Creating Curved Art with Straight Lines and Perspective

Mark Mercer

Art creates a visual play between positive and negative spaces. Mary Boole's curve stitching—a precursor to string art and similar forms of line art—encourages this type of exploration. These forms address geometric concepts as artists count points and lines. In this challenge, students will create math objects that disappear after certain patterns appear—a tension between necessity and prominence.

Ultimately, this challenge encourages students to create beautiful images. As their skills and imagination develop negative spaces, the focus can shift to an exploration of art, revealing some beautiful mathematics.

Two problems are presented here. The individual problem encourages students to illustrate images using curves and presumes some developed skill. The group problem involves a quilting-type project in which students stitch together images illustrating a sort of dynamic. The group problem allows students to develop the skills needed for the individual problem. This fits nicely into current assessment paradigms, in which group work is needed for individual practice, solitary imagination and demonstration.

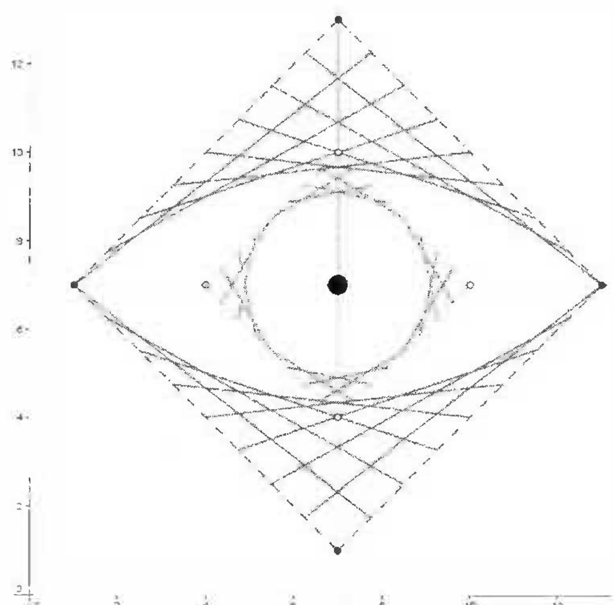
This article encourages a larger perspective in math education. For the teacher, this challenge requires an understanding of various math storytelling methods, as well as grade-specific knowledge about how to guide students through their mathematical progression—a shift in resource development.

Individual Problem

Create an image of simple shapes by using basic shape edges as axes and straight lines as grid patterns. The image should appear to have curved lines and artistic depth.

Consider the image below layered on a coordinate plane (Figure 1). It begins as two related squares rotated from an archetype position, and straight lines create the simple shapes of an oval and a circle, resembling an eye. Note that the image is created using only straight lines, and that the curved lines and depth are an optical illusion.

Figure 1



Group Problem

Create a set of images based on one simple shape, using a variety of patterns forming lines that result in an image that appears to have curves. These shapes and images narrate movement and a surprising fact about these curves, a visual proof. To view this visual proof, the images should be organized and presented with verbal or written explanations.

Because of the scope of this challenge, an example is given below, followed by material lists and other tips.

A Teacher's Preview

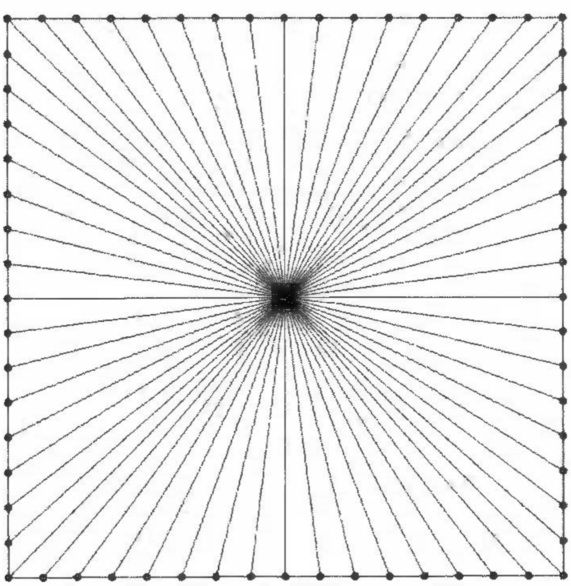
The following progression of images within a basic shape—a square—illustrates the group problem.[1] Taken together, the images reveal a pattern of change: a cross-pattern slowly opens at its centre (or intersection) to reveal a square that turns into a circle. Notice that this revealed square and circle are not accurate. The intersections and line segments creating the images are straight, not curved. As the intersections (or line segment distances) become closer, related to the original connecting dots, the lines of the revealed shape become more curved. Thus, the revealed shape approaches more accuracy. This illustrates some properties of a manufactured curve (made of intersections and line segments between segments).

Using a ruler is strongly recommended in order to be able to see endpoints, follow lines through various intersections and count how many line segments occur between connected points. (See the following section for an explanation of counting.) Working with this technique also provides experience when students struggle with visual patterns or are overwhelmed by visual stimuli (intersections and line segments).

This progression should also help develop students' geometric skills for the individual problem. Students are able to see the curved lines and depth while practising necessary line drawing and counting skills. With reflection, they also view others' attempts as meaningful and can stitch together a progression, enabling the mental imaging necessary for the individual problem.

Figure 2 shows the image created within a square when lines are drawn straight across, using a ruler.

Figure 2



In Figure 3, a ruler has been used to draw lines almost straight across—to the point on either side of the point that's straight across, making two lines. The intersections begin at the axes points and then repeat at decreasing intervals. The intersections and negative space form square patterns.

Figure 3

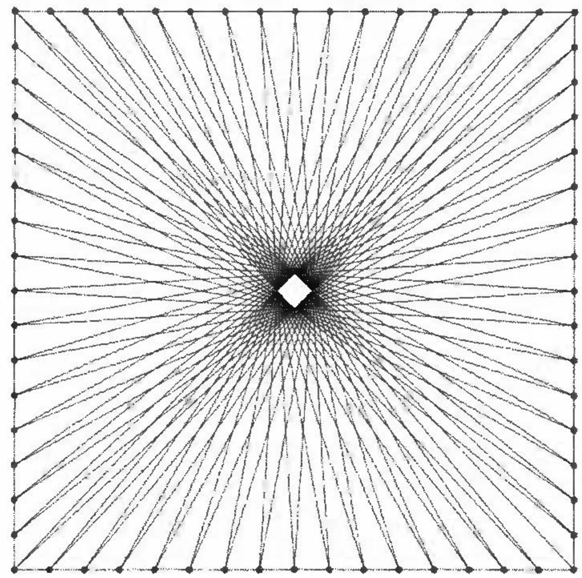
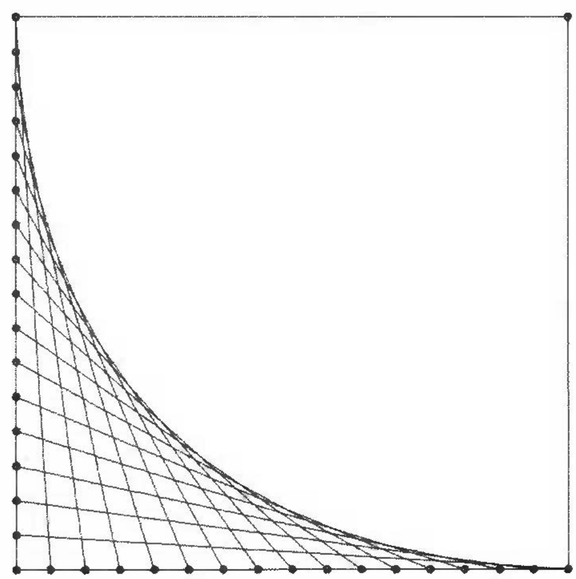


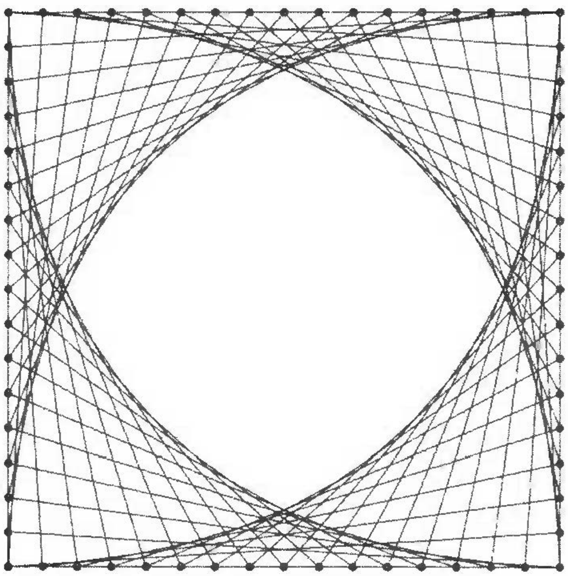
Figure 4 is an intermediary image in which the lines are connected to consecutive points on the perpendicular side of the square.

Figure 4



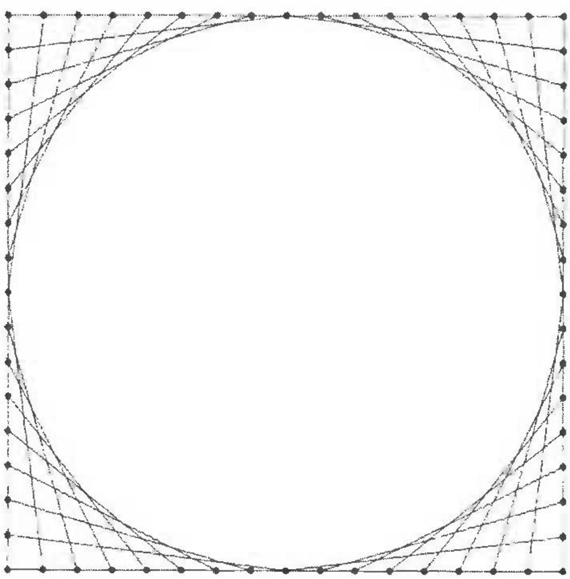
In Figure 5, the pattern from Figure 4 is repeated on each side of the square.

Figure 5



In Figure 6, the pattern from Figure 4 is followed—starting from the point in the middle of each side of the square.

Figure 6



This progression of images can be applied to other basic shapes—any closed regular or irregular polygon—to understand other progressions.

An Interlude: Counting

Figures 7 and 8 repeat Figures 2 and 6, respectively. The numbered line segments and emphasized lines serve as a visual for how to count in curve stitching. Notice that counting is made easier by attending to line segments rather than points, since endpoints are sometimes included and other times not.

Figure 7

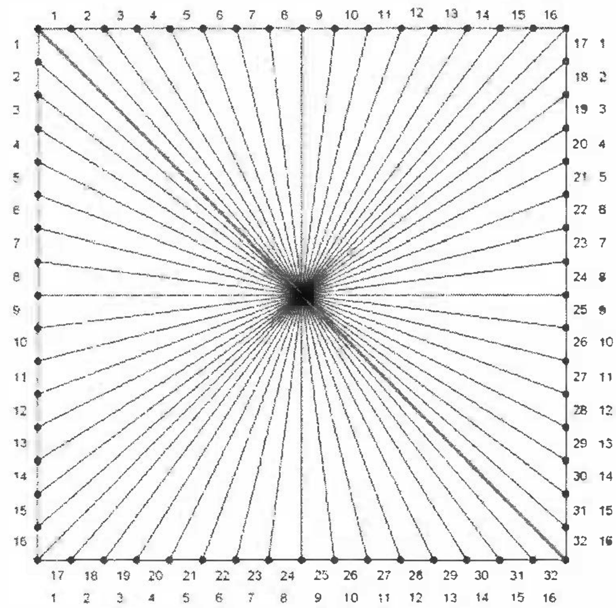
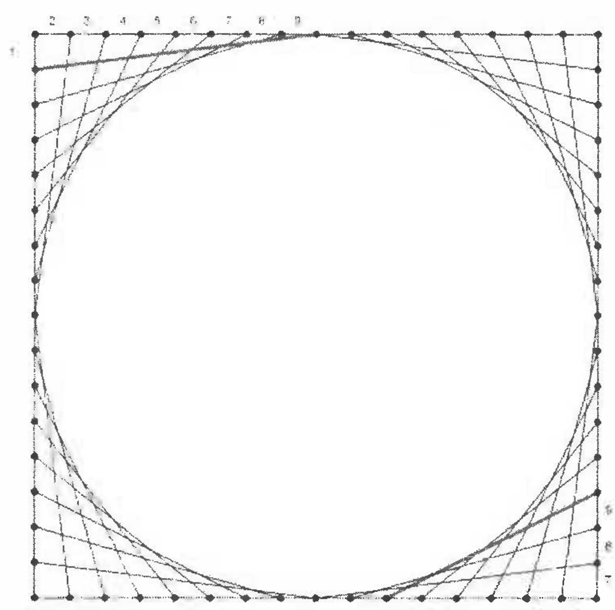


Figure 8



Variations and Extensions

Various shapes, ways of constructing curves and ways of adding illusions are possible. The Internet offers a plethora of images teachers can use to create progressions.

This article provides just one example of a progression that enables students' imaginations. What is possible and the kind of images needed to develop students' thinking and creativity depend on how the teacher facilitates this activity.

For younger or less experienced students, templates can be downloaded from various websites (see the section Useful Websites).

Older and more adventurous students, who can use a ruler with accuracy and precision, should be encouraged to create their own designs. Simple shapes can be made with points plotted and patterns. Students can also try connecting points by skip-counting (rather than counting every point). One recreational mathematician has shared some curve-stitch designs he created using technology (www.deimel.org/rec_math/curve_stitch.htm). These designs are complicated and are not intended for elementary students. I mention them here as an example of the detailed designs possible.

Classroom Set-Up

A group curve-stitching activity requires age-appropriate materials; student groupings based on leadership, cooperation and expertise; and a plan for progressing through the problem.

The following materials are needed:

- Templates on paper
- Pencils
- Ruler
- Coloured pencils (optional)

For students who are able to use a ruler, this challenge can be done with pencil and paper (no template). The challenge can also be adapted for the computer.

For younger students or those who require a more structured template, a template can be fixed to a flat piece of wood, with nails as points. String is then weaved around the nails. The string should be long enough for an adequate weave and can be wrapped around a small (pencil-sized) dowel. Each student should be given two or three choices of string.

For slightly older students, templates can be photocopied onto cardstock, with holes punched where points would be. The string is then sewn through the holes with plastic needles (available from the craft store) and affixed to the cardstock with tape.

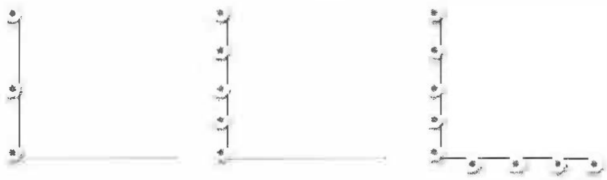
The appendix provides a sample template for student practice in which the intersections and line segments creating envelopes are clearly visible.

To progress through this challenge, prior student knowledge must be evaluated. Students should have experience creating simple curves or images (similar to those presented in this article or to professionally created templates). For specific ideas, refer to the section Beginning to Draw Curves. Then, students can begin creating their own templates—sharing ideas and working through difficulties creatively. Afterward, celebrate the finished artworks.

Beginning to Draw Curves

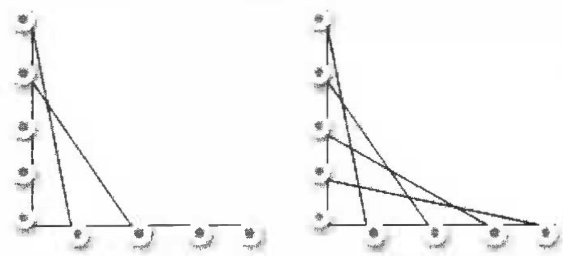
Once two axes are chosen, add equally spaced points. Plot a sequence of midpoints along each axis, as shown in Figure 9. Plotting midpoints assists with equal spacing. Different but equal spacing is also possible.

Figure 9



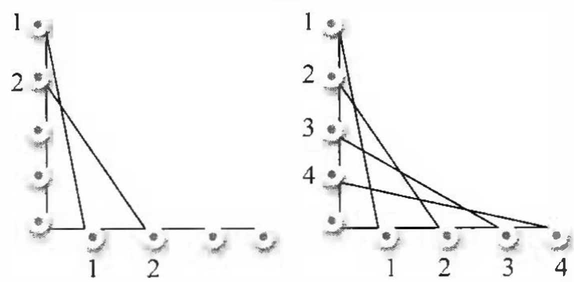
Add straight lines by following a predetermined pattern or rule. The pattern in Figure 10 creates the curve of the eyelid in Figure 1.

Figure 10



Some students have difficulty recognizing a visual pattern with many lines showing intersections and grids. Another way of counting, mimicking a ruler, is illustrated in Figure 11. The first point connects to the first point on the other axis. The second point connects to the second point, the third to the third, and so on. Notice that this does not follow a coordinate plane and may require explanation.

Figure 11



Mathematics in This Challenge

Separating the challenge into stages can help distinguish the mathematics used in this problem. Teachers can address concepts when preparing some space to create a curve, seeing manufactured curves, and generalizing curves through technology or various mathematical notations.

In the figures here, lines resembling coordinate plane axes prepare angled spaces. Some tick marks are counted by beginning with the endpoints and repeatedly finding midpoints. Others are counted by changing the distance of the spaces between the tick marks. The former method seems best for fixed line lengths drawn on paper, and the latter for programming software, where line lengths might consider other variables. "Midpointing" or measuring lines can be applied to various curricular outcomes. However, other ways of preparing spaces, like curved or circular lines, change this sort of vocabulary. Furthermore, searching for geometric art yields shapes as these prepared spaces. Understanding how to count in angled spaces seems a common math place to begin.

This article uses graphing terms such as *intersections* and *line segments*. Other terms are available, depending on the sort of visual mathematics developed. For example, in this article, points on lines (intercepts) are counted and lines are drawn to manufacture a curved line. This first set of mathematics is necessary for the construction of a curve, and its prominence immediately changes. Once this prominence changes, more mathematics can be used to observe and describe the result. Mathematics often works in the following way: ignoring some set of beginning mathematics and emphasizing another set, while existing as causal. Broadly speaking, students require these shifts in math contexts, especially when they are highly creative and intrinsically motivated, so that other shifts in math contexts are accessible.

Mathematical notations generalizing students' curves and art serve to attribute meaning to these mathematical lenses that are often studied purely. For example, the curves we see do not exist as smooth lines, ever. These might be tangents to an imaginary curved line that our brains process as a result of lower-level neuron processing. Thus, these curved lines can be explored and their properties viewed in relation to the process of creating art. Once viewed as tangents, calculus and postsecondary mathematics (like curve envelopes) enter what elementary students can develop with their sophistication of counting and tool usage. In between these imagined students' sophistication is mathematics requiring mathematical storytelling appropriate to whoever creates their art—what a teacher might do. This article encourages this type of facilitation.

Problems and challenges like this shift resource recording and development for these sorts of activities. Grade school teachers and other educators might become a community of storytellers, all possessing different perspectives but all responsible to the same plot. With the breadth and depth of this challenge, considering the sorts of mathematics available, various people contribute and someone, perhaps an editor, assembles this information for more natural methods of pedagogy.

Biology in This Challenge

This problem can also be connected to science. Like a piece of paper, the human retina is a two-dimensional space where three-dimensional sight begins. Before stimulating light receptors, light passes through layers of nerve cells. These nerve cells process changes of light between receptors into lines and overlapping shapes. These are the same shapes video game programmers and artists of various media use to create three-dimensional effects. This information is then received by the brain for further processing.

This means that the intersections creating curved lines, three-dimensional grids and other patterns in the positive and negative spaces can be known with experience instead of being simply more aesthetic. The nature of optical illusions can be explored.

Useful Websites

Curve-Stitch Designs
www.deimel.org/rec_math/curve_stitch.htm
Detailed curve-stitch designs created by a recreational mathematician using technology

GeoGebra
www.geogebra.org/cms/
A free drawing program that can be used for curve stitching

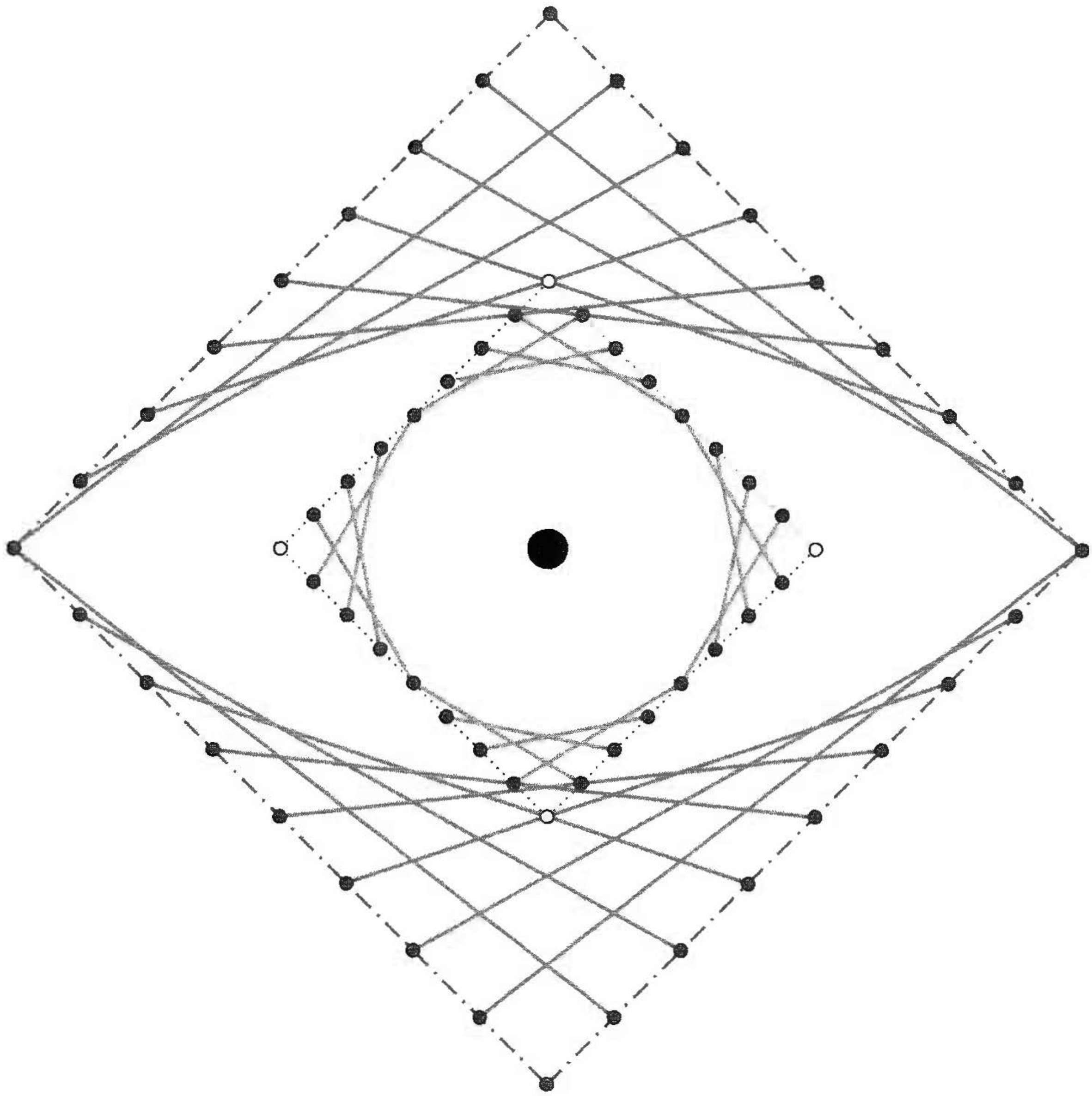
String Art Fun
www.stringartfun.com
Free templates, as well as templates for purchase

String Art Is My Craft
http://stringart.ismycraft.com
Free templates, templates for purchase and an opportunity to share information

String Art (Math Cats)
www.mathcats.com/crafts/stringart.html#patterns
Simple printable templates and a good explanation for getting started

Appendix

Sample Template



Note

1. These images were created with GeoGebra, a free drawing program available at www.geogebra.org.

Mark Mercer teaches math at Ross Sheppard High School, in Edmonton. He is interested in finding cross-curricular approaches to mathematics. Through art, he attempts to help students practise graphing strategies (physical practice) that logically connect to beautiful constructions (notions). With this play, he hopes to address misconceptions within the broad, creative space of a student's journey. If you have any questions or would like to share student work, e-mail Mark at mmercer@ualberta.ca.