead of what we wrote above.

Now, the second equation (X – Y = Z) can be written as Z + Y = X, which is the same form as the first equation. So we need to find two equations of the form Z + Y = X using only numbers from 1, 6, 7, 8, 9, 11 and 13. One of these equations cannot use the number 1, so it must be $6 + 7 = 13$. Then the only possibility for the other equation is $1 + 8 = 9$. So the first two equations must use the numbers 6, 7, 13, 1, 8 and 9 in some order. The missing number must be 11.

3. Let a be the number of X-monkeys and b the number of X-hypercows. Since X-monkeys have two legs, they contribute $2a$ legs to the total. There are $3b$ legs from the X-hypercows, for a total of 87 legs. This gives the equation

$$2a + 3b = 87.$$

Counting heads, we get the equation

$$a + 4b = 86.$$

So we can write $a = 86 - 4b$ and substitute this into the first equation to get

$$2(86 - 4b) + 3b = 87.$$

Then $5b = 85$, so $b = 17$ and then $a = 86 - 4 \times 17 = 18$. So the herd has 18 X-monkeys and 17 X-hypercows.

Another way to proceed is to note that taking one X-monkey and one X-hypercow gives five heads and five legs. Now, $85 = 5 \times 17$, so 17 X-monkeys and 17 X-hypercows would give 85 heads and 85 legs. So you need one more head and two more legs, which is another X-monkey. So there are 18 X-monkeys and 17 X-hypercows.

4. From the information, the size of each slice from the first cut is $1/a$ of the whole pie, the size of each slice from the second cut is $1/ab$ of the whole pie, and the size of each slice from the third cut is $1/abc$ of the whole pie. So we have

$$\frac{1}{a} + \frac{1}{ab} + \frac{1}{abc} = \frac{3}{5}.$$

Multiplying both sides by $5abc$, we get the equation

$$5bc + 5c + 5 = 3abc.$$

Now, c is a factor of all the summands except 5, so c must divide into 5. Since 5 is prime and c is bigger than 1, $c = 5$. Using this in the equation, we obtain

$$25b + 25 + 5 = 15ab,$$

so $5b + 6 = 3ab$. This tells us that b must divide into 6. Let us look at the possibilities. Trying $b = 2$ gives $10 + 6 = 6a$, which isn't possible since 6 doesn't divide into 16. Next, try $b = 3$. This gives $15 + 6 = 9a$, which doesn't work since 9 doesn't divide into 21. So $b = 6$. Then $a = 2$. The answer is $a = 2$, $b = 6$ and $c = 5$.

5.

	Total Points
A	6
B	5
C	4
D	3
E	2

	Winner (or Tie)
A vs B	B
A vs C	A
A vs D	A
A vs E	A
B vs C	T
B vs D	T
B vs E	T
C vs D	C
C vs E	T
D vs E	D

Note that if we total the points for each match, we obtain 2, so the point total recorded for the 10 games is 20. Since B was the only team that did not lose a game, A lost at least one game, making its maximum possible score 6. Its score could not be 5 since $5 + 4 + 3 + 2 + 1 = 15 < 20$. Thus, A has three wins and one loss. Since B has no losses, the game A lost must have been to B. Then B must have a tie in all three of its other games (otherwise, it has at least 6 points, at least as many as A). All the teams but E won some games, so both C and D won some game. Neither of them could win both games (excluding those with A and B, about which we already know) because that would give a score of 5, which B got. Thus, C won one of the other games and had one tie, and D won one game and had one loss, which means that E had one loss and one tie in the remaining games. Thus, E lost the game with D and had a tie with C.

6. Note that triangle ABC is equilateral, so the medians of the three sides intersect at the centre O of the triangle. Let D be the midpoint of side AB, so that CD is a median of the triangle. Then triangle ADO is a right triangle. This triangle is similar to triangle CDA, so OD : AD = AD : AC = 1 : 2, and AO = 2OD. So, since AO = CO, you get OD = (1/3)CD. Also,

$$CD = \sqrt{6^2 - 3^2} = 3\sqrt{3}.$$

Then,

$$OA = \frac{2}{3} \times 3\sqrt{3} = 2\sqrt{3}.$$

Let V be the other vertex of the pyramid, and S the centre of the sphere. Then triangle SOA is a right-angled triangle, and if h (= 10) is the height of the pyramid and r the radius of the sphere, we get from Pythagoras' theorem that

$$OA^2 = SA^2 - SO^2.$$

This becomes

$$\left(2\sqrt{3}\right)^2 = r^2 - (h - r)^2$$

and

$$12 = 2rh - h^2 = 20r - 100.$$

Thus, $112 = 20r$, and the radius of the sphere is 5.6 cm.

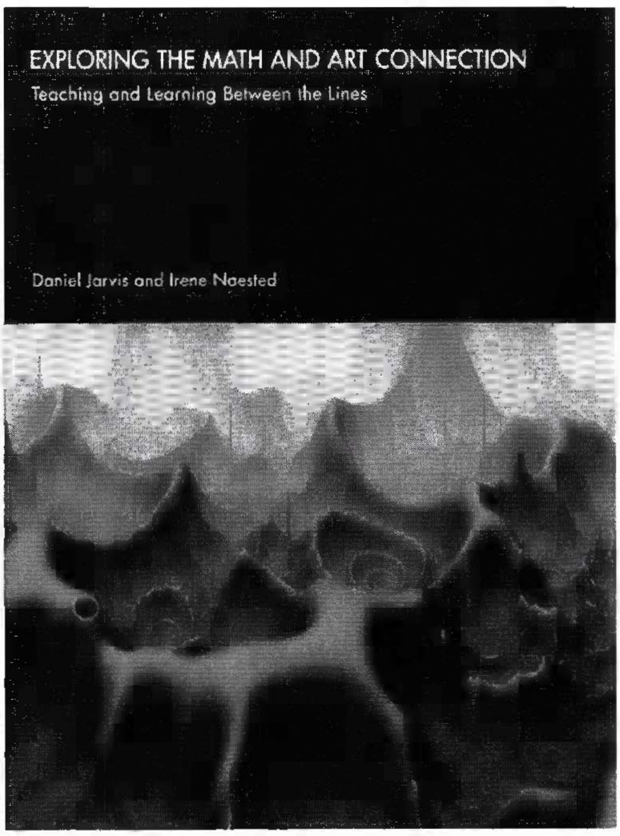
Exploring the Math and Art Connection: Teaching and Learning Between the Lines, by Daniel Jarvis and Irene Naested

Brush Education, 2012

Reviewed by Roberta La Haye

Exploring the Math and Art Connection: Teaching and Learning Between the Lines is a recent publication by Daniel Jarvis (Nipissing University) and Irene Naested (Mount Royal University), two education professors with expertise in teaching art and mathematics. The book is intended as a teaching resource for educators, especially at the elementary level. It is built on the principles that both mathematics and visual arts provide valuable tools with which to understand the world around us and that connecting the two disciplines can enhance students' appreciation and understanding of both.

I had the pleasure of teaching a new course for future elementary educators with Dr Naested. The course dealt with the integration of mathematics and the visual arts for pedagogical reasons. This book was both a valuable resource for the course and an eye-opener for me. As a professor of mathematics with an interest in visual art and experience with math–art outreach activities, I thought I had a decent understanding of the math–art connection. I was wrong! Many learning experiences that I thought of as math–art are actually math activities with crafts added on to make mathematics more appealing. These "math crafts" give no consideration to artistic principles or the true value of integrated learning to both math and art.



Chapter 1 outlines the history of the math–art connection and discusses a breadth of educational theories and strategies that support connecting the two disciplines. The intention is to show readers the value to students of exploiting these connections in the classroom.

Chapter 2 highlights the major elements of both the mathematics curriculum and the art curriculum and finishes up with a discussion about planning integrated activities. A single chapter can't make a math teacher an art expert, or an art teacher a math expert. Instead, the chapter emphasizes that both disciplines have substantial and meaningful curricula and that there are links between them.

Chapters 3–7 get down to the nuts and bolts of exploiting connections between the two subjects through teaching and learning experiences. The authors chose not to organize the chapters using the math or art curricula. Instead, they have organized the topics according to the world around us. There are chapters related to flora, fauna, the human figure, architecture and designed objects. The authors also bring in connections to other disciplines, including science and sociology.

The following are examples of the learning experiences outlined:

- Linking grid drawings and distorted grid drawings to measurement and area
- Measuring angles and using symmetry to construct kaleidoscope patterns
- Making data collection a part of the artistic process of realistically capturing the human figure
- Problem solving with ratios to get a "life-sized" depiction of a sasquatch

Imagine discussing math and art topics not because you hit that section in the textbook but because you and your students were looking at the world around you and saw them there!

Finally, in Chapter 8 the authors further discuss the why of curriculum integration and get into a few specifics about how it can be achieved. They warn that to do a good job of integrating mathematics and art curricula in planning learning experiences, the teacher must have a good understanding of both subjects and must carefully plan the lessons. The references and resources at the back of the book are also an asset.

Overall, the book puts a little more emphasis on art—probably because the authors have more combined experience in that discipline than in mathematics. From the mathematics viewpoint, it is interesting to see experts in another discipline also lamenting how little respect their discipline gets and how its goals are being watered down.

There are some really nice ideas in these chapters but, for the sake of breadth and to appeal to a wider audience, the book just outlines the learning experiences. The onus is on the reader to flesh out these ideas and customize them to their individual goals in the curriculum. This is not necessarily an easy task, but it has the potential to be a rewarding one. I'd recommend this book to any educators who are both open to the idea of truly integrated math and art activities and willing to put in the time and effort to expand their expertise and apply the ideas. It will not be your only resource, but it is a great start.

Roberta La Haye is an associate professor of mathematics at Mount Royal University, where she teaches courses in calculus, algebra and statistics and has helped develop and run a course in general education, as well as a course connecting mathematics and art for future educators. She has a PhD in group theory and, in the past few years, has developed an interest in the ties between mathematics and art. This interest has manifested itself in outreach activities to elementary and middle school children, as well as in research.

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