Edmonton Junior High Math Contest 2013

Part A: Multiple Choice

1. If a stack of five dimes has a height of 6 mm, then what would be the value, in dollars, of a 1.5 m high stack of dimes?

   (a) \$1.25 (b) \$12.50 (c) \$125.00 (d) \$125.50 (e) \$1,250.00

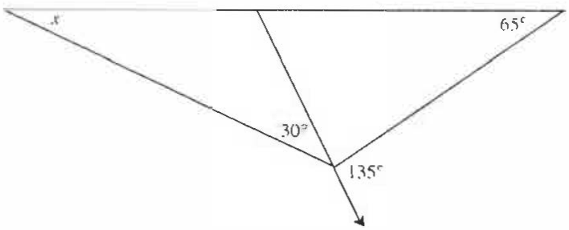
2. There are about 7.06 billion people in the world, and there are about 35 million people in Canada. What percentage of the world population is in Canada?

   (a) 0.005% (b) 0.05% (c) 0.5% (d) 5.0% (e) 5.5%

3. A large soup pot is in the shape of a right circular cylinder, and it has no lid. When filled to the top, it can hold 9.42 L of soup. The height of the pot is 30 cm. Approximately how many square centimetres of metal are needed to make the pot? Round the answer to the nearest whole square centimetre. (1 L = 1,000 cm^3, use $\pi = 3.14$ for all your calculations)

   (a) 2,198 (b) 2,218 (c) 2,838 (d) 3,010 (e) 3,140

4. Without a protractor, determine the number of degrees for x. Note: The diagram is *not* drawn to scale.



   (a) 30° (b) 40° (c) 45° (d) 60° (e) 65°

5. Robert wanted to buy Mandy a gold bracelet while it was on sale for \$160 off the regular price. He planned to pay it off with two equal monthly payments of \$340. Instead, it went on sale for only \$75 off the regular price, and he paid for it with five equal monthly payments. How much was each of his monthly payments? (Assume that there is no interest or GST.)

   (a) \$89 (b) \$136 (c) \$151 (d) \$153 (e) \$168

Solutions

1. Five dimes have a height of 6 mm. Therefore,

$$5d = 6$$
$$d = 1.2.$$

Therefore, one dime has a height of 1.2 mm.

$$1.5 \text{ m} = 1{,}500 \text{ mm}$$
$$1{,}500 \div 1.2 = 1{,}250 \text{ dimes}$$
$$1{,}250 \times 0.1 = 125$$

The value of 1,250 dimes is \$125.00. The answer is **(c)**.

2. $$35{,}000{,}000 \div 7{,}060{,}000{,}000 = 0.00495$$
$$0.00495 \approx 0.5\%$$

The answer is **(c)**.

3. $$9.42 \text{ L} = 9{,}420 \text{ cm}^3$$
$$9{,}420 = \pi r^2 h$$
$$9{,}420 \div (30\pi) = r^2$$
$$r = 10 \text{ cm}$$

Find the surface area of the bottom and the lateral side.

$$SA = \pi r^2 + 2\pi rh$$
$$SA = \pi(10)^2 + 2\pi(10)(30)$$
$$SA = 100\pi + 600\pi = 700\pi = 2{,}198 \text{ cm}^2$$

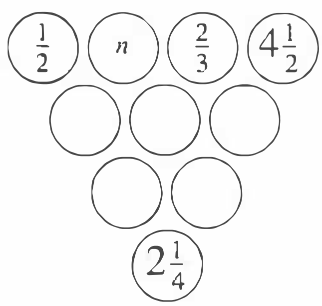
The answer is **(a)**.

4. The angle marked as 135° forms a supplementary pair with a 45° angle. The missing angle is 180 – (65 + 45 + 30), or 40°. The answer is **(b)**.

5. The regular price of the bracelet is 160 + 2(340), or \$840. The sale price with the \$75 discount is 840 – 75, or \$765. The monthly payment is \$765 ÷ 5 = \$153. The answer is **(d)**.

Part B: Short Answer

6. The number in each circle is the product of the two numbers above it. What is the value of n?



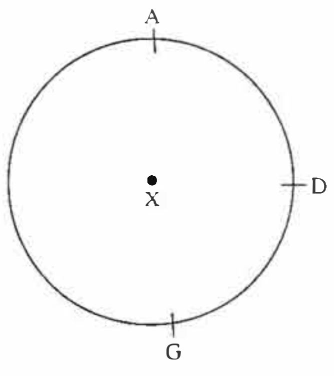
7. The sum of eight consecutive odd integers is −32. By how much does the median exceed the minimum number?

8. What fraction of the numbers from 1 to 100, inclusive, is prime? Express your answer in lowest terms.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

9. The three dimensions in centimetres (length, width and height) of a right rectangular prism are all natural numbers. The volume of the prism is 770 cm³. What is the least possible sum that the three numbers can have?

10. Twelve points are equally spaced on a circle with centre X. Points are labelled sequentially clockwise around the circle using the letters A–L. To the nearest degree, and without the use of a protractor, calculate the measure of ∠AFX.

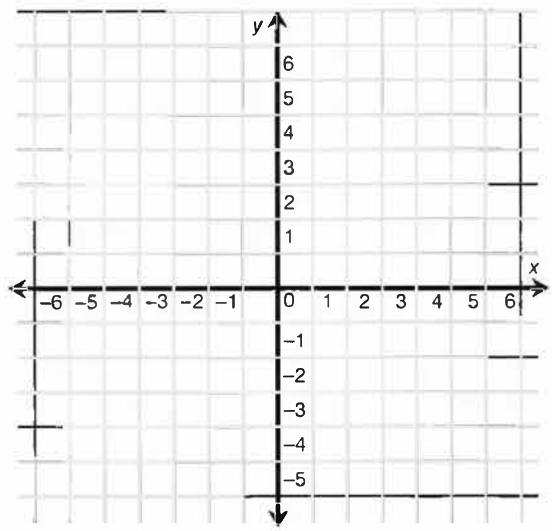


11. Kylee has a set of five cards numbered 1–5. Kassidy has a set of 10 cards numbered 1–10. If they each pick one card from their deck at random, what is the probability that the product of the two chosen numbers will be odd? Write your answer as a percentage.

12. A three-digit number has the following properties. The hundreds digit is a composite number, the tens digit is a prime number, and the units digit is greater than 2 but less than or equal to 6. How many such three-digit numbers are there in total?

13. Svitlana takes 1½ h to cycle to her friend's house if she averages 340 m/min. How many minutes should it take her to make the same trip if she travels at an average speed of 54 km/h in her car? Express the answer rounded to the nearest whole number of minutes.

14. Points A (−5, 5), B (5, 3) and C (−3, −3) are vertices of a triangle. The perimeter of ΔABC is between which two whole numbers?



Solutions

6. Start with the first two numbers in the first row.

$$\frac{1}{2}n = \frac{n}{2}$$

The value of the left circle in the second row is $n/2$.

Next, find the product of n and 2/3.

$$n \times \frac{2}{3} = \frac{2n}{3}$$

The value of the middle circle in the second row is $2n/3$.

Find the product of the last two numbers in the first row.

$$\frac{2}{3} \times \frac{9}{2} = 3$$

The value of the last circle in the second row is 3.

Find the product of $n/2$ and $2n/3$.

$$\frac{n}{2} \times \frac{2n}{3} = \frac{n^2}{3}$$

The value of the left circle in the third row is $n^2/3$.

Find the product of $2n/3$ and 3.

$$\frac{2n}{3} \times 3 = 2n$$

The value of the right circle in the third row is $2n$.

Find the product of $n^2/3$ and $2n$.

$$\frac{n^2}{3} \times 2n = \frac{2n^3}{3}$$

$$\frac{2n^3}{3} = \frac{9}{4}$$

Solve for n.

$$n = \frac{3}{2}$$

The answer is **3/2**, or **1.5**.

7. Let x = the first odd number, $x + 2$ = the second odd number, $x + 4$ = the third odd number, $x + 6$ = the fourth odd number, $x + 8$ = the fifth odd number, $x + 10$ = the sixth odd number, $x + 12$ = the seventh odd number and $x + 14$ = the eighth odd number.

$$8x + 56 = -32$$
$$x = -11$$

The eight consecutive odd numbers are −11, −9, −7, −5, −3, −1, 1 and 3. The median is (−5 + −3) ÷ 2, or −4. The median exceeds the minimum value of −11 by −4 − (−11) = 7. The answer is **7**.

Alternative solution: Let the eight consecutive odd numbers be $n - 8$, $n - 6$, $n - 4$, $n - 2$, n, $n + 2$, $n + 4$ and $n + 6$. The median is $(n + n - 2)/2 = n - 1$. The difference is $(n - 1) - (n - 8) = 7$.

8. There are 25 prime numbers between 1 and 100: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

$$\frac{25}{100} = \frac{1}{4}$$

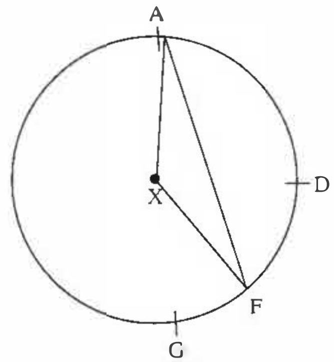
The answer is **1/4**.

9. The following table lists some of the possible dimensions and the sum of the dimensions.

Length	Width	Height	Sum
1	77	10	88
2	5	77	84
7	11	10	28
5	7	22	34
5	14	11	30
2	11	35	48
2	7	55	64

The least sum is 28. The answer is **28**.

10.



Since there are 12 points spaced equally on the circle, all 12 arcs are equal. Each arc has a central angle of 360° ÷ 12, or 30°. ∠AXF subtends five of these arcs and has a measure of 30° × 5, or 150°. ΔAXF is an isosceles triangle; therefore, ∠AFX = (180° − 150°)/2 = **15°**.

11. The sample space consists of 50 ordered pairs. Fifteen of these—(1, 1), (1, 3), (1, 5), (1, 7), (1, 9), (3, 1), (3, 3), (3, 5), (3, 7), (3, 9), (5, 1), (5, 3), (5, 5), (5, 7), (5, 9)—have an odd product. The probability is 15 ÷ 50, or **30%**.

12. The hundreds digit could be 4, 6, 8 or 9. The tens digit could be 2, 3, 5 or 7. The ones digit could be 3, 4, 5 or 6. There are 4 × 4 × 4, or 64, possible three-digit numbers. The answer is **64**.

13. Find the distance travelled.

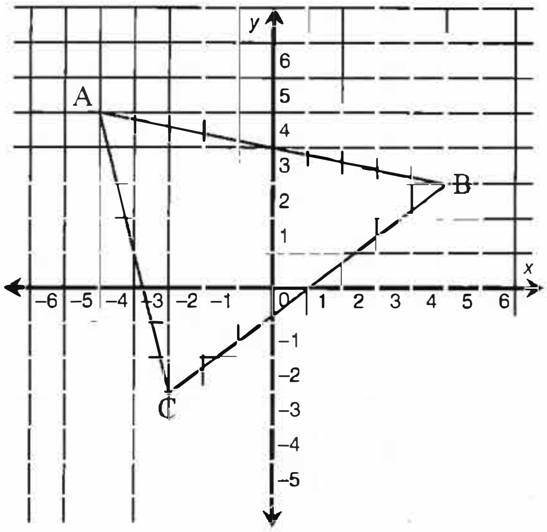
$$d = rt$$
$$d = (340 \text{ m/min})(90 \text{ min})$$
$$d = 30{,}600 \text{ m, or } 30.6 \text{ km}$$

Find the time for the rate of 54 km/h.

$$54 \text{ km/h} = 0.9 \text{ km/min}$$
$$t = \frac{d}{r}$$
$$t = \frac{30.6 \text{ km}}{0.9 \text{ km/min}}$$
$$t = 34 \text{ min}$$

The answer is **34**.

14.



The distance between AC is $\sqrt{8^2 + 2^2} \approx 8.246$. The distance between AB is $\sqrt{10^2 + 2^2} \approx 10.198$. The distance between BC is $\sqrt{6^2 + 8^2} = 10$. The perimeter ≈ 8.246 + 10.198 + 10 ≈ 28.444, which is between 28 and 29.

Part C: Short Answer

15. The digits A, B, C, D, E, F, G, H and I, not necessarily all different digits, are arranged in a three-by-three configuration. The first two rows, ABC and DEF, are three-digit prime numbers. The third row, GHI, and the first column, ADG, are three-digit cubes. The last two columns, BEH and CFI, are three-digit squares. What is the value of digit E?

16. In triangle ABC, AB = 25 and CA = 24. E is a point on CA and F is a point on AB such that EF cuts ABC into two regions of equal areas. If CE = 4, what is the length of BF?

17. How many numbers between 100 and 1,000,000 have all digits the same and are divisible by 3?

18. What is the largest number whose digits are all different and which is *not* divisible by 9?

19. There exist two prime numbers, p and q, such that $2p + 3q = 99$. The sum of p and q is also the product of two other prime numbers: m and n. Find m and n.

Solutions

15. This is the configuration:

A B C
D E F
G H I

There are five three-digit cubes: 125, 216, 343, 512 and 729. ADG and GHI are cubes. If ADG is 125, then GHI is 512. If ADG is 512, then GHI is 216. No other combinations will work. A three-digit square number cannot end in 2, so eliminate ADG: 125 and GHI: 512. Therefore, the value for ADG is 512 and the value for GHI is 216.

5 B C
1 E F
2 1 6

There are four three-digit square numbers that end in 1: 121, 361, 441 and 841. There are four three-digit square numbers that end in 6: 196, 256, 576 and 676. If 5BC is a prime number, it cannot end in 2, 5 or 6. This gives CFI = 196.

5 B 1
1 E 9
2 1 6

If BE1 = 121, then 1E9 = 129. However, 129 is not prime. If BE1 = 361, then 5B1 = 531. However, 531 is not prime, either. That leaves 441 or 841 for BE1. Both give E = 4. Therefore, the value of digit E is **4**. Also note that 841 won't work, as this would give 5B1 = 581, which is not prime.

16. Connect CF. Let x represent the area of triangle CEF. Using CE and EA as the bases, the two triangles CEF and AFE have the same height. Since the area of triangle CEF = x, then the area of triangle AFE = $5x$. The area of triangle BFC = $5x - x = 4x$. The area of triangle ACF = $x + 5x = 6x$. Let m = length of BF and $(25 - m)$ = length of AF. Using FB and FA as the bases, the two triangles FCB and FAC have the same height, H.

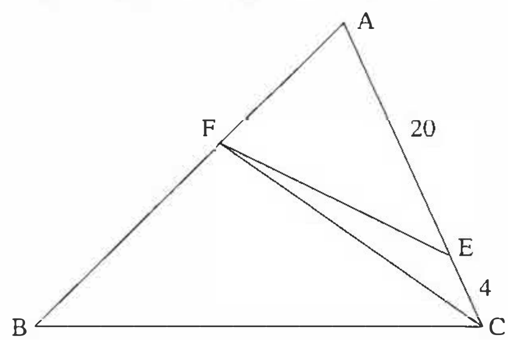
$$\text{Area of } \Delta FCB = \frac{m(H)}{2} = 4x$$

$$\text{Area of } \Delta FAC = \frac{(25-m)(H)}{2} = 6x$$

Rewriting both equations in terms of H, we have

$$\frac{8x}{m} = \frac{12x}{25-m}.$$

Solve for m, we have $m = 10$. The answer is **10**.



17. The number can have three, four, five or six digits.

If the number has three digits, it has the form *aaa*, with $1 \le a \le 9$. The sum of the digits is $3a$, which is always divisible by 3. There are nine three-digit numbers that satisfy this condition.

If the number has four digits, it has the form *aaaa*, with $1 \le a \le 9$. The sum of the digits is $4a$, which is divisible by 3 only when a is 3, 6 or 9. There are three four-digit numbers that satisfy this condition.

If the number has five digits, it has the form *aaaaa*, with $1 \le a \le 9$. The sum of the digits is $5a$, which is divisible by 3 only when a is 3, 6 or 9. There are three five-digit numbers that satisfy this condition.

If the number has six digits, it has the form *aaaaaa*, with $1 \le a \le 9$. The sum of the digits is $6a$, which is always divisible by 3. There are nine six-digit numbers that satisfy this condition.

In total, there are $9 + 3 + 3 + 9 = 24$ such numbers.

The answer is **24**.

18. Since the number has distinct digits, it has at most 10 digits. If the number has 10 digits, then its digits must be exactly 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 in some order. But then the sum of the digits is 45, and the number is divisible by 9. Thus, the number cannot have 10 digits.

If the number has nine digits, then one of the 10 digits must be missing. The sum of the digits then is 45 – (the missing digit). In order for this number not to be divisible by 9, the missing digit can be anything except 0 or 9.

Since we are looking for the largest nine-digit number, the missing digit must be as small as possible. Therefore, it must be 1.

This shows that our number has exactly the digits 0, 2, 3, 4, 5, 6, 7, 8 and 9. Since the largest number is wanted, the digits must be decreasing. Therefore, the number is **987,654,320.**

19.
$$2p + 3q = 99$$
$$2p = 99 - 3q$$
$$2p = 3(33 - q)$$

The p must be divisible by 3. Since p is prime, $p = 3$.

Substitute $p = 3$ into the equation:

$$2(3) + 3q = 99.$$

Solving for q, $q = 31$.

The sum of p and q is 34, which can be factored only two ways: $1 \times 34 = 34$ and $2 \times 17 = 34$.

The numbers 1 and 34 are not prime, but the numbers 2 and 17 are prime. Therefore, m and n have the values of **2 and 17**.