delta-k

Journal of the Mathematics Council of the Alberta Teachers' Association

Volume 50, Number 3 June 2013

Celebrating 50 years: 1962–2012

Guidelines for Manuscripts

delta-K is a professional journal for mathematics teachers in Alberta. It is published twice a year to
- promote the professional development of mathematics educators, and
- stimulate thinking, explore new ideas and offer various viewpoints.

Submissions are requested that have a classroom as well as a scholarly focus. They may include
- personal explorations of significant classroom experiences;
- descriptions of innovative classroom and school practices;
- reviews or evaluations of instructional and curricular methods, programs or materials;
- discussions of trends, issues or policies;
- a specific focus on technology in the classroom; or
- a focus on the curriculum, professional and assessment standards of the NCTM.

Suggestions for Writers

1. *delta-K* is a refereed journal. Manuscripts submitted to *delta-K* should be original material. Articles currently under consideration by other journals will not be reviewed.
2. If a manuscript is accepted for publication, its author(s) will agree to transfer copyright to the Mathematics Council of the Alberta Teachers' Association for the republication, representation and distribution of the original and derivative material.
3. All manuscripts should be typewritten and properly referenced. All pages should be numbered.
4. The author's name and full address should be provided on a separate page. If an article has more than one author, the contact author must be clearly identified. Authors should avoid all other references that may reveal their identities to the reviewers.
5. All manuscripts should be submitted electronically, using Microsoft Word format.
6. Pictures or illustrations should be clearly labelled and placed where you want them to appear in the article. A caption and photo credit should accompany each photograph.
7. References should be formatted consistently using *The Chicago Manual of Style*'s author-date system or the American Psychological Association (APA) style manual.
8. If any student sample work is included, please provide a release letter from the student's parent/guardian allowing publication in the journal.
9. Articles are normally 8–10 pages in length.
10. Letters to the editor or reviews of curriculum materials are welcome.
11. Send manuscripts and inquiries to the editor: Gladys Sterenberg, 195 Sheep River Cove, Okotoks, AB, T1S 2L4; e-mail gsterenberg@mtroyal.ca.

MCATA Mission Statement

Providing leadership to encourage the continuing enhancement of teaching, learning and understanding mathematics.

Cover photo courtesy of the Edmonton Public School Board archives.

delta-K

Volume 50, Number 3 June 2013

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From the Editor's Desk

Gladys Sterenberg

Looking back over the past 50 years has been an incredible journey for me. In this issue, I have included the entire second issue of the *Math Council Newsletter* (it was renamed *delta-K* in the 1970s) because the articles seemed to represent much of what happened throughout the beginning years of the Math Council of the Alberta Teachers' Association (MCATA). Pedagogical experimentation and risk taking were evident in the form of new teaching topics and approaches. I continue to be impressed by the vibrancy of MCATA over the years and am grateful for the leadership math teachers have shown in our province.

In compiling this issue, I am reminded of the importance of community, a common theme throughout the years. Whenever I select the conference photographs, I feel a bit like I'm "finding Waldo" as I try to see how many colleagues I know. The images remind me of the significance of learning together as we share common challenges and teaching highlights with one another. I have included photographs of many award winners; missing are those for the 2012 Friends of MCATA who have served on the executive and made great contributions to our community; namely, Christine Henzel, Shauna Rebus and Christopher Smith.

The feature articles and teaching ideas in this issue continue traditions of pedagogical innovation and insight into mathematics education. I encourage you to become part of the next 50 years by contributing your ideas to our publication. It is through this sharing with our community that we remain vibrant. I hope you enjoy this collection of readings.

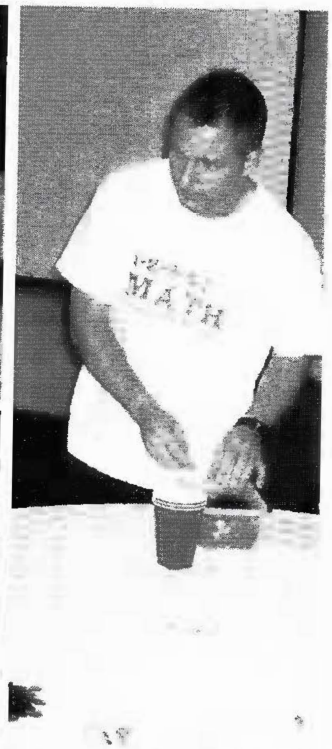
Conference Committee 2012

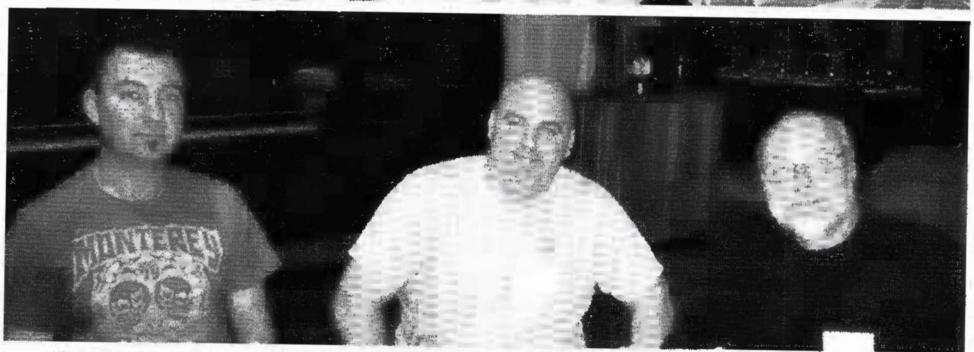
Back row (l–r): Rod Lowry, David Martin, Alicia Burdess, Marj Farris, Daryl Chichak,
Front row (l–r): Carmen Wasylynuik, Tancy Lazar, Mark Mercer, Donna Chanasyk
Missing: Robert Wong and Pat Chichak

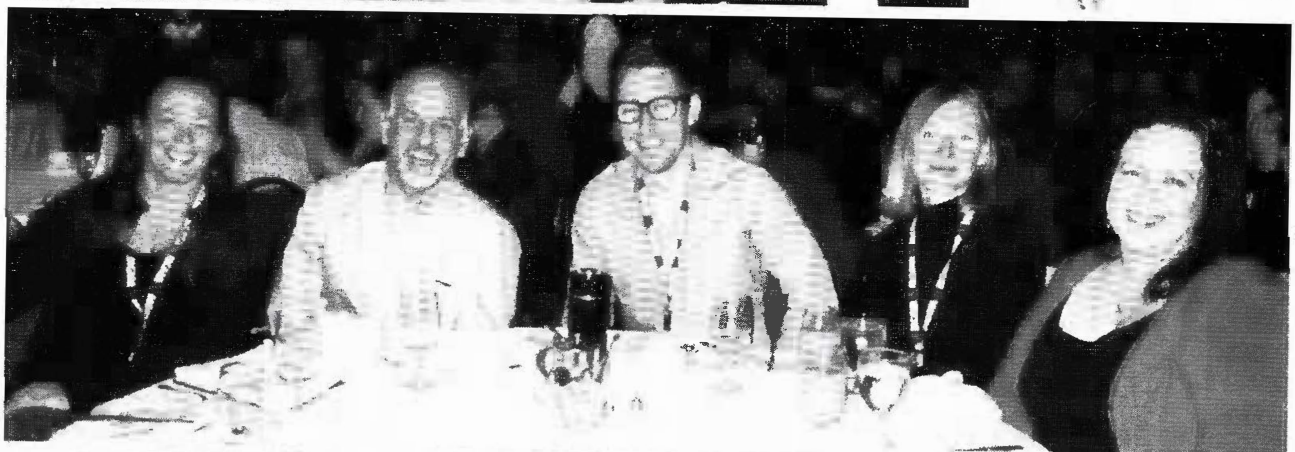
Cochairs: Tancy Lazar and David Martin

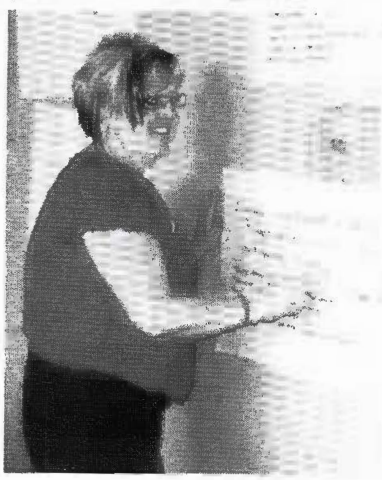
Conference Fun 2012

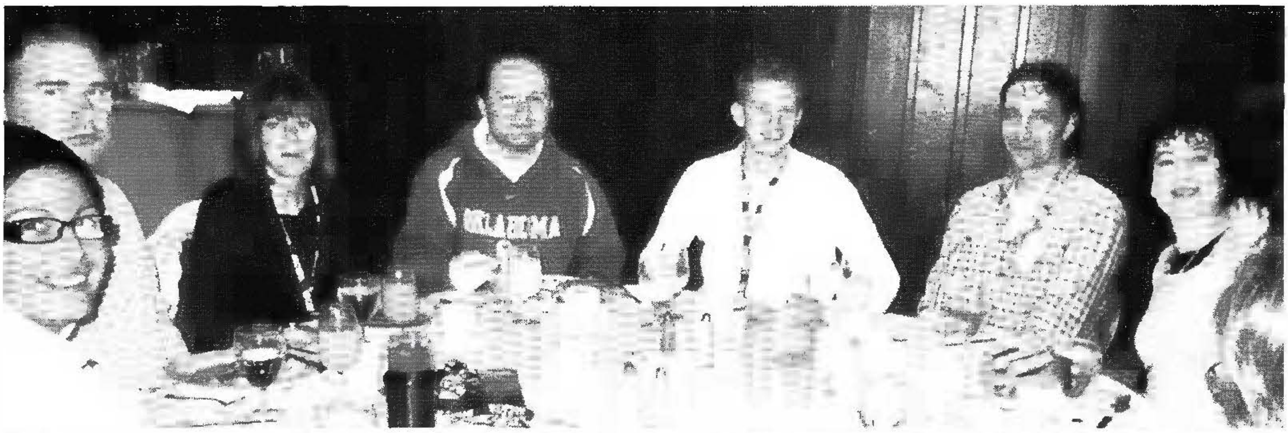
Executive, Registration and Preservice Teacher Volunteers







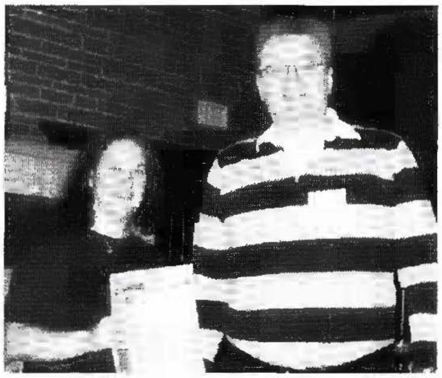


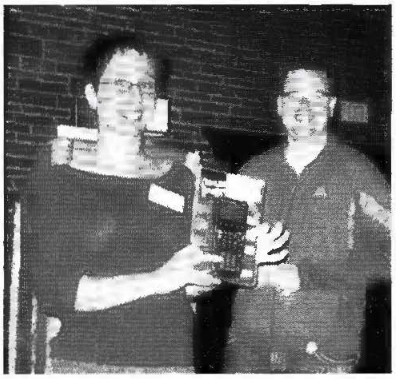

OKLAHOMA

Keynote speaker Alec Couros

Keynote speaker Tom Schimmer (left)

Door Prizes





Awards

Alberta Mathematics Educator Award recipient Benita Greenwood (left)

Bernadette McMechan receives the Dr Arthur Jorgensen Chair Award from Jens Jorgensen, the son of the late Dr Jorgensen.

New Book Announcement

Selected Writings from the Journal of the Mathematics Council of the Alberta Teachers' Association

Celebrating 50 years (1962–2012) of *delta-K*

Edited by Egan J Chernoff (University of Saskatchewan) and Gladys Sterenberg (University of Alberta)

The teaching and learning of mathematics in Alberta has a long and storied history. An integral part of the past 50 years of history has been *delta-K*, the journal of the Mathematics Council of the Alberta Teachers' Association (MCATA).

This monograph, which presents 50 memorable articles from the journal (10 from each of the past five decades), shares this rich history with a wide range of people interested in the teaching and learning of mathematics and mathematics education. The section for each decade begins with an introduction that provides the historical context and concludes with a commentary from a prominent member of the Alberta mathematics education community. As a result, the monograph provides both a historical account and a contemporary view of trends and issues (such as curriculum and technology) in the teaching and learning of mathematics. It is intended as a resource for a variety of people (including teachers of mathematics, mathematics teacher educators, mathematics education researchers, historians, and undergraduate and graduate students) and as a celebratory retrospective on the work of MCATA.

Selected Writings from the Journal of the Mathematics Council of the Alberta Teachers' Association: Celebrating 50 years (1962–2012) of "delta-K" is a volume in the series The Montana Mathematics Enthusiast: Monograph Series in Mathematics Education, edited by Bharath Sriraman (University of Montana).

The monograph is in development and will be available from Information Age Publishing.

For more information, go to www.infoagepub.com/products/Mathematics-Council-of-the-Alberta-Teachers-Association or call the IAP office at 704-752-9125.

Second Issue of the *Mathematics Council Newsletter* (April 1962)

THE ALBERTA TEACHERS' ASSOCIATION

MATHEMATICS COUNCIL

NEWSLETTER

Volume I, Number 2, April, 1962

Contents

INTRODUCING CHILDREN TO TOPOLOGY, by Pierre de Latil

Editor's Note: This article comes to us from UNESCO. It is a report of a daring pedagogical experiment - daring in that instead of a cautious approach to the teaching of a new subject, a radical approach was used. The old was completely abandoned and replaced by a new system. Members may find this article of interest.

When she had finished handing round red or blue paper hats, some of them with cockades, to the little girls, the nursery school teacher said to them: "At the signal, all the red hats form together on this

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side! All the blue hats on the other side!" And all the little girls scampered happily to their places when she clapped her hands. "Now, we'll put a rope around those who are together!" And each of the groups was roped in. Then the teacher said: "Next, we'll tie all those wearing a cockade together".

This was more difficult because there were children with cockades on their hats in both groups, and the cord around them had to cut across the other two ropes.

Is this a new game for the kindergarten? Not quite. We've just imagined a little science fiction story about a nursery school class in advanced mathematics, where young children are put in contact with a fundamental concept of human reasoning.

For the theory of groups illustrated by this very simple example is located in the vanguard of modern mathematics, on the peaks where generalizations merge arithmetic, algebra, geometry, topology, and even logic. At present, it is almost never approached before the university.

A "group" may be defined as a "neighborhood or assembly of several entities which together become elements of a new group entity". Here, the "entities" were the little girls. But they could equally well have been hairs on a head, books in a library, the claws of a cat, figures of a multiplication table, points located inside a rectangle, or lines of light rays converging on the same point.

From Concrete Games to Abstract Reasoning

In the example above, the formation of a group of little girls with cockades on their hats from the two other groups was designed to illustrate another fundamental concept, the idea of "intersection".

With slightly older pupils, the teacher might draw colored lines on the blackboard to represent the cords enclosing the groups. She would then ask each of the pupils in turn to come to the board and mark her place inside one of the circles. "No, Mary, you're not there! You are wearing a blue hat, and you don't have a cockade. So you're not in the intersection of the 'blue' group with the 'cockade' group. You're here!"

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And so, gradually, the children would progress, effortlessly, from concrete games to the most abstract conceptions, considered today as belonging to the realm of advanced mathematics.

Papy, the Pioneer

Perhaps, our story is not so farfetched after all. Today, the teaching of mathematics dominates the scientific future of all countries; it is decisive in training engineers and scientists, in short supply everywhere. Already, several countries are moving towards a large-scale reform of mathematics teaching. And one of them, Belguim, is determinedly charting the way. A university professor, Georges Papy, who teaches "modern algebra" at Brussels University, has done pioneering work to introduce these modern theories into secondary, and even primary, schools. And top officials at the Belgian Ministry of Education are supporting his efforts by encouraging a large-scale experiment with the new methods.

To proceed cautiously in a matter like this is to court failure. Usually, when schools decide to teach new subjects, they offer them as optional courses which the pupils take - or don't - without any great interest. Or else, a few notions, so summary that they are useless, are added to the old course. The fact is, one cannot add new material to old without dangerously overloading the syllabus.

The solution consists of abandoning the old system completely and replacing it by the new. But if this were done, would not pupils run the risk of flunking entrance examinations and finding themselves barred from higher education? And how would they be able to pass examinations in subjects which they have stopped studying but which are still being taught? Changing the entire mathematics syllabus of the national school system has been suggested. But one can't launch out into such a reform without prior testing; and further, the subject is so new that it would be impossible to find sufficient teachers right away to take all the new classes.

The Test Laboratory

The Belgian solution is remarkable in that it takes account of all these difficulties. To test the new teaching, schools were chosen which are an end in themselves, since they do not prepare for higher

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studies. They are the Ecoles Normales Gardiennes which train young girls to be nursery school teachers. These girls are not particularly gifted for mathematics; they are not even especially bright pupils. Therefore, if the experiment were to succeed, it would be highly convincing, and no one would be able to say that the instruction was too difficult for an average class. And even if it should fail, the careers of the young girls would in no way be harmed.

What are the results to date? After two years of experimentation, Professor Papy finds them eminently satisfactory. These girls of 14 to 16 are perfectly at home in the abstruse universe of groups. They handle problems the mere statement of which is incomprehensible to most adults.

The Belgian authorities fully realize this. In choosing the Ecoles Gardiennes as a testing ground for their new methods, they are pursuing what may develop into a long-term project. By educating future teachers in the theory of groups, they are paving the way for the introduction of these concepts into nursery schools. That may explain Professor Papy's quip: "Perhaps the crowning point of my career will be to teach one day in a nursery school."

Spare the Young

But why so much importance attached to this famous theory of groups? It was first outlined some 80 years ago by the Russian-born German-educated mathematician, George Cantor, who suggested a new conception of geometry in which every figure was conceived as a "group of points" Later the French mathematician Maurice Fréchet generalized the theory, extending it to "abstract groups" comprising any kind of object. Since then, groups have become increasingly important in mathematical thinking. Today, they are considered the common trunk of all branches of mathematics; and the principles of the theory are regarded as the very foundations of reasoning which works best starting from groups of objects rather than from an object taken singly.

By dint of abstractions, through the discovery of principles which are common to all mathematical disciplines - the principles of logic - mathematics have succeeded in achieving an extremely high degree of generalization. This took centuries of groping and hesitation. But now that the discoveries are made, young people should be spared the

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tortuous paths followed by past generations. They should be launched from the start on the straight road of modern mathematics.

CALGARY HIGH SCHOOL STUDENTS ATTEND A COMPUTING CONFERENCE, by B. A. Hodson

Editor's Note: B. A. Hodson, a graduate of the University of Manchester, England, is supervisor of technical computer programming for Imperial Oil. He teaches two classes on computer programming at the University of Alberta, Calgary.

The Calgary Computing and Data Processing Society was inaugurated in January of 1961 and the first board of directors was elected to office in May of that year. At the suggestion of the new president, [the author], it was decided to form a committee to study the possiblity of holding a conference on computing for high school students later in the year. A committee of three was formed under the chairmanship of Dr. J. E. L Peck of the University of Alberta. The result of this committee's work was a conference held at the University of Alberta, Calgary, January 13, 1962.

Registration of students began at ten o'clock with campus students and members of the Society assisting in registering some 140 students before 10:30. Each student was given a program of the day's activity and an identification badge supplied by the Alberta Wheat Pool. Each badge was one of seven colors, a code to establish which computer installation was to be visited later in the day.

At 10:30 the students were welcomed by the president on behalf of the Computing Society and by Dr. Peck on behalf of the University. The meeting was then handed over to the chairmanship of Bill Taylor of CES Computing Centre in Calgary. Mr. Taylor explained the purpose of the conference, to make students familiar with electronic computers and also to introduce to them the possibility of a career in this rapidly expanding profession. This was followed by an address by this author entitled "Introduction to the Electronic Computer".

He explained that an electronic computer is made up of five basic elements: input units, memory, arithmetic unit, control unit, and output units. He explained how the computer works internally by means

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of electronic devices akin to switches and illustrated the binary code used by many machines. The purpose of the input units was to convert the everyday language of business and science into the language of the computer. This could be done by means of paper tape, punched cards or magnetic tape. Samples of paper tape and punched cards were given to each of the students, who were also shown a reel of magnetic tape such as is used on the larger computers. The output units are to convert the internal language of the computer back into the everyday language of business and science. In addition to punched cards, magnetic and paper tape for output, there is also a high speed printing device and cathode ray display device. The memory is made up of magnetic cores and is divided into characters and words. It is used for storing instructions for the computer and also for storing the data the instructions are to work upon. The order in which the instructions are performed is under the direction of the control unit. If the instruction involves arithmetic then this is carried out in an arithmetic unit. Students were then introduced to P. Brown of Shell Oil Company who spoke on "Working and Playing with a Computer".

Mr. Brown told the gathering that computers were not intelligent in the least but rather the slaves of a group of people known as programmers. These people tell the computer by means of coded instructions exactly how the machine is to solve a particular problem. He pointed out how these instructions are stored within the memory device of the computer and how it is able to perform several thousand of these instructions every second. Examples of instructions for a particular computer were then given and with these instructions a simple computer program was illustrated. It was emphasized that before writing down these instructions it was necessary to analyze a problem in great detail, spelling out minutely how the answers would be derived from the given input. The results of this minute analysis were expressed in a detailed flow chart from which the actual machine instructions were then constructed.

After a brief recess Dr. J. E. L. Peck spoke on "Careers in Computing". In the next five years it is expected that the computing profession will employ some 375,000 persons in North America, in all categories. Dr. Peck outlined some of the job categories with the qualifications for each. He illustrated the jobs of machine operator and coders, for which a high school education alone would be sufficient. Operators press the buttons to make the computer operate whi coders convert the detailed flow chart prepared by programmers into computer language code. Above these categories he listed programmers whose job is to prepare the detailed flow charts by which a problem would be solved on the computer. Training for this would be any general degree with mathematics and science, while not a necessity, being of advantage.

Systems analysts usually require an honors degree in some subject and will usually analyze problems for computer solution in that particular subject, although they could analyze problems in related fields. They usually outline the problem solution in sufficient detail that the work can then be carried on by a computer programmer who will prepare the detailed flow charts. The numerical analyst he listed as the cream of the computing profession. For this an honors mathematics degree is a necessity The numerical analyst takes the various integrals, differential equations, business mode, and the like and develops numerical methods with which to solve them on the computer. In the area of electronics there is a vast field of work in computer maintenance, for which an interest in electronics but not necessarily a degree is required. Requiring an honors degree, preferably in mathematics of physics, is the field of computer design. This is the development of the logic of computers. The actual design of components and machine "hardware" requires another group of personnel specializing in electronic and electrical engineering.

During the lunch break, the computing society members mingled with the students to answer any questions that may have arisen during the morning sessions and the group reconvened at 1:15 p.m. at which time Mr. Taylor introduced D. Wehrhahn of International Business Machines Company to talk on "The Future of Computing".

At this lecture the audience heard of the development of high speed magnetic tape readers, able to read 150,000 characters per second. Document readers that read typewritten documents were mentioned with the further development of machines to read handwritten documents directly into the computer memory. Computer memory devices would become larger in the number of characters that they could store but smaller in the volume of space that they would occupy. Scientists are now working in the field of cryogenics to develop circuits that operate close to the absolute zero of temperature which can change their state is less than a billionth of a second, requiring very

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little electrical power. Present machines can perform almost one million operations per second and computers under development will be even faster than this. Machines are under study that will understand the human voice, while in existence already is a machine that will talk to you.

After this final lecture students split into groups to visit computer installations in Calgary They visited an LGP 30 at CES Computer Services, an RPC 4000 at Texaco Oil Company and at the Royal McBee Company, a 1620 at International Business Machines, an IBM 650 at Shell Oil Company, and an IBM 1410 at Imperial Oil Company. At each installation the group was shown the computer and its peripheral equipment Demonstrations were also seen showing the computer at work and at play A questionnaire completed by the students indicated that the conference was well accepted. Many indicated that they would like to see something similar for other professions. It is hoped that this conference will become an annual event.

A SUMMER AT STANFORD, by H. F. McCall

> Editor's Note: Dr. McCall, principal of Seba Beach School, was awarded the Shell Merit Fellowship last year. We plan to include an article by him in our June issue.

The chance to be a Shell Fellow and participate in the Stanford activities of this special group of science and mathematics teachers chosen by the Shell Oil Company does not come to everyone, but it did come to me. It might come to you too if you entice Lady Luck a little say, by showing your interest in this program and making inquiries from Shell Merit Fellowships, School of Education, Stanford University Stanford, California.

The eight-week program at Stanford is designed specifically for the Shell Fellows, a total of about 50 science and mathematics teachers, five of whom are from Western Canada and the rest from the United States west of the Mississippi.

For part of the day all of the group were together. Then for the rest of the time we were separated into three groups - the physics, chemistry, and mathematics sections, for specialized work in those fields. While together, we dealt with many things from all three fields.

The value of the specialized work in the various fields will be obvious to anyone. But the time spent together, whether in semi-formal discussions with outstanding world figures in education, science or mathematics, or whether in informal discussions with each other, had values of many different kinds - values which, in some cases, would be more difficult to measure, but which, in practically all cases, were tremendous. No effort was spared to bring us leaders in every field, Nobel Prize winners where possible; nor was any effort spared to take us to the finest research laboratories. These included the High Energy Physics Laboratory (Stanford), Radiation Laboratory (University of California at Berkeley), Plant Biology Laboratory (Carnegie Foundation), Computer Laboratory, Radio Astronomy Institute, and others no less exciting and inspiring in various areas of chemistry.

The opportunity to participate in such stimulating study and exchange of ideas is one which would serve Alberta teachers well.

EXPERIMENTAL CLASSES IN GRADE VII MATHEMATICS, by L. C. Pallesen

> Editor's Note: Mr. Pallesen is supervisor for Division III, Calgary Public School Board.

Under the direction of the Junior High School Mathematics Subcommittee of the Department of Education, ten Calgary Grade VII classes took part in experimental work in mathematics. They were part of a group of 30 classes over the whole province using a new text, Seeing Through Mathematics, published by W. J. Gage, Ltd. The publishers have designed this text to be a sequel to the Seeing Through Arithmetic series which has recently been authorized for the Alberta elementary schools.

Because of the authorization of this new series in the elementary schools the Junior High School Mathematics Subcommittee feels it must consider the changing of the junior high school text. It would be the hope of the subcommittee that a text might be found which would continue to develop ideas along the lines of the elementary texts and would at the same time introduce some of the ideas of the "new" or

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"modern" mathematics. Because of the emphasis on change in mathematics at the present time many of the publishers are working on revisions, most of which are not yet published. The subcommittee is examining all these publications as soon as they are available, and hopes to try in classrooms any texts that seem on examination to be suitable to the Alberta situation. The current Calgary experiments are part of this program.

Seeing Through Mathematics differs markedly from the text presently authorized. Rather than start Grade VII with a review of the basic operations of mathematics, it introduces a completely new area - the symbolism of sets. Once this symbolism is introduced it is consistently used to deal with geometric ideas and algebraic topics. A great deal of stress is placed on exactness of expression with emphasis placed on the distinction between numbers and numerals, on the meaning of open and closed sentences and on inequalities as well as equalities. The authors claim that, in their treatment, students become much more proficient in problem-solving than in traditional courses.

To attempt to yield a maximum amount of information the ten Calgary classes were chosen to include all levels of ability. Several of the classes were heterogeneous groups including students of all abilities others were homogeneous with both the top end and the bottom end of the ability scale being represented. Furthermore, approximately half the teachers had recently completed a university summer school course on modern mathematics, including the set theory, while the other half of the teachers had had no recent courses in mathematics.

The experimental use of this text was planned to continue for a four-month period ending about January 31. At that time most of the teachers returned to the traditional text for the balance of the year However, four Calgary classes have been granted permission to continu the use of the text until June, 1962. The number of classes which continued would have been larger had it not been for the administrative difficulties caused by pupil transfers, since new students could not conveniently be placed in these classes.

During the period of the experiment teachers have met twice monthly to discuss any mutual problems and general progress. Each of the participating teachers is being asked to complete a full questionnair

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dealing with pupil progress and pupil reaction to the course as well as stating their own appraisal of the strengths and weaknesses of the course. Inasmuch as all these questionnaires will not be examined until the next meeting of the subcommittee no general reaction is yet available. As might be expected the enthusiasm of the ten participating teachers varied considerably. To judge the text on the basis of the four-month experiment will be difficult, not only because of the shortness of the period of time and the newness of the material, but also because the text is designed for graduates of an elementary program using the Seeing Through Arithmetic series.

Present plans in the city of Calgary schools include the introduction of the Seeing Through Arithmetic texts for Grades I to IV in all schools in September, 1962. The series will be extended to Grade V in September, 1963 and to Grade VI in September, 1964. This suggests that before September, 1965 it would be desirable that the junior high school subcommittee reach a decision. The present Calgary experiment is an attempt to assist in this task.

ELEMENTARY ARITHMETIC FILMS

Donald in Mathmagic Land, T-1397, (30 minutes)
A general interest film showing many applications of mathematics.

Today's Need in Arithmetic, (14 minutes)
An extremely well-prepared film but highly commercialized - gives a reasonably good general introduction to the Seeing Through Arithmetic series.

The following five films were prepared by Scott, Foresman and made available to the Audio-Visual Aids Branch by W. J. Gage Ltd. The production standard in these films is rather poor. The main personality in the films, Mr. George Russell, was asked to speak on the Seeing Through Arithmetic series at a teachers' institute in New Mexico. On his arrival he discovered that arrangements had been made to record all his talks on film. Mr. Russell was not prepared for this kind of an assignment and, as a result, the finished product is not the best from a technical point of view. It is true, however, that the content of Mr. Russell's talks will prove very valuable to teachers in inservice programs.

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Basic Mathematical Ideas, T-1449, (29½ minutes)
Computation, T-1450, (27 minutes)
Division, T-1451, (29½ minutes)
Problem Solving, Part 1, Equations of Numbers (29½ minutes)
Problem Solving, Part 2, Equations of Ratios (27 minutes)

MCATA NOTES

1. Hi-Lites from MCATA Executive Committee Meeting, December 29, 19(

Membership fees in the MCATA will cover the term September 1 to August 31.

A Mathematics Seminar on the "new mathematics" at the elementary level will be held at Alberta College, Edmonton, from July 3 to July 10.

A seminar planning committee, consisting of T. Atkinson, E. Wasylyk, M. Sillito and J. Cherniwchan was formed.

The date for the annual conference was set for July 11, 12, and 13. A tentative program was drawn up.

The executive committee will meet again on April 26 in Calgary.

2. MCATA Conference

The dates, July 11, 12, and 13 have been set aside for the conferenc We may be able to hold the conference at the University of Alberta, Edmonton.

An effort is being made to include in the program topics dealing wit curriculum revision: (a) experience with the STA series, (b) how much should be retained from our present mathematics curriculum, (c) results of Grade VII mathematics experimentation in 1961-62, and (d) an overall view of a mathematics curriculum.

We hope to include other topics: Programmed Learning, Programming for a Computer, Role of a Statistician. A more detailed account of the program will be given in the June newsletter.

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3. Membership

We have now 97 paid up members.

4. Programmed Instruction

Are you following the articles on programmed learning that are appearing in periodicals? Note that a monograph entitled, "Programmed Instruction: An Outline of Developments in Teaching Machines, Programmed Notebooks, and Scrambled Textbooks", has been published by The Alberta Teachers' Association and is available to teachers.

5. MCATA Mathematics Seminar

July 3 to July 10 inclusive are the dates for the Mathematics Seminar to be held this summer at Alberta College in Edmonton. Sponsored by MCATA, this seminar is designed to train resource people for inservice work in the "new mathematics". Delegates to the seminar will be expected to take part in this work. The seminar will be devoted to work in the elementary school but attendance is not necessarily restricted to elementary teachers. It will deal with basic concepts and teaching methods in modern elementary mathematics and is expected to provide teachers with new ideas and new approaches in arithmetic teaching. An outstanding staff, including both experts who have organized such seminars throughout the United States and Albertans who have taken a leading role in experimental work with the "new mathematics", will provide the instruction.

The costs of the seminar will be: accommodation, $28; fees, $5; and transportation, variable.

Applications are invited from interested teachers. If you are interested, complete the form on the last page of this newsletter; if you know of someone who might be interested, please pass on the form to them.

Editor - J. M. Cherniwchan, 276 Evergreen Street, Sherwood Park

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Mail under cover to:

M. T. Sillito
Executive Assistant
The Alberta Teachers' Association
Barnett House
11010 - 142 Street
Edmonton, Alberta

I am interested in attending the Mathematics Seminar in Edmonton, July 3-10, 1962. Please consider my application and send further information as it becomes available.

Name ______________________

Address ______________________

____

Grade Taught 1 [ ] 2 [ ] 3 [ ] 4 [ ] 5 [ ] 6 [ ]

7 [ ] 8 [ ] 9 [ ] 10 [ ] 11 [ ] 12 [ ]

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An Induction Proof of the Factorization of $a^n - b^n$

Ron Persky

Editor's Note: Ron Persky is a long-time contributor to delta-K *and a strong supporter of mathematics education in the United States. This proof is included in this issue as a mathematical prompt because, in Persky's words, it is "not what you call advanced math but I just think it is nice." I agree.*

We establish that $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + ab^{n-2} + b^{n-1})$. (1)

To do this, the Second Principle of Finite Induction will be used as well as the following identity:
$a^{n+1} - b^{n+1} = (a+b)(a^n - b^n) - ab(a^{n-1} - b^{n-1})$.

For n = 1, (1) yields $a - b = a - b$.

Assume (1) is true. Then for n+1, we have, using the above identity:
$a^{n+1} - b^{n+1} = (a+b)(a^n - b^n) - ab(a^{n-1} - b^{n-1})$.

By the induction hypothesis and the Second Principle of Finite Induction

$$\begin{aligned} a^{n+1} - b^{n+1} &= (a+b)(a-b)(a^{n-1} + a^{n-2}b + \ldots + ab^{n-2} + b^{n-1}) + \\ &\quad -ab(a-b)(a^{n-2} + a^{n-3}b + \ldots + ab^{n-3} + b^{n-2}) \\ &= (a-b)\{(a+b)(a^{n-1} + a^{n-2}b + \ldots + ab^{n-2} + b^{n-1}) - ab\,(a^{n-2} + \\ &\quad a^{n-3}b + \ldots + ab^{n-3} + b^{n-2})\} \end{aligned}$$

Multiply the two terms inside the braces.

$$(a-b)\left\{\begin{array}{c}[a^n + a^{n-1}b + \cdots + a^2b^{n-2} + ab^{n-1}] + [a^{n-1}b + a^{n-2}b^2 + \cdots + ab^{n-1} + b^n] - \\ {[a^{n-1}b + a^{n-2}b^2 + \cdots + a^2b^{n-2} + ab^{n-1}]}\end{array}\right\}$$

In the first bracket, [ ], the second term, $a^{n-1}b$, through the last term, ab^{n-1}, is cancelled by the terms in the third bracket. This leaves
$(a-b)\{a^n + a^{n-1}b + a^{n-2}b^2 + \cdots + ab^{n-1} + b^n\}$ and gives formula (1) for $a^{n+1} - b^{n+1}$ which is what is required to complete the induction proof.

Exploring Children's Literature in the Elementary Mathematics Classroom

Gregory Bryan and Ralph Mason

The project reported here involved the collaborative design and implementation of a "math-lit" approach to mathematics and language arts instruction in four classrooms from three school divisions. In literature-triggered mathematics, a teacher uses sets of children's trade books with mathematical elements to lead students into contextually rich, active and interactive mathematics activities. After appreciating each trade book as a literary experience, students engage with the mathematics in the story through various activities. The classrooms contained heterogeneous student groups from Grades 3 to 5 with widely varying mathematics and reading achievement levels. The purpose of the research was to explore the mathematical experiences of children engaged in literature-triggered mathematical activities.

The teachers in this project demonstrated a willingness to follow the leads of their students. They encouraged the creativity of the children, who participated in various activities for two months, working within the framework of the project. When given the freedom to respond to their reading and to the math in the books, some students selected traditional options like creating number sentences to reflect the math. Other students embraced the opportunity to demonstrate and enhance their learning with such diverse activities as literature-circle discussions, dramatic interpretations and performances of roles depicted in some of the picture books, (for example, a Grade 5 performance of *Minnie's Diner* [Dodds and Manders 2004], and a Grade 3 performance of *The Wolf's Chicken Stew* [Kasza 1987]) and the creation of reader's theatre scripts based on and inspired by the various books.

The teachers were pleased that the children approached their mathematics learning with excitement and enthusiasm. One teacher felt sure that she was on the right track when a Grade 5 student exclaimed, "This story is so cool. It has math in it!"

We, Ralph Mason and Greg Bryan, are the authors of this article. Mason is a professor of mathematics education, and Bryan is a professor of children's literature and literacy education. This study was a good marriage of our interests and expertise. We began the project by meeting with the teachers involved in the study. We shared our perspectives and discussed the possibilities that the project could bring to the classrooms. Above all else, we encouraged the teachers to use the books as they pleased—we wanted to see how they and the students would use them.

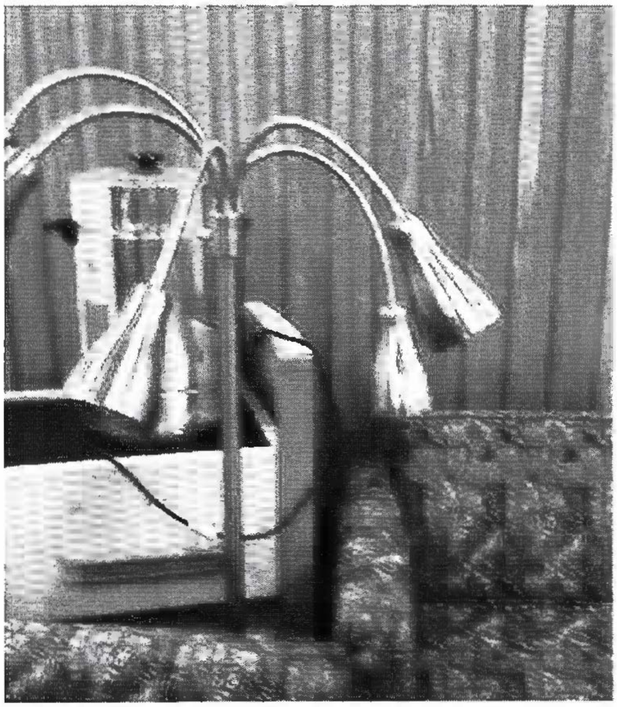
We assembled five or six sets of different picture books. In some of the books, the math was more explicit than in others. Some mathematical concepts in a story were less obvious and, seemingly, less deliberate. The text sets were arranged in particular mathematical themes: addition and subtraction, repeated doubling, and multiplying and dividing.

Introductory Activity

When the teachers took the math-lit books to their classrooms, the initial approach was to share with students what we had identified as suitable introductory texts: Stephen Johnson's (1998) *City by Numbers* and Scieszka and Smith's (1995) *Math Curse*. The teachers used these two books to raise awareness of the idea of math being all around us.

In *Math Curse* a teacher says to the class, "You know, you can think of almost everything as a math problem." This leads one child to realize how prevalent math is in the world. *City by Numbers* depicts city scenes where the shape of the urban environments portrays the shapes of numerals from 0 to 21. For instance, the twin smoke stacks of an old factory look like the numeral 11, a crumbling brick wall appears to contain a 15, while a bridge looks like a 4.

With Johnson's example of New York City scenes in mind, students dispersed across their schools to look for hidden numerals. In one classroom, students took digital photographs depicting the numerical shapes they found embedded in their school. For instance, the following photographs illustrate the location of a 6 that appeared in the teachers' lounge when an electrical cord blended with a light stand.



Through the students' interactions with the books and teacher-guided actions with the books' ideas, students' feelings about mathematics and their visual creativity became legitimate elements of their engagement with mathematics.

Addition and Subtraction Text Set

One book in the addition and subtraction set was a counting book entitled *My Little Sister Ate One Hare* (Grossman and Hawkes 1996). The book begins:

My little sister ate 1 hare.
We thought she'd throw up then and there.
But she didn't.

The little sister then eats two snakes and one hare. Then she eats three ants, two snakes and one hare. After the little sister is depicted eating a series of items, the book ends:

My little sister ate 10 peas.
But eating healthy foods like these
Makes my sister sick, I guess.
Oh, my goodness! What a mess!

This book can be an important addition to mathematics classrooms because it can add fun to the math experience. It also serves as a starting point for various mathematical explorations. Of all the things the little sister ate, the students were asked what they would most like and dislike. They wrote their names on Post-it notes and voted by adding their names to bar graphs depicting the most and least popular items. In that particular class, eating nine lizards was the most resisted meal, while, fortunately, eating the 10 peas was preferred to eating worms, mice and bats.

One Grade 4 group created a readers' theatre script inspired by this same book. In the student-created script, a mother asked her children to clean their bedrooms, so they proceeded to eat the mess in their rooms: one spider, two books, three dirty socks and so on. Keeping to the pattern of the original text, eventually the children vomited everything. When the mother came to the bedrooms to check on the progress, she said, "Oh, my goodness! What a mess!"

A necessary step to understanding arithmetic as sensible is seeing addition as an extension of counting. When children's literature enables students to fold back again, to look at their prior counting experiences in the light of addition and subtractions, the students can form links between the arithmetic they are studying and the acts of counting up and down. The arithmetic they are learning can be a sensible extension of the counting they understand, literally, backward and forward.

Repeated Doubling Text Set

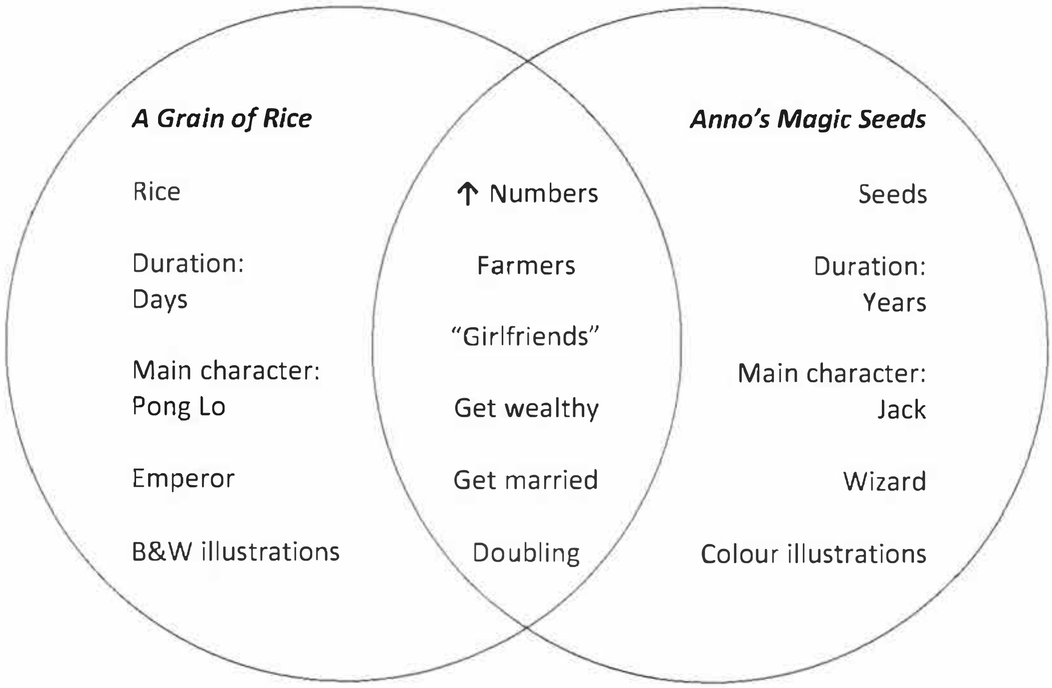
One text set included books that involved the repeated doubling of quantities. This set deliberately included several different versions of the same folk tale of a ruler and his rice. In Birch and Grebu's (1988) version of this story, *The King's Chessboard*, a wise man helps an Indian king. Not wanting to be indebted to anyone, the king wishes to repay the debt.

> The wise man replies, "I wish no other reward than to serve you."
>
> After some pressure from the king, the wise man eventually says:
>
> I only ask this: Tomorrow, for the first square of your chessboard, give me one grain of rice; the next day, for the second square, two grains of rice; the next day after that, four grains of rice; then, the following day, eight grains for the next square.

The king sees this as an odd but easily satisfied request so he agrees. The teachers had their students use manipulatives to represent the mathematics within the story. Because the numbers got large quickly, students needed to use different manipulatives to represent place value units. Although they started with individual grains of rice, the grains were soon spilling beyond the edges of the chessboard squares with which they worked. Therefore the children began representing tens, hundreds and thousands of grains with coins and bottle caps. Just as with the king in the story, the children were amazed to see the amounts owed each day. Within just two weeks, the number of grains of rice owed for that 14th day had grown to 8,192. Through the process of doubling, by the end of four weeks, the number of grains of rice owed for the 28th day was 134,217,728. It was a surprising discovery for the children, just as it had been for the teachers when first demonstrated by Ralph during the planning meetings.

In the repeated doubling text set, there were five different versions of the same multiplying rice story: *The Rajah's Rice* (Barry and Perrone 1994), *The King's Chessboard* (Birch and Grebu 1988), *One Grain of Rice* (Demi 1997), *The Token Gift* (McKibbon and Cameron 1996) and *A Grain of Rice* (Pittman 1999). Although illustrated in black and white, Pittman's book is a short novel and differs in appearance and format from the other books. The different versions of the same story allowed the teachers and students to explore the notion of critical reading, comparing and contrasting the books. Students identified their favourite version of the story. This led to discussions of preferences and reasons for those preferences. In turn, these reading discussions led to exploration of the mathematical concept of sets, incorporating the use of Venn diagrams to identify similarities and differences.

In one classroom, Pittman's novella was compared with another book from the doubling set, *Anno's Magic Seeds* (Anno 1995). The figure below illustrates elements identified by one group of Grade 5 students and the manner in which that group classified each element.



People may wonder why repeated doubling experiences would be provided to young students. On the one hand, doubling is a fundamental act of arithmetic, linked to adding an amount to itself. One plus one is two, two plus two is four, four plus four is eight and so on. It is a little more complicated but just as natural as skip-counting by two. Yet, on the other hand, repeated doubling is represented mathematically by powers of two; for example, eight is two to the third power, which means two is multiplied by itself three times. Surely powers are not an elementary-years mathematics topic, yet all the students engaged with repeated doubling in their own way when they were exposed to these big ideas with the stories in the books.

In time, powers as repeated multiplication must make sense to students, just as addition as repeated counting makes sense in the early grades, and multiplication as repeated counting makes sense in Grades 3–5. Literature-based experiences enable students to engage with the core idea of repeated doubling before they need to think of the process as the repeated multiplication that can be symbolized with exponents as powers. As they express and extend the idea in ways of their own choosing, they build foundational experiences for the formal symbols they will learn later. When they do learn the symbol system, the children will have referents in the varied experiences they have had over the years, such as the literature-triggered mathematics described here.

Multiplication and Division Text Set

The multiplication and division text set included two books by Dayle Ann Dodds. One was especially popular with the students. At the end of the project, many students nominated Dodds' (2004) book, *Minnie's Diner*, as their favourite. In *Minnie's Diner*, a succession of brothers goes to the diner to eat. As each brother arrives, he looks at what his younger brother is eating and says, "I'll have what he's having but make mine a double." This book could also have been included in the doubling set of books. In *The Great Divide* (1999), a long distance race is under way. The number of competitors keeps reducing by half as racers get lost or fall by the wayside. In one literature-circle discussion of the book, six Grade 5 students wondered aloud whether the book involved division or subtraction. This discussion helped the children to articulate and solidify their understandings of these two processes and how they can be related. At the same time, the students enrich their understanding of multiplication as accumulated sets with the same number of elements in each set. Similarly, as larger numbers are split into equal sets in different ways, students can learn that amounts can be split into many groups each with the same number of members. Students can then see division as a formal mathematical idea for describing things that they have imagined and created, drawn and described.

As a professor of mathematics education, Mason described McElligott's (2009) book, *The Lion's Share*, as perhaps the best math-lit book he had ever read. The book features members of the animal kingdom who each divide the lion's special cake into halves, one of which they eat. Splitting into two equal parts—or dividing by two—provides an accessible starting point for the concept of division.

The book begins:

> Every year at the start of spring, the lion invited a small group of animals to join him for a special dinner. The ant had never dined with the king before. She was very nervous and wanted to make a good impression.

At the dinner, Lion produces a cake. Elephant thinks, "I could eat this in one bite, but that might seem greedy," so Elephant takes only half the cake. In turn, as the cake reaches them, each guest takes half of the remaining cake. When the cake finally gets to the ant, there is so little left that it crumples when the ant tries to slice it in half. All of the other guests criticize the ant for being greedy and selfish. Mortified, the ant humbly offers to bake another cake for the host. Not to be outdone, each of the other animals promises to bake double the number of cakes nominated by the previous animal. As it turns out, while all the other animals are busy baking their cakes, the ant is able to complete its one cake and dine in peace and harmony with the Lion King.

The students were asked to identify and discuss connections they could make to the book. Several students made text-to-text connections and discussed the similarities between this book and other books including *One Grain of Rice* and *Minnie's Diner*. Others made connections to movies like *The Chronicles of Narnia* and Disney's *The Lion King*. Some children made text-to-self connections and discussed seeing lions when they went to the zoo.

Making Connections

Students made personal connections to the content of the stories but less so to the mathematics. For instance, in *Spaghetti and Meatballs for All* (Burns and Tilley 1997), the story involves arranging tables for a family gathering. This reminded one student of a family gathering to celebrate the birth of a newborn baby. "When I went to my auntie's house," the student said, "we helped her get some more chairs because there was a party for my other aunty who had a baby." The student easily recalled needing to get more chairs for the guests; however, she and her peers had considerable difficulty connecting to instances from their own lives from the idea of splitting a number into equal sets, like the restaurant patrons being seated at multiple tables. We value such limitations in students' responses as valid feedback about their readiness to perceive aspects of their lives in terms of the mathematical ideas they are studying, in this case multiplication. The book provides an opportunity for students to be guided to engage with the mathematics of arrangements of equal sets, so that, in time, multiplication can be real life to them.

Occasionally, a student could connect to the numerical values within the stories. In *The Wolf's Chicken Stew* (Kasza 1987), from the addition and subtraction text set, the wolf is trying to fatten up a chicken for his stew so he bakes 100 pancakes. This reminded one student of her sister: "My sister had to make a hundred cupcakes because it was the 100th day of school," she said. This and others, however, were rather basic connections to the math in the books. Just as teachers often encourage students to make connections to the texts that they read, teachers might do well to encourage students to think about the personal, individual connections they can make to the mathematics they are doing in class. When have they done some halving outside the classroom? When did they need to multiply or divide outside of the classroom? Such encouragement might help children to more fully recognize the benefits of their mathematics learning.

Participant Comments

After several weeks of working with children's trade books during math class, students were asked what they thought about including books in the math class. Below are some of the Grade 3 students' comments:

> When we read a book, it gives us an idea of the math that we're going to do.
>
> When there's a story, you get more details about what you're going to start off with.
>
> The book gives us a little bit of a hint.

A Grade 4 boy mentioned that the story "tells you more things." One Grade 5 student said that the story "teaches you why it is" while her classmate said that the story "tells you how to do it."

These interesting comments suggest that, like us, the students saw literature-triggered math as offering a kind of life raft or floatation device for children's exploration of mathematics in that it provides contextualized opportunities for learning. The books provide assistance to the children. After all, a Grade 3 student said, "When we just do math [without stories], it's just a bit harder for me." At the same time, a Grade 5 student reported, "You get confused [doing math without a story], and in the stories, you don't."

When further asked for their opinions, some of the Grades 4 and 5 students replied:

> You can enjoy the story.
>
> Whenever I do math [without a story] it's always hard for me to think.
>
> When there's a storybook, you could explain [the mathematics] better.
>
> It's easier to understand [when there's a story].

These comments suggest the students' focus wasn't on just getting right answers. The books assisted them with arriving at correct answers, but the verbs used (enjoy, think, explain and understand) suggest that the products (the correct answers) were really just an outcome of a desirable process of enjoying, thinking, explaining and understanding. Incidentally, these verbs are the type of words emphasized in the NCTM standards for talking about success in mathematics.

Concluding Comments

The use of children's literature trade books as a mathematics resource can represent a significant departure from the traditional use of math textbooks. We are not suggesting this literature-triggered approach as a replacement for more traditional approaches to math instruction. Rather, we see literature-triggered mathematics as a worthwhile supplement, believing that the mathematics that matters happens as children visualize, represent, represent and extend math beginning with the mathematics in the books.

Reflecting on the events of the previous several weeks, the Grade 5 teacher summed up the success of the project with the comment, "It's great to see them excited and thinking about math." In elementary classrooms where this excitement and incisive thinking are absent from mathematics periods, perhaps teachers would do well to consider including children's literature trade books when next they sit down to prepare their mathematics instruction. Especially for teachers who are confident in their abilities to engage students productively in activities that build on the literature they experience, literature-triggered mathematics may offer a potent supplement to textbook-based mathematics.

Texts Sets Used in the Study

Introductory Texts (Math Everywhere)

Johnson, S T. 1998. *City by Numbers*. New York: Viking.

Scieszka, J, and L Smith. 1995. *Math Curse*. New York: Viking.

Addition and Subtraction

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Murphy, S J, and S Bjorkman. 2002. *Safari Park*. New York: HarperCollins.

Palatini, M, and B Moser. 2005. *The Three Silly Billies*. New York: Simon & Schuster.

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Multiplication and Division

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Murphy, S J, and C Jabar. 2003. *The Sundae Scoop*. New York: HarperCollins.

Napoli, D J, and A Walrod. 2001. *How Hungry Are You?* New York: Atheneum.

Neuschwander, C, and L Woodruff. 1998. *Amanda Bean's Amazing Dream*. New York: Scholastic.

Pinczes, E J, and B Mackain. 1993. *One Hundred Hungry Ants*. Boston, Mass: Houghton Mifflin.

——. 1995. *A Remainder of One*. Boston, Mass: Houghton Mifflin.

Other Resources for Further Support

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Gear, A L. 2012. "A Cultural Introduction to Math." *Teaching Children Mathematics* 18, no 6: 354–60.

Gerretson, H, and B C Cruz. 2011. "Museums, Mysteries, and Math." *Teaching Children Mathematics* 17, no 7: 404–09.

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Goals of the New Elementary Mathematics Curriculum: The Power of Open-Ended Questions and Tasks

Werner Liedtke

Introduction: The Problem and Assumptions

According to the *Common Curriculum Framework for K–9 Mathematics* (Ministries of Education 2006, 4), the main goals of mathematics education include preparing students to use mathematics confidently to solve problems and to communicate and reason mathematically. Students who have met the goals of instruction should, among other things, exhibit a positive attitude toward mathematics, engage and persevere in mathematical tasks, be confident and take risks, and exhibit curiosity.

How could these important goals be reached? Hiebert (2000, 437), a mathematics educator interested in the quality of teaching and learning in schools, suggests that "although research can rarely prove that a particular course of action is the best one for all people and for all time, it can help boost the level of confidence with which decisions are made." Based on his research, Hiebert has developed guidelines for designing mathematics classroom environments. Two of those guidelines have implications for the goals set out in the *Common Curriculum Framework* (CCF): students need opportunities to engage directly in the kind of mathematics that are stated in the goals, and instruction can emphasize the conceptual understanding inherent in these goals without sacrificing skill proficiency.

A project that involved Grades 1 and 2 students led Spungin (1996) to conclude: "Problems and tasks that have multiple solutions or methods of solution are best in stimulating discussion, debate, creativity and risk-taking." According to the author, open-ended tasks promote curiosity, provide opportunities for students to wrestle with difficult ideas to revise their thinking, and demonstrate respect for all ideas by encouraging contributions from all students. A replication of Spungin's project with Grade 1 students led to similar conclusions (Liedtke, Kallio and O'Brian 1998). These results clearly indicate that open-ended problems and tasks are conducive to reaching the goals set out in the CCF for students.

Two challenges exist. Because the majority of problems and questions that are included in references used by students lack open-endedness, they need to be modified. Presenting open-ended tasks to groups of students requires the orchestration of discussions that accommodate all types of responses, provide opportunities for learning to respect different ideas and allow students to revise their thinking.

The examples described in this article illustrate the challenges that can exist and show the power of open-ended problems. I will also show how tasks that lack open-endedness can be detrimental because they can discourage willingness to take risks.

Tasks and Settings with Multiple Solutions

Sorting Tasks

Sorting is a thinking strategy that is related to many aspects of mathematics learning. For example, elementary students might be asked to look at four different blocks, figures or animals and respond to these questions: Which of these is not like the other? Which of these is not the same? For many students this request implies that one choice is different, and they must determine which one the person who designed the task had in mind. Students who guess correctly are rewarded. Students who guess differently may believe that their choices are incorrect.

The following example is taken from a reference for students. One choice is identified as correct: Which of the following numbers does not belong in the set? 8 17 25 33. (Which number would you pick and why?)

How can this problem be changed to foster students' confidence, encourage them to take risks and develop their ability to communicate and reason mathematically? Possible options include the following:

- Questions like, Which of these do you think does not belong? Why do you think so? will result in having students verbalize the reason or reasons for their choice. As they listen to others state their reasons, new ways of thinking about what has been presented may be discovered. The ability to defend one's choice makes a response correct.
- Students could be told that someone selected one option; that is, 17 for the example above, and they are asked to think of possible reasons for choosing this number. Asking students to think of more than one reason has advantages. However, the question must be posed carefully.

For example, some questions try to encourage flexible thinking by asking, Can you think of another way? (Spungin 1996). This can be detrimental. Some students will opt out and respond quickly with a definite no, and any attempt to collect further diagnostic or assessment information has to stop.

During an observation in one classroom the students were using the benchmarks 0, ½ and 1, and attempting to place fractions on this part of the number line. When the teacher asked a student if he can place ⅔ on the number line, the student immediately responded with, "Yes, what else?" After a brief pause and a smile, the lesson continued. Questions that can be answered with one word; for example, Can you...? or Do you know of another way...? are of little value. Equally ineffective are questions like, In how many different ways can you sort these things? How many ways can you come up with? or How many solutions are there? Should full marks be assigned to such responses as none, zero or any number that students record?

The following types of requests encourage students to talk:

- Try to think of more than one possible reason or way for....
- Try to think of at least two ways to....
- How would or could you....
- Try to....
- Show me what you would do to....

Meanings of words and ideas may be internalized and valuable diagnostic and/or assessment information becomes available as explanations are shared.

One student's reason for selecting 17 was that it is the only one with straight lines, and he was unable to think of another possible reason for this choice. This response supports Peck, Jencks and Connell's (1998) findings that without a follow-up question or a brief interview, more than one-half of the time students may be misjudged. Data from my collection of interviews indicate that children may also state unpredictable logical reasons for choices that are identified as correct. Then there will always be justifications that are too good to be true. For example, a child was asked to sort objects from a box that somehow went together. When he was asked to explain his sorting technique, he pointed and stated: "These are interesting; these aren't." To foster flexible thinking students can be asked, working alone or with a partner, to think of how each member in a list of items might be different from the others.

If having students identify one odd member in a collection is the goal, the request must be specific. For example, as students face four different animals, the types of requests could be as follows:

- Think about what animals eat.
- Think about where animals live and show me the animal that does not belong.
- Tell me why that is the case.

However, a specific request may not guarantee that the desired response will be elicited. For the choices, S T 7 L, and the request, I am thinking of letters of the alphabet. Which one does not belong? yielded the following responses from two children: "This one (T) because it has a roof on it, and this one (S) because it is curvy."

Tasks with Patterns

I agree with those who suggest that some published programs for the primary grades devote too much time to activities with patterns. My observations lead me to conclude that too many tasks in the early grades are similar. They do not connect to any aspects of mathematics learning, and in some settings students spend too much time on the low-level activities of colouring or cutting and pasting. I agree with a former NCTM president's conclusion that, "Work with patterns is probably overemphasized in some quarters as the defining component of algebra for younger learners" (Fennell 2008).

During interviews I have asked students who have looked at patterns during their first three years of school what they think a pattern is. The majority of them were unable to make up their own definition. My collection includes the following responses from four students:

- A pattern is one shape after another.
- A pattern is different colours in a pattern.
- A pattern is very neat.
- A pattern is good and fun for you to practise. (Liedtke 2010)

Activity sheets and assessment items about patterns that are provided to students typically include the question: What comes next? or the request: Extend in a logical way. The majority of authors who design tasks for this question and request have one correct response in mind. Without instructions that are much more specific and detailed, many students will not produce the response the authors had in mind. A common misconception is that a sequence is determined by the first few terms. However, a sequence is not defined by the first several terms, but by a function or a context. The question, What comes next: 1, 3, 5, 7? allows for anything to be a correct answer, and thus the students' explanation of their thinking becomes essential.

Patterns can be extended in many ways. For example, it is easy to change a repeating pattern into a growing or increasing pattern and vice versa. An answer key that identifies one response as correct can be detrimental for students. Because different correct responses are possible, the need for a follow-up question or brief interview suggested by Peck, Jencks and Connell (1998) is even greater than for the sorting tasks that were described. The following illustrate a few of the possible dilemmas that can arise in response to sorting tasks:

- For the pattern of different two-dimensional figures: triangle–square–triangle–square–triangle–square, a five-year-old boy selected a circular shape in response to, What comes next? His reason for this choice was, "I want to see a circle." It might have been tempting to conclude that this boy lacks an understanding of repeating patterns. However, that was not the case. When the question, What would you put next? was posed several times, he created a repeating pattern that included the circle (Liedtke and Thom 2009).
- As part of an interview with a Grade 1 student the request, What comes next? was made for 1 2 3.... Without any hesitation he recorded 6. Was a high level of thinking involved in arriving at the response? His response to, Why did you choose six? reinforced the need for a follow-up question. The reason, "I am six years old," was logical according to him. The look on his face seemed to suggest, "Let's put down a number that is important to me and then go from there."
- The problem, Extend in a logical way: M T W T, was sent to me over the Internet, and it identified one response as being correct. I have presented this problem to a number of students and adults and have several responses that differ from what the author had in mind—the first letter of the days of the week. Each of these responses is logical, according to those who created it, and I agree.
- After a presentation to parents at a local school, one mother told me that her Grade 5 son had brought home a page of tasks about patterns. The question for the whole page was, What comes next? and every response he had recorded was marked by the teacher as being incorrect. After he explained his answers to his mother she asked, "So what do you think?" He responded, "I think my teacher has lost her imagination." This powerful scenario points to the need to dream up new ways to ask students to work with patterns.

What are some possible options for tasks?

1. The instructions can be specific and detailed. For example, As you decide what you think should come next in the shown sequence, think of a repeating pattern. However, this may not guarantee that the desired response is elicited.
2. To encourage students to be confident and take risks, an open-ended question can be posed: What do you think could come next and why? Explain your reason or reasons.
3. Students can be challenged to use their imagination by making the request to try and generate many different patterns for a given sequence.

During a workshop with teachers, the point was made that one of the most effective ways to probe understanding of patterns is to present a pattern, hide one or more of the members and ask students, What do you think is hidden? How do you know? In retrospect, it was unfortunate that the pattern that was displayed consisted of seven two-dimensional figures and the member in the middle was hidden. One very observant teacher was quick to point out that she thought there existed several possible responses to the question. She cited the example of the boy who wanted to see a circle, stated that she could insert a circle and then continue the pattern by flipping the members about the circle. She also suggested that two or more figures, one above the other, could be hidden. Unforeseen scenarios can turn into valuable learning experiences.

Guessing and Estimating

Making and sharing a guess requires confidence and involves some risk taking. Since that is the case, whenever an opportunity arises young students should be asked to guess. Students should realize that all guesses are equally valued. If that goal is reached, they will participate with confidence whenever they are asked for a response.

Let's assume a group of students is asked to guess how many jelly beans are in a jar. It must be assumed that every recorded number represents the quantity a student visualized while looking at the jar. If that assumption is made, all guesses have to be evaluated in the same way; that is, as "good guesses." It could be very discouraging for some students and it may affect their future willingness to take risks if one or two responses are identified as "very good" or as "excellent." It is more appropriate to use the descriptors "lucky" and/or "a little luckier" since there is little evidence of the students' thinking.

Young students learn the difference between guessing and estimating. The estimation strategies students develop involve the use of referents or benchmarks. During the early stages of developing estimation strategies, different degrees of familiarity with units will result in estimates that will vary from student to student. These differences should not result in different evaluation categories. All estimation results should be acknowledged in the same way.

In some student references the request is made to record estimates for calculations or measurements. For example, students may be asked to estimate the sum of 24 and 17 or to estimate the perimeter of a rectangle that is 35 cm × 3 cm. Sometimes students are asked to record estimates before answers are calculated or objects are measured. It is easy to guess why some students reverse the order of this request or change their estimates after their calculation or measurement. Neither of these types of tasks will give any insight into students' ability to estimate. Assessment about estimation requires an oral or a written explanation; that is, What would you do to estimate the number of jelly beans in the jar?

Teachable Moments

Whenever young children make mistakes or have difficulty expressing ideas in their own words, some teachers will take advantage of what they think is a teachable moment, correct the mistakes and/or tell children what to say and/or think. These teachers have the false belief that they are providing a shortcut to cognitive development. They forget that ideas grow slowly and, along with the meanings of words, are internalized when children have the opportunity to express them in their own words at their own level of ability. An opportunity to talk about what has been learned is essential for learning how to communicate and reason mathematically. Mistakes and unexpected responses should lead to an exchange of questions and a discussion of ideas.

The goals of the CCF can be reached by attending to the power of open-ended questions and tasks that encourage students to explain their thinking. I conclude by presenting a conversation as told to me by a teacher in one of my courses. After his Grade 2 son had looked at his digital watch and stated, "Dad, it's 5:41, 22 minutes to *Scooby Doo*" the following exchange took place:

"How many minutes is it from 41 to 50?"

"Nine minutes."

"How many minutes is it from 50 to 60?"

"Ten minutes."

"How much is nine plus ten?"

"Nineteen …, but dad, there are two minutes of commercials first."

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An Exploration of Per Cents and Fractions Through a Study of Fractals

Michael Jarry-Shore

In the winter of 2011, four classes of Grade 8 students at the Calgary Girls' School, a public charter school for girls in Calgary, Alberta, explored the fascinating world of fractals. Alberta's *Mathematics Program of Studies, Grades K–9*, states that students are to "gain an understanding and appreciation of the contributions of mathematics as a science, philosophy and art" (Alberta Learning 2007, 4). At the Calgary Girls' School, significant efforts are made to connect students' mathematics learning to other subject areas to help students see the applications of concepts addressed in class. What follows is a discussion of an integrated project involving fractals that sought to develop students' understanding of several mathematics concepts, while simultaneously requiring them to call on their emerging understanding to create fractal art.

Two specific outcomes listed in the number strand of the *Mathematics Program of Studies, Grades K–9* (2007) at the Grade 8 level are as follows:

- Demonstrate an understanding of percents greater than or equal to 0%, including greater than 100%.
- Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

When faced with the task of developing an engaging learning activity intended to assist students in meeting these outcomes, the Grade 8 mathematics teachers at the Calgary Girls' School saw an opportunity in the study of fractals. Fractals are repetitive geometric patterns that go on infinitely; they consist of increasingly smaller and smaller shapes that are similar to preceding shapes in the fractal pattern. Objects that are similar have the same shape, but different size, and the same interior angles. The Sierpinski triangle (see Figure 1), for example, consists of increasingly smaller, similar equilateral triangles.

Preceding Work

Prior to our work re-creating fractals, students developed their ability to solve authentic problems involving percentages with the assistance of percentage benchmarks (see Figure 2). For example, students used their emerging understanding of benchmarks to calculate 30 per cent of $80.

Figure 2. Percentage-benchmark example

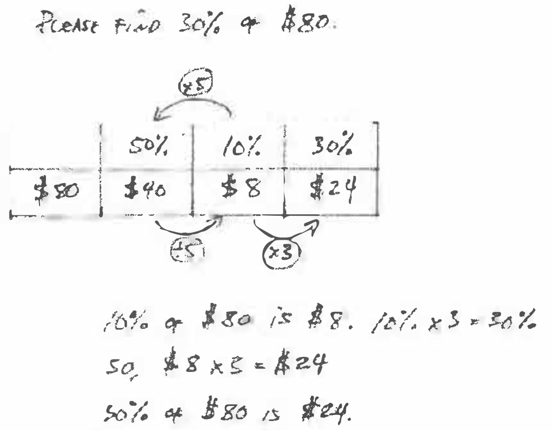
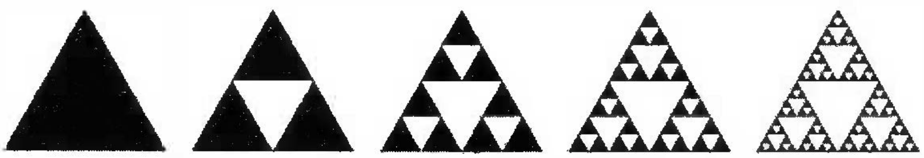


Figure 1. The Sierpinski triangle



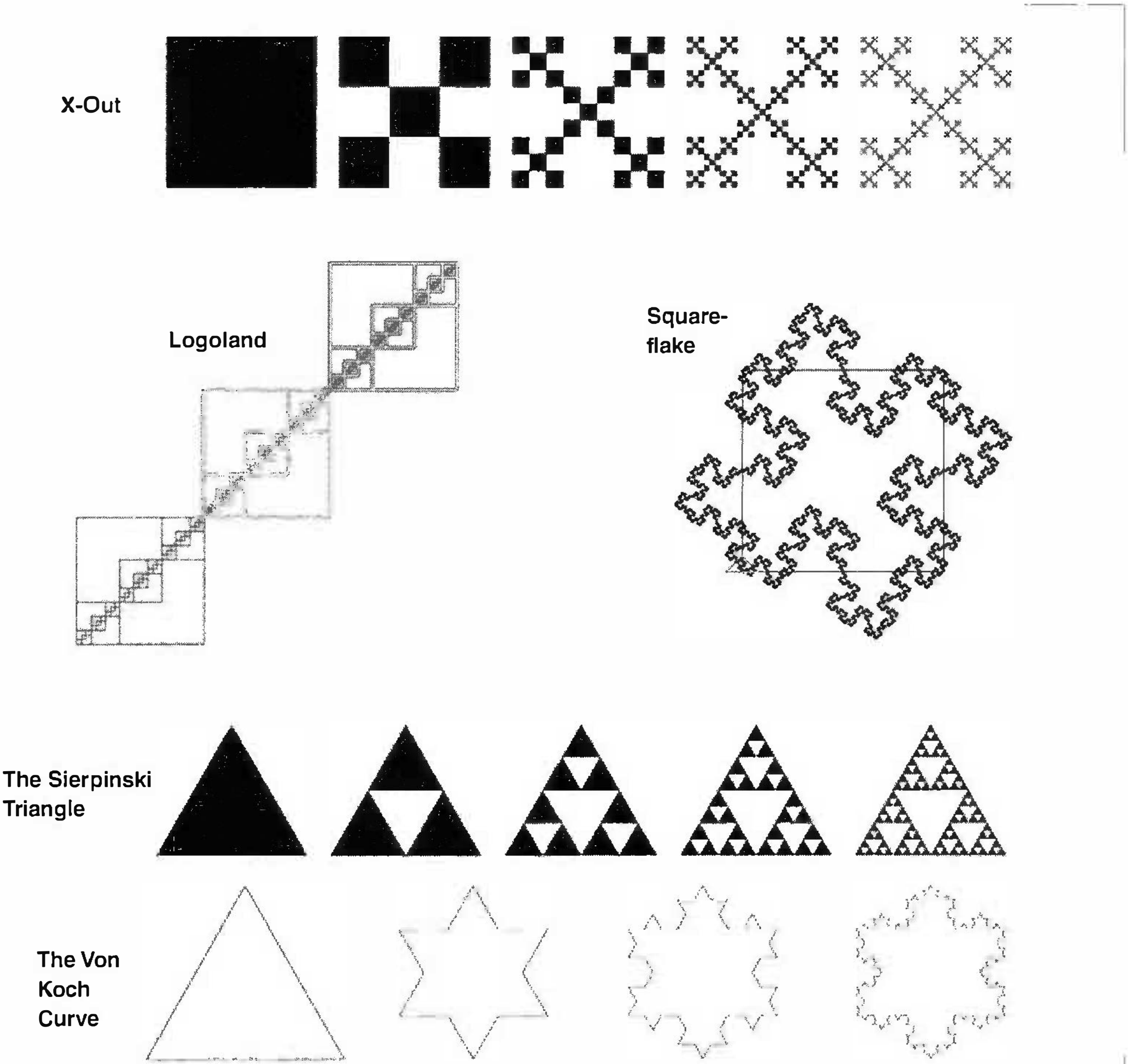
The Fractal Patterns

For our study of fractals, we selected five fractal patterns for students to analyze, investigate and ultimately re-create. We referred to them as X-Out, Logoland, the Sierpinski triangle, the Von Koch curve and Squareflake (see Figure 3).

These fractal patterns exhibited differing levels of complexity. The Sierpinski triangle and Von Koch curve consisted of similar equilateral triangles; each of the remaining three fractals consisted of similar squares.

After selecting these fractal patterns, the Grade 8 teachers placed students in each class of 25 into groups of two to three students, then assigned each group a specific fractal pattern whose complexity aligned with the collective attributes of those in the group (collaborative skill, fine-motor skill, perseverance, interest level and so on). Student groups were then given graph paper, pencils, rulers, right angles and protractors, and were required to work collaboratively in an effort to re-create a rough design of their assigned fractal. It should be noted that at the Calgary Girls' School, great emphasis is placed on collaborative group work, and students are regularly expected to engage in relevant dialogue with one another as they seek to construct their own meaning of the various mathematics concepts addressed in class.

Figure 3. Fractal patterns



Mathematical Concepts Addressed

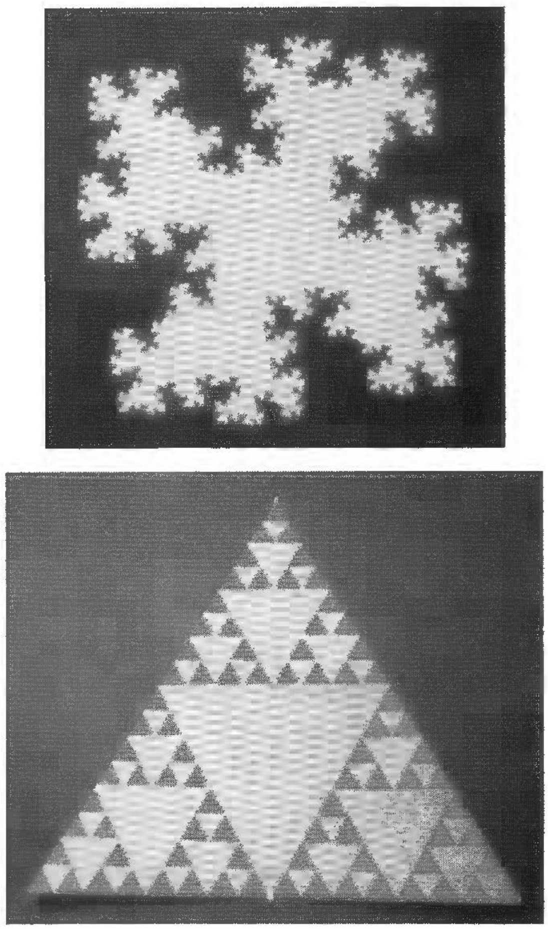
Once student groups were confident in their ability to re-create their assigned fractal pattern, they met with their mathematics teacher to share sketches of and dimensions for necessary shapes. For example, in meeting with their teacher, the majority of students who re-created the X-Out fractal-pattern first shared their observation that the largest square in their pattern required dimensions that were a multiple of three, as each subsequent square in the pattern had dimensions that were one-third that of the preceding square's dimensions. Most students in the X-Out groups went on to share their observation that the dimensions of the square in the first stage of their pattern needed to be a fairly large multiple of three or else they would quickly end up with decimal-number dimensions after several reductions in size, which many students wanted to avoid.

During these meetings, students in each group also worked with their teacher in developing their understanding of percentages, scaling, converting fractions into equivalent percentages and vice versa, as well as the multiplication of whole numbers by fractions. Students were asked to identify the key fraction at play in the re-creation of their assigned fractal (for example, 1/3 for X-Out) and to convert this fraction into an equivalent fraction with a denominator of 100 (the fraction's equivalent per cent). Students also pictorially modelled whole number by fraction multiplication pictorially by first identifying the dimensions of their given shapes on graph paper then calculating the dimensions of subsequent shapes in their fractal patterns after scaling down by some fraction. As an example, one X-Out group started their fractal pattern with a 27×27 cm square, then figured out the dimensions of the next squares in their pattern by calculating 1/3 of 27 cm. Last, students in these meetings were required to use their understanding of percentage benchmarks to calculate the length and width of a shape whose dimensions were either 50 per cent or 200 per cent that of some given shape. As an example, students re-creating the Sierpinski Triangle were asked to identify the side length of an equilateral triangle whose dimensions were 50 per cent that of an equilateral triangle whose side length was 32 cm; they were then asked to use benchmarks to figure out the dimensions of this shape if a scale factor of 200 per cent were used instead.

Building the Fractals

After meeting with their teachers, students in each group used rulers, right-angle tools, protractors, pencils, construction paper of varying colours, scissors and glue to assemble their assigned fractal pattern. While this phase of the work was quite time-consuming and could certainly be sped up with the use of technology, requiring students to actually measure, draw and cut out the shapes needed for their fractal re-creation enhanced their measurement skills and developed their appreciation for the exponential growth in the number of shapes at each successive stage of their fractal. One of the groups re-creating the Squareflake found the process too tiresome and pursued the re-creation of a simpler fractal pattern, while another Squareflake group was presented with the same opportunity, but chose to soldier on, later finding themselves highly satisfied with their finished product (see Figure 4). A few students became visibly frustrated when re-creating shapes in the later stages of the fractal pattern, but spoke with confidence about both the exponential growth and the patterns present within fractals.

Figure 4. Student fractal re-creations: Squareflake and Sierpinski triangle



Formative and Summative Assessment

To assess students' understanding of concepts addressed in our work with fractals, we required each of them to take part in a one-on-one interview/performance assessment with their mathematics teacher. It should be noted that though students at the Calgary Girls' School complete a great deal of work in collaboration with their peers, group work is never assessed as such. Instead, students are assessed independently following collaborative group-work projects and activities. During the interview/performance assessments, students were presented with graph paper, a right angle, a protractor, a pencil and a ruler, and were asked to show how they re-created their fractal, discuss the reason behind their choice of dimensions, explain what made their pattern a fractal, and identify which fraction/percentage was applied in the re-creation of their pattern and how. Students were also asked to use benchmarks to identify the length and width of a square whose dimensions were 25 per cent or 300 per cent that of a 12 × 12 cm square. Although these interviews were time-consuming, the insight they provided into students' mathematical reasoning and conceptual understanding were well worth the effort. As a result of their work in re-creating fractals, it was apparent that many students not only solidified their understanding of percentage benchmarks but also developed their proficiency in working with fractions and their ability to identify patterns.

Along the way to these summative one-on-one assessments, students completed several formative assessments known as thinking-logs (see Figure 5), in which they were required to use benchmarks to solve authentic problems involving percentages. On these thinking logs, students were also required to assess their own understanding of the concept at hand by identifying themselves as red, yellow or green (see rubric in Figure 5 for additional details).

Figure 5. Percentages thinking log

Tom's Shoes Thinking-Log

TOMS

Shoes for Tomorrow

A pair of Tom's shoes originally-priced at $50 are on sale for 20% off. GST of 5% will be charged on the purchase of the shoes. How much will the shoes cost?

Please explain how you went about solving this question:

Please describe any challenges, if applicable, that you encountered in solving the question:

Identify your level of understanding by circling beside each question (How confident do you feel about your answers to these questions?)

[illegible] *Not confident*	YELLOW *Somewhat confident*	[illegible] *Confident*
• I don't get it • I don't know where to start • I cannot explain my answer • I need help • I am confused	• I am on my way to understanding • I can start but have some questions along the way • I need some help to explain my answer • I think my answer is correct but would like it to be checked	• I get it • I can solve it by myself • I can explain my answer • I feel confident in helping others

Curriculum Connections at Other Grade Levels

Although this project was given to students in Grade 8, it might make for a good fit in Grade 9, where students are required to "demonstrate an understanding of similarity of polygons" (Alberta Learning 2007, 152). Moreover, fractal patterns could do much to illustrate the nature of exponential growth to Grade 9 students. For example, the Sierpinski triangle in Figure 1 consists of 1 equilateral triangle in white in the second stage of the fractal, 3 smaller equilateral triangles in white in the next stage of the pattern, 9 smaller triangles in the third stage, 27 in the fourth stage and so on. Hence, the Sierpinski triangle provides an authentic example of the 1, 3, 9, 27 base-3 exponential pattern. The number of shapes at each successive stage of the X-Out fractal pattern is as follows: 1, 5, 25, 125 and so on (base-5 exponential pattern). Congruent shapes in several of the fractals discussed in this paper can also be related through translations, reflections and rotations, and so, this project could be adapted to fit an exploration of transformational geometry in any mathematics class at just about any level.

Although fractals are not explicitly identified as a topic of study in the Alberta program of studies for mathematics, an investigation of these complex, visually appealing mathematical patterns presents the opportunity to explore a wide variety of concepts. Our own particular exploration addressed scaling and the use of percentages greater and less than 100 per cent, converting fractions into equivalent percentages and vice versa, as well as the multiplication of fractions by whole numbers. This specific study was welcomed by both teachers and students alike for its hands-on nature and its ability to place students at the centre of their mathematics learning. Our investigation of fractals led to much discussion and the exploration of numerous other concepts (exponential growth, pattern recognition and extension, measurement, angles, similarity and congruence), which we hadn't planned on addressing with this work, yet were brought up along the way by our curious group of young mathematicians.

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Michael Jarry-Shore has taught mathematics and science for seven years to Grades 7–9 students in at the Calgary Girls' School. He is currently enrolled as a graduate student at McGill University, where he is working to complete his master's degree in education. The focus of his research is preservice teacher education in the area of mathematics. He is particularly interested in exploring the acquisition and development of mathematical knowledge for preservice elementary teachers. He is also interested in the establishment of formal coaching initiatives intended to support teachers new to the profession.

Catch Billy Miner

Trevor Pasanen and Krista Francis-Poscente

Are you looking for fresh ideas to challenge your students? Catch Billy Miner is a great game to play in class. We call this game unfair because there is a way to always win. By always winning you can have some fun with your students demonstrating your exceptional skills at game play. Once your students learn the winning strategy, they will have gained skills in solving problems with numbers and patterns.

The Catch Billy Miner Game

The Mountie starts in Edmonton, and Billy Miner in Charlottetown. One player moves the Mountie and another moves Billy. A move consists of sliding the chip from one city to an adjacent city along the dashed line. The Mountie tries to capture Billy by moving onto the city occupied by Billy. Billy tries to evade

Figure 1: Starting positions on the game board

imprisonment. If the Mountie captures Billy in five moves or fewer—that is, five of Billy's moves—then the Mountie wins. If the Mountie fails to capture Billy in five moves, then Billy wins and the criminals take over Canada. The Mountie goes first.

Catch Billy Miner in the Classroom

One way to introduce this problem is with an interactive whiteboard. Load the template (see Appendix). Have two volunteers come up to the interactive whiteboard to demonstrate how to play the game. Assign one to be the North West Mounted Police (NWMP) and the other to be Billy Miner, the robber. The Mountie goes first and tries to catch Billy. Once students understand the rules, hand out game boards (see the Appendix) and small chips to represent the Mountie and Billy. The children will be excited to play for several minutes.

Move around the classroom and challenge a student to a game. You play the Mountie; let the student be Billy. Make sure you win. Ask the students if they think you were just lucky. Play enough times with a few students so that the classroom buzz will be that you know how to win. When we played this game with students, they squealed with delight and exasperation each time we won and begged us for our secret.

Some students might figure out how to win. If they do, ask them to come up to the front and explain their strategy. If not, play a student on the interactive whiteboard and win. Prompt the students to be observant. Many will notice that you always head to Yellowknife. Most will not notice your second and third moves.

On your second game on the interactive whiteboard, ask the students to tell you how to move the Mountie. You will lose the game if they did not observe your second and third moves. Tell them to remember what was done last time and what could be different in the next game. Repeat this process until they figure out the strategy.

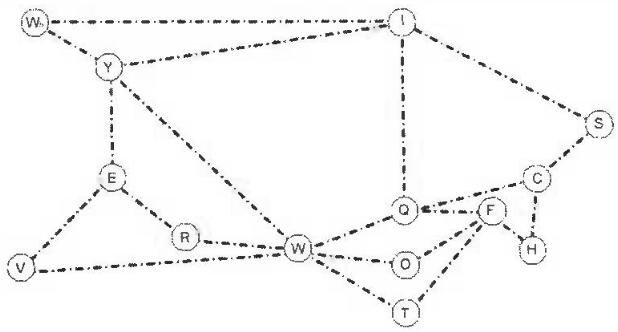
Solution to the Game

The solution to the problem comes from graph theory and discrete mathematics, which are emerging fields with applications to computer programming. They aren't specifically mentioned in the program of studies (Alberta Education 2007); however, many concepts are encompassed by graph theory.

Graphs in the program of studies are defined in terms of linear relations in Cartesian space positioned along the x and y axis. However, graph theory provides a different definition with abstract representation of objects connected by lines: A graph is a collection of dots and lines, with every line terminated by a dot at each end. The dots are called vertices and the lines are called edges.

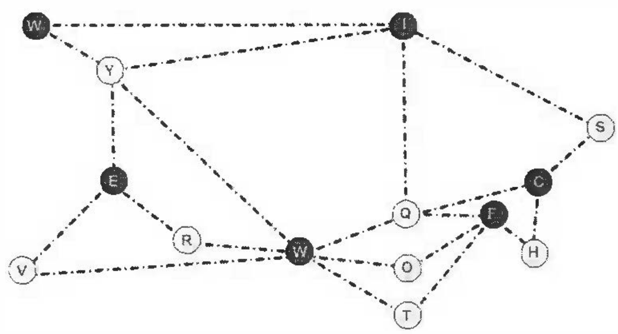
The graph below is a representation of the map above. Each city is represented as a vertex. Edges join the city.

Figure 2: Graph of the game board



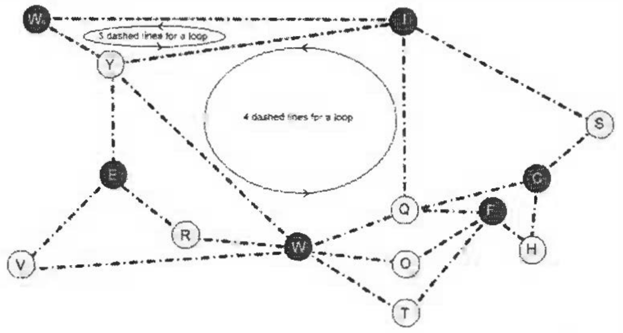
Colour Yellowknife yellow. Colour every city that is connected to Yellowknife purple. Colour the cities attached to those cities yellow. Keep going until the cities are yellow or purple.

Figure 3: Color-coded graph of game board



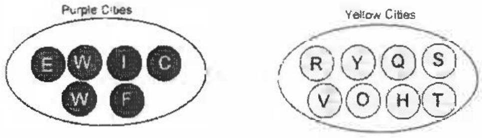
Notice that all the loops between cities have four edges with the exception of the Yellowknife-Iqaluit-Whitehorse loop. That loop has only three edges. Also notice that Whitehorse and Iqaluit are adjacent and are both purple. Without four edges alternating colours is not possible.

Figure 4: Loops between cities



The cities can be divided into purple and yellow groups.

Figure 5: Cities that have purple vertices and cities that have yellow vertices



A Common Strategy for the Unaware Mountie

Billy Miner starts in Charlottetown, and the Mountie from the NWMP starts in Edmonton. If the Mountie moves to Regina and Winnipeg, Billy Miner will stay in the same coloured group as the Mountie.

Start:

Figure 6: Start of the game

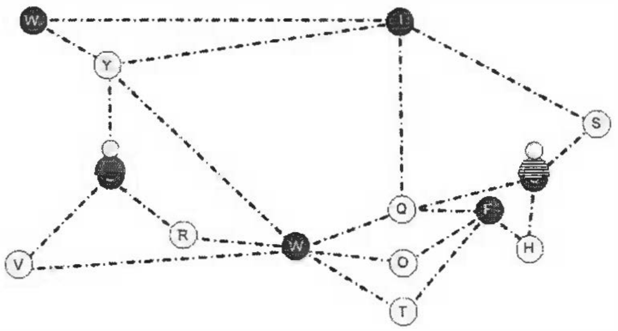
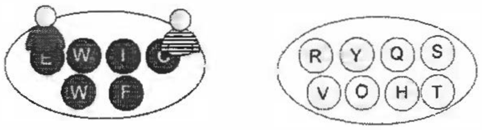


Figure 7: Billy the Miner and the Mountie are both on purple vertices



Move 1: The Mountie moves to Regina and Billy runs to Halifax. Both are in yellow cities now.

Figure 8: A possible first move

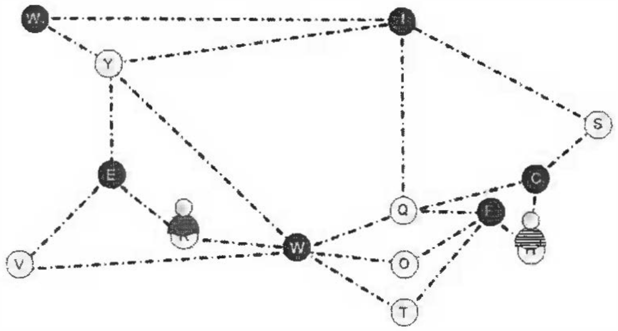
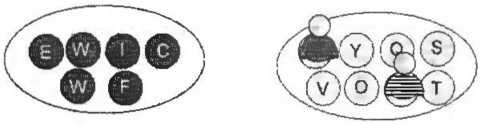
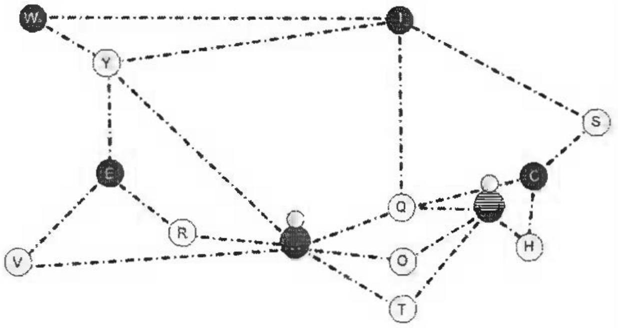


Figure 9: Billy and the Mountie are both on yellow vertices



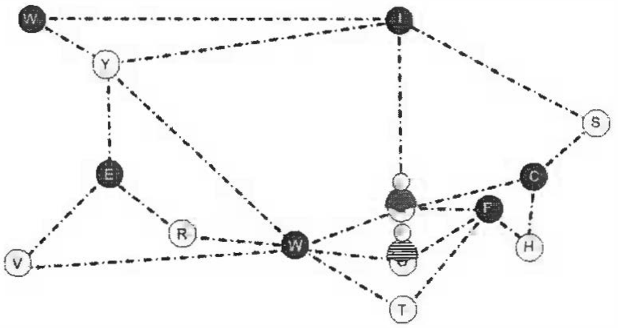
Move 2: The Mountie moves to Winnipeg and Billy hides in Fredericton. Both are in purple cities.

Figure 10: A possible third move



Move 3: The Mountie moves to Quebec City and Billy runs to Ottawa. Billy is free.

Figure 11: Still chasing Billy



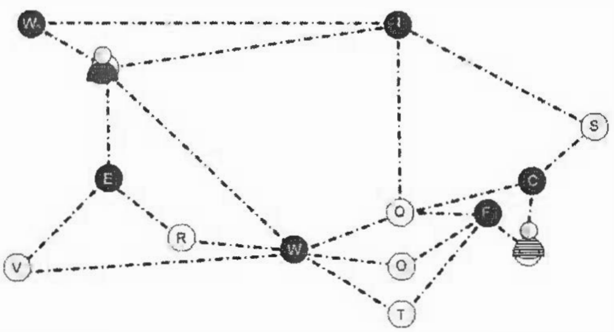
Billy will always be safe from the Mountie if the Mountie stays in the loops with four edges. When the Mountie travels along the loops of four, Billy and the Miner will always be on the same coloured vertices. They will always have another vertex between them.

Unfair Strategy for the Aware Mountie

The situation changes if the Mountie moves from Whitehorse to Iqaluit and into the loop with three vertices.

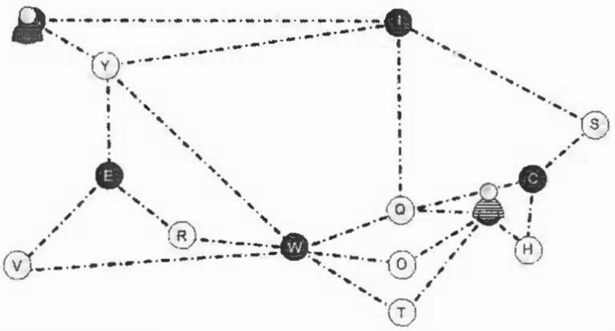
Move 1: Instead of moving to Regina, the Mountie moves to Yellowknife.

Figure 12: A different first move



Move 2: The Mountie moves to Whitehorse and Billy runs to Fredericton. Both are still in purple cities.

Figure 13: The next move. The Mountie is in the loop with three vertices.



Move 3: The Mountie moves to Iqaluit and Billy moves to Ottawa. Notice the colour of the vertices. The Mountie is now on a purple vertex. Billy is on a yellow vertex.

Figure 14: Move 3. The Mountie is in the loop with three vertices and has a different coloured vertex than Billy

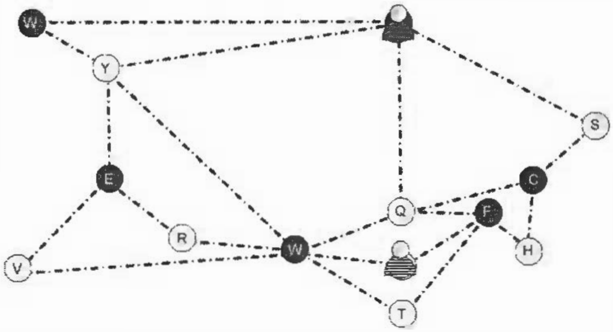
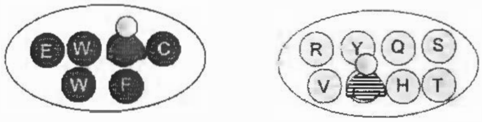
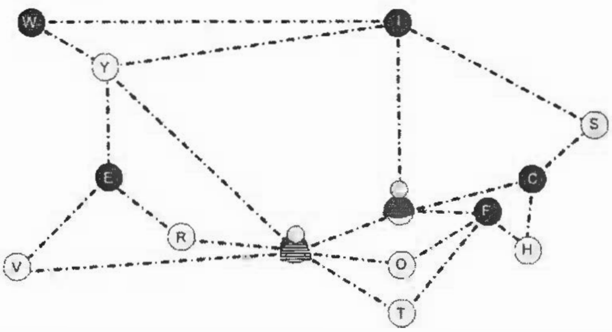


Figure 15: Billy and the Mountie are in different coloured cities.



Move 4: The Mountie moves to Quebec. Billy runs to Winnipeg.

Figure 16: The Mountie can taste his victory

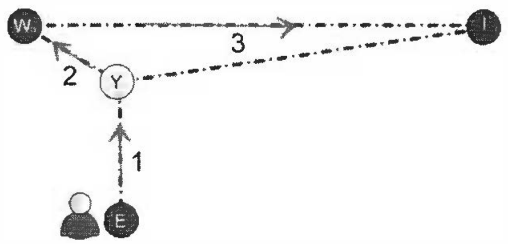


The Mountie catches Billy in the next move. By moving into the loop with three lines, the Mountie was able to occupy a city with a different colour than Billy Miner. Being on a different coloured city enabled the Mountie to get to an adjacent city.

How the Mountie Can Always Win

The Mountie can always catch Billy in five or less moves. Surprisingly the most important move in this game is from Whitehorse to Iqaluit, think about what happens if the Mountie races to make that move. The Mountie starts the game by making the following three moves:

Figure 17: The Mountie can always win with these moves



Meanwhile, while avoiding the Mountie, Billy's first three moves follow the pattern of going to a yellow city, then to a purple city and then back to a yellow city. This means the Mountie's fourth move starts in Iqaluit while Billy is in one of the eight yellow cities. Now, investigate each situation in three cases:

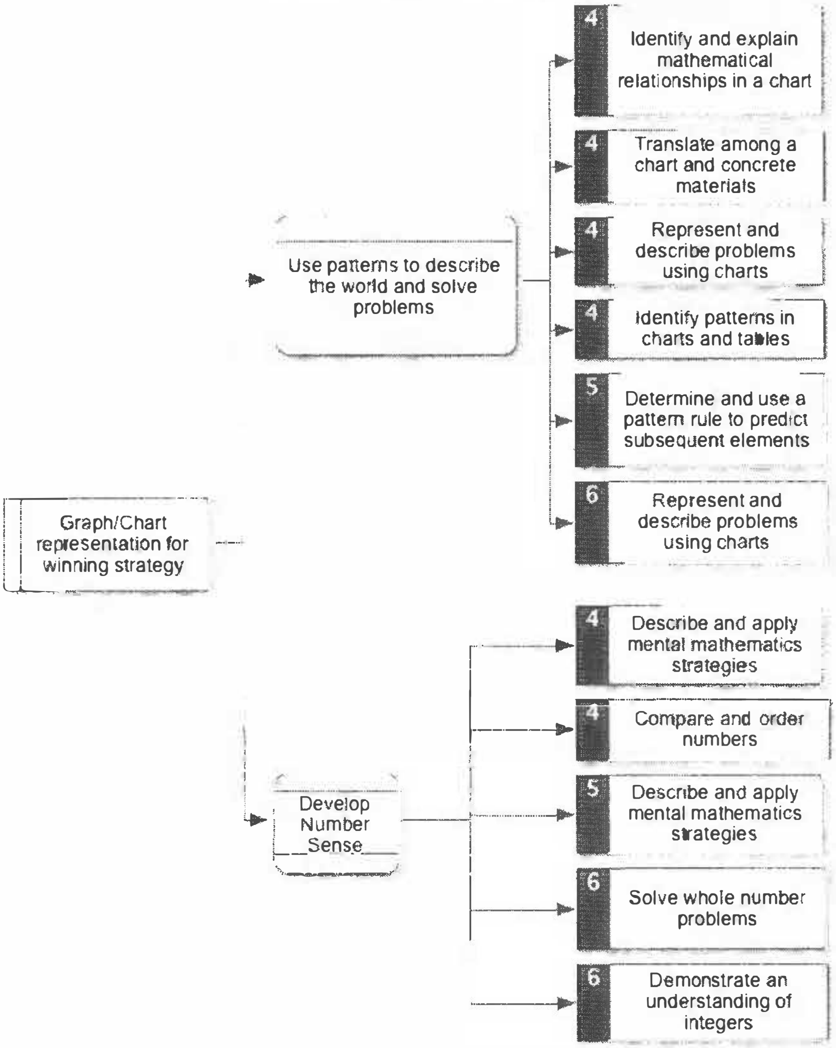
- Case 1: Billy is in Quebec, St John's or Yellowknife. In this case the Mountie moves toward Billy and catches him on the fourth move.
- Case 2: Billy is in Halifax, Ottawa or Toronto. In this case the Mountie's fourth move is to Quebec City and Billy's fourth move is to one of Winnipeg, Fredericton or Charlottetown.
- Finally, the Mountie moves toward Billy and catches him on the fifth move.
- Case 3: Billy is in Regina or Victoria. In this case the Mountie's fourth move is to Yellowknife and Billy's fourth move is to Winnipeg or Edmonton. Again, the Mountie moves toward Billy and catches him on the fifth move.

Relevance to Program of Studies

The game of Catch Billy Miner addresses all of the mathematical processes listed in the program of studies: communications, connections, mental estimations and mental mathematics, problem solving, reasoning, technology and visualization. Learning how to apply graph theory to win at a game is mathematical problem solving at its best. Strategic game play provides relevance and motivation to learn mathematical concepts.

Within the General Outcome: Develop Number Sense, this problem helps children develop mental strategies for solving problems. Understanding how a different-sized loop changes the position of the Mountie relative to Billy involves number comparison and problem solving with whole numbers and integers. Within the General Outcome: use patterns to describe the world and solve problems. Knowing how the Mountie can always win reinforces understanding of mathematical relationships within a chart to solve a problem. Similarly translating and applying rules for prediction are also reinforced. The following figure illustrates the relevant Specific Outcomes:

Figure 18: Relevant outcomes of Catch Billy Miner to the Alberta program of studies



There are opportunities for cross-curricular exploration. Notice that the cities are the capitals of Canada. Also Billy Miner was a famous stagecoach robber in the early 1900s. According to Wikipedia, Billy Miner, who was also known as the Gentleman Bandit, was responsible for British Columbia's first train robbery (Wikipedia 2012).

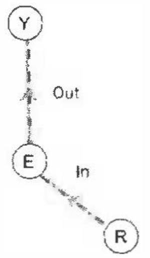
Problem Extension

Consider a different problem where only the Mountie moves through the map. Billy has set up a camp along one of the roads between two capital cities. The Mountie needs to find Billy's camp by checking all the roads on the map. Billy may move his camp soon; there is enough time for the Mountie to check each road only once. Plan a route that guarantees that the Mountie will find Billy's camp in time.

Extension Solution

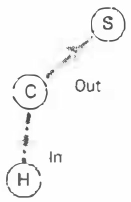
Notice, there are 12 cities with an even number of roads (call them even cities) but only two cities (Edmonton and Charlottetown) with an odd number of roads. When starting in an even city, the Mountie will get stuck in Edmonton or Charlottetown before checking all the roads. For example, consider the Mountie's route starting in Winnipeg. The first pass through Edmonton would check two of the three roads:

Figure 19: Edmonton start



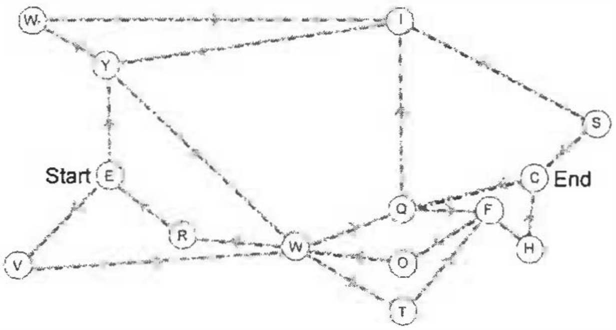
Now the Mountie does not want to check the last road leading into Edmonton until all other roads are checked. But the first pass through Charlottetown (See Figure 20 below) would look similar:

Figure 20: Charlottetown start



At this point there are two or more roads left to check. Travelling along an unchecked road leading into Edmonton or Charlottetown will trap the Mountie before he can check all the roads. Therefore there is no way for the Mountie to check all the roads exactly once when starting in an even city. This means the Mountie's route must start in Edmonton or Charlottetown. One possible route starting in Edmonton is illustrated below:

Figure 21: One solution to the extension



How Many Routes Are There?

Notice when starting in Edmonton, the Mountie cannot get trapped in Edmonton. In addition, the Mountie cannot get trapped in any even city, because every time the Mountie goes into an even city there is an unchecked road leading out of that city. In conclusion, the Mountie's route will always be successful if the Mountie starts in Edmonton and checks all the roads before the second visit to Charlottetown. For this reason there are hundreds of other routes.

In current mathematical research the problem of finding all such paths in every graph is still unsolved.

Summary

A healthy dose of competition will bring energy and excitement into your classroom. Not only is this game fun, but the strategies lead to very interesting mathematics. Winning games against your students should not be so much fun, but it is. The benefits of playing mathematical games in class include creating a positive atmosphere for exploration. The Horizon Report (New Media Consortium 2012) considers gaming to be part of the future in K–12 education. Playing this game with your students will make you a leader of innovative practices.

Make sure you let the students take home a copy of the game board to challenge their parents. They will relish in knowing how to beat their parents and siblings every time.

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Appendix

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Interactive Whiteboards in Early Childhood Mathematics

Strategies for Effective Implementation in Pre-K–Grade 3

Sandra M Linder

In a first-grade classroom, children rotate through a variety of centres. A group of four children approaches the math centre, which is stationed at the interactive whiteboard (IWB) this week. The children have been investigating money during the past few weeks, and this centre provides an extension to these investigations. At the top of the IWB, there is a picture of a quarter labelled "25 Cents." Below it are pictures of pennies, nickels, and dimes that can be moved around the board and duplicated, depending on the activity. One child presses a picture of a horn, activating an audio file that asks the children to make 25 cents in a variety of ways using the coins on the IWB. The children work together to use the pennies, nickels and dimes to create sets of 25.

Teachers are using technological innovations—including interactive whiteboards—in pre-K–Grade 3 classrooms across the country. An IWB is a wall-mounted, touch-sensitive flat screen. When connected to a computer (or another electronic device) and a projector, it displays enlarged instructional content (such as a math word problem, pictures or graphics, or an excerpt from a story). Teachers and children can manipulate this content. Many early childhood teachers are incorporating this technology in their mathematics instruction. This article will help educators use IWBs and other technologies in ways that coincide with best practices in early childhood math instruction. It also shares examples of how to integrate other digital tools into mathematics instruction.

Math lessons in early childhood should use child-centred practices to develop children's conceptual understanding of a variety of topics. However, teachers sometimes use IWBs merely to complete electronic worksheets or to show examples of problems to be solved during the lesson. By making small tweaks to their approach, teachers can alter the focus of a task to promote children's active learning. For example, a second-grade teacher might bring up place value in the number representing the day on the calendar ("Today is January 31. How many ones are in 31? How many tens are in 31?"). Instead of ending the discussion here and showing a representation of the tens and ones on the IWB, have the children form small groups and draw a representation of the place values for the 3 and the 1. Ask the groups to share their representations by redrawing them on the IWB.

The current literature on interactive white boards is limited; however, there are examples of early childhood teachers incorporating IWBs effectively. Murcia (2010) relates a case study describing how elementary school teachers integrate IWBs into their science curriculum. Murcia's findings show that the use of multimodal representations—such as tables and graphics that children can manipulate on the IWB—increase the richness of lesson plans and the collaboration and communication among students. In a study of kindergartners working with an IWB and other forms of technology during math lessons on fractions, Goodwin (2008) finds that children who use these forms of interactive technology have more complex understandings of fractions than children in settings where the technology is not incorporated into lessons.

Essential Characteristics of Early Childhood Mathematics Lessons

The table below describes characteristics that should be present in every early childhood math lesson. These characteristics reflect the process standards of the National Council of Teachers of Mathematics (2006). Although the list does not include all effective practices, it can be helpful when designing mathematics instruction. By including these practices in every lesson, you can help children be active, rather than passive, learners of math concepts.

Essential Characteristics of Early Childhood Mathematics Lessons

Building communities and communication	Create activities that build a *community of learners* in your classroom. Have children work collaboratively. Structure discussions so that interactions occur between children, between you and the children, and between the children and you (meaning that they ask you questions or pose thoughts rather than you eliciting information from them). For example, ask children multiple open-ended questions as they work together on a math task, and incorporate time in each lesson for whole-group discussions so that children can reflect on their task and describe their strategies for solving it. Structure your lessons so that all children feel ownership of a task and so they can all engage in the discussion.
Making connections	Make *connections* between mathematics and other content areas, and between mathematics and real-world situations. Children need to understand why they are engaging in tasks. Insert discussions about math concepts into informal situations, such as during free play or while setting the table for snack or lunch. When math concepts are meaningful for children, they can see the value of mathematics and relate the information to what they already understand.
Representing understanding	Provide opportunities for children to *represent their thinking* in a variety of ways. For example, ask kindergartners to show their strategies for breaking up (decomposing) the number 5 into groups, using both pictures and numbers as well as discussion. Ask second-graders to show their strategies for solving a two-digit addition problem in pictures, numbers and written sentences. These different representations help children move from concrete to abstract understandings.
Exploring with materials	Provide opportunities for children to use different *materials* or *manipulatives* to help solve math tasks. Remember, mathematics instruction should not be just hands-on; it should be minds-on—meaning that children should use the manipulatives as a tool to help represent their understanding. Instead of asking them to use materials in prescribed ways, allow for flexibility in how children decide to use the materials.
Child-centred tasks	Design math *tasks* in which children can approach a challenge in a variety of ways. There should be no one way to solve a problem. Avoid telling children how you would solve the problem. Allow them time to explore the task, which will give you an opportunity to ask questions and understand and build on their thinking.

Integrating Technology and Mathematics Using an IWB

When building math lessons around the essential characteristics in the table, it is important that the IWB not be the only tool children interact with during lessons. The best way to use an IWB is either before or after a small-group task in which children use concrete materials, such as plastic cubes that children can link together and pull apart. Use the IWB to introduce a topic, to stimulate discussion or to connect math concepts to real-world situations. Avoid using the IWB to show children how to complete the task.

The following are examples from practising teachers showing how to incorporate an IWB in each early childhood mathematics content area (number and operations, data analysis, measurement, algebra and geometry).

Number and Operations

In a pre-K classroom (with 4-year-olds), Ms Carlin is teaching counting skills and the concept of cardinality (understanding a set of objects as a total quantity or sum rather than as individual parts). Children gather on the rug in front of an IWB where Ms Carlin displays pictures of 10 elephants, all slightly different (in terms of height, colour, trunk length). She asks the children to talk about the different ways to count the elephants (count them all, count just the grey ones, count just the tall ones). They then discuss how to tell if an elephant meets the criterion set by the group.

Ms Carlin: How can you tell if this elephant is tall or short?

Quoila: It is large when it is next to this elephant, and it is short when it is next to this elephant.

Phin: It is large when it is bigger than the things around it.

Following this discussion, Ms Carlin gives each child an opportunity to count a set of elephants and tell how many there are in all. The children go to the IWB and use their fingers to drag the elephants from one side of the board to the other to represent the action of counting, and to demonstrate their understanding of one-to-one correspondence. Once they have all taken a turn with the IWB, they work with a partner to group and count sets of plastic animals in different ways.

Quoila: We counted them by colour.

Phin: We counted them by number of legs.

LaMont: We put them in groups of two and then counted by twos.

Following every turn of counting, the pairs draw each set on a piece of paper and label it with a number.

Data Analysis in Kindergarten

When introducing the parts of a pictograph in a kindergarten class, Ms Nocenti uses the interactive whiteboard to do a class survey of favourite types of apples (red, green or other). Children begin the lesson on the carpet in front of the IWB, where the teacher holds up a bundle of apples and asks the children to come up with questions to ask about them.

Ms Nocenti: What are some questions we can ask about these apples?

Malik: How big are they?

Sera: How many apples are there in the bundle?

Marzuk: Which one do you like best?

Matt: What is your favourite one?

From the list of questions generated by the children, they choose "What is your favourite type of apple?" to explore. Ms Nocenti uses this question to guide the creation of, and conversation about, a pictograph on the IWB.

Ms Nocenti: How can we find out what everyone's favourite type of apple is?

Matt: We can each take a bite of apple and then choose.

Malik: We can raise our hand when you say the type we like best.

Sera: We can draw a picture of the apple we like best.

Throughout this discussion, Ms Nocenti records on the IWB children's responses about ways to learn which type of apple they like best. After recording everyone's responses, she asks children to draw a picture of their favourite apple. Ms Nocenti then calls each child up to the board to draw a circle (in the appropriate colour), representing his or her choice. The drawings are scattered on the IWB, with no apparent method of organizing the data.

Ms Nocenti: Now that you have each decided which type of apple you like best, how can we organize this information so it is easy to see?

Matt: We can put all of our apple drawings together and then put [same-colour apples] next to each other.

Following this conversation, each group receives a bundle of apples and comes up with a different question to ask about them, creates a pictograph on paper to represent their findings and shares their graph with the class. As the groups share their graphs, Ms Nocenti re-creates them on the IWB. As she records them on the IWB, the whole class gains an understanding of how each group created its pictograph. In addition, she saves each re-created pictograph from the IWB so that she has documentation of the children's thinking.

Measurement in First Grade

Money is often a difficult concept for young children to grasp. In his first-grade class, Mr Jimenez introduces a lesson on pennies, nickels and dimes by having a picture of each coin on the IWB. The children compare and contrast the characteristics of the coins and create a class list on the IWB of each coin's attributes.

Mr Jimenez: How are each of these coins similar?
Annie: They all can be used to buy things.
Niranjan: They are all round.
Tasunke: They all have faces on them.
Mr Jimenez: How are each of these coins different?
Niranjan: They are different sizes. This one is bigger than that one.
Akiko: The dime is ten cents, the nickel is five cents, the penny is only one.
Annie: The penny is a different colour.
Niranjan: You can get more with the dime.

After the class creates its list, the children break up into small groups and create money amounts using as many coin combinations as possible. Following this task, the children gather back at the IWB and each group demonstrates how they used a combination of coins to make a certain money amount, using their fingers to drag each coin into a set that represents the amount.

Akiko: We used 2 dimes and 2 nickels to make 30 cents. Then we used 10 pennies and 2 dimes to make 30 cents. Then we used 15 pennies and 3 nickels to make 30 cents.

Mr Jimenez facilitates a whole-group discussion by asking questions about the children's decision making when choosing certain coin combinations, and by having children compare these decisions to their own thinking. For example, when Akiko describes the three ways that her group made 30 cents, Mr Jimenez asks the rest of the groups to share other ways to make 30 cents. Mr Jimenez develops the centre activity described at the beginning of this article as an extension of this lesson.

Algebra in Second Grade

In a lesson on generalizing patterns, second-grade teacher Ms Romita uses the IWB during the reflection period following a math task. Children begin the lesson by discussing the different repeating patterns they see in their classroom and whether they had seen any of those repeating patterns outside of school. For example, one child sees a colour pattern of stripes on a classmate's T-shirt and then recognizes that the same pattern is present in a bed of flowers in the yard.

The children then work in pairs to create the same repeating pattern in three different ways (with a picture, with pattern blocks and with movement). For example, one pair creates a pattern of red square, yellow hexagon, green triangle with pattern blocks, and then represents this pattern with a star, a cat and a heart. Once they create these representations, they use movement to create a third representation of the same pattern with hop on two feet, clap, and raising both hands above their heads. Next, they write a general statement about each of the three representations.

Following this task, the children gather at the interactive whiteboard, where Ms Romita has each pair share one of their representations. The pairs come up to the IWB and show the movement representation of their pattern and then draw the picture version of the pattern on the IWB. The teacher then asks a child from outside the pair to make another representation of the pattern using colour tiles on the IWB.

Ms Romita: How did you represent your pattern?
Gwen: We used sounds. We did clap, stomp, stomp, clap; clap, stomp, stomp, clap.
Ms Romita: Then what did you do?
Jorge: We made the same pattern with triangle, square, square, triangle.
Ms Romita: Boys and girls, how can you use the colour tiles to show this pattern?
Ciria: Well, you could put the yellow tile first, then put the red tile, and then another red tile, and then a yellow tile, and then keep going.

Geometry in Third Grade

In a third-grade lesson on intersecting and parallel lines, Ms Talamantes uses the IWB during the first part of the lesson to show pictures of intersecting and parallel lines in the real world, including photos of roads and buildings in their community. Children draw over the pictures—using the interactive pen that comes with the IWB—to show where the parallel or intersecting lines occur. She asks children to explain the differences between the examples, which eventually leads the children to come up with their own definitions for intersecting lines and parallel lines.

Ms Talamantes: How are these two lines here and these two lines here similar?
Malik: They are all straight.
Soo Jin: They are on the edges of the buildings.

Ms Talamantes: How are they different?
Yasmine: These cross and these don't.
Soo Jin: The ones that cross make a corner on the building.
Malik: The ones that don't cross on either side of the building are connected by another line that crosses over both of them.

Following this whole-group discussion, children work in pairs to draw a picture of a space in their school (for example, the lunchroom, playground, gym, classroom). After they draw their picture, each pair writes a description of their space on paper, giving attention specifically to where they found parallel or intersecting lines. During the week, the children take turns visiting the spaces to determine if there are any examples they had missed.

Other Technologies for Teaching Math

The following are examples of other forms of technology that teachers can use to enhance mathematics lessons—with or without an IWB. Remember to include the essential characteristics (see the chart on page 47) in your math lessons when implementing these examples.

Virtual Manipulatives

Rosen and Hoffman (2009) define *virtual manipulatives* as "interactive, web-based, computer-generated images of objects that children can manipulate on the computer screen" (p 26). They are available to teachers for free through a variety of websites (see "Resources," p 52). These manipulatives often look similar to the concrete forms you may already use in your classroom (for example, place-value blocks, pattern blocks, colour tiles). However, if you don't have access to such materials, virtual manipulatives are an option.

Websites with virtual manipulatives often offer specific tasks for children to complete. Teachers can easily incorporate these tasks into a math centre when an IWB is not available. Invite children to work in pairs at a computer to complete a math task using virtual manipulatives. For example, on the National Library of Virtual Manipulatives website (http://nlvm.usu.edu), pairs can access an *attribute train*, enabling them to identify and complete patterns by analyzing attributes of shapes. Often, these tasks require children to simply answer the questions. (For example, if a task involves using place-value blocks to solve an addition problem, the website might not require the children to describe their strategy for using the blocks.) Enhance these tasks by asking the pairs of children to create another example or to represent their strategies for solving the original problem on a separate piece of paper. For example, after pairs interact with the attribute train, have one child in the pair create her own example of an attribute train on paper and then ask the other child to complete the pattern. Once the pattern is complete, have the pair switch roles. After all of the children have interacted with the virtual manipulatives at the mathematics centre, gather the class as a whole group to discuss their strategies.

With an IWB, you can optimize the use of virtual manipulatives by combining them with concrete manipulatives. For example, during a lesson on attributes of shapes, ask children to form small groups and use concrete manipulatives, such as pattern blocks or tangrams (seven individual shapes that, when combined without overlapping, can form a variety of larger shapes), to make a representation of an animal or a monster. Allow the children to design their own representations rather than telling them what to make. Following this exploration, gather the children together and have them share their representations using the virtual manipulatives on the IWB. Display the virtual manipulatives and ask children to click and drag the shapes to recreate their representation. During discussion, children can explore how to use more shapes to create the same animal. Teachers can also use other computer programs, such as Kidspiration (www.inspiration.com/Kidspiration; a free trial is available to download, but the program must be purchased) or Microsoft Word (by inserting shapes into a document), to create their own virtual manipulatives.

Webquests

Webquests—Internet-based explorations in which children visit teacher-selected websites to solve a problem or complete a task—are a great way to make connections in mathematics lessons. A variety of websites enable teachers to easily create webquests for any content area for free (see "Resources," p 52). For classrooms without IWBs, children can pursue webquests on computers in math centres as long as they can easily navigate the sites. Instructions should be succinct, and links should be easy to find so children do not spend more time figuring out the technology than they do working on the math task. Ideally, children work in pairs or small groups to complete the webquest, and they have opportunities to make connections between the webquest and the classroom. For example, if children are completing a webquest on identifying three-dimensional shapes on various websites, ask them to identify the same shapes in their classroom and to represent them in drawings.

Incorporate webquests into whole-group lessons in classrooms with IWBs by developing a math task that can be solved only by exploring a variety of websites as a group. Ms Romita developed a webquest for her second-grade class that connected mathematics and social studies by following an explorer's travels as he visited different communities around the world. Children explore the sites he visited and, as a group, figure out how far he travelled. To encourage more child-to-child interaction, have children work together in smaller groups during the lesson to complete the tasks included in the webquest (such as adding together the miles from one location to another) and then meet back as a whole group to discuss and compare findings.

Recordings and Photographs

It can be difficult to formatively assess all children as they work together during a math lesson. Use a video or voice recorder to capture discussions and interactions while children engage in math tasks. The information gathered from these tools will help inform your assessment and planning for the subsequent lesson. A video or voice recorder can also enhance a lesson by providing a way for children to record their thoughts so they can think about them later. For example, in a second-grade lesson on addition with regrouping, children work in groups to solve a word problem, showing three solving strategies (pictures, words and numbers). Following this task, the teacher gathers children for a whole-group discussion of the different strategies. The teacher records the discussion, but there is time for only half the class to share their strategies. The next day, the teacher plays back the recording so the children can remember the discussion. Often, due to time restrictions, the reflection component—which is critical to mathematics lessons—is shortened. By recording their thinking, children can return to this information later to help refocus them on the task.

In classrooms with an IWB, use a video or voice recorder to document assessment data from children in small groups, and then play the recording during whole-group discussions at the IWB. Allow children to watch another small group working, for example, to build a structure with three-dimensional shapes. After watching the small group work, the teacher can ask all the children specific questions about how the small group completed the task, helping children think about mathematical processes. For example, kindergartners building structures with three-dimensional shapes watch a video of another group building a castle and begin to identify castles they had seen in the real world or on television. One of the children in the video struggles to find a place for a sphere, and as the children watch him try out various spots, they predict whether the sphere will fall and volunteer alternative suggestions for where to place it.

Digital photography can also enhance instruction in mathematics. Using a digital camera, children in pairs can collect data during mathematical tasks. For example, if children take a pattern walk (identifying examples of repeating or growing patterns as they walk around the school), have them use a digital camera with their partner to capture examples of these patterns along the way. Digital photographs can easily be shown on an IWB or a computer, or printed and displayed on a board as a means to encourage whole-group discussion and make math connections to the real world. For example, displaying pictures of flowers that children grew in a community garden can encourage a discussion on symmetry.

Conclusion

Technology can be a vital tool in enhancing mathematics instruction for young children. However, if a teacher is standing next to the interactive whiteboard throughout an entire mathematics lesson, or if children's only interaction with an IWB is to come up one at a time to answer a question, then it is not being used in the most effective manner. When early childhood teachers design lessons by integrating the forms of technology discussed here with the essential characteristics for teaching early childhood mathematics, children are more likely to develop conceptual understandings and positive dispositions toward mathematics at a young age.

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Resources

Websites Offering Virtual Manipulatives

Illuminations. National Council of Teachers of Mathematics site featuring pre-K–12 lesson plans and activities related to all content areas. The activities link provides free access to early childhood virtual manipulatives. http://illuminations.nctm.org

Kidspiration. Software designed for grades K–5. Originally a language arts program, it now includes a mathematics component, allowing teachers to use pictures, text and numbers to create math problems and tasks. A free trial version is available, but eventually the program must be purchased. www.inspiration.com/Kidspiration.

Math Forum. Reviews and links to websites that provide virtual manipulatives or sample lessons using virtual manipulatives. http://mathforum.org

National Library of Virtual Manipulatives. Virtual manipulatives for all pre-K–12 math content areas (number and operations, algebra, geometry, measurement, data analysis, and probability). A free trial version is available. http://nlvm.usu.edu

Websites for Webquests

Discovery Education. Provides Mac and PC templates for creating webquests, plus information on creating and implementing webquests. Sample webquests from practicing teachers are included. http://school.discoveryeducation.com/schrockguide/webquest/webquest.html

Education World. Features a detailed description of qualities and components to include in a webquest, plus downloadable webquest templates and links to other webquest resources. www.educationworld.com/a_tech/tech/tech011.shtml

TeacherWeb. Information on using webquests across all content areas, and a template for creating webquests. Sample webquests are also provided. www.teacherweb.com

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MCATA Mission Statement

Providing leadership to encourage the continuing enhancement of teaching, learning and understanding mathematics.

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