

Edmonton Junior High Math Contest 2012

Part A: Multiple Choice

- The product of four positive integers is 1365. Which of the following could not be the sum of any three of the integers?
(a) 15 (b) 21 (c) 23 (d) 24 (e) 25
- Eight cards are placed face down. Each has one of the following numbers: 2, 3, 6, 7, 8, 9, 15 or 18. If you and your friend each turn over one card, what is the probability, to the nearest whole percentage point, that the sum of the pair of turned-up cards will be odd?
(a) 47% (b) 50% (c) 57% (d) 60% (e) 67%
- Five different integers have a sum of -6 . The first integer is 2 greater than the third. The second and fifth integers are opposite. The fourth integer is 2 greater than the second, and it is double the third. Which of the following must be one of the integers?
(a) 7 (b) 6 (c) 5 (d) 4 (e) 3
- A cube is $3\text{ cm} \times 3\text{ cm} \times 3\text{ cm}$. The complete surface is painted green. It is cut into congruent cubes that are $1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}$ (unit cubes). What is the ratio of unit cubes with exactly two green faces to unit cubes with exactly three green faces?
(a) 5:27 (b) 4:3 (c) 2:3 (d) 3:4 (e) 3:2
- A square picture has a side length of 28 cm, and a circular picture has a diameter of 30 cm. They each have a uniform 5 cm border enclosing the exposed area of the photo. For which picture is the ratio of the exposed area of the photo to its border area the greatest, and what is that ratio?
(a) The square picture, with a ratio of 0.70:1
(b) The circular picture, with a ratio of 0.80:1
(c) The square picture, with a ratio of 2.07:1
(d) The circular picture, with a ratio of 2.27:1
(e) Both have the same ratio of 0.75:1
- There are four even numbers and four odd numbers. Odd plus odd is even. Even plus even is even. Odd plus even is odd. There are 8 out of 14 pairs that are odd plus even. Therefore, the probability of an odd sum is $8/14$, or 0.57—which is 57%. The correct answer is (c).
- Let x be the third number. Then, $x + 2$ is the first number, $2x$ is the fourth number, $2x - 2$ is the second number and $-2x + 2$ is the fifth number. Then,
$$x + x + 2 + 2x + 2x - 2 + -2x + 2 = -6$$
$$x = -2.$$
Therefore, the integers are 0, -6 , -2 , -4 and 6. The correct answer is (b).
- Of the 3^3 , or 27, unit cubes, 12 have green on exactly two faces and 8 have green on exactly three faces. Therefore, 12:8 or 3:2 is the ratio. The correct answer is (e).
- A square with side length of 28 cm would have a square of 18 cm within since there is a border 5 cm wide around the inner square. The ratio would be
$$18^2:28^2 - 18^2$$
$$324:460$$
$$0.70:1.$$
A circle with diameter 30 cm would have a circle within with diameter 20 cm since there is a border 5 cm wide around the inner circle. The ratio would be
$$10^2:15^2 - 10^2$$
$$100:125$$
$$0.8:1.$$
Therefore, the circular picture has the greatest ratio, which is 0.8:1. The correct answer is (b).

Solutions

- The prime factors of 1365 are 3, 5, 7 and 13.

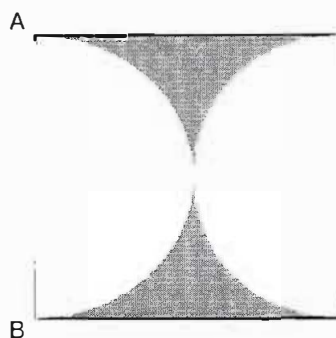
$$3 + 5 + 7 = 15$$
$$3 + 7 + 13 = 23$$
$$3 + 5 + 13 = 21$$
$$5 + 7 + 13 = 25$$

It is not possible to make a sum of 24. The correct answer is (d).

Part B: Short Answer

- Connor walked an average of 6 km/h from his home to a park. When he returned along the same route, he averaged 6 km/h. At the halfway point from the park to home, he remembered that he'd left his watch behind and he ran back to the park at 9 km/h. Once he found his watch, he walked back home at 6 km/h. What was Connor's average speed, rounded to the nearest tenth, for the whole trip?

7. An artist mixes a shade of paint that is $33\frac{1}{3}\%$ red, 15% yellow and the rest blue. He then mixes a second shade of paint, using only red and blue in a 3:1 ratio. If the artist then combines 300 mL of the first shade with 120 mL of the second shade and 85 mL of blue, what percentage of the resulting shade of paint will be blue? (Express your answer to the nearest whole percentage point.)
8. In a collection of 48 coins, one is counterfeit, and it has a mass that is slightly less than all of the other identical coins. What is the minimum number of mass comparisons required, using a balance scale, to ensure that the counterfeit coin is discovered?
9. The square root of $1/25$ of a composite number is 4 less than the sixth prime number. What is the composite number?
10. What is the area of the shaded region in the diagram below, to the closest whole square centimetre, if the length of semicircle AB is 18.85 cm?



11. A pet shop had 1500 cats, dogs and birds. Of this total, 55% were cats and 20% were dogs. A group of cat lovers bought cats only, until just 40% of the remaining pets were cats. How many cats were bought by the cat lovers?
12. In the real number system of mathematics, how many of the following statements are true?
- 1 is a prime number
 - $\sqrt{-25} = -5$
 - $\pi = 3.14$
 - $\frac{x^3}{x^0-1} \geq 0, x > 0$
 - $-n^4 = (-n)^4$
 - $\sqrt{x^{100}} = x^{10}$
 - If a three-dimensional object is a right square prism, it must be a cube
13. Wong's Fortune Cookies are sold in four package sizes. Compared to the largest size, the three smaller sizes contain, respectively, $\frac{1}{5}$ as many cookies, $\frac{1}{2}$ as many cookies and $\frac{3}{4}$ as many cookies. If you buy one of each of the four package

sizes, you will get 62 cookies. How many cookies are in the second-smallest package?

14. What is the tens digit of the smallest multiple of 13 whose units digit is greater than 0 but less than its hundreds digit?

Solutions

6. Choose the distance from home to the park to be the lowest common multiple of 6 and 9, which is 18. Therefore, it took Connor 3 h to walk from his home to the park. At the halfway point, he walked for 1.5 h and ran for 1 h, after which he walked back home for 3 h. The total time spent was $3 + 1.5 + 1 + 3$, or 8.5 h. The total distance was $3(18)$, or 54 km. So, Connor's average speed was $54/8.5$, or approximately 6.4 km/h.
7. The first shade of paint is $100 - 33\frac{1}{3} - 15$, or $51\frac{2}{3}\%$ blue. The second shade is 25% blue. The total amount of blue paint (in millilitres) is as follows:

$$\begin{aligned} 300(51\frac{2}{3}\%) + 120(25\%) + 85 \\ = 155 + 30 + 85 \\ = 270. \end{aligned}$$

The total amount of paint (in millilitres) is $300 + 120 + 85 = 505$.

The percentage of the resulting shade of paint that is blue is

$$\frac{270}{505} = 53.465\%.$$

Rounding to the nearest whole percentage point gives us 53%.

8. Divide the 48 coins into three piles of 16 coins each. For the first weighing, put 16 coins on the left pan, 16 coins on the right pan and 16 coins on the table. Divide the 16 coins on the table into three piles. For the second weighing, put 5 coins on the left pan, 5 coins on the right pan and 6 coins on the table. For the third weighing, now there are piles of coins of 2, 2, 1 or 2, 2, 2. For the fourth weighing, 1 coin will be on each pan. Thus, a minimum of four weighings are necessary.
9. The sixth prime number is 13, so

$$\begin{aligned} \sqrt{\frac{x}{25}} &= 13 - 4 \\ \sqrt{\frac{x}{25}} &= 9 \\ \frac{x}{25} &= 81 \\ x &= 2025. \end{aligned}$$

The composite number is 2025.

10. Since the length of the semicircle is 18.85 cm, the circumference of the circle is 37.7 cm. Divide the circumference by π :

$$\frac{37.7}{\pi} = d$$

$$12 = d.$$

Therefore, the diameter of the circle is 12 cm. To find the area of the shaded region, subtract the area of the circle with radius 6 cm from the area of the square with side of 12 cm:

$$12^2 - (6^2)\pi = 30.9.$$

Rounding to the closest whole number, the answer is 31 cm.

11. For the cats, 55% of 1500 is 825. For the dogs, 20% of 1500 is 300. For the birds, $1500 - 825 - 300 = 375$. Let n be the number of cats purchased. Then,

$$825 - n = 0.4(1500 - n)$$

$$n = 375.$$

There were 375 cats purchased.

12. All of the statements are false. Therefore, the answer is 0.
13. Let x represent the largest package size. Then, $\frac{1}{3}x$ represents the smallest size, $\frac{1}{2}x$ represents the second-smallest size and $\frac{3}{4}x$ represents the third-smallest size. Then,

$$x + \frac{1}{3}x + \frac{1}{2}x + \frac{3}{4}x = 62$$

$$x = 24.$$

Therefore, the second-smallest size of package contains $\frac{1}{2}(24)$ or 12 cookies.

14. The hundreds digit must be at least 2. The first such multiple of 13 is 208, but the units digit is greater than 2. The next one is 221, which satisfies the conditions of the problem. The tens digit is 2.

Part C: Short Answer

15. John wrote a computer program that changes numbers. If he enters a number x , the program outputs the number

$$\frac{1}{1-x}.$$

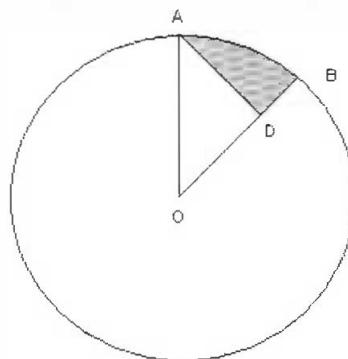
For example, if John enters the number 3, the output is

$$\frac{1}{1-3} = -\frac{1}{2}.$$

The first number John enters is 2, which results in an output of -1 . John will then take this output

and use it for his next input. What will be the computer's 2012th output?

16. How many positive integers up to 2012 have a ones (units) digit of either 1 or 8?
17. A level road of length 16 km links A and B. From B, the road goes uphill to C. Kelly and Kerry both ride their bikes at 8 km/h on level road, 5 km/h uphill and 10 km/h downhill. Kelly starts from A and goes toward C, while Kerry starts from C and goes toward A. They meet at a point on the level road 4 km from B. They continue to their respective destinations and turn around immediately. Where will they meet the second time?
18. Three regular polygons of equal side lengths fit perfectly around a point. If the first polygon has five sides and the second has ten sides, what is the number of sides of the third polygon?
19. In the diagram below, O is the centre of a circle of radius 1. A and B are two points on the circle, and $\angle AOB = 45^\circ$. D is a point on radius OB so that segment AD is perpendicular to OB. Find the area of the shaded region. Write the answer in decimal form and round it to the nearest hundredth.



Solutions

15. The following table shows input values and output values for the first six inputs:

Number of inputs	Input value	Output value
1	2	-1
2	-1	$\frac{1}{2}$
3	$\frac{1}{2}$	2
4	2	-1
5	-1	$\frac{1}{2}$
6	$\frac{1}{2}$	2

The output values repeat for every three inputs. In 2010 inputs, there would be 670 repeats of the outputs of -1 , $\frac{1}{2}$ and 2. Therefore, the 2011th

input would yield -1 , and the 2012th input would yield $\frac{1}{2}$.

16. The numbers that have a ones digit of 1 are 1, 11, 21, ..., 2011. If we erase the 1 at the end, we obtain exactly the numbers 0, 1, 2, ..., 201. Thus, there are exactly $201 + 1 = 202$ such numbers (the extra number coming from the fact that we start our counting at 0). The numbers that have a ones digit of 8 are 8, 18, 28, ..., 2008. If we erase the 8 at the end, we obtain exactly the numbers 0, 1, 2, ..., 200. Thus, there are exactly $200 + 1 = 201$ such numbers (the extra number coming from the fact that we start our counting at 0). Thus, in total there are $202 + 201 = 403$ such numbers.
17. When Kelly and Kerry meet the first time, Kelly has travelled 12 km on level ground. Therefore, she has travelled for $12/8$ or 1.5 h. The distance from B to C is unknown, but the fact that both Kelly and Kerry have travelled for 1.5 h can be used to find the distance from B to C:

$$1.5 = \frac{4}{8} + \frac{d}{10}$$

$$10 = d.$$

The distance from B to C is 10 km. The following chart can be used to determine when Kelly and Kerry will meet again.

	Time from starting points (h)	Distance travelled (km)	Location
Kelly	1	8	Halfway between A and B
Kerry	1	10	B
Kelly	2	8	B
Kerry	2	8	Halfway between B and A
Kelly	3	5	Halfway between B and C
Kerry	3	8	A
Kelly	4	5	C
Kerry	4	8	Halfway between A and B
Kelly	5	10	B
Kerry	5	8	B

Therefore, Kelly and Kerry will meet again at point B.

18. The measure of each interior angle of a regular polygon with ten sides is $180(n - 2)/n = 180(10 - 2)/10 = 144^\circ$. The measure of each interior angle of a regular polygon with five sides is 108° . To fit perfectly around a point, the sum of the interior angles of the three polygons must be 360° : $360 - 144 - 108 = 108^\circ$. Therefore, the third polygon is a regular pentagon, which has five sides.
19. To find the area of circle O use πr^2 , where $r = 1$; therefore, the area of circle O is π . Sector AOB represents $\frac{1}{8}$ of the circle, so the area of sector AOB is $\pi/8$. Triangle AOD is an isosceles right triangle with hypotenuse 1, so the area of triangle AOD is $\frac{1}{4}$. The area of the shaded region is found by subtracting the area of triangle AOD from the area of sector AOB:

$$\frac{\pi}{8} - \frac{1}{4}$$

or

$$\frac{\pi - 2}{8}.$$

Written as a decimal, the answer is 0.14.