delta-k

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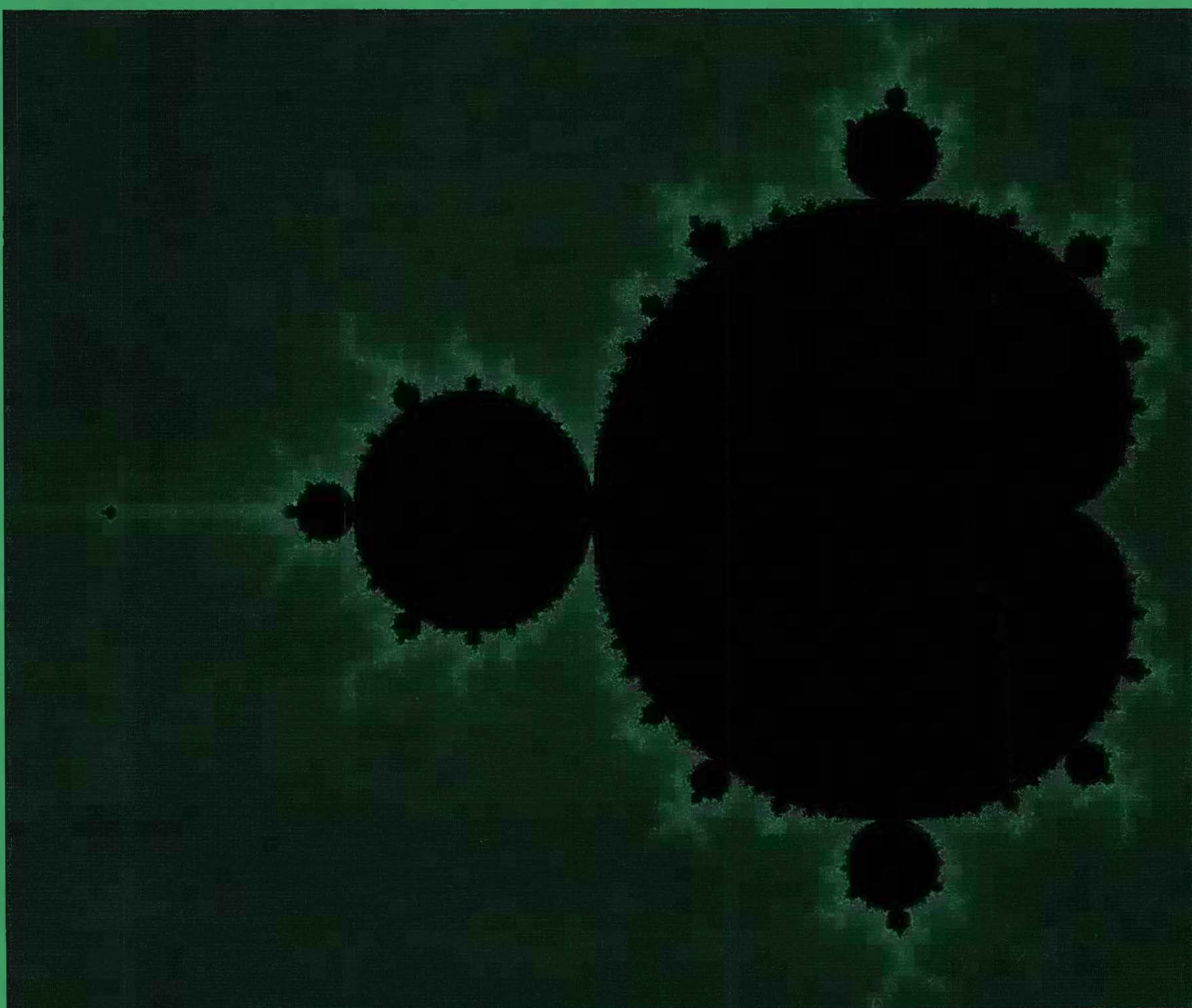

Guidelines for Manuscripts

delta-K is a professional journal for mathematics teachers in Alberta. It is published twice a year to

- promote the professional development of mathematics educators, and
- stimulate thinking, explore new ideas, and offer various viewpoints.

Submissions are requested that have a classroom as well as a scholarly focus. They may include

- personal explorations of significant classroom experiences;
- descriptions of innovative classroom and school practices;
- reviews or evaluations of instructional and curricular methods, programs or materials;
- discussions of trends, issues or policies;
- a specific focus on technology in the classroom; or
- a focus on the curriculum, professional and assessment standards of the NCTM.

Suggestions for Writers

1. *delta-K* is a refereed journal. Manuscripts submitted to *delta-K* should be original material. Articles currently under consideration by other journals will not be reviewed.
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4. The author's name and full address should be provided on a separate page. If an article has more than one author, the contact author must be clearly identified. Authors should avoid all other references that may reveal their identities to the reviewers.
5. All manuscripts should be submitted electronically, using Microsoft Word format.
6. Pictures or illustrations should be clearly labelled and placed where you want them to appear in the article. A caption and photo credit should accompany each photograph.
7. References should be formatted consistently using *The Chicago Manual of Style*'s author–date system or *The American Psychological Association (APA)* style manual.
8. If any student sample work is included, please provide a release letter from the student's parent/guardian allowing publication in the journal.
9. Articles are normally 8–10 pages in length.
10. Letters to the editor or reviews of curriculum materials are welcome.
11. Send manuscripts and inquiries to the editor: Gladys Sterenberg, 195 Sheep River Cove, Okotoks, AB T1S 2L4; e-mail gladyss@ualberta.ca.

MCATA Mission Statement

Providing leadership to encourage the continuing enhancement of teaching, learning and understanding mathematics.

delta-K

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From the Editor's Desk

Gladys Sterenberg

January brings with it the tendency to look back on the past year. For me, I am reminded of the mathematicians and math educators who have made a great impact on me. I thought it would be appropriate to share a few of my memories and experiences and invite you to share with me the memories you cherish.

As a mathematics teacher, I often encounter students' views of mathematics as certain, unchangeable and absolute. My own views of mathematics shifted several years ago when I read *Chaos* (Gleick 1987). I remembered encountering the images: the Lorenz attractor, the Koch curve, the Mandelbrot set, fractal clusters. Drawn into the stories, I felt excited by these new mathematical possibilities for describing our world. Nonlinearity, pluralism and the dynamic nature of chaos resonated with my view of the nature of mathematics. Yet it was the computer-generated pictures of fractals that held my attention. I was enthralled by the colour and fluidity of shape. I began to play with my understanding of the universe. I imagined tree branches, coastlines and blood vessels as fractals and was fascinated by the harmonious arrangement of order and disorder occurring in natural contexts. This view of mathematics was relational: I was relating to the beauty, elegance and imagery of mathematics. The work of Benoit Mandelbrot, who coined the term *fractal* and worked extensively on describing the Mandelbrot set, had an enormous effect on my view of mathematics. Benoit Mandelbrot passed away on October 14, 2010.

On a more personal note, my work as editor of *delta-K* was greatly influenced by a previous editor, Art Jorgensen. About four years ago, I had the pleasure of meeting him and his wife, Ivy. While I didn't know him well, I was impressed by the encouragement he provided to me both professionally and personally. Some of you are aware that my husband went through high-impact cancer therapy a few years ago. When Art found this out, he regularly phoned my husband to speak with him and find out how he was. He also contacted me to provide feedback about my work. These acts of care were greatly appreciated and will be profoundly missed.

Art has greatly affected math education in our province and, years ago, MCATA created the Dr Arthur Jorgensen Chair Award in honour of his long-time interest, involvement and support of our organization. This award is presented to a student in a degree program at a faculty of education in Alberta who has demonstrated academic excellence and a clear commitment to mathematics education. Art spent more than 50 years involved with education. I had the pleasure of interviewing him about his experiences and, in his memory, excerpts from our conversation are reprinted in this issue of *delta-K*. Dr Arthur Otto Jorgensen, of Edson, Alberta, passed away on February 19, 2011, at the age of 83.

Great teachers continue to provide leadership in our community. This issue contains a discussion by Sherry Matheson on problem solving, research on initiating conversations, by Greg Belostotski, and an examination of the use of a workbook for Math 10C, by Richelle Marynowski. Each of these articles was selected to provide a glimpse of issues facing teachers in the classroom. Some practical teaching ideas for patterns are explored by Krista Francis-Poscente, Sharon Friesen and Trevor Pasanen. Veselin Jungic and Jamie Mulholland present their findings about teaching math in an online environment. Finally, *delta-K* presents a problem by Gregory Akulov—I encourage you to submit teacher and student solutions.

As always, I welcome your feedback and submissions. Guidelines for manuscripts can be found on the inside front cover of this issue. I am more than willing to help you with the writing process.

At this time of celebration and remembrance, I encourage you to reflect on the impact of your colleagues and former teachers. Perhaps you will have the opportunity to write a brief note in acknowledgement of how they have touched you. Live well and remember Art's words, "You teach children, not math. Math can't learn a damn thing!"

Reference

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MCATA Executive, 2011/12

Back row: Indy Lagu, Daryl Chichak, Robert Wong, Lisa Everitt, Christopher Smith, Debbie Duvall, Christine Henzel, Carmen Wasylynuik, Donna Chanasyk

Front row: Tancy Lazar, Marj Farris, Karen Viersen

Missing: Olive Chapman, Carol Henderson, Rod Lowry, David Martin, Shauna Rebus, Gladys Sterenberg

A Conversation with Dr Jorgensen

Dr Arthur Jorgensen has greatly influenced mathematics education in our province. He has been a principal and teacher in various schools in northern Alberta, an instructor at Grande Prairie College and the University of Lethbridge, a consultant in Jamaica, a Justice of the Peace, and an assistant superintendent. He served on the MCATA executive as director, secretary, vice-president, journal editor and newsletter editor, and most recently has been a contributor to delta-K. *MCATA's annual Dr Arthur Jorgensen Chair Award is created to honour his work in education and his passion for mathematics. I wanted to know what he was thinking about current educational issues in mathematics. Here are excerpts from our conversation.*

—Gladys Sterenberg

G: Tell me a little bit about who you are and your background.

Dr Jorgensen: I've really been interested in mathematics ever since I was just a child. In elementary school the teacher and I got along very well. I often had ideas, and I really supported her. Then when I got into high school, I often helped many of the students with their math. And then of course I got into university and I took some math there. As far as training is concerned, I got my BEd, which was general; I got a bachelor of arts in psychology and I got my master's in education, which dealt primarily with administration. And then my doctorate in curriculum and instruction in mathematics.

I have really, really been concerned about how children are taught mathematics. I don't think, historically, children have been taught mathematics well. They are so turned off because the teachers themselves are turned off. When people say, "Well, I teach mathematics," that's a disaster. You have to teach *children* mathematics and when you are teaching children, you've got to realize they are individuals. They don't all learn at the same pace, and they don't all learn in the same way. And as a result you have to treat them as individuals.

Mathematics should be enjoyable for children and for anybody who's taking it. Some people would say to me, "It's fun," and I'd say, "No, I wouldn't say it's fun, I'd say it's enjoyable." I don't consider it fun but I consider it enjoyable, and that's the way it should be for children. But that doesn't happen sometimes. I think of giving a test to a little boy in Grade 3. I tell him on Friday that on Monday there will be a test. He knows he's going to fail the test, his friend knows he's going to fail the test, his teacher knows he's going to fail the test, his parents know he's going to fail the test. And what kind of weekend does he have? I myself know that if I have a test tomorrow, I won't sleep very well tonight. And the same with this little boy; he's not going to have a very good weekend. So what really was the point of the test? And if this little boy had worked hard, it's like running a race. Somebody will be first and somebody will be last. And the one that was last maybe worked harder than the one that was first. And the same thing applies to mathematics. It really disturbs me. I know of a professor. He knew some math but he didn't have any teaching skills, and students had taken his course and they were crying. I wish I could have helped them in the classroom.

G: Talk a little bit about how you envisioned a classroom that would be different from that.

Dr Jorgensen: First of all, I get to know my students right from the start. From Grade 1, 2, 3, I would want them to become involved with mathematics. What we do now is get them to regurgitate answers. 6×56. All we want is an answer. ***I*** want that child to actually get involved with 6×56—I can teach a dog 6×56. Let's look at something like $2 + 3$. What's $2 + 3$?

G: Five.

Dr Jorgensen: Is that the only answer that you'll accept?

G: What would you suggest?

Dr Jorgensen: I say that there are many answers to that question: $3 + 3 = 6$, and $6 - 1 = 1+1+1+1+1$. You get all kinds of answers. Something like $11 + 2$ is 13. But what if I say $11 + 2$ is 1? And we do it every day on a clock. Everybody knows 11 and 2 is 13. But 1 and 1 can be 0 if I'm working in base 2. I think what's important is that children get an opportunity to see how math works in all these different ways. And I think then they will enjoy mathematics.

G: So, how can we work with teachers?

Dr Jorgensen: I think that teachers should have some good workshops on how to teach children mathematics. When I was in Jamaica, I had teachers who were able to run workshops in mathematics, good workshops, and feel confident in doing it. But we've got to spend more time on teaching teachers how to teach children. People think there's just one answer to mathematics. There are all kinds of answers to mathematics. Let's get our teachers involved in creativity and get them involved with things.

Get the teachers involved with things and show them different ways of doing things. And when you ask our teachers why they did something, they might not know. All they know is that it works. Well, show them *why* something works and show them other ways to make it work. It would make more sense to me to teach children that way. We want our teachers to enjoy the process because many teachers today do not enjoy teaching kids mathematics.

G: You said earlier that you've enjoyed math for a very long time. How did you grow to enjoy it?

Dr Jorgensen: I guess we've all got our areas of interest, but right from the start I could work with numbers, from the time I was very young. And a teacher I had in school would often ask me, "How did you do this?" So I showed her how I would do things. When I got into high school, I often helped other students.

G: Tell me what you would do with teachers that don't have that natural ability or that natural interest. You said you would get them involved, but often they really struggle with their own confidence and with their own abilities and past experiences.

Dr Jorgensen: I don't think all teachers should be expected to teach children mathematics any more than they expect to teach them art or physical education, for example. I think we should have teachers that teach children mathematics who have a good understanding of it. I think children should adjust. Often one teacher is expected to teach children all the subjects, and I disagree with that because there are some teachers who will not be able to teach children mathematics.

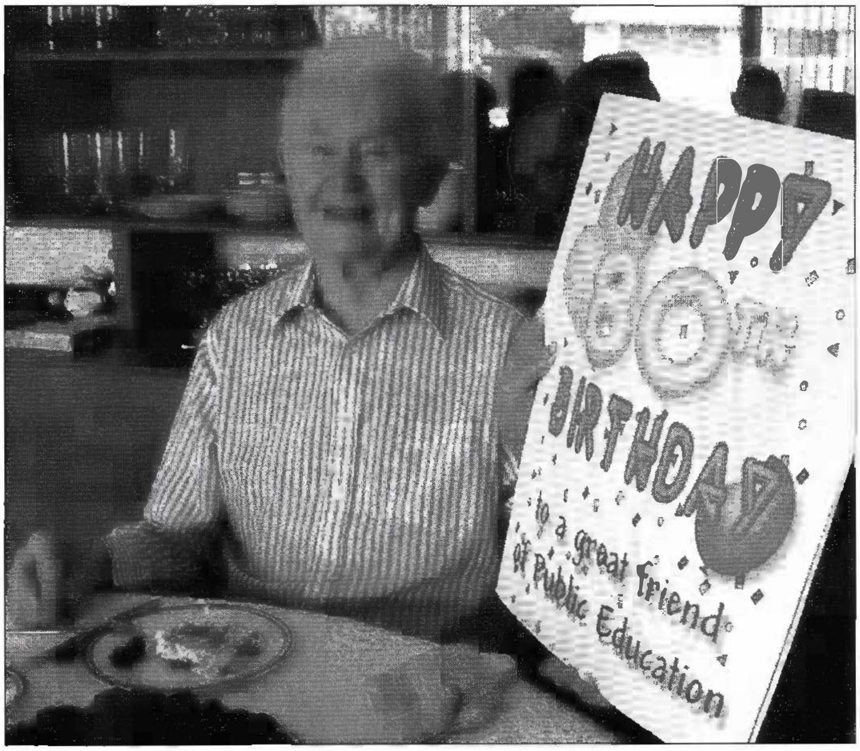
G: So, you are kind of making an argument that we should have specialists in our elementary schools.

Dr Jorgensen: Yes, I am. Absolutely. Who will teach piano lessons? You expect teachers in school to do this but not all teachers can play the piano. If I'm having problems with my heart, I go to a heart specialist, and if I'm having problems with my brain, I go to a brain specialist. These doctors don't know everything about our health, and we don't expect them to. And I think we can do the same thing for children in school. I think we should have people who are really interested in mathematics teaching math.

G: How did you become involved with MCATA?

Dr Jorgensen: I became involved with MCATA, I guess 45 years ago. I felt this council made a difference. And I guess I kept bumping into some very interesting people in MCATA. I was hoping that I could have some influence on how to teach children mathematics.

This article is reprinted from delta-K *volume 45, number 2, June 2008.*



The Problem of Problem Solving

Sherry Matheson

Teachers in Alberta are required to submit yearly professional growth plans to their school administration. Each goal undertaken by the teacher for that school year must align with a descriptor, set by Ministerial Order #016/97, of knowledge, skill or attribute required of teachers who possess a permanent teaching certificate in this province. One of the descriptors states, "Teachers are career-long learners."[1] My own career-long learning has focused on primary children's learning. It has taken me most of my thirty-year teaching career to work through the problem of problem solving.

The problem started much earlier than that. I was a reasonably strong student in elementary school, and though I never really had any difficulties with computation, I was quite anxious when we were assigned problems. I just never could read those problems and actually know what to do. When I faced a word problem such as

> At the school store Harry bought a textbook for which he paid 10 per cent less than the regular price of $1.40. How much did Harry pay for the book?

I found I was stumped. I couldn't make sense of the Physics 10 question that demanded to know how wide was the river based on the height of the tree that I could see on the other side. I managed to get my first degree without ever taking a university-level mathematics course.

As a young teacher of primary school children, I could pick and choose which problems I would ask my students to solve. If I thought the students would find it too difficult, I would just leave it out. That worked for the first twelve years of teaching, but then I was placed in a Grade 3 classroom. My task was to prepare these students for their first experience with the provincial achievement exam in mathematics—and it was *all word problems!*

I equated the mathematics test with a reading test. I believed that if my students could *read* the problem, they could *solve* the problem. We worked with a traditional textbook that contained lessons for practising beginning addition or subtraction that were followed with either words or pictures that gave the students the chance to apply the algorithm that they had just learned. These were often called story problems. If the lesson was about subtracting one-digit numbers from two-digit numbers, with regrouping, the problem might be

> Tyler had 23 hockey cards. He gave 6 to his younger brother. How many hockey cards does Tyler have now?

For the first two or three years in this teaching assignment I would create wall charts that I believed helped the students know which operation to apply by looking for the *hint words* in the actual word problem. If the question used the word *altogether*, the students knew that they were to add. If the question used the word *left*, the students knew that they were to subtract. The words were the key to the finding the correct answer. I was convinced that I had finally learned how to solve problems! But then a word problem like this came along:

> Joyce had 17 apples altogether. She had 8 red apples. How many were green?

It didn't fit the pattern. The students added according to the classroom chart, and although the hint word *altogether* was there in the question, the answer, 26, wasn't correct! I had a hard time explaining to the students why, for this problem, they had to subtract instead of add. They lost faith in my charts. They had a procedure but no understanding. "Students who memorize facts or procedures without understanding often are not sure when and how to use what they know, and such learning is often quite fragile" (Bransford, Brown and Cocking 1999). My students did not understand the procedure that I had created for them.

Not too long after, I tripped over the problem-solving strategies put forth by Polya. The four steps were logical. Paraphrased, they are (a) understand the problem, (b) devise a plan, (c) carry out the plan and (d) look back (Polya 1988). A new chart went up in the classroom. I used the words from my father's old math textbook (Banting, Banting and Brueckner 1936), which had been authorized by the ministers of education in Alberta and Manitoba.

I. What does the problem ask for?
II. What must be done to solve the problem if all the facts are not clear?
III. What is a reasonable answer?
IV. What checks should be used? (p 14)

The students were unsure of the second step. They did not have clear ideas of what they were meant to do without input from me, the teacher. I searched some more and began using *The Problem Solver 2* (Hoogeboom and Goodnow 1987), which was based on introducing a variety of strategies that the students could apply when solving a word or story problem. The strategies fit into Polya's second step, devising a plan: (a) act out or use objects, (b) make a picture or diagram, (c) use or make a table, (d) make an organized list, (e) guess and check, (f) use or look for a pattern, (g) work backwards, (h) use logical reasoning, (i) make it simpler, and (j) brainstorm (Hoogeboom and Goodnow 1987, viii). The binder of worksheets gave specific examples to work through with the students and a multitude of worksheets that allowed the students to practise the strategy that they had just been taught but had not really learned. This followed the traditional approach where

> the teacher demonstrates or leads a discussion on how to solve a sample problem. The aim is to clarify the steps in the procedure so that students will be able to execute the same procedure on their own. ... students practice using the procedure by solving problems similar to the sample problem. (Stigler and Hiebert 1997, 18)

At about this same time, elementary teachers in Alberta were being introduced to a new mathematics program of studies (Alberta Learning 1997). This new curriculum presented a shift from the manner in which mathematics had traditionally been taught. It suggested that the students would not be given prepared worksheets with algorithms to be completed silently and individually. Rather, it stated, "Problem solving, reasoning and connections are vital to increasing mathematical power and must be integrated throughout the program. A minimum of half the available time within all strands needs to be dedicated to activities related to these processes" (p 13). To help teachers recognize the areas where problem solving could be effectively added to the concepts being taught, a bold PS appeared after the actual specific outcome in the following manner: "6. Recognize, build, compare and order sets that contain 0 to 1000 elements. [PS, R, V]" (p 15).

This was the first time that I had been exposed to the idea that problem solving was an important component of mathematics. I had not understood that the role of problem solving was to actually develop the basic mathematical computational skills that I had been asking the students learn by rote or through the practice worksheets that I dutifully copied and distributed daily. I railed against the new textbooks that did not have practice pages before the introduction of a problem. I could not imagine how the students could solve a problem before they knew how to complete an algorithm correctly. Sadly, there was no one to help me understand this challenging change. My school district purchased the textbooks, but there was no assistance to help me change my own understanding. I was alone and I could rail against this change with indignation. What did *they* know about teaching kids? I had been successful and I did not need to change! "One cannot expect teachers to change their teaching practice simply because they have been told to" (Mewborn 2003, 49). I ignored the changes and for many years left those new textbooks, bindings uncracked, on the classroom shelves, using them only to press leaves in the fall.

It is now nearly fifteen years later. There are times when I marvel at how far I have come. Problem solving is not a problem for me or for my students any more because we no longer attempt textbook-generated word and story problems. The students no longer sit at their desks and work individually at worksheets with rows and rows of algorithms. The students and I are engaged in rich problem-solving activities. I enjoy mathematics, both the teaching and the learning, that occurs every day in the classroom. I see my students making sense of what they are doing and constructing their own knowledge. But how could this shift have taken place?

Just over five years ago, the schools were abuzz over another mathematics curriculum change, but this time it was different. Teachers were encouraged to attend professional development opportunities to help them understand the changes. I attended the first workshop at Barnett House, in Edmonton, led by a former teacher. Throughout the day, this teacher brought the curriculum changes into focus, explaining how they were intertwined and how the activities she shared could be done with all students. She espoused the same philosophy as Clements and Sarama (2009), who believe that "especially for younger children, mathematic topics should not be treated as isolated topics; rather, they should be connected to each other, often in the context of solving a significant problem or engaging in an interesting project" (p 207). The words of the new curriculum (Alberta Education 2007) began to have more meaning.

> Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type *How would you ...?* or *How could you ...?*, the problem solving approach is being modeled. ... A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement ... Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers. (p 6)

Tentatively, I took a chance and attempted an activity from the workshop. It meant that the students and I would be talking about the possibilities offered by the problem posed. It meant that we would work in groups, we would share our ideas and we would be willing to make mistakes. Luckily, it was April, so trust had already been established, and the students knew that I would not introduce an activity that they could not succeed at. Using wooden pattern blocks, the students were introduced to increasing patterns. A red square block was set down with two cream-coloured rhombus blocks placed on opposite sides to represent legs. This created a caterpillar-like creature. As each red square was added, two rhombi were added. After creating a creature with 4 squares (body) and 8 rhombi (legs), I challenged the students to use what they knew and predict the number of legs the creature would have when there were 6 body parts, 8 body parts and 10 body parts. With the students working in pairs, with manipulatives, the challenge began. As I walked about and listened to the students' conversations and attempts at a solution, I marvelled at the feeling in the air. It was electric! These students had never been so engaged. They didn't need me. They didn't want me. They wanted to work! I knew that I could never go back to the old way of teaching problem solving again.

> Once one accepts that the learner must herself actively explore mathematical concepts in order to build the necessary structures of understanding, it then follows that teaching mathematics must be reconceived as the provision of meaningful problems designed to encourage and facilitate the constructive process. In effect, the mathematics classroom becomes a problem-solving environment in which developing an approach to thinking about mathematical issues, including the ability to pose questions for oneself, and building the confidence necessary to approach new problems are valued more highly than memorizing algorithms and using them to get the right answers. (Schifter and Fosnot 1993, 9)

The challenge continues—not the challenge of *how to teach* using the new approaches required by the new mathematics curriculum, but *how to engage the students in meaningful investigations* and recognize opportunities to bring meaningful problems into the classroom. "[T]eaching is not just about starting with mathematically rich problems, even ones connected to what students are thinking. And it is also not just about listening to students and asking them to describe their thinking" (Franke, Kazemi and Battey 2007, 226). It is through these problems, which must be thought out, that new learning is encouraged. The problems must reach all of the students at the level at which they are currently constructing their own understanding of the mathematics being presented. The solutions must be their own. The problems must scaffold from students' prior knowledge and move into the next level of investigation. The problems must represent what is important about mathematics and illustrate real-world situations; they must engage and delight and offer opportunities for pondering, discussion, strategies, failure and success. They must allow a community of learners to work cooperatively and find solutions that are acceptable, not because the teacher says so, but because the community has looked for and found an acceptable explanation. "Classrooms need to be places where teachers and students are engaged in rigorous mathematics in ways that both parties learn" (Franke, Kazemi and Battey 2007, 228).

I think that next year, when I submit my professional growth plan, I might just write "Teachers are career-long learners." Period. Being able to learn is more powerful than having learned. It is the gift I wish to give my students.

Note

1. Teaching Quality Standard Applicable to the Provision of Basic Education in Alberta 1997 (Ministerial Order #016/97), 6

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Breaking Silence: Initiating Conversations in Mathematics Classrooms

Gregory Belostotski

Introduction

The character of the boring economics teacher played by Ben Stein in John Hughes's 1986 film *Ferris Bueller's Day Off* briefly crosses my mind as Mr Paavi (all names used here are pseudonyms) first asks the class "Any questions?" and moments later inquires if students have heard of a recent "pay it forward" advertising campaign by a local bank:

> Mr P: Any of you guys hear about this yesterday? ... Anyone? Right ...

Like the students depicted in the film, Mr Paavi's Grade 12 applied mathematics students stare back in silence. But unlike Ben Stein's monotonal character, Mr Paavi is a dynamic speaker who is sufficiently loud and commands attention. He is not afraid to challenge and engage with his students. Mr Paavi does not accept the silence:

> Mr P: Who has a heartbeat?! Does anyone have a heartbeat today? Let's start with that ... Who has a heartbeat? (Sam, smiling, holds his hand up. Geoff follows reluctantly.) OK. Oh, OK, so, all right, just checking, OK, good, thank you ...

As the class begins with an extracurricular chat about the local bank, charitable acts and advertising, the students and the teacher are explicitly exchanging their expectations. Through inaction, the students are communicating their reluctance to participate. For the teacher, on the other hand, the class needs a pulse. It has to be alive and responsive. Mr Paavi needs some reassurance that what he tells his students over the next 80 minutes of instructional time will not die from inattention. "Who has a heartbeat?" is a rhetorical question, but it is also a message that in this class participation is important.

Several days after I watched Mr Paavi ask his students, "Who has a heartbeat?" the following interaction in Mr Brodiew's classroom (Grade 12 pure or precalculus mathematics) caught my attention:

> Mr B: It's, it's going to be, right here (pointing to an exponential function written on the board), when you plug ... yeah ... when you plug negative three in, the negative three is going to reciprocate that and it will be two cubed, or eight.
> Mr. B: This will be?
> S1: Oh, four!
> Multiple students: Four, four...
> Mr B: Four!
> (Teacher points to the successive values of x as a number of students recite in unison.)
> Multiple students: Two. Two!
> Mr B: Two!
> Multiple students: One! Negative ... one over two! One over four! One over eight! One over sixteen!
> Mr B: Yeah! You're getting it! There is excitement! You're doing math!
> (Some students in the back playfully high-five each other, while several others smile.)

As students recite the answers, the spontaneous display of energy reverberates through the room. It is perhaps this type of excitement—the loud heartbeat of the classroom—that Mr Paavi, Mr Brodiew and so many other mathematics teachers strive to experience.

Study Background

In an attempt to explore student questions in secondary mathematics classrooms, I have collected a large amount of data that includes narrative accounts of students' questioning experiences, responses from a focus-group discussion among three experienced mathematics teachers on the topic of student questions, and observations and video recordings of 69 mathematics classes. Two of these classes are Mr Paavi's Grade 12 applied mathematics course and Mr Brodiew's Grade 12 pure mathematics course.

All video recordings, in particular classroom videos from these two classes, have been reviewed, and video clips containing instances of student participation (primarily student questions) were created. These clips were then coded thematically with keywords identifying particular aspects of each clip. Some of the keywords identified elements of student participation (eg, formal requests to speak), others focused on the types of questions asked (eg, clarification), and still others described turn taking (eg, precipitating utterances), teacher moves (eg, delegation of response) or teaching style (eg, lecture). The observations of these classrooms and my analysis of student questions inform the ideas discussed in this work.

Introducing Mr Paavi and Mr Brodiew

Mr Paavi and Mr Brodiew are two experienced, well-respected and well-liked mathematics teachers. They are both dynamic speakers who can create a highly interactive dialogue with their students and their colleagues. Mr Paavi and Mr Brodiew are also two of four participating teachers who make extracurricular conversations, such as the interaction set out at the beginning of this paper, part of the classroom routine. Mr Paavi is well aware that he frequently spends significant amounts of instructional time discussing issues that parallel the topic of the lesson. He explains to his students that he cannot resist discussing the issues of finance, business and government.

Mr Brodiew, on the other hand, has a penchant for mathematics and movies. His lessons are full of references to popular culture and cartoons. These references are made in passing, interrupting conversations that otherwise focus on the lesson. Both teachers, and Mr Brodiew in particular, are skilful in eliciting laughter with jokes, impersonations and social commentary. In fact, laughter is a prominent fixture in Mr Brodiew's classroom.

Most of Mr Brodiew's students have agreed to appear on camera and to take part in my observations. A number of students routinely ask questions or reply to teacher inquiries. At the same time, a significant group of students seldom or never participate during my visits. The class is held in the final block of each school day.

Mr Paavi's class, which I featured in the opening quote, is small (20 mostly Grade 12 students) and held every morning. Only 10 to 15 students are in attendance on any given day. The total number of students who have agreed to participate and who have chosen to sit in view of the camera is even smaller. Students in this class are very reserved, and only a handful of "target students" (Tobin and Gallagher 1987) make occasional contributions through questions or answers to teacher inquiries.

I write this article for all the teachers who identify with Mr Paavi and Mr Brodiew in their need to hear their students; it examines the challenges in initiating mathematical conversations with students. In the concluding section, I make some practical recommendations that are not tested through practice but are born out of observation, through thoughtful reflection and a review of relevant literature. In making the recommendations, I recognize the individual differences of the many mathematics classrooms.

Theoretical View of the Role of Student Classroom Participation and Conversation

Many researchers consider student involvement and on-topic conversation in the classroom an important learning strategy (eg, Turner and Patrick 2004). Consequently, several mathematics curricula accept and promote student participation primarily through student communication of ideas. The Principles and Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM] 2000) and the Western and Northern Canadian Protocol (WNCP 2008) documents describe communication of mathematical ideas as a significant area of student development. Both documents explicitly expect students to use conversation to become precise and to show reason in their discussion of mathematical concepts, and to form links between various representations of mathematical ideas.

Although the pedagogical aim to promote and sustain student participation may be reasonable, student participation in mathematics and other classes is a complicated matter in middle and secondary school classrooms (Daly, Kreiser and Roghaar 1994; Patchen 2005; Turner and Patrick 2004). Problems with student participation are further exacerbated by the presence of English language learners (ELLs) (Patchen 2005; Yoon 2007), students with diverse abilities and special needs, the gender composition of the classroom, and teacher approach to student participation, among other issues (Daly, Kreiser and Roghaar 1994; Patchen 2005).

Curiously, even reports that aim to quantify the problem of student participation, such as the one by Daly, Kreiser, and Roghaar (1994), report that the majority of students appear to be comfortable asking questions and, by extension, participating in class. That study has collected information from 24,599 students between the ages of 13 and 16. The authors report that the mean question-asking comfort score is close to 12 and the standard deviation is approximately 2.5 on a task with a score range from 4 to 16. However, they find that question-asking comfort does correlate inversely with the age of students and directly with gender (males report greater comfort than females), socioeconomic status, personal goals, language ability and perception of teacher helpfulness.

Two questions arise for me:

- If the majority of students are comfortable participants, why at times don't they participate?
- What do we do about students who report discomfort with classroom participation?

Sfard et al (1998) raise a similar question, but they focus on teacher practice. In their exploration of the role of conversation in mathematics education they conclude

> In short, the question is not whether to teach through conversation, but rather how. Since learning mathematics may be equated to the process of entering into a certain well defined type of discourse, we should give much thought to the ways students' participation in this special type of conversation might be enhanced. (p 50)

The greater education community has been preoccupied for some time with similar challenges. To enhance opportunities for student conversation and participation, many innovations have been introduced but have had variable success. Classroom activities (for example, brainstorming and group work) and curriculum modifications with an emphasis on communication and a constructivist learning framework are being tried by teachers across Canada. It is now not unusual to find various classroom technologies such as the Student Response Systems (or clickers) and interactive whiteboards across North American classrooms (Anderson et al 2003; Dufresne et al 1996; Nocente, Belostotski and Brooks 2009; Penuel, Abrahamson and Roschelle 2006; Roschelle, Penuel and Abrahamson 2004). However, as Judson and Sawada (2002) point out, any success with the implementation of new technologies and practice that leads to an increase in student on-task participation still rests largely on the shoulders of the teachers who build a classroom culture conducive to participation.

Patchen (2005) expands on the general call for teachers to improve student participation with the following five recommendations, which focus on recent immigrant adolescents but include all educational settings: (1) deepen personal understanding of students' cultural background, (2) establish relationships, (3) diversify participation structures, (4) ask answerable questions and (5) solicit student feedback (pp 45, 46).

But how does one "deepen personal understanding" and "establish relationships"? What space—be it time or curricular—is available to meet these suggestions?

An Interactive Exchange: One Example from a Mathematics Classroom

To provide one example of a teacher having some success in drawing students into participating, I presented a sequence in the introductory section, in which a number of students join in a chorus listing the answers one by one. It is important to recognize that the unison recitation included some students who had not participated in classroom conversation in my presence before. At the same time, it is important to note that not all students participated. Still, I would like to propose that through laughter and lighthearted, often extracurricular, conversation, Mr Brodiew has created a class culture that enables student participation.

Consider how the conversation unfolds, as set out below in "The Mathematical Chorus and the Soloist." The transcript lines are numbered by speaking turn for later reference; overlapping speech is included in square brackets and formatted to vertically overlap.

The Mathematical Chorus and the Soloist

1. Mr B: It's, it's going to be, right here (pointing to an exponential function written on the board), when you plug ... yeah ... when you plug negative three in, the negative three is going to reciprocate that and it will be two cubed, or eight.
2. Mr B: This will be?
3. S1: Oh, four!
4. Multiple students: Four, four ...
5. Mr B: Four!

(Teacher points to the successive values of x as a number of students recite in unison.)

6. Multiple students: Two. Two!
7. Mr B: Two!
8. Multiple students: One! Negative ... one over two! One over four! One over eight! One over sixteen!
9. Mr B: Yeah! You're getting it! There is excitement! You're doing math!

(Some students in the back playfully high-five each other, while several others smile.)

10. S2 (off camera): {Inaudible}
11. Mr B: What's that?
12. S2: Why isn't it the square root ... like {inaudible} in the second {inaudible} ...
13. Mr B: OK. OK. Why would, why would I flip this? (The teacher points to a number with a negative exponent.)

(3.8 seconds)

14. S3: The negative?
15. Mr B: The negative in the exponent. What does the negative do?
16. S3: It flips [it]
17. Mr B: It flips it, so negative three is going to be ... see that?

(Several students are now talking among themselves.)

18. S4: Mr Brodiew?
19. (Mr B reacts to some suggestion he hears.)
20. Mr B: Oh, oh, hold on, don't do that, {inaudible} don't do that.
21. S5: Wait!

(Noisy)

22. S4: Do you mean the negative exponent flips [the fraction?]
23. S5: [What do you] mean, "Don't do that"?

(Noisy)

24. Mr B: Aha, so this becomes, this basically becomes two to the positive three. Exactly! Good! Now, now look at this ...

Though the transcript might create an impression of uniform participation, the classroom video clearly shows that many students in this large class do not participate. These students include those who perform well academically and those who do not.

The recitation in lines 3 to 9 clearly demonstrates how some students are able to recite the terms of a geometric sequence or, perhaps, continue the most likely number pattern without much consideration for the topic (note the desire by some students on line 8 to recite the wrong pattern continuing from one to the negative numbers).

Lines 10, 11 and 12, on the other hand, deserve additional consideration. Student S2 refuses to be swept away with the excitement and questions the simple patterning. In the process, she rescues the moment for some of her classmates who might have not understood what was being recited. In asking her question, the student risks being exposed as perhaps the only student—or one of the very few—who did not understand the process and thus facilitates a valuable learning opportunity for her classmates.

The decision to ask is selfless and brave. The risk of exposure and the social ramifications of this act cannot be understated, but they can be mediated by building an appropriate class culture where student participation is not evaluated, interruptions are accepted, the atmosphere is friendly, student participation is welcomed and students are part of an ongoing conversation. In short, students need to feel safe enough to, on occasion, stand against the flow of the class.

I argue that in the case of Mr Brodiew and Mr Paavi, the element of safety comes from frequent extracurricular commentary and as a greater social comfort of being in a friendly environment. Students of Mr Brodiew and Mr Paavi are drawn into discussion—become a part of the conversation—even before they actively participate in learning about mathematics. These opportunities appear to create the necessary conditions for keeping students involved in conversation as it shifts from extracurricular chat toward a discussion of the concept of the day.

Discussion

Earlier I asked the following two questions:

- If the majority of students are comfortable participants, why at times don't they participate?
- What do we do about those students who report discomfort with classroom participation?

One plausible answer to both questions lies in providing opportunities for all students to be part of a discussion. Be it joining a chorus of peers or a chat about the publicity programs by a local bank, the immediate relevance of the conversation in itself is irrelevant.

The Mathematical Chorus

Not all students reciting the numbers in "The Mathematical Chorus and the Soloist" understood what was being recited—perhaps only the ones who correct the pattern in line 8 did. However, such understanding of this particular pattern may be secondary to the value of participation. In the excitement, many students became a part of the living classroom. Several students who normally don't say a word finally added the sound of their voice to the classroom. They said things aloud. In a conversation some years ago, I asked a group of preservice teachers about classroom participation. One of the students said that she used to be shy about the sound of her own voice. As a result, she avoided speaking in class. A recital such as this, similar to singing in a choir, would provide an appropriate medium for students like her to join in and add the sound of their voices.

Besides creating a medium where even the shy students can add their voices, this group recital is a self-correcting process. The initial desire to follow the wrong path is not as important as the realization that their contribution was sufficiently close. Van der Meij (1990) describes several hypotheses and studies that suggest that perplexity—the first stage of questioning—arises from various internal or external events. Van der Meij writes:

It is believed that the most likely condition leading to such a perplexity occurs when a stimulus resembles something well-known but is also distinct enough to be interesting. If it is too remote from experience, or too familiar, the reaction will be one of indifference ...). (p 141)

Consequently, the initial making of an error may offer a significant opportunity to confront and repair a student's own misunderstanding.

Extracurricular Chat

An opportunity to talk about a variety of topics offers the possibility that conversation in the classroom is not just for the satisfaction of the teacher—a means to determine the level of alertness of the students. Saying things aloud in a classroom provides an entry for students to become part of the classroom and join an ongoing conversation. Once part of a conversation, the students participate in the classroom conversation about mathematics amongst other things.

I cannot offer a guarantee that all students will remain in conversation. For example, student S2, following the initial utterance on line 10, was compelled to participate in the conversation she initiated only once—in line 12. It is also unreasonable to expect two groups of students to respond to the teacher in exactly the same way. However, in this instance, student S2 felt safe enough in the class to engage the teacher. Students reciting the numbers felt the safety of the chorus to speak. At the end of the interaction, students talked about mathematics with each other and the teacher.

My Recommendations

Having considered the issues of classroom participation and the literature on classroom participation, my recommendations for drawing students into participation include the following:

- Use Patchen's (2005) suggestions and promote talk that encourages the students to share their background and interests.
- Ask students to repeat terminology such as *dispersion*, *deviation* and *reciprocal*, because not knowing how to pronounce a word should not stand in the way of talking about concepts.
- Create opportunities for students to say things out loud, be it the mathematics term of the day (eg, on a count of three say *numerator*) or the part of the lesson you have found the most difficult (on a count of three say your name).
- Look for alternatives to speaking (such as classroom communication and presentation technologies).
- Invite students to answer questions, make suggestions and speak mathematically together, perhaps even at the same time, taking some of the risk out of the participation equation.
- Visit the classrooms of your colleagues and to see what their students are doing, saying, or not saying.

Join in the conversation and let us all know what has worked for you in giving your students a voice in your mathematics classroom.

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An Analysis of Question Types in a Workbook for Mathematics 10C

Richelle Marynowski

In this article I analyze the *Foundations of Mathematics and Pre-Calculus Grade 10 Workbook (for Math 10 Combined)*[1] (2010) and focus on the philosophy of mathematics education as presented in the Alberta Grades 10–12 Mathematics Program of Studies (2008). I completed this analysis to satisfy my curiosity about whether using a workbook to plan from, teach from and learn from would completely reflect the intent of the program of studies.

I have used a workbook by the publisher of *Foundations* for a previous mathematics program of studies. I found that workbook designed more for a drill-and-practice type of classroom than an investigative and understanding-building classroom in which students contribute to knowledge generation that would be reflective of the 2008 Alberta program of studies. According to Silver et al (2009), "the adoption of new curriculum materials, especially those designed to embody innovative ideas and practices, can catalyze changes in teachers' instructional practice and enhance students' opportunities to lean mathematics" (p 245). I am not positive that *Foundations* will be representative of innovation in teaching or of enhancing students' experiences in mathematics learning. In my analysis, I consider how well *Foundations* reflects the program of studies with respect to allowing for and expecting individual representation of knowledge and understanding.

Program Philosophy

The philosophy for mathematics education of the Alberta Grades 10–12 program of studies (2008) centres on individual differences in students. Students should be encouraged to develop their own understanding of the mathematical concepts and their own personal strategies for solving problems and answering questions. The front matter of the program of studies refers to students "taking intellectual risks, asking questions and posing conjectures" (p 2). I feel that a major change in the program is in allowing students to express their mathematical understanding in their own way and "that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood" (p 2). Many teachers have previously taught that there are only one or two acceptable ways to solve a problem or approach a task. The change in focus from the teacher being the giver of knowledge to the student being the creator of knowledge may prove to be challenging for some teachers. In my experience as a high school mathematics teacher, I have struggled with stepping back and letting my students create their understanding; I anticipate that other mathematics teachers will experience similar struggles.

Allowing students the freedom to create their own solutions and use their personal strategies in approaching problems was a major focus in my analysis of *Foundations*. My analysis centred on how the lessons and activities in the workbook demonstrate that the creation of knowledge and the use of personal strategies by students is valued and expected. I have been working closely with the program of studies over the last couple of years, and I believe that one of the most important features of the program of studies document is the front matter. Unfortunately, I suspect that teachers often pass over the front matter to focus on the specific outcomes, not on the philosophy of mathematics education that the program was built on.

Format of the Workbook

Foundations is organized into ten chapters, each consisting of seven to twelve lessons of which the last in each chapter is a practice test. Each lesson follows a similar pattern: class examples for the teacher to go through with the students, a set of assignment questions on the lesson and an answer key for the assignment questions. The lessons may also have other components, such as definitions, references to previously learned material and previous lessons in the workbook, how to access certain features on a calculator, warm-up activities and investigations. Several of the lessons include an "Extension" section that introduces students to material that is beyond the scope of the program of studies and sets out practice questions on the extension material. The "Assignment" section of each lesson contains a variety of question types that include short answer (or completion questions), long answer, true/false, matching, multiple choice and numerical response.[2] The practice tests at the end of each lesson include multiple-choice, numerical-response and written-response questions.

Foundations is designed so that teachers use the examples provided, and students copy down the answers that the teachers give. There is room in the workbook for students to write their solutions to the questions. At the end of each lesson and each practice test there is an answer key for each of the assignment and practice test questions. Answers, but not the worked-out solutions, are provided for the questions in the workbook; the solutions are provided in a separate solutions manual that the students can purchase if they choose. The solutions manual is a condensed version of the teacher's manual. Both manuals provide a solution to each of the questions.

What I noticed in previous versions of *Foundations* is that the teacher and student solution manuals provided only one method of completing a question. Teachers often followed this solution regardless of what might be best for their students. When I have used my professional judgment in class and have strayed from using the method demonstrated in the solutions manual, students question me, stating that what I have done is not in the book and, therefore, it is not correct. According to Christiansen and Walther (1986), teachers need to respond to their students and their students' needs and also to the principles of pedagogy that the teacher believes in. This type of manual may stifle teacher and student creativity, present one correct way to do math and take the decision making out of the teacher's hands.

Method

Foundations is organized into sections titled "Lessons," "Assignments" and "Practice Tests." Approximately 36 per cent of the pages in *Foundations* contain lessons, 48 per cent contain assignments, 9 per cent are practice tests, 5 per cent have only answer key content on them and 2 per cent of the pages are blank. I analyzed 33 pages (approximately 5 per cent) in *Foundations*; each section was represented proportionally in the sample. I used a random number generator (www.graphpad.com/quickcalcs/randomn1.cfm) to generate 55 numbers to represent page numbers in the workbook to analyze. I used 54 of the generated numbers, because 5 of the generated numbers were repeated in the sample, 4 corresponded to answer key pages, 1 was blank, and 14 were skipped because they corresponded to sections that already had enough pages in the sample. In the selection of the pages analyzed, 14 of the pages were from lessons, 18 were from assignments and 3 were from practice tests. Two of the pages contained both lesson material and assignment questions and were therefore counted in each category and included in the count for both lessons and assignments above.

The goal of my analysis was to see how the material presented in *Foundations* reflects the philosophy of mathematics education in the 2008 Alberta program of studies as presented in the previous section. I focused on the words that were used when eliciting a response to a question in the three different sections of the workbook as listed above. The questions that were posed were either for the teachers to use as examples for students or for students to answer as a part of an investigation or an assignment. I also considered the material that was provided in the lessons that was either used as an explanation or definition of a concept or process. I broke down the questions that I saw in *Foundations* into five categories: (1) instances where students/teachers are asked to explain, describe or state a rule, (2) instances where the method to be used to solve the problem or answer the question is given, (3) instances where the method to be used to solve the problem or answer the question is not given, (4) instances where alternate representations are given or expected as a response and (5) instances where a unique question is asked or a unique response is expected.

The focus in my analysis is to consider how *Foundations* encourages students to consider alternative methods of approaching problems. I wondered if the style of the questions in *Foundations* would encourage students to explore, think critically or consider alternative solutions. This query arose from my experience with a previous workbook produced by the same publisher. The following sections contain the details of my analysis, broken down by *Foundations* section and expectation, and a conclusion based on the analysis.

Lessons

As mentioned previously, *Foundations* contains ten chapters consisting of several lessons each; 14 of the 33 pages analyzed were from lessons. The letter that accompanies the workbook states that "the class examples are designed to be teacher led" and that "each unit contains some exploration or investigative work which allows students the opportunity to develop new mathematical techniques or formulas."

After reading this I was eager to see how the exploration and investigation were handled in *Foundations*.

The results of my analysis of these 14 pages with respect to the categories of questions asked are shown in Table 1 below.

Of the 41 instances of questions or directions, I consider that only 5 asked the students to "communicate and reason mathematically" (Alberta Education 2010, 2). According to the philosophy of the program of studies, I would expect that one focus of instruction in the course would be to have students communicate and explain their understanding of the concept. What I found most interesting is that approximately 46 per cent of the instruction expects teachers and students to complete questions or tasks in a specific way. Of the 14 pages analyzed in the lesson category, none contained an investigation, as was mentioned in the *Foundations* supporting materials. Nor did I see any questions engaging the students in discovering mathematical concepts for themselves.

I do understand that a large portion of examples specifically mention a method of solution because that particular method is taught in that lesson, and therefore the students need to have exposure to that method. I saw very little opportunity in these lessons for students to take risks or think and reflect independently (Alberta Education 2010). Even in the cases where the specific method was not given for completing the task, the method was implied by the lesson that included the task.

One example, on page 530, asks for an analysis of student work where at least two of the three solutions are incorrect. The errors in the provided work are to be described and the correct answer is to be found. I appreciate this type of question because it gives students an opportunity to think about solutions and analyze possible errors. This skill is necessary for students to reflect on their own solutions and possible errors. I found only two other instances in the pages analyzed where students and teachers are expected to describe and think about what they are doing and why.

Table 1

Category	Number of Instances	Examples
Instances where students/teachers are asked to explain, describe or state a rule	2	"Which of the calculations above is the easier method for..." p 94 "Explain each of their significance ..." p 477
Instances where the method to be used to solve the problem or answer the question is given	19	"Convert ... using ..." p 145 "Write ... using ..." p 101 "Use ... to ..." pp 7, 94, 477, 540 "The method of ... can be applied to ..." p 358 "Estimate mentally ... use a calculator to find ..." p 25 "Complete" (part of the solution is already given) pp 63, 599 "Evaluate" (part of the solution is already given) p 94
Instances where the method to be used to solve the problem or answer the question is not given	16	"State" p 7 "List" p 399 "Determine" p 477 "Estimate" pp 25, 145 "Calculate" pp 228, 599 "Write the equation" p 572
Instances where alternate representations are given or expected as a response	0	
Instances where a unique question is asked or a unique response is expected	3	"Write in words the meaning of ..." p 477 "Describe all errors which have been made" p 530

Assignments

I performed a similar analysis with the assignment pages that accompanied the lessons. Of the 33 pages analyzed, 18 contained what were designated as assignment questions. According to the documentation accompanying *Foundations*, "the assignments are intended to be done by the students individually, in pairs, or in small groups" (Appleby and Ranieri 2010, 1). There is no mention of projects, extended assignments or investigations that students are expected to complete on their own.

The results of my analysis of the 18 assignment pages with respect to the categories of questions asked are shown in Table 2 below.

The majority of the assignments in *Foundations* consist of question styles that either give the students the method to use or do not give a method, but ask students to calculate or determine. Many of the assignment questions were the straightforward, do-the-question-the-way-you-were-just-taught type of question. There were only two instances, both on the same page, that asked students to explain their thinking. The multiple choice and numeric-response questions were also basic complete-and-get-the-answer style. There was one multiple-choice question that required students to match an item with its corresponding value.

I appreciated the two questions that had a unique question style. One of the questions, on page 474, asked the students to explain and correct two errors in a given statement. This question would challenge students more than simply determining answers to similar questions would. The second question was

Table 2

Category	Number of Instances	Examples
Instances where students/teachers are asked to explain, describe or state a rule	4	"Describe" pp 421, 508 "Write a rule" p 540 p 637—After having students complete one question in two different ways: "Which method do you prefer?"
Instances where the method to be used to solve the problem or answer the question is given	14	"Solve … by …" p 637 "Use … to …" pp 421, 358 "Determine … using …" p 540 "Without using technology, graph …" p 617 "Estimate the value mentally" then use the calculator to verify—p 30
Instances where the method to be used to solve the problem or answer the question is not given	53[3]	"Simplify" p 101 "Sketch" pp 453, 498 "Write … as …" p 101 "Determine" pp. 498, 574 "Write the equation" p. 574 "Calculate" p. 230, 498, 508 "Verify the solution." p. 617 "Arrange the following" p. 270 pp. 150, 226, 255, 421 contained multiple-choice and/or numeric-response questions
Instances where alternative representations are given or expected as a response	1	"Provide two sets of answers to the problem" p 474
Instances where a unique question is asked or a unique response is expected	3	"Explain two errors" p 474 "Explain clearly how to use the graph to determine …" p 474 "Match each item in List 1 … with the equivalent item in List 2 ... Each item in list 2 may be used once, more than once, or not at all." p 240

the matching question on page 240. This question was also challenging in that one set of characteristics to be matched contained more choices than the other set, so not all of the items were to be used. The question also stated that "each item in List 2 may be used once, more than once, or not at all" (p 240). Including this statement makes the question even more challenging, because students would have to consider each option several times before matching it to the appropriate choice.

Practice Tests

As the practice tests represented a small portion of my overall sample (only 3 of 33 pages), there was only a limited variety of questioning for analysis. According to the documentation accompanying *Foundations*, "the last lesson in each unit is a practice test which the students can complete at home or in class if time allows" (Appleby and Ranieri 2010, 1). The questions that make up the majority of each practice test are multiple choice and numeric response that ask for an answer to be chosen or given. On each practice test there is one written-response question that consists of multiple parts.

The results of my analysis of the three practice test pages with respect to the categories of questions asked are shown in Table 3 below.

In the questions on these pages, there was no expectation of different solution methods or strategies, though the students could use whatever strategy they chose to answer the multiple-choice or numeric-response questions. The sample contained one written-response question from the practice tests, which asked the students to explain why a particular card in a card game was valued at a specific value. There was no evidence that alternative strategies to complete the questions were valued or expected.

Conclusion

My analysis of the *Foundations of Mathematics and Pre-Calculus Grade 10 Workbook (for Math 10 Combined)* (Appleby and Ranieri 2010) answered my query whether the workbook is reflective of the intent of the program of studies. I did not see strong evidence that this resource fully supports the philosophy of the Alberta Mathematics 10–12 program of studies (2008). The most common question styles in the analyzed pages asked the teacher or the student to provide a solution either by a designated method or by a method that was assumed to have been taught in the lesson. There are few instances of students being asked to communicate their mathematical understanding or to express why. I found very few questions that challenged students to think about what they were being asked to do or to question the validity of the processes and procedures they were being asked to use. My conclusion is that the use of the *Foundations of Mathematics and Pre-Calculus Grade 10 Workbook (for Math 10 Combined)* (2010) for instruction would not support the philosophy of mathematics

Table 3

Category	Number of Instances	Examples
Instances where students/teachers are asked to explain, describe or state a rule	0	
Instances where the method to be used to solve the problem or answer the question is given	1	"Susan solves … by …" p 660
Instances where the method to be used to solve the problem or answer the question is not given	8	pp 58, 610, and 660 contained multiple-choice and/or numeric-response questions
Instances where alternative representations are given or expected as a response	0	
Instances where a unique question is asked or a unique response is expected	0	

education as presented in the Alberta mathematics Grades 10–12 program of studies.

Notes

1. For ease of reading, I will use the word *Foundations* to refer to *Foundations of Mathematics and Pre-Calculus Grade 10 Workbook (for Math 10 Combined)* (2010).

2. This is not meant to be an exhaustive list of the types of questions in the assignments; it is just a sampling of the assignment questions students would encounter.

3. Many of the questions included in these pages contained multiple parts that had the same instruction and thus were not counted separately.

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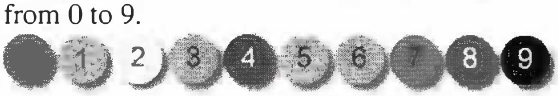
Irritating Things

Krista Francis-Poscente, Sharon Friesen and Trevor Pasanen

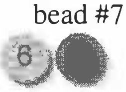
Beading with numbers can be an exciting way to engage children's mathematical thinking and develop their proficiency in addition. In this paper, we will explore a hands-on problem, "Irritating Things" (Galileo Educational Network Association [GENA] 2009), which combines colour coding with integers from 0 to 9, to create colourful beaded bracelets. In solving this problem, students will use logical thinking and addition to translate the numbers into beads. The patterns and relationships in this problem can be represented and described with words, tables, numbers and bracelets. The multiple representations will help students organize and communicate their ideas. By working with multiple representations, students will gain flexibility in their thinking and develop their proficiency in addition. The problem has multiple solutions; therefore, students can experience different ways to solve a problem. Working with problems with multiple solutions and representations gives students opportunities to discuss and learn each other's problem-solving strategies and solutions. This might help them gain deeper understanding of addition, pattern and their own reasoning ability. "Irritating Things" easily lends itself to differentiated instruction, giving all students opportunities for challenges and success. While the problem has considerable depth and can extend into computer programming, coding theory and discrete mathematics at a university level, the following activities are targeted to Grades 4 to 6.

"Irritating Things" Problem

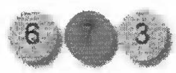
There are 10 beads of different colours, numbered from 0 to 9.



1. Pick a first and second bead. They can be the same number, or not. For example, pick bead # 6 and bead #7.



2. To get the third bead, add the numbers on the first and second beads. If the sum is more than 9, just use the last (ones) digit of the sum. Adding 6 + 7 equals 13. Using the rules, drop the first digit and use 3.



3. To get the next bead, add the numbers on the last two beads used, and use only the ones digit.

4. Keep going until the first and second beads repeat, in that order.
5. Tie them in a loop to make a bracelet. (Don't use the last two beads, since they ust repeat the first two beads.)

Instructional Strategies

Day 1: Creating a Colour-Coded Addition Key

Before students begin beading, we suggest that students make their own colour addition chart to help them with the beading addition. In the top row, start with the + sign in the uppermost left-hand corner. Sequentially, place the numbers 0 through 9 in each column of the top row. In the left-most column, sequentially place the numbers 0 through 9. Then fill in the table with the accurate operations and colour.

Developing this colour-coded addition key should take one 45-minute class. It is important that all the students work with the same colour codes—for example, a red bead should always be 0. If everyone has the same colour code, finding addition errors will be much easier. Also, in order that patterns in the bracelets be recognized, all beads have to use the same number representation. See the appendix for a template that can be used in class.

Figure 1: Example of a colour-coded addition key

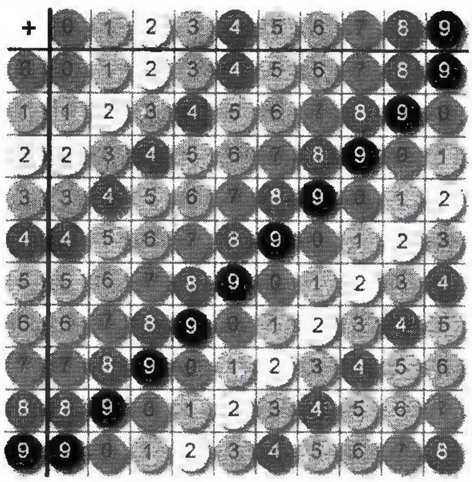
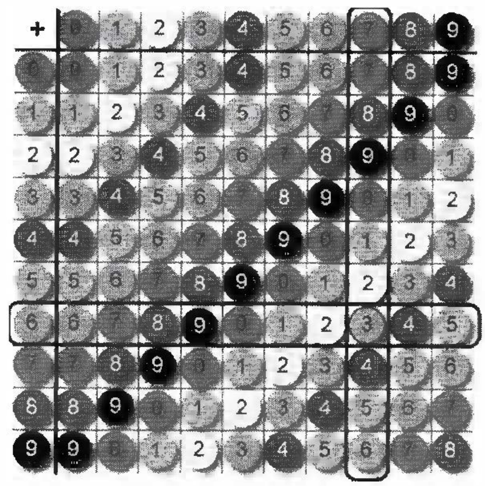


Figure 2: Example of adding 7 + 6 on the colour-coded addition key



Day 2: Introducing the Problem

Materials needed:

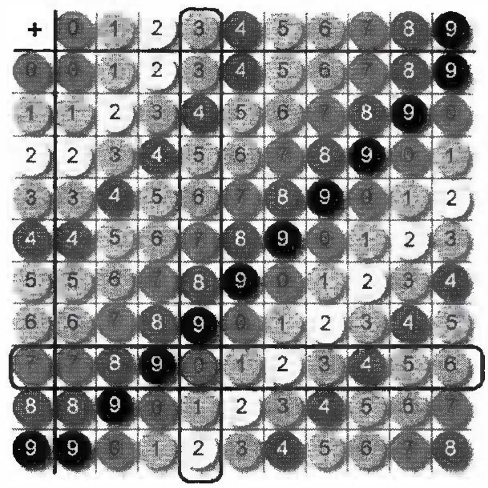
- Numerous beads with at least 10 different colours. Each group will need approximately 150 different-coloured beads. Inexpensive opaque pony beads from craft stores work well.
- Thick cording or shoelaces to thread beads—five strings per pair of students.

Demonstrate to the students how to begin to create their bracelet. Pick two beads, for example, . . Add the numbers on the beads together: + = 13. There are no beads that represent 13, so there is one more step to find the next bead. The rule is to drop off the first digit. When the first digit is removed from 13, the next bead is . Reinforce how to find the next bead with the colour-coded addition key. A SmartBoard is useful for demonstrations with the colour-coded addition key. Circle the row beside the 6 and the column under the 7. The number 3 is the bead that is where the 6 row and 7 column intersect. Students can follow their colour-coded addition key with their fingers.

Next, string the third bead on the shoelace .

To find the fourth bead, add the last two beads together: + = 10. There is no bead that represents 10, so when the first digit is dropped, a remains. Demonstrate how to find the fourth bead with the colour-coded addition key. Circle the row beside the 7 and the column under the 3. The number 0 is the bead where the 7 row and the 3 column intersect.

Figure 3: Example of adding 7 + 3 with the colour-coded addition key



String the fourth bead on the shoelace.

Encourage students to work in pairs, checking their work as they put each new bead on the bracelet. Pairing students together encourages collaboration and makes the problem easier to tackle. Persuade each student to be responsible for the accuracy of the addition. Plan for the bracelet making to take a couple of classes to complete.

Day 4 or 5: Finding the Patterns in Class Discussion

Find bracelets that are the same size. Have students compare their bracelets. Lead them to discover that bracelets of the same size are actually the same bracelet. Starting with any two beads on the bracelet eventually results in the same bracelet. For instance, if a student picked 4 and 2, the bracelet would be the same bracelet with the same length if they picked 8 and 4.

Another interesting pattern is that when the total lengths of all the bracelets are added: 1 + 3 + 4 + 12 + 20 + 60 = 100. 100 equals the total number of beads in the rows and columns of the colour coded addition key. 100 equals 10 rows times 10 columns for multiplication—the number of different ways beads can pair together.

Solutions to "Irritating Things"

There are exactly six different bracelets, with the numbers of pairs of beads of 1, 3, 4, 12, 20 and 60. Expect many errors in addition. If the students have created a bracelet with a total number of pairs beads that is not equal to 1, 3, 4, 12, 20 or 60, you can be certain that an addition error has been made. Have the students find their error. If available, a few parent volunteers could help students find the error. Once an error has been found, remove all the beads past the error. In our experience, students express dismay when the error is found and they have to pick up from the mistake. However, they very quickly go back to beading again.

One idea for differentiating is to encourage a group with slower addition skills to start with one of the smaller bracelets. Pick out the beads for 9 and so that students will find a bracelet that is 12 beads long, or pick out the beads 2 and 8 for a pair to find a bracelet that is 20 beads long. They will find success more quickly with a smaller bracelet.

When students find their bracelet, there are several more to be found. The students with quicker addition skills can find more bracelets. Ask them to find a different-sized bracelet. With differentiation, the entire class can be engaged.

Mapping to the Program of Studies

"Irritating Things" addresses the mathematical processes discussed in the front matter of the mathematics program of studies (Alberta Education 2007). "Irritating Things" connects addition to pattern; encourages the development of fluency with addition, visualization of addition and pattern; and develops mathematical reasoning through problem solving. The colour-coded addition key is a guide that can correspond to developing strategies for mental addition.

Students solve this problem primarily by using whole-number addition to discover a repeating pattern. Throughout the problem solving, students are translating whole numbers to colour-coded beads. The chart provides another representation to facilitate the translation. The representation of whole-number addition symbolic beads and a pictorial chart address many specific outcomes in the program of studies. The following figure outlines the specific outcomes by grade.

Algebraic Extension

The steps for addition and dropping the 10s column for addition provide an excellent exploration for algebra and the distributed property. The following explanation is beyond the elementary levels of the activities, but might be useful for further understanding of the problem.

This is an algebraic explanation of the solution. Consider the first bead as a and the second as b. The first bead can be represented as $1a + 0b$, and the second as $0a + 1b$. When you add them together—$1a + 0b + 0a + 1b$, you get the third bead $1a + 1b$. The sequence of subsequent beads follows:

$1a + 0b$
$0a + 1b$
$1a + 1b$
$1a + 2b$
$2a + 3b$
$3a + 5b$
$5a + 8b$

Solution:

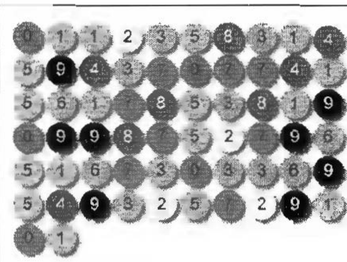
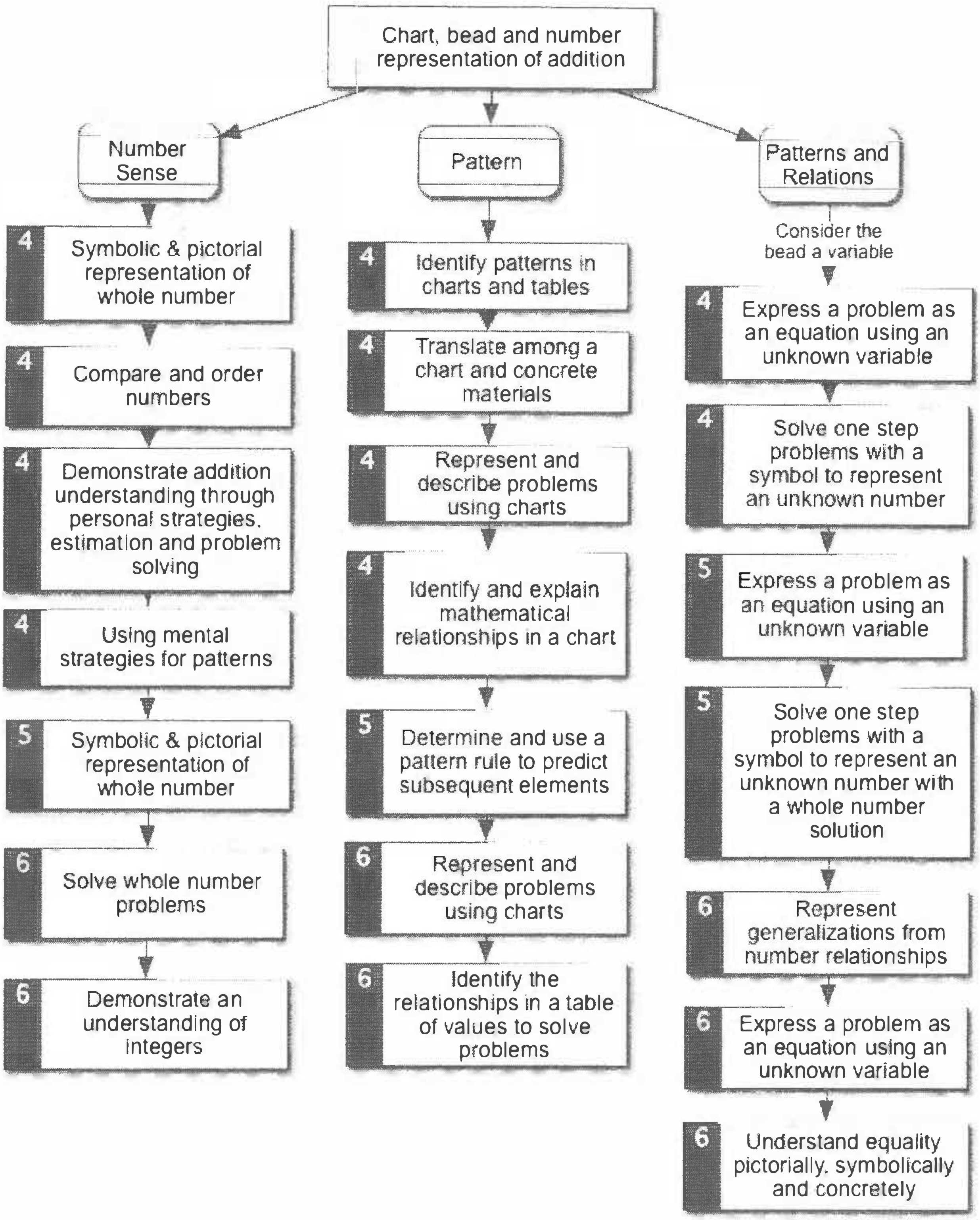
# of pairs	1	3	4	12	20	60
Sequence	0	0-5-5	4-2-6-8	2-1-3-4-7-1-8-9-7-6-3-9	2-2-4-6-0-6-6-2-8-0-8-8-6-4-0-4-4-8-2-0	

Figure 4: Specific outcome curriculum mapping for Grades 4 to 6 (Alberta Education 2007)



The next bead requires the special condition of dropping the 10 digit. We need to apply the distributed formula to see how this works. The next bead in the sequence is $8a + 13b$. Using the distributive formula for b, we get $8a + (10 + 3)b = 8a + 10b + 3b$. However, the rule means that $10b$ is equal to zero. Thus, the next bead is $8a + 3b$. The sequence continues as follows:

$8a + 3b$
$3a + 1b$
$1a + 4b$
$4a + 5b$
$5a + 9b$
$9a + 4b$
$4a + 3b$
$3a + 7b$
$7a + 0b$
$0a + 7b$
$7a + 7b$
...

Continuing with this algebraic sequence would eventually lead to 60 pairs of beads before the sequence repeated: the maximum size of the bracelet. This is one algebraic proof of the maximum size. Notice that if a or b equals 0, then the bracelet will be the maximum bracelet size. If a and b both equal 0, then the minimum bracelet is found. What bracelets are found with other values of a or b?

Conclusion

"Irritating Things" provides an exciting opportunity to explore addition and pattern with problem solving. Creating a colour-coded addition key reinforces the correspondence of number and addition. Building the bracelets provides a non-pencil-and-paper method for developing proficiency with addition. Finding that each bracelet of the same size is the same bracelet is an exciting discovery of pattern. Importantly, this set of activities is rigorous, demanding and fun.

"Irritating Things" is a rich problem that can be explored in different ways. For instance, what happens when a different number of beads is used? How about 8 beads? How about 12 beads? Are there more bracelets? Or are there fewer? How long is the longest bracelet? How long is the shortest? Using different numbers of beads takes students into operations with a non-10 base. Most of all, have fun exploring this investigation into addition and pattern.

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Appendix: Colour-Coded Addition Key Template

+	⓪	①	②	③	④	⑤	⑥	⑦	⑧	⑨
⓪	○	○	○	○	○	○	○	○	○	○
①	○	○	○	○	○	○	○	○	○	○
②	○	○	○	○	○	○	○	○	○	○
③	○	○	○	○	○	○	○	○	○	○
④	○	○	○	○	○	○	○	○	○	○
⑤	○	○	○	○	○	○	○	○	○	○
⑥	○	○	○	○	○	○	○	○	○	○
⑦	○	○	○	○	○	○	○	○	○	○
⑧	○	○	○	○	○	○	○	○	○	○
⑨	○	○	○	○	○	○	○	○	○	○

Online Calculus Course: Combining Two Worlds

Veselin Jungic and Jamie Mulholland

Introduction

The purpose of this article is to describe our experience in creating, promoting and running a web-based differential calculus course that has been offered through the Centre for Distance Education (CODE) at a Canadian university.

In May 2007, we were asked to design an online version of one of the courses offered by the department of mathematics at a Canadian university. At that time, none of the department's mainstream mathematics courses was being offered by distance education. The most natural place to start was the first-semester calculus course, Math 150: Calculus I with Review. Math 150 covers standard topics in introductory differential calculus. It is designed to go through the required material at a somewhat slower pace, giving enough time for the instructor to do examples in more detail and to spend more time communicating the important ideas that form the base of this mathematical field. Both of us have taught variations of this course a number of times over the past several years. Over those years we developed all class material together: notes, online assignments, paper assignments with solutions, a repository of exam questions, exam checklists, demos and so forth. All of this material was created and later edited in electronic form.

There are two main reasons why we accepted the challenge of creating a web-based calculus course:

- We felt that while building an online course we could create additional material that might be used in teaching our live courses.
- We wanted to experiment with the available technology and technological support provided by CODE to enhance the course material that we had created over the years.

From the very beginning our approach was to create an online version of the course that would be as similar as possible to our live offerings. The reasoning behind this approach was based on our belief that the ultimate responsibility of the mathematics instructor, even at the lowest level, is to lead each student through the course in a reasonable way, making sure that the student gets a fair chance, with an appropriate amount of work, to complete the course to the best of his or her abilities. In other words, we believe that the instructor's role is to be a demonstrator, a motivator, a moderator and a (fair!) evaluator. Hence, our starting premise in teaching mathematics is that a motivated student, in the appropriate learning environment and with the right support, has a chance to develop his or her mathematical talent to its fullest. Since the structure of our live classes is based on this premise, our view was that the online course should keep the same structure.

We note that mimicking live courses is not a common approach in teaching distance education math courses. For example, Akdemir (2008) claims that, "online learning requires a radical change in the way educators do business."

Clearly the main difference between any live course and its online version is in how lectures are delivered. Delivering mathematical content in a video lecture is not new. Academic Earth, iTunes University, Algebra 2 Go, WatchKnow and YourOtherTeacher, to name a few, host numerous video lectures at all levels of mathematics. Delivery of content in these lectures, however, remains somewhat uniform. They are either videotaped live lectures or video screen captures of a computer screen with a voice-over. In our view, there is value in a hybrid of live lecture coupled with computer screen captures, even though this approach is still in its infancy. In creating our course we focused on this hybrid approach and also on improving navigation through the recordings.

A seemingly simple fact—that the level of involvement of the instructor in an online course is substantially different from the level of involvement of the instructor in a face-to-face class---came as a surprising discovery for us during the first offering of the course. This discovery has led us to better appreciate our everyday interaction with students in our live classes. Also, it became clear to us that the instructor's role is the single biggest obstacle in an attempt to truly mimic a live offering of a course in its online version.

Creating the Course

In addition to the problem of delivering mathematical content, creators of an online lower-division university math course, in our view, have to deal with the following two important issues:

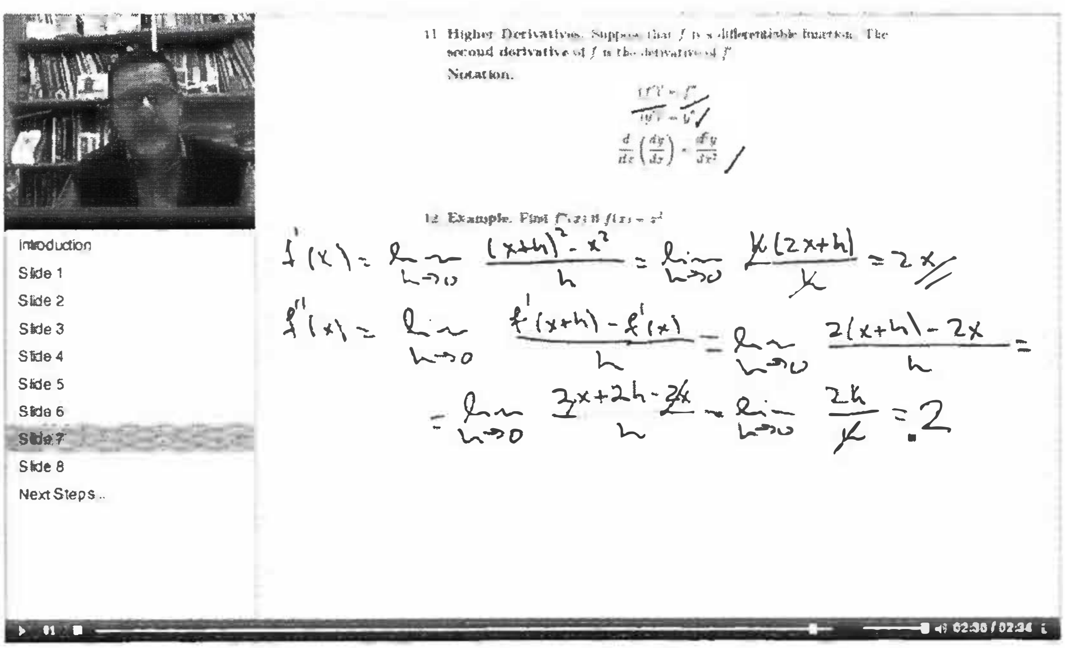
- Structuring the course in a way that each student has to do a fair amount of work during each week of classes on his or her own—as the saying goes, "Mathematics is not a spectator sport."
- Taking full responsibility to ensure that a student who completes the online course is ready to take the next math course in the sequence.

To meet these two challenges, we abided by the following principle: to provide the students taking our distance education course with an experience as close as possible to the live classroom. Thus, our online course has the same structure as the live course and uses the same material: the same class notes are used in lectures, the same online assignments and the same concept of paper assignments are used to check students' weekly progress, and the two midterms and the final exam are created from the same already existing repository of exam questions. The main and obvious difference is that we deliver the content of our lectures using video lectures. Our videos feature two windows on the same screen: one shows the instructor's image and the other serves as a notepad for the instructor's writing or as a screen for various demonstrations (see Figure 1). Students are directed to download a skeleton outline of the notes and follow along with the video lecture to fill in the details. Having the instructor's face (and upper body) in the video means that we don't have to constantly be writing; we can underline what we are saying by using body language, making gestures and facial expressions, much as we do in class. Also, in this video we include animations and applets that have been created over the last few years to help students build their conceptual understanding of the material. All this is synchronized with the audio and video recordings of the instructor's comments and explanations.

As mentioned above, throughout the semester the students in the course have to do a significant amount of work on their own. For example, students are assigned weekly readings from the textbook, weekly practice problems from the textbook, and weekly paper and online assignment questions. The online questions are made in such a way as to encourage students to carefully go through each lecture and the course notes and/or use the textbook (see Figure 2 for a sample question). The paper assignment questions

Figure 1: A video lecture features two windows: one contains the instructor's face and upper body; the other contains course notes or demonstrations.

Section 2.8: The Derivative as a Function



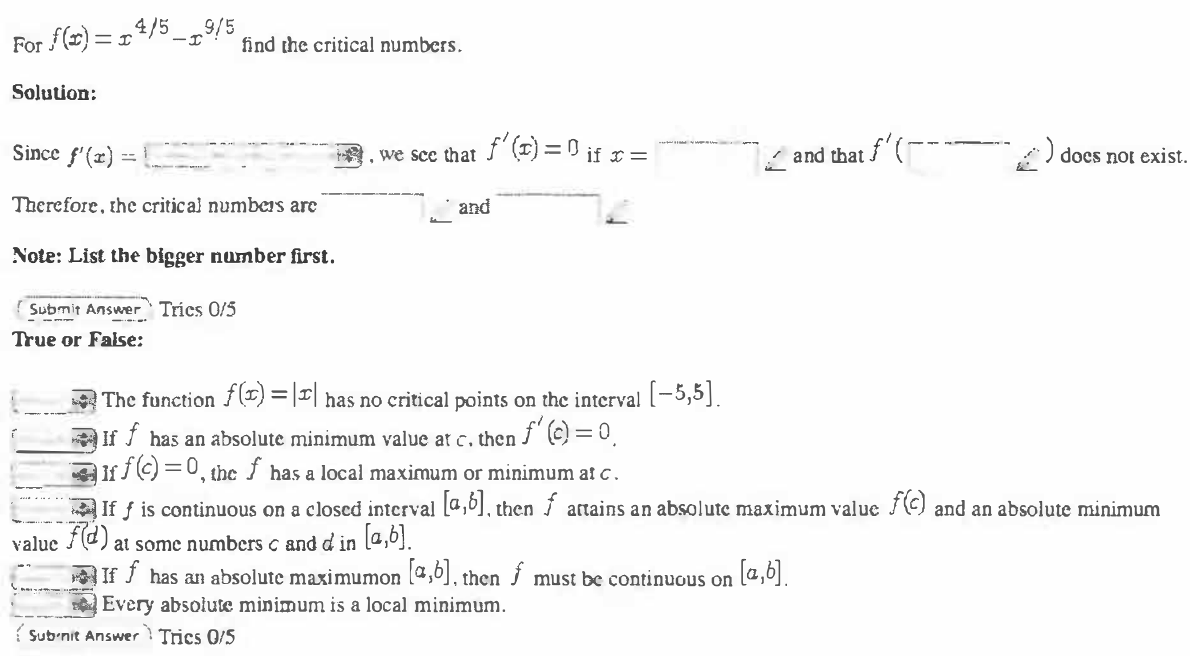
are chosen from various sources, including other textbooks and old exams, or they are constructed by the instructor, and are generally more challenging than practice problems. This means that students are expected to do 25 to 30 problems per week on their own. On average about 10 of those problems are submitted and fully marked, either online or by a teaching assistant. In our view, an important aspect of this process of students' learning is that they can discuss course content and assigned problems among themselves in chat rooms and on discussion boards on the website. Much valuable discussion takes place on discussion boards attached to each online assignment question, and since each student receives a different (randomized) question, the discussion is focused more on conceptual understanding of the material rather than on identifying the right answer. Both of these boards are constantly monitored and moderated by the teaching assistants and instructors. Students might contact the teaching assistant by e-mail or phone during teaching assistant's office hours to ask for advice regarding homework or a practice question.

This is similar to how our live course is taught. Again, we underline the fact that all the material—the skeleton outline of the notes, applets, animations, paper and online assignments—is the same as we use in the live class.

The ultimate dream of any math instructor is to have his or her students actively involved in lectures. This is an everyday challenge in our classrooms, and it seems to be another big obstacle in delivering an online math course. We believe that our concept, with captured audio and video images of the instructor explaining concepts to the viewer and completing the notes that are on the paper in front of the viewer, demands that the online student be an active participant in the lecture. We closely tie all our assignment and midterm exam questions to the course lectures to emphasize the importance of attending each lecture and using all the provided additional material.

As we have already mentioned, in our view the course instructor has full responsibility for all aspects of the course. One of the important aspects in teaching mathematics is the instructor's role as a moderator and a mediator. By *mediator* we mean the math instructor's role as a link between students and the mathematical ideas and techniques that students need to grasp.[1] By *moderator* we mean the math instructor's role as one who directs the learning process.[2] In a live classroom the instructor talks to a group of students and, based on the group's reaction (a question

Figure 2: A sample question on the online homework assignment. Parameters of the question are randomly generated, and the submission is computer graded.



For $f(x) = x^{4/5} - x^{9/5}$ find the critical numbers.

Solution:

Since $f'(x) =$ ______, we see that $f'(x) = 0$ if $x =$ ______ and that $f'($ ______ $)$ does not exist.

Therefore, the critical numbers are ______ and ______

Note: List the bigger number first.

Submit Answer Tries 0/5

True or False:

The function $f(x) = |x|$ has no critical points on the interval $[-5,5]$.

If f has an absolute minimum value at c, then $f'(c) = 0$.

If $f(c) = 0$, the f has a local maximum or minimum at c.

If f is continuous on a closed interval $[a,b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a,b]$.

If f has an absolute maximumon $[a,b]$, then f must be continuous on $[a,b]$.

Every absolute minimum is a local minimum.

Submit Answer Tries 0/5

during the lecture or sudden silence in the classroom or puzzled expressions on students' faces, for example), she can usually perceive a problem and intervene accordingly. Hence, the instructor's mediation of the particular math topic and its moderation in a live classroom are subject to the interaction between the instructor and students in the class. On the other hand, we make our recordings as though we are talking directly to the viewer, as explained earlier. We are convinced that this is the right method to use when recording math lectures for online courses, but we are aware that this implies that we are taking a one-size-fits-all approach. We acknowledge this important limitation of our recorded lectures. Regarding the instructor's role as a mediator and a moderator, this limitation stresses the significance of the other elements of the course (notes, readings, assignments and discussion boards) and the importance of the quality of the recordings, what was done and said, which applets were used, and so forth.

We faced a contradictory situation during the first two offerings of the online course. As the course instructors and creators, we felt responsible for everything that was related to the course, from checking that all resources were posted in a timely fashion on the course website to assigning final marks. At the same time, we realized that the nature of an online course requires that a whole team of people works behind the scenes making sure that

- the website is running properly,
- paper assignments are collected on time and passed on to the teaching assistant for marking, and
- multiple sites, together with invigilators, are booked for writing midterm and final exams, and so forth.

This coexistence (rather than collaboration) between the instructor of an online math course and anonymous administrative and technical helpers is not without its negative consequences. For example, to put paper assignments provided by the creators of the course into the standard CODE format and not being familiar with LaTeX,[3] we had a CODE employee convert the original .pdf files into .doc files, edit them and convert them back into .pdf files. In this process, the assignment questions got mixed, the notation got lost, and the beauty of LaTeX got destroyed. Another problem is that CODE expects that the main contact for students during the semester is the teaching assistant in the online course—thus the contradiction between the level of responsibility that we as the course instructors assumed and the fact that we were not expected to be too involved in the day-to-day running of the course.

Here we mention a few pitfalls. Prerecorded lectures make it impossible to ask and answer questions. Not having students sitting beside each other to confirm understanding is a drawback (however, the benefit of the videos is that there is a rewind button, so this has an advantage over the live class—we used this fact to add a bit of humour to one of our promotional videos). Another pitfall is that our live course is serviced by a drop-in tutorial centre where students have access to teaching assistants five days a week. Since the vast majority of students enrolled in the online course are not on campus, they are not in position to use the drop-in centre. This implies that students in the online course lack opportunity for their work to be checked and corrected (or praised!) while they are completing the assignment. Even though we and the teaching assistant monitor discussion boards on a regular basis, we find that it has been difficult to match the communication aspect of the course with its equivalent in our live courses. For example, in our first offering we had a mature student working full-time and taking our online course. After scoring low on the first midterm the student expressed his disappointment and frustration by posting a message on the course discussion board. We sent him an e-mail to encourage him to keep studying, but this caused even more frustration on the student's part. In our experience, situations like this in a live course would be dealt with in a one-on-one conversation between the instructor and the student, and a resolution satisfactory to both sides would be more likely.

The hardware we used to create the videos was a tablet PC and an external webcam and microphone. We created the note templates with LaTeX and used PowerPoint to annotate the notes during the lecture. Video screen capture and audio processing were done using Camtasia. After the technician at CODE gave us a quick tutorial on using Camtasia, we were left to do all the recordings ourselves. This was a long and, sometimes, very painful process. Here are some points to keep in mind.

- Our estimate is that for each hour of recorded lecture we had to spend about four hours preparing/rehearsing, recording, viewing and reworking.
- We believe making recordings of this kind requires the presence and involvement of at least three people: the lecturer, a technician and another mathematician. The role of the second mathematician would be to spot mistakes, either spoken or written, and alert the lecturer to correct them right away.[4]
- A professional should manage the recording technology. In our experience, manipulating even relatively simple technology distracts the lecturer and

causes unnecessary mistakes, both in what is said and in the functioning of the technology.

- A technician from CODE did all the editing of the recordings. We believe the editing process should be a joint project between the lecturer and the professional technician. We have witnessed that for this generation of students even small editorial glitches or an abrupt transition between slides, for example, can cause frustration.

During the first two offerings of the online Math 150 course the results on assignments and midterm exams matched results from our live courses.

To promote the first offering of the course, we created three short videos and posted them on websites of the Department of Mathematics and CODE. The clips are also posted on YouTube. Two of those clips (Jungic and Mulholland 2009a, 2009b) humorously promoted the convenience of an online course (see Figure 3). The third clip (Jungic and Mulholland 2009c) explains in the detail how the course works and what the course website contains. The clips attracted a significant level of interest from the university community, and we see that they have inspired some of our colleagues to present their courses in a similar fashion.

Conclusion

We conclude by describing our experience using the recorded lectures as a supplement to our live courses. After each live lecture we posted our recording of the same lecture on the course website. We are aware of the risk that there might be students who would decide not to come to the lecture (which is normally held at 8:30 AM), but our experience has been that the vast majority of students use the recordings in the way that we anticipated. The following quote, from a student in the live class taught by the second author in the fall semester of 2009, supports this claim. "I remember a few lectures ago you mentioned your online lessons and I figured I would give them a shot. Personally, I found them very helpful (I watched them all already). They allowed me to fill in any notes I missed and gave me a handy review to help me through my homework. Nearly every question I had regarding my notes was easily solved by simply going to the respective video. This is a great idea. I think every teacher should do this."

Notes

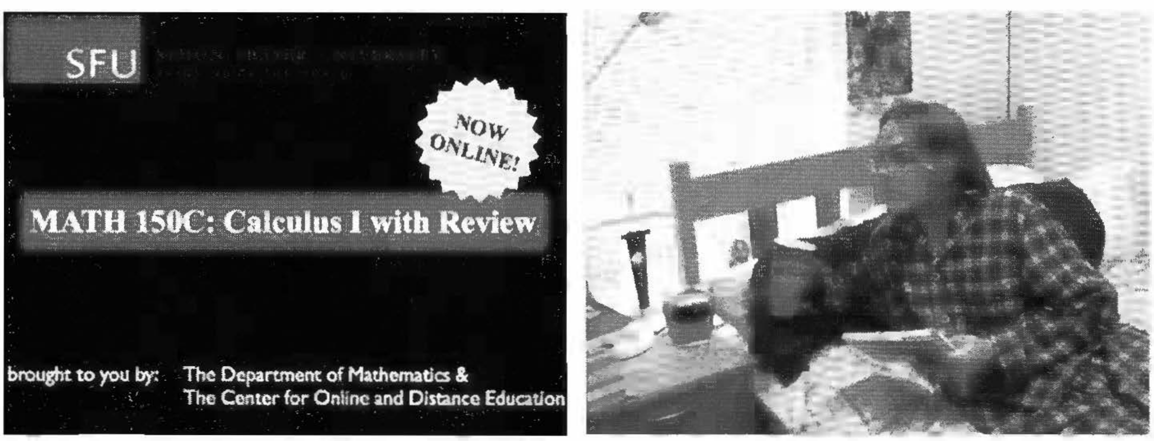
1. Each math instructor brings his or her own knowledge, understanding (or interpretation) and emotions into teaching a particular math topic. Thus, two instructors might mediate the same material to their students in different ways.

2. An instructor who needs to introduce (mediate) the idea of the limit of a function to his 8:30 am calculus class for engineers and his 11:30 am calculus class for social science students will probably moderate the topic in two different ways.

3. Editor's note: LaTeX is a typesetting system that is most often used for the production of technical and scientific documents. More information is available at www.latex-project.org.

4. We learned the hard way that the camera has no mercy; misprints, dysfunctional technology, stumbling, or a phone ringing in the background might momentarily destroy a recording of the best lecture the world was about to witness.

Figure 3: Scenes from the promotional video, which features a student struggling to get to his 8:30 am class on time. The student then finds the online course a convenient alternative.



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The Alberta High School Mathematics Competition
Part I, November 16, 2010

1. The number of positive integers n such that the number $4n$ has exactly two digits is

   (a) 21 (b) 22 (c) 23 (d) 24 (e) 25

2. A 4×6 plot of land is divided into 1×1 lots by fences parallel to the edges of the plot, with fences along the edges as well. The total length of fences is:

   (a) 58 (b) 62 (c) 68 (d) 72 (e) 96

3. The greatest common divisor and least common multiple of two positive integers are 1 and 10 respectively. If neither of them is equal to 10, their sum is equal to

   (a) 3 (b) 6 (c) 7 (d) 11 (e) none of these

4. The number of pairs (x, y) of non-negative integers such that $3x + 2y = 27$ is

   (a) 4 (b) 5 (c) 8 (d) 9 (e) 10

5. In the sequence 1, 2, 3, 4, 6, 7, 8, 9, ..., obtained by deleting the multiples of 5 from the sequence of the positive integers, the 2010th term is

   (a) 2511 (b) 2512 (c) 2513 (d) 2514 (e) none of these

6. Alice, Brian, Colin, Debra and Ethel are in a hotel. Their rooms are on floors 1, 2, 3, 21 and 40 respectively. In order to minimize the total number of floors they have to cover to get together, the floor on which the get-together should be is

   (a) 18 (b) 19 (c) 20 (d) 21 (e) none of these

7. A square pigeon coop is divided by interior walls into 9 square pigeonholes in a 3×3 configuration. Each of two pigeons chooses a pigeonhole at random, possibly the same one. The probability that they choose two holes on the opposite sides of an interior wall is

   (a) $\frac{1}{18}$ (b) $\frac{1}{9}$ (c) $\frac{4}{27}$ (d) $\frac{8}{27}$ (e) $\frac{1}{3}$

8. The set of all values of the real number x such that $\frac{1}{x} \leq -3 \leq x$ is

   (a) $\{x \leq -1/3\}$ (b) $\{-3 \leq x \leq -1/3\}$ (c) $\{-3 \leq x < 0\}$

   (d) $\{-1/3 \leq x < 0\}$ (e) none of these

9. In the quadrilateral $ABCD$. AB is parallel to DC, $DC=2AB$. $\angle ADC=30^\circ$ and $\angle BCD=50^\circ$. Let M be the midpoint of CD. The measure of $\angle AMB$ is

(a) 80° (b) 90° (c) 100° (d) 110° (e) 120°

10. We are constructing isosceles but non-equilateral triangles with positive areas and integral side lengths between 1 and 9 inclusive. The number of such triangles which are non-congruent is

(a) 16 (b) 36 (c) 52 (d) 61 (e) none of these

11. In each of the following numbers, the exponents are to be evaluated from top down. For instance, $a^{b^c} = a^{(b^c)}$. The largest one of these five numbers is

(a) $2^{2^{2^{2^3}}}$ (b) $2^{2^{2^{3^2}}}$ (c) $2^{2^{3^{2^2}}}$ (d) $2^{3^{2^{2^2}}}$ (e) $3^{2^{2^{2^2}}}$

12. A gold number is a positive integer which can be expressed in the form $ab+a+b$, where a and b are positive integers. The number of gold numbers between 1 and 20 inclusive is

(a) 8 (b) 9 (c) 10 (d) 11 (e) 12

13. The edges DA, DB and DC of a tetrahedron $ABCD$ are perpendicular to one another. If the length of DA is 1 cm and the length of each of DB and DC is 2 cm, the radius, in cm, of the sphere passing through A, B, C and D is

(a) $\frac{3}{2}$ (b) $\frac{\sqrt{5}+1}{2}$ (c) $\sqrt{3}$ (d) $\sqrt{2}+\frac{1}{2}$ (e) none of these

14. Let $f(x) = x^2$ and $g(x) = x^4$. We apply f and g alternatively to form

$$f(x) = x^2,\ g(f(x)) = g(x^2) = (x^2)^4 = x^8,\ f(g(f(x)) = f(x^8) = (x^8)^2 = x^{16},$$

and so on. After we have applied f 50 times and g 49 times, the answer is x^n where n is

(a) 148 (b) 296 (c) 2^{148} (d) 2^{296} (e) none of these

15. Triangle ABC has area 1. X, Y are points on the side AB and Z a point on the side AC such that $XY = 2AX$, XZ is parallel to YC and YZ is parallel to BC. The area of XYZ is

(a) $\frac{1}{27}$ (b) $\frac{2}{27}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$ (e) $\frac{1}{3}$

16. The number of integers n for which $n^3 - 3n + 2$ is divisible by $2n + 1$ is

(a) 3 (b) 4 (c) 5 (d) 6 (e) 8

The Alberta High School Mathematics Competition
Solution to Part I, 2010

1. Since $10 \leq 4n \leq 99$, $3 \leq n \leq 24$. Hence there are $24 - 3 + 1 = 22$ such values. The answer is **(b)**.

2. There are 7 fences of length 4 and 5 fences of length 6. The total length is $7 \times 4 + 5 \times 6 = 58$. The answer is **(a)**.

3. Since the least common multiple is even, at least one number is even. Since the greatest common divisor is odd, exactly one number is even. We can show in a similar manner that exactly one of the two numbers is divisible by 5. Since neither is 10, one of them is 2 and the other is 5, yielding a sum of 7. The answer is **(c)**.

4. Note that x must be odd, and $x = 9 - \frac{2y}{3}$. Since $y \geq 0$, $x \leq 9$. Thus there are 5 triples $(x, y) = (9, 0)$, $(7, 3)$, $(5, 6)$, $(3, 9)$ and $(1, 12)$. The answer is **(b)**.

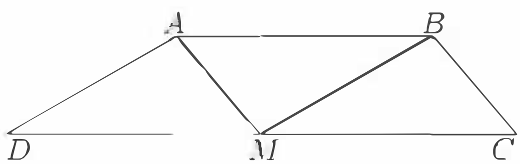
5. Note that $2010 \times \frac{5}{4} = 2512.5$. There are 502 multiples of 5 from 5 to 2510 inclusive. Hence 2512 is the $(2512 - 502)$-th or 2010-th number in the punctured sequence. The answer is **(b)**.

6. Alice and Ethel are 39 floors apart, and as long as the get-together floor is in between, the total number of floors they cover is 39. Similarly, the total number of floors Brian and Debra cover is 19. The minimum number of floors Colin covers is 0, when they get together on floor 3. The answer is **(e)**.

7. The pigeons can choose the pigeonholes in $9 \times 9 = 81$ ways. There are 12 pairs of rooms separated by an interior wall. Since the pigeons can choose these two rooms in 2 ways, the desired probability is $\frac{2 \times 12}{81} = \frac{8}{27}$. The answer is **(d)**.

8. Since $\frac{1}{x} \leq -3$, $x \leq 0$. Multiplying by $-\frac{x}{3}$, we have $-\frac{1}{3} \leq x$. Hence the set of all values of x is $\{-\frac{1}{3} \leq x < 0\}$. The answer is **(d)**.

9. Since $AB = DM$ and AB is parallel to DM, $ABMD$ is a parallelogram. Similarly, $ABCM$ is a parallelogram. Therefore, $\angle AMD = \angle BCD = 50^\circ$ and $\angle BMC = \angle ADC = 30^\circ$. Therefore, $\angle AMB = 180^\circ - \angle AMD - \angle BMC = 180^\circ - 50^\circ - 30^\circ = 100^\circ$. The answer is **(c)**.



10. We first count the triangles in which the equal sides of length k are longer than the third side, which can be of length from 1 to $k-1$. Summing from $k = 1$ to 9, we have $0+1+2+\cdots+8 = 36$ such triangles. We now count the triangles in which the equal sides of length k are shorter than the third side, which can be of length from $k+1$ to $2k-1$. Summing from $k = 1$ to 9, we have $0 + 1 + 2 + 3 + 4 + 3 + 2 + 1 + 0 = 16$. The total is $36 + 16 = 52$. The answer is **(c)**.

11. The first three choices are equal respectively to $2^{2^{256}}$, $2^{2^{512}}$ and $2^{2^{81}}$. Clearly, the second one is the largest among them. The fourth number is equal to $2^{3^{16}}$. Since $2^{512} = 4^{256} > 3^{256} > 3^{16}$, the second number is larger than the fourth one. The fifth number is equal to $3^{2^{16}}$. Clearly, $2^{2^{256}} = 4^{2^{255}} > 3^{2^{255}} > 3^{2^{16}}$. Hence the second number is the largest among the five choices. The answer is **(b)**.

12. Note that $ab + a + b + 1 = (a + 1)(b + 1)$. Every composite number can be written in this form and no prime number can be written in this form. Therefore, the positive integers that are not gold numbers are those that are one less than a prime. By simple counting, we see there are 8 primes from 2 to 21. Therefore, the number of gold numbers between 1 and 20 inclusive is $20 - 8 = 12$. The answer is **(e)**.

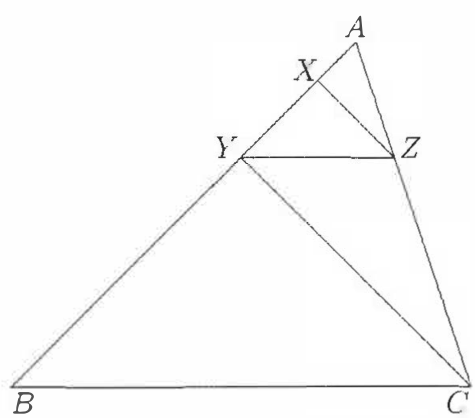
13. The sphere which passes through A, B, C and D also passes through the other four vertices of a $1 \times 2 \times 2$ block having A, B, C and D as four of its vertices. Since the space diagonal of this block is of length $\sqrt{2^2 + 2^2 + 1^2} = 3$, the radius of the sphere is $\frac{3}{2}$. The answer is **(a)**.

14. Each application of f doubles the exponent while each application of g quadruples the exponent. After 50 applications of f and 49 applications of g, the exponent has been doubled $50 + 2 \times 49 = 148$ times so that $n = 2^{148}$. The answer is **(c)**.

15. Denote the area of triangle T by $[T]$. Since triangles AXZ and AYC are similar, $ZC = 2AZ$. Since triangles AYZ and ABC are similar, $YB = 2AY$. It follows that

$$[XYZ] = \frac{2}{3}[AYZ] = \frac{2}{9}[AYC] = \frac{2}{27}[ABC] = \frac{2}{27}.$$

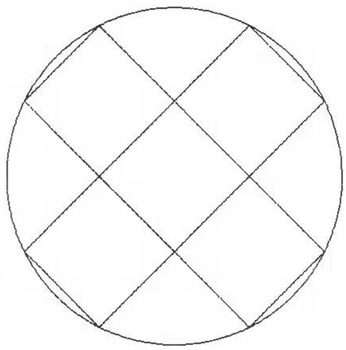
The answer is **(b)**.

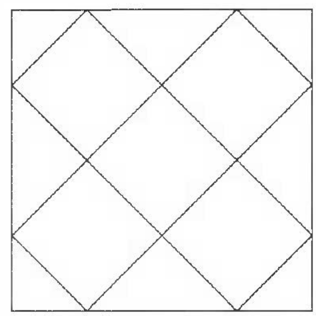


16. When $n^3 - 3n + 2$ is divided by $2n + 1$, the quotient is $\frac{n^2}{2} - \frac{n}{4} - \frac{11}{8}$ and the remainder is $\frac{27}{8}$. Hence $8(n^3 - 3n + 2) = (2n + 1)(4n^2 - 2n - 11) + 27$, so that $2n + 1$ divides $n^3 - 3n + 2$ if and only if it divides 27. The set of all values of n for which $2n + 1$ divides 27 is $\{-14, -5, -2, -1, 0, 1, 4, 13\}$, and there are 8 such values. The answer is **(e)**.

The Alberta High School Mathematics Competition
Part II, February 2, 2011.

Problem 1.
A cross-shaped figure is made up of five unit squares. Determine which has the larger area, the circle touching all eight outside corners of this figure, as shown in the diagram below on the left, or the square touching the same eight corners, as shown in the diagram below on the right.





Problem 2.
There is exactly one triple (x, y, z) of real numbers such that $x^2 + y^2 = 2z$ and $x + y + z = t$. Determine the value of t.

Problem 3.
On the side BC of triangle ABC are points P and Q such that P is closer to B than Q and $\angle PAQ = \frac{1}{2}\angle BAC$. X and Y are points on lines AB and AC, respectively, such that $\angle XPA = \angle APQ$ and $\angle YQA = \angle AQP$. Prove that $PQ = PX + QY$.

Problem 4.
Determine all the functions f from the set of integers to the set of positive integers such that $f(n-1) + f(n+1) \leq 2f(n)$ for all integers n.

Problem 5.
Seven teams gather and each pair of teams play one of three sports, such that no set of three teams all play the same sport among themselves. A triplet of teams is said to be *diverse* if all three sports are played among themselves. What is the maximum possible number of diverse triplets among the seven teams?

The Alberta High School Mathematics Competition
Solution to Part II, 2011.

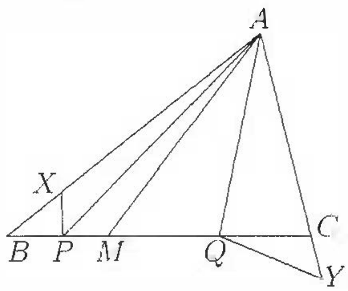
Problem 1.
The diameter of the circle, being the diagonal of a 1×3 rectangle, is $\sqrt{10}$, so the area of the circle is $\pi(\sqrt{10}/2)^2 = \frac{5\pi}{2}$. The diagonal of the square is 4, so the side of the square is $\frac{4^2}{2} = 8$. Since $\pi < 3.2 = \frac{16}{5}$, we have $\frac{5\pi}{2} < 8$. Thus the square has greater area than the circle.

Problem 2.
Eliminating z, we have $x^2 + y^2 = 2(t - x - y)$ so that $(x+1)^2 + (y+1)^2 = 2(t+1)$. In order to have a unique solution for x and y, we must have $2(t+1) = 0$ or $t = -1$.

Problem 3.
Let M be the point on PQ such that $\angle MAP = \angle BAP$. Then

$$\begin{aligned} \angle MAQ &= \angle PAQ - \angle MAP \\ &= \frac{1}{2}(\angle ABC - \angle MAB) \\ &= \frac{1}{2}\angle MAC \\ &= \angle CAQ. \end{aligned}$$

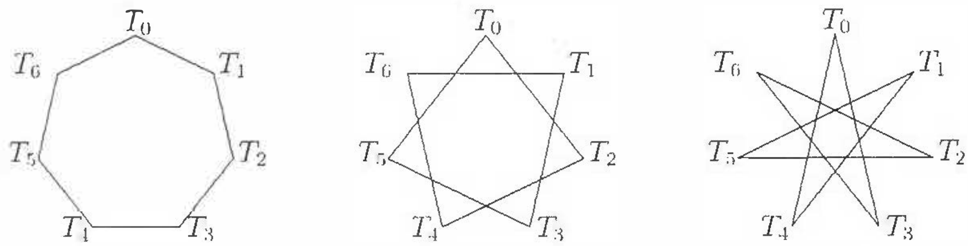
Since $\angle XPA = \angle MPA$, triangles XAP and MAP are congruent by the ASA Postulate, so that $PX = PM$. Similarly, we can prove that $QY = QM$, so that $PX + QY = PM + QM = PQ$.



Problem 4.
Since $f(n)$ takes on only positive integral values, it has a minimum value m. Let n be such that $f(n) = m$. Then $2m \le f(n-1)+f(n+1) \le 2f(n) = 2m$, which implies that $f(n-1) = f(n+1) = m$ also. It follows easily that $f(n) = m$ for all integers n.

Problem 5.

We first show that the conditions of the problem can be satisfied. Construct a graph where the teams are represented by vertices T_i, $0 \leq i \leq 6$. In the diagram below, we partition the graph into three subgraphs. Two teams play each other in the first sport if and only if the vertices representing them are joined by an edge in the first subgraph, the second sport in the second subgraph and the third sport in the third subgraph. None of the subgraphs contains a triangle.



The edges in the same subgraph have the same length, and those in different subgraphs have different lengths. In geometric terms, a diverse triple is a scalene triangle. There is basically one such triangle, namely $T_0T_1T_3$. Six others can be obtained from it by rotation, and seven more by reflection. Thus we may have as many as 14 scalene triangles.

We now prove that there are at most 14 diverse triples. Construct a complete graph on 7 vertices which represent the 7 teams. Paint an edge in the i-th colour if the teams represented by its endpoints play each other in the i-th sport, $1 \leq i \leq 3$. A triangle is diverse if all three sides are of different colours, and non-diverse otherwise. Since there are no monochromatic triangles, a non-diverse triangle has two sides of the same colour. Call the vertex at the junction of the two sides of the same colour its *pivot*. The number of pivots is equal to the number of non-diverse triangles. There are six edges incident with each vertex. If at least 3 of them are of the same colour, then this vertex is the pivot of at least 3 non-diverse triangles. If not, then exactly 2 edges are of each colour, so that the vertex is the pivot of exactly 3 isosceles triangles. Hence each vertex is the pivot of at least 3 non-diverse triangles. Since there are 7 vertices, this brings the total to at least 21, so that the maximum number of diverse triangles or diverse triples is 14.

Print ID #: ______________

Total = ___ / 90

School Name: ____________________ Student Name: ____________________

(Print: First Name & Last Name)

2011 Edmonton Junior High Math Contest

Multiple Choice

(PRINT neatly using CAPITAL letters)

Numeric Response

Part A: (4 pts each)	**Part B: (6 pts each)**	**Part C: (8 pts each)**
1. E	6. D	11. 1
2. B	7. B	12. 1005
3. E	8. D	13. 15
4. C	9. D	14. 28
5. B	10. B	15. 77 777 779 779

Instructions:

1. Grid paper, scrap paper, and non-programmable calculators **ARE** permitted. You may write on the booklet.
2. Programmable calculators, cell phones, and wireless devices **ARE NOT** allowed.
3. To avoid others from seeing your answers, **DON'T** print your answers **TOO LARGE**, and be sure your answers are **HIDDEN FROM VIEW** at all times.
4. Each **CORRECT ANSWER** in:
   - Part A is worth 4 points,
   - Part B is worth 6 points,
   - Part C is worth 8 points.
5. Each **BLANK** in:
   - Part A is worth 1 point,
   - Part B is worth 2 points,
   - Part C is worth 0 points.
6. Each **INCORRECT ANSWER** is worth 0 points.
7. You have 60 minutes of writing time.
8. When done, carefully **REMOVE** and **HAND IN** only this **COVER PAGE**.

MARKER USE ONLY

Part A: ________ (# Correct) × 4 + ________ (# Blank) × 1 = ______

Part B: ________ (# Correct) × 6 + ________ (# Blank) × 2 = ______

Part C: ________ (# Correct) × 8 = ______

Total: = ______

2011 Edmonton Junior High Math Contest

Part A: Multiple Choice. Place the letter that corresponds to the correct answer on the blank provided. Each correct answer is worth 4 points. Each unanswered question is worth 2 points, up to a maximum of 3 blanks in parts A and B combined.

1. The square root of half Mitchell's age in years is half the sum of the first 3 prime numbers. What is Mitchell's age in years?

A) 9

B) 10

C) 18

D) 25

E) 50 ←

Let a = Mitchell's age

$$\sqrt{\frac{a}{2}} = \frac{2+3+5}{2}$$

$$\sqrt{\frac{a}{2}} = 5$$

$$\frac{a}{2} = 25$$

a = 50

Mitchell is 50 years old.

2. A bag contains red, yellow, and green gumdrops. Of the total, $\frac{1}{4}$ are red, $\frac{1}{3}$ are yellow, and the remaining 70 gumdrops are green. How many gumdrops are in the bag?

A) 120

B) 168←

C) 192

D) 204

E) 210

Let x = # of gumdrops

$$\frac{1}{4}x + \frac{1}{3}x + 70 = x$$

$$70 = \frac{5}{12}x$$

$$168 = x$$

The bag contained 168 gumdrops.

3. Kendra has a basket containing 4 types of fruit. She has 3 times as many bananas as apples. There are 4 more pears than bananas, and 2 less lemons than apples. What is the least number of pieces of fruit that could be in Kendra's basket?

A) 10

B) 14

C) 18

D) 20

E) 26 ←

x = # of apples

$3x$ = # of bananas

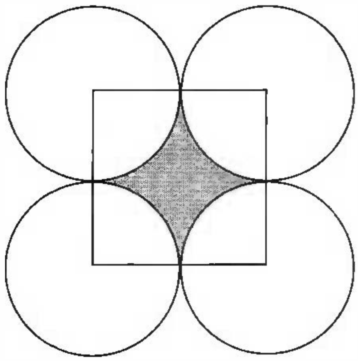
$3x + 4$ = # of pears

$x - 2$ = # of lemons

$x + 3x + 3x + 4 + x - 2 =$ ______________

Since there is at least one of each type of fruit, in order for there to be one lemon, x must be 3. By substitution, this will yield 26 fruits.Therefore, the answer E) 26, x= 3, which yields a positive integer for each type of fruit, is the answer.

4. In the figure shown, at the right, the radius of each circle is 3 cm. The centres of the circles represent the vertices of a square. What is the area of the closed shaded region, to the nearest square centimetre?



A) 4

B) 6

C) 8 ←

D) 16

E) 18

The area of the square – 4(one-fourth of a circle) = the shaded region.

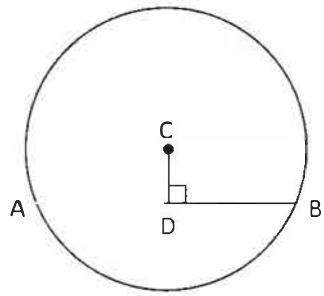
$s^2 - \pi r^2 =$

$6^2 - \pi 3^2 =$

7.72

To the nearest square centimetre the answers is 8.

5. In the figure shown at the right, the length of segment AB = 16 cm and the length of segment CD = 6 cm. What is the radius of Circle C, to the nearest centimetre?



A) 8

B) 10 ←

C) 14

D) 17

E) 20

Draw in the radius from A to C. A right triangle is formed. Use the Pythagorean property to find the radius, r.

$r = \sqrt{6^2 + 8^2}$

$r = \sqrt{100}$

r = 10

The radius is 10 cm.

Part B: Multiple Choice. Place the letter that corresponds to the correct answer on the blank provided. Each correct answer is worth 6 points. Each unanswered question is worth 2 points, up to a maximum of 3 blanks in parts A and B combined.

6. What is the sum of the first 63 terms of the following sequence?

1, -2, 3, -4, 5, 1, -2, 3, -4, 5, 1, -2, 3, -4, 5, 1, -2, 3, -4, 5, ...

A) 34

B) 36

C) 37

D) 38 ←-

E) 40

The pattern 1, -2, 3, -4, 5 occurs 12 times in the first 63 terms.

1 + (-2) + 3 + (-4) + 5 = 3.

The 61^{st} term is 1, the 62^{nd} term is -2, and the 63^{rd} term is 3.

(12)(3) + 1 + (-2) + 3 = 38.

The sum is 38.

7. The first 13 terms of a number pattern are shown below, What is the 15^{th} term?

1, 1, 2, 2, 4, 6, 3, 9, 12, 4, 16, 20, 5, ...

A) 25

B) 30 ←-

C) 35

D) 36

E) 38

The following table shows a pattern to the sequence of numbers:

Starting number	Square the starting number	Starting with 1, add the next consecutive integer to the previous number.	Divide the previous number by the same number that was added in the previous step
1	$1^2 = 1$	$1 + 1 = 2$	$2 \div 1 = 2$
2	$2^2 = 4$	$4 + 2 = 6$	$6 \div 2 = 3$
3	$3^2 = 9$	$9 + 3 = 12$	$12 \div 3 = 4$
4	$4^2 = 16$	$16 + 4 = 20$	$20 \div 4 = 5$
5	$5^2 = 25$	**25 + 5 = 30**	$30 \div 5 = 6$
		(This is the 15th term.)	

Or

There are three separate patterns here.

1 1 2 2 4 6 3 9 12 4 16 20 5

Red is 1, 2, 3, 4, 5 consecutive numbers

Green is 1, 4, 9, 16 square numbers

Blue is 2, 6, 12, 20 … we can say it's +4, +6, +8, + 10 in between terms. This would give 20 + 10 = 30. Or a more elegant solution would be $n(n+1) = 5(6)$ where n is the term number

8. A large cube with an edge of 8 cm is made from the least possible number of centicubes. Although the object looks solid, it is hollow inside. How many centicubes are needed to make the object?

A) 96

B) 169

C) 216

D) 296 ←

E) 384

If the large cube was solid it would be made up of 8^3 centicubes. The hollow portion inside has a volume of 6^3 cm^3. So, the numbers of cubes needed is $8^3 - 6^3 = 296$.

9. If (1, 2) and (-1, -2) are two vertices of a square, which of the following points could not be another vertex of the square?

A) (-2, 1)

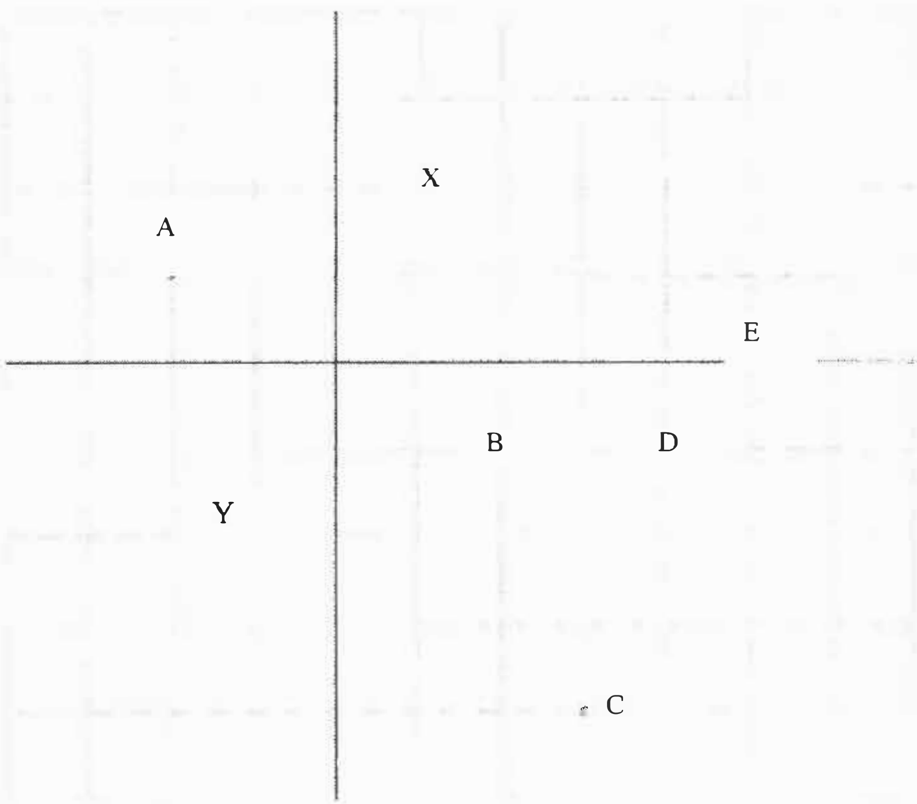
B) (2, -1)

C) (3, -4)

D) (4, -1) ←

E) (5, 0)

Label point (1, 2) with X. Label point (-1, -2) with Y. Label the points in the answer with their corresponding letter, A, B, and so on. Plot and label the points.



Use the origin as the turn center, rotate point X 90 ° three times, and the images of X will be at points B, Y, and A. Use the point (2, -1) as the turn center, rotate point X 90° three times, and the images of X will be at point E, C, and Y. Therefore, only point D (4, -1) could not be another vertex of a square.

10. Let ABCD be a quadrilateral with AB parallel to CD and CD = 2AB. Let E be a point on CD so that AE is parallel to BC.

Find the ratio of the areas of ADE to ABCD.

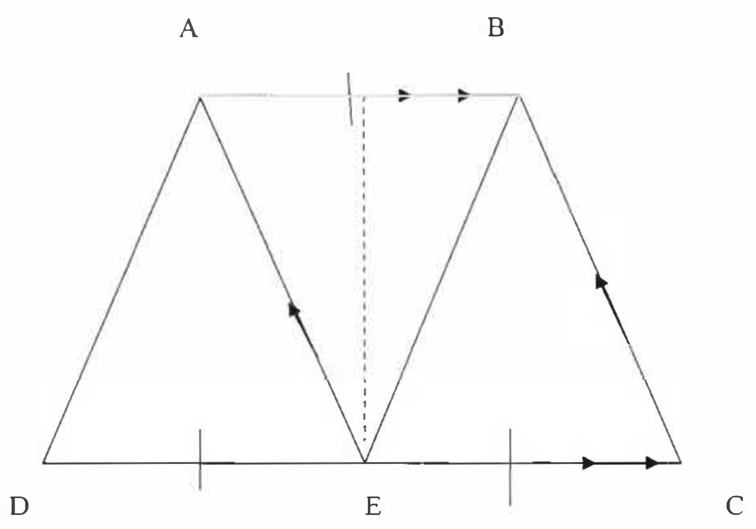
A) 1 : 4

B) 1 : 3 ←

C) 1: 2

D) 2: 3

E) 3: 4



Since AB is parallel to CD and AE is parallel to BC, quadrilateral ABCE is a parallelogram. Therefore, side AB is equal to segment EC. Since CD = 2AB, CE = ED. Since AB is parallel to CD, all three triangles have the same height. Therefore, the three triangles are congruent and are equal in area. The ratio of the areas of triangle ADE to quadrilateral ABCD is 1 : 3.

Part C: Numeric Response. Place the correct answer on the blank provided. Each correct answer is worth 8 points. Each unanswered question is worth 0 points.

11. Find all natural numbers $n \geq 1$ for which $n(n-1)(n+1) + 3$ is prime.

n, n-1, and n+1 represent 3 consecutive integers. Therefore, $n(n - 1)(n + 1)$ is a multiple of 3. The sum of a multiple of 3 and 3 cannot be prime. Therefore, $n(n - 1)(n + 1)$ must be equal to zero. By the zero property, three cases occur: $n = 0$, or $n - 1 = 0$, or $n + 1 = 0$. The first and third cases yield numbers that do not satisfy the condition that $n \geq 1$, therefore, the case, $n - 1 = 0$, is the only one that works. There is only one answer, and that is **n = 1.**

12. The numbers between 1 and 2011 are written on a piece of paper. Logan circles the even numbers with red circles and Miranda circles the multiples of 5 with blue circles. How many numbers are circled with only one color?

Logan circle 1005 numbers, Miranda circled 402 numbers. The only numbers which are circled by two colors are the multiples of 10; thus there are 201 numbers circled both with red and blue colors. There are 1005 – 201 = 804 numbers circled only with red, and 402 – 201 = 201 numbers circled only with blue. In total there are 804 + 201 = **1005 numbers circled with only one color.**

Or

Using a venn diagram, we have {multiples of 2} + {multiples of 5} – 2 x {multiples of 10}

$= (2011/2) + (2011/5) - 2 \times (2011/10)$

$= 1005 + 402 - 2(201)$

$= 1005 + 402 - 402$

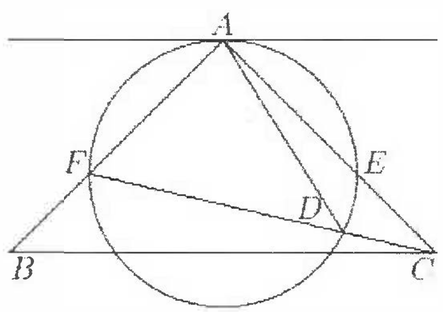
$= 1005$

13. The table below shows the integers from 1 to 25 in a 5 by 5 array. Choose five of the numbers, with one in each row and one in each column, such that the smallest of the five chosen numbers is a large as possible. What is the largest possible value for this number?

11	17	25	19	16
24	10	13	15	3
12	5	14	2	18
23	4	1	8	22
6	20	7	21	9

At the most four of the five numbers chosen can be from those on the border of the array. That means that at least one number among the nine at the centre can be chosen. The largest number there is 15. Therefore, the smallest of the chosen numbers cannot be greater than 15. By choosing 15, 25, 18, 23, and 20, there are five numbers with no two in the same row or column, and **the smallest of them is 15.**

14. In the diagram below, triangle ABC has a right angle at A and AB = AC. A circle passing through A cuts AC at E, AB at F and CD at D, with AE = AF. If the measure of angle CAD is 17°, what is the measure of angle ACF?



Angle A is a common angle to both triangle AFE and ABC. Angle AFE = angle ABC = angle AEF = angle ACB = 45°. Thus, triangle AFE is similar to triangle ABC.

17° = Angle CAD = angle EAD = angle EFD as the two angles are subtended by the same arc DE.

FE is parallel to BC. Angle EFD = angle EFC = angle FCB because they are alternate interior angles.

Angle ACF = angle ACB - angle FCB = 45° – 17° = 28°.

The measure of angle ACF is 28°.

15. What is the smallest positive integer which is divisible by both 7 and 9, each digit is 7 or 9, and there is at least one 7 and at least one 9?

To be divisible by 9, the sum of the digits must also be divisible by 9. Since 7 and 9 have no common factors, the number of copies of 7 must be a multiple of 9 as well. Since there is at least one 7, there are at least nine of them. As for divisibility by 7, all the digits 7 may be replaced by 0 and digits 9 by 2, so we are looking for the smallest positive multiple of 7 whose digits are 0 and 2. This is 2002. Putting everything together, **the number we want is 77777779779.**

THE CALGARY MATHEMATICAL ASSOCIATION

35th JUNIOR HIGH SCHOOL MATHEMATICS CONTEST

May 4, 2011

NAME: SOLUTIONS GENDER: ☐ M ☐ F

PLEASE PRINT (First name Last name)

SCHOOL: ______________________ GRADE: ____ (7,8,9)

- You have 90 minutes for the examination. The test has two parts: PART A — short answer; and PART B — long answer. The exam has 9 pages including this one.
- Each correct answer to PART A will score 5 points. You must put the answer in the space provided. No part marks are given.
- Each problem in PART B carries 9 points. You should show all your work. Some credit for each problem is based on the clarity and completeness of your answer. You should make it clear why the answer is correct. PART A has a total possible score of 45 points. PART B has a total possible score of 54 points.
- You are permitted the use of rough paper. Geometry instruments are not necessary. References including mathematical tables and formula sheets are **not** permitted. Simple calculators without programming or graphic capabilities **are** allowed. Diagrams are not drawn to scale. They are intended as visual hints only.
- When the teacher tells you to start work you should read all the problems and select those you have the best chance to do first. You should answer as many problems as possible, but you may not have time to answer all the problems.

MARKERS' USE ONLY	
PART A ×5	
B1	
B2	
B3	
B4	
B5	
B6	
TOTAL (max: 99)	

BE SURE TO MARK YOUR NAME AND SCHOOL AT THE TOP OF THIS PAGE.
THE EXAM HAS 9 PAGES INCLUDING THIS COVER PAGE.

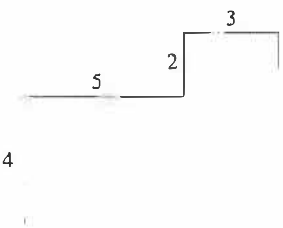
Please return the entire exam to your supervising teacher at the end of 90 minutes.

PART A: SHORT ANSWER QUESTIONS

A1 A store sells pies. Each pie costs the same price and two pies cost \$8. How much do three pies cost?

\$12

A2 Nahlah's living room is as shown in the diagram, with all distances in metres and with all angles 90°. What is the area (in square metres) of her living room?



38

A3 Doan mixes together 1 litre of 1% butterfat milk, 2 litres of 2% butterfat milk and 4 litres of 4% butterfat milk. What percentage of the resulting seven litres of milk is butterfat?

3%

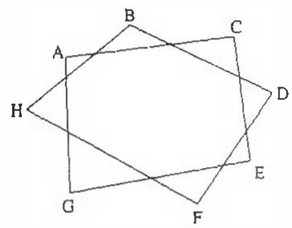
A4 Nine people, all with different heights, are sitting around a circular table. What is the greatest possible number of people that could be taller than both persons sitting next to him/her?

4

A5 The number 111...1 has 102 ones, and the number 222...2 has 101 twos. Suppose you do the subtraction 111...1 − 222...2 to get a whole number. What is the sum of the digits of this whole number?

809

A6 In the following 8-pointed star, what is the sum of the angles A, B, C, D, E, F, G, H?

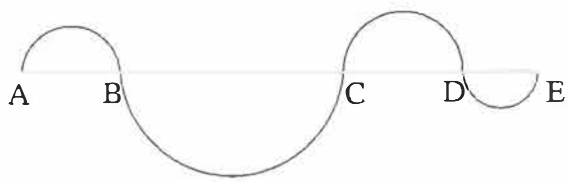


720 degrees

A7 Sixteen coins, numbered 1 to 16, are each red on one side and blue on the other side. Initially, all coins have their red sides facing up. The coins that are multiples of 2 are turned over. Then the coins that are multiples of 4 are turned over. Then the coins that are multiples of 8 are turned over. Then the coins that are multiples of 16 are turned over. Afterwards, how many of the coins have the red side facing up?

11

A8 Five points A, B, C, D, E lie on a line segment in order, as shown. The segment AE has length 10cm. Semi-circles with diameters AB, BC, CD, DE are drawn, as shown. The sum of the lengths of the semicircles $\widehat{AB}, \widehat{BC}, \widehat{CD}, \widehat{DE}$ can be written in the form $k\pi$ for some number k. What is k?



5

A9 Suppose that a and b are positive integers, and the four numbers

$$a + b, \quad a - b, \quad a \times b, \quad a \div b$$

are all different and are all positive integers. What is the smallest possible value of $a + b$?

8

PART B: LONG ANSWER QUESTIONS

B1 Ariel purchased a certain amount of apricots. 90% of the apricot weight was water. She dried the apricots until just 60% of the apricot weight was water. 15 kg of water was lost in the process. What was the original weight of the apricots (in kg)?

Solution 1: Let x be the original weight of the apricots (in kg). Then in the original apricots, $\frac{9x}{10}$ kg of the apricots is water. Since 15 kg of the water is lost during the drying process, $\frac{9x}{10} - 15$ kg of the water remains and the apricots weigh $x - 15$ kg. Since 60% of the dried apricot weight is water, we have

$$\frac{\frac{9x}{10} - 15}{x - 15} = \frac{60}{100} = \frac{3}{5}.$$

Cross multiplying yields $5(\frac{9x}{10} - 15) = 3(x - 15)$, or equivalently, $\frac{9x}{2} - 75 = 3x - 45$. Therefore, $\frac{9x}{2} - 3x = 30$, which simplifies to $\frac{3x}{2} = 30$. Hence, $3x = 60$. Solving for x yields $x = 20$.

Therefore, the original weight of the apricots is 20 kg.

Solution 2: In the dried apricots, the water is $\frac{60}{100} = \frac{3}{5}$ of the apricots. Therefore, the ratio of the water to the non-water part of the dried apricots is $3 : 2$. The ratio of the water to the non-water part of the original apricots is $9 : 1 = 18 : 2$. The non-water weight remains the same throughout the drying process. Hence, we can let $2x$ be the weight of the non-water part of the dried apricots. Then $18x$ is the weight of the water before drying and $3x$ is the weight of the water after drying. Since 15 kg of the water is lost, $18x - 3x = 15$, i.e. $15x = 15$, which yields $x = 1$. The original weight of the apricots is $18x + 2x = 20x = 20$.

Therefore, the original weight of the apricots is 20 kg.

B2 A group of ten friends all went to a movie together. Another group of nine friends also went to the same movie together. Fourteen of these 19 people each bought a regular bag of popcorn as well. It turned out that the total cost of the movie plus popcorn for one of the two groups was the same as for the other group. A movie ticket costs \$6. Find all possible costs of a regular bag of popcorn.

Solution 1: Since more people are in the first group than in the second group and the costs of the two groups are equal. there must be more people in the second group that ordered popcorn than in the first group. Since a total of 14 people ordered popcorn. but only 9 people are in the second group. there are two possibilities as to how many people from each group ordered popcorn.

Case 1: Five people from the first group ordered popcorn and all nine people from the second group ordered popcorn.

Case 2: Six people from the first group ordered popcorn and eight of the nine people from the second group ordered popcorn.

In the first case. the four extra popcorns for the second group must be equal in cost to the one extra movie ticket for the first group. Therefore. four popcorns must cost \$6. so one regular popcorn must cost \$1.50.

In the second case. the two extra popcorns for the second group must be equal in cost to the one extra movie ticket for the first group. Therefore two popcorns must cost \$6. so one regular popcorn must cost \$3.

Therefore. the possible costs of a regular bag of popcorn are \$1.50 and \$3.

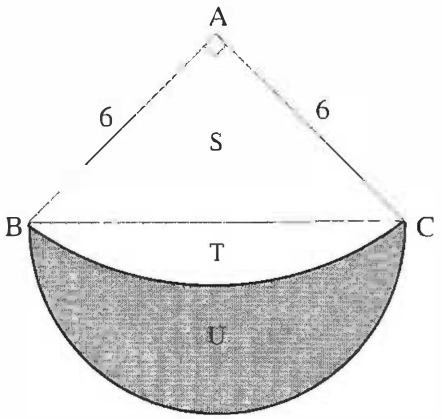
Solution 2: We proceed up to the two cases of Solution 1. Let x be the cost of one regular popcorn.

In the first case. the first group paid $10 \times 6 + 5x$ dollars and the second group paid $9 \times 6 + 9x$ dollars. Therefore. $10 \times 6 + 5x = 9 \times 6 + 9x$. Hence. $60 + 5x = 54 + 9x$. Therefore. $6 = 4x$. which yields $x = \frac{6}{4} = \frac{3}{2}$. Therefore. a regular popcorn costs \$1.50.

In the second case. the first group paid $10 \times 6 + 6x$ dollars and the second group paid $9 \times 6 + 8x$ dollars. Therefore. $10 \times 6 + 6x = 9 \times 6 + 8x$. Hence. $60 + 6x = 54 + 8x$. Therefore. $6 = 2x$. which yields $x = 3$. Therefore. a regular popcorn costs \$3.

Therefore. the possible costs of a regular bag of popcorn are \$1.50 and \$3.

B3 In the diagram, $AB = 6$ cm, $AC = 6$ cm and $\angle BAC$ is a right angle. Two arcs are drawn: a circular arc with centre A and passing through B and C, and a semi-circle with diameter BC, as shown.



(a) (1 mark) What is the area of ΔABC?

The area of the triangle is $\frac{1}{2} \times AB \times AC = \frac{1}{2} \times 6 \times 6 = 18$ cm^2.

(b) (2 marks) What is the length of BC?

By Pythagorean Theorem, the length of BC is $\sqrt{AB^2 + AC^2} = \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2}$. Therefore, the length of BC is $\sqrt{72}$ cm, or equivalently, $6\sqrt{2}$ cm.

(c) (6 marks) Find the area between the two arcs, i.e. find the area of the shaded figure in the diagram.

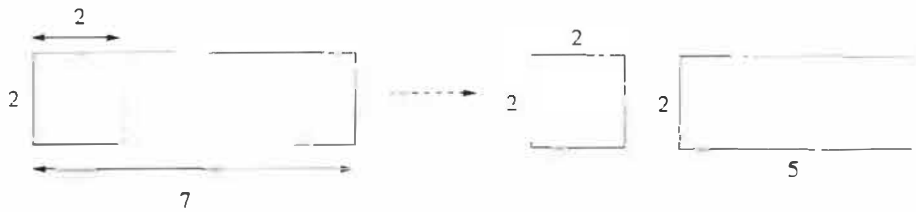
Solution: Let S, T, U be regions labeled in the above diagram, i.e. S the area of the triangle, T the area between side BC and the arc with centre A passing through BC and U the area between the two arcs.

We first find the area of the semi-circle with diameter BC. This is the area of T and U. The radius of the circle is half of BC, which is $3\sqrt{2}$. Hence, the area of the semi-circle is $\frac{1}{2}(\pi(3\sqrt{2})^2) = \frac{1}{2} \times 18\pi = 9\pi$ cm^2. Therefore, the area of T and U is 9π cm^2.

We now find the area of the quarter-circle with centre A passing through B and C, i.e. the area of S and T. The area of S and T is $\frac{1}{4} \times \pi \times 6^2 = 9\pi$ cm^2. Therefore, the area of S and T is 9π cm^2. Since the area of T and U is also 9π cm^2, S and U have the same area. The area of S is 18 cm^2 by (a), the area of U is also 18 cm^2.

Hence, the area of the shaded figure is 18 cm^2.

B4 Given a non-square rectangle, a *square-cut* is a cutting-up of the rectangle into two pieces, a square and a rectangle (which may or may not be a square). For example, performing a square-cut on a 2×7 rectangle yields a 2×2 square and a 2×5 rectangle, as shown.



You are initially given a 40×2011 rectangle. At each stage, you make a square-cut on the non-square piece. You repeat this until all pieces are squares. How many square pieces are there at the end?

Solution: We first cut as many squares with side 40 as we can. Upon division of 2011 by 40, the quotient is 50 and the remainder is 11. Therefore, we will have 50 squares with side 40 and what remains is a 11×40 rectangle.

With the resulting 11×40 rectangle, we use the similar idea as in the previous paragraph. The quotient of $40 \div 11$ is 3 and the remainder is 7. Therefore, we will have 3 squares with side 11 and what remains is a 7×11 rectangle.

The quotient of $11 \div 7$ is 1 and the remainder is 4. Therefore, we will have one square with side 7 and what remains is a 7×4 rectangle.

The quotient of $7 \div 4$ is 1 and the remainder is 3. Therefore, we will have one square with side 4 and what remains is a 4×3 rectangle.

The quotient of $4 \div 3$ is 1 and the remainder is 1. Therefore, we will have one square with side 3 and what remains is a 3×1 rectangle.

The quotient of $3 \div 1$ is 3 and the remainder is 0. Therefore, we will have three squares with side 1. Now, every piece is a square.

Counting all of the squares we have cut, the number of squares is $50+3+1+1+1+3 = 59$ squares. Therefore, there are 59 square pieces at the end.

B5 Five teams A, B, C, D, E participate in a hockey tournament where each team plays against each other team exactly once. Each game either ends in a win for one team and a loss for the other, or ends in a tie for both teams. The following table originally showed all of the results of the tournament, but some of the entries in the table have been erased.

Team	Wins	Losses	Ties
A	3		
B	1		1
C	1		
D	0		
E			4

The result of each game played can be uniquely determined. For each game in the table below, if the game ended in a win for one team, write down the winner of the game. If the game ended in a tie, write the word "Tie".

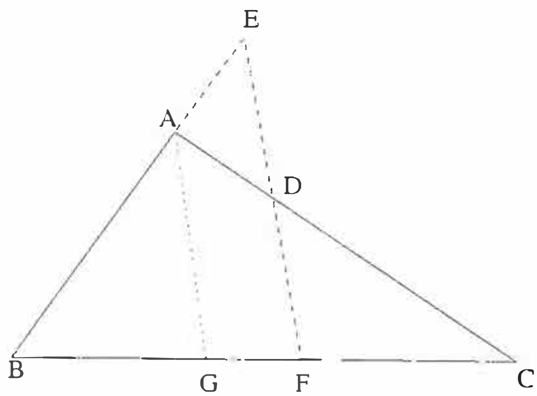
Solution:

Team A vs Team B	Team A
Team A vs Team C	Team A
Team A vs Team D	Team A
Team A vs Team E	Tie
Team B vs Team C	Team C
Team B vs Team D	Team B
Team B vs Team E	Tie
Team C vs Team D	Tie
Team C vs Team E	Tie
Team D vs Team E	Tie

Each team played four games. Therefore, team B lost 2 games. Furthermore, team E did not win or lose any game and team E tied with every team. Since team A won 3 games, and tied one game (with team E) and won every other game. Team A won against Teams B, C and D.

We now figure out the result amongst the games between teams B,C,D. Note that the total number of wins in the tournament is 5. Therefore, five games ended in a win/loss, which implies that the other five games ended in a tie. Therefore, the sum of the number of ties for the five teams is $5 \times 2 = 10$. Since all four games involving team E ended in a tie, there is another tied game in a game played amongst A,B,C,D. Since each of teams A,B has only one tie (with team E), then teams C and D tied in their game. Hence, the number of ties teams C and D each have is 2. Therefore, team C lost 1 game and team D lost 2 games. We know team C lost to team A and has one win unaccounted for. Therefore, team C won against team B. Finally, team B has one win unaccounted for. Therefore, team B won against team D. This completes the table.

B6 A triangle ABC has sides $AB = 5, AC = 7, BC = 8$. Point D is on side AC such that $AB = CD$. We extend the side BA past A to a point E such that $AC = BE$. Let the line ED intersect side BC at a point F.



(a) (2 marks) Find the lengths of AD and AE.

Since $DC = 5$, $AD = AC - DC = 7 - 5 = 2$. Since $BE = AC = 7$, $AE = BE - AB = 7 - 5 = 2$.

Therefore, AD has length 2 and AE has length 2.

(b) (7 marks) Find the lengths of BF and FC.

We draw a line passing through A parallel to the line EF, as shown. Let this line intersect the side BC at a point G. Then by similar triangles and parallel lines, we have

$$\frac{BG}{GF} = \frac{BA}{AE} = \frac{5}{2}, \text{ and } \frac{GF}{FC} = \frac{AD}{DC} = \frac{2}{5}.$$

Hence, $BG : GF : FC = 5 : 2 : 5$. Therefore,

$$\frac{BF}{FC} = \frac{7}{5}.$$

Since $BC = 8$, $BF = \frac{7}{12} \times BC = \frac{7}{12} \times 8 = \frac{14}{3}$. Hence, $FC = 8 - BF = 8 - \frac{14}{3} = \frac{10}{3}$.

Therefore, the length of BF is $\frac{14}{3}$ and the length of FC is $\frac{10}{3}$.

Sense Making in an Era of Curriculum Change

Gladys Sterenberg

Making Mathematics Meaningful for Students in the Intermediate Grades: Fostering Numeracy.
W W Liedtke 2010. Bloomington, Ind: Trafford.

Werner Liedtke has been a long-time contributor to *delta-K*, so it was with great anticipation that I read his book *Making Mathematics Meaningful for Students in the Intermediate Grades: Fostering Numeracy*. This book offers strategies for working with our new elementary math curriculum and suggestions for addressing the process outcomes.

Drawing on his vast experience in math education, Liedtke describes sense making as including "number sense, spatial sense, measurement sense, statistical sense, sense of relationships, and developing and applying new mathematical knowledge *through problem solving*" (p 12) [italics in original]. While addressing all these senses is an ambitious undertaking, Liedtke clearly delineates background research on cognition, goals for students, assessment and reporting, and diagnosis and intervention that informs changes in our understanding of teaching and learning mathematics. Each chapter following the introduction provides a rationale for the importance of the specific math strand, presents practical suggestions for activities and problems, and includes questions to prompt reflection.

Although other books that address mathematical content for teaching and learning exist, what is specifically helpful about Liedtke's writing is his insertion of practical examples from his experience working with children. These examples are based on observations in classrooms, excerpts from videotaped lessons and assessment interviews conducted with students. I was impressed by how his practice has informed his writing to a great extent.

The author addresses the number strand in great detail. However, I believe that the integration of the various senses might contribute to a more comprehensive understanding of making sense of mathematics. For example, the author writes, "There exists a similarity between the act of counting sets of discrete objects and the act of using measuring to describe parts of continuous quantity" (p 145). Yet, he organizes the book into separate chapters without a more detailed explanation of such similarities.

For me, the strength of this book is the practical teaching ideas that are presented alongside research and student examples. The content is arranged in a consistent format that allows for a thorough investigation of math teaching and learning through problem solving. I believe that teachers will find this book to be a practical addition to their library of curriculum resources. Certainly its strong connection to the Alberta Mathematics K–9 program of studies prompts us to think deeply about the tasks we engage our students in.

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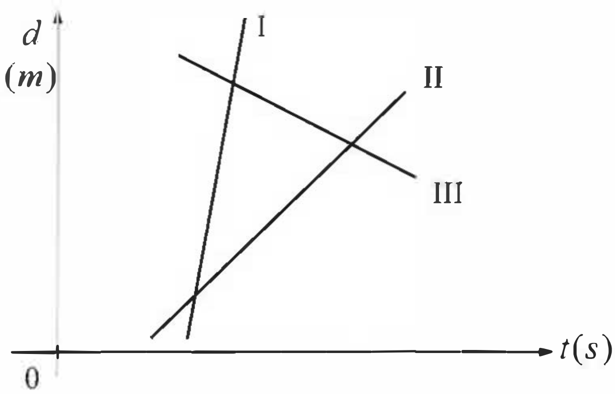
Three Speedy Whales Solutions

Editor's note: This problem appeared in the December 2010 issue of delta-K. *The author's solution and teachers' solutions are presented below.*

Once upon a time three whales swam along a north–south shoreline in the ocean near the island. Children at the island lighthouse observed the whales' motion and graphed their coordinates. They noticed that the intersections of three position-time graphs form an isosceles triangle (see Figure 1 below). If the first whale swam at 7 m/s [N] and the second at 1 m/s [N], with what velocity did the third whale swim?

Figure 1



Author's Solution

Gregory V Akulov

Certainly, there are several different ways to solve this problem. I believe that the shortest, most beautiful solution uses the slope of the angle bisector formula.

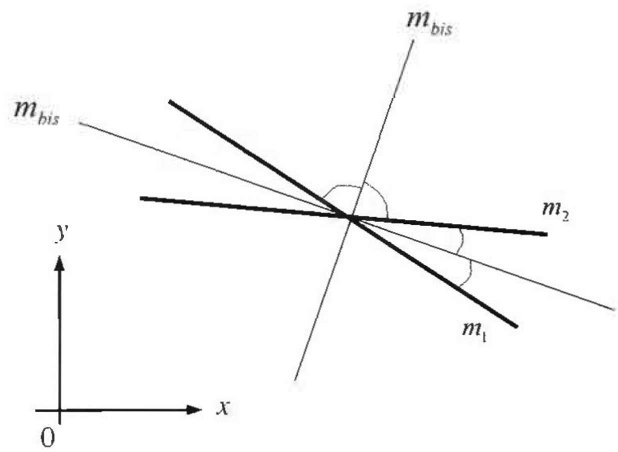
Since the triangle is isosceles, then graph III is parallel to the one of two angle bisectors between lines I and II. Noticing that the slope is equal to velocity, and using a formula for the slope of the angle bisector for $m_1 = v_1 = 7$ m/s and $m_2 = v_2 = 1$ m/s, we get that the third whale swam at

$$v_3 = m_{bis} = \frac{8}{-6-\sqrt{8^2+6^2}} = -0.5 \text{ m/s, or } 0.5 \text{ m/s[S]}.$$

If m_1 and m_2 are the slopes of two intersecting lines, and m_{bis} is the slope of the line that bisects angle between them (as shown on Figure 2), then

$$m_{bis} = \frac{a}{b \pm \sqrt{a^2+b^2}}, \text{ where } a = m_1 + m_2,\ b = 1 - m_1 m_2 .$$

Figure 2



This relationship was found and first published in Canada in 2009.

Teachers' Solution

Darryl Smith

Percy Zalasky and I collaborated on this problem, and we both began by writing multiple equations that were of little use. The key to doing the problem is to realize that the slope of any position–time graph gives the velocity, so conversely, if we know the velocity, we can determine the slope of the corresponding line. Since the velocity associated with line I is 7 and the velocity associated with line II is 1, we can determine the angle at the vertex of the isosceles triangle by using $tan^{-1}(7) - tan^{-1}(1) = 36.9$ *degrees*.

The rest of the solution follows from there, and we get an answer of about 0.5 m/s south.

The solution is fairly elegant, and even a Grade 10 class could do this, albeit perhaps with a bit of guidance.

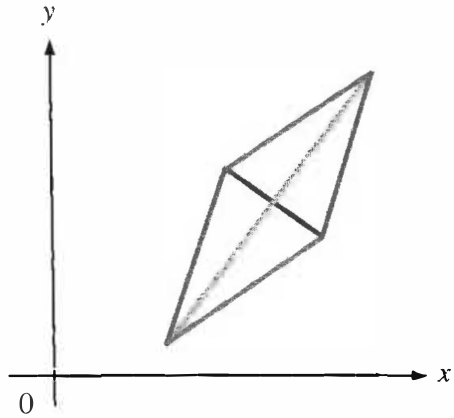
Diamond Slopes Problem

Gregory Akulov

If $d_{1,2}$ are the slopes of rhombus' diagonals (see Figure), and $s_{1,2}$ are the slopes of its sides, then

$$d_{1,2} = \frac{a}{b \pm \sqrt{a^2 + b^2}}$$

where $a = s_1 + s_2$, $b = 1 - s_1 s_2$. Prove it.



Figure

Dr Gregory Akulov teaches mathematics and physics at Luther College High School, in Regina, Saskatchewan. He has a PhD in mathematics (with specialization in probability theory) from Kyiv National Taras Shevchenko University, in Kyiv, Ukraine. His research interests are also in theory of functions, foundations of geometry and mathematics curriculum development.

Diversity • Equity • Human Rights Diversity • Equity • Human Rights

Specialist councils' role in promoting diversity, equity and human rights

Alberta's rapidly changing demographics are creating an exciting cultural diversity that is reflected in the province's urban and rural classrooms. The new landscape of the school provides an ideal context in which to teach students that strength lies in diversity. The challenge that teachers face is to capitalize on the energy of today's intercultural classroom mix to lay the groundwork for all students to succeed. To support teachers in their critical roles as leaders in inclusive education, in 2000 the Alberta Teachers' Association established the Diversity, Equity and Human Rights Committee (DEHRC).

DEHRC aims to assist educators in their legal, professional and ethical responsibilities to protect all students and to maintain safe, caring and inclusive learning environments. Topics of focus for DEHRC include intercultural education, inclusive learning communities, gender equity, UNESCO Associated Schools Project Network, sexual orientation and gender variance.

Here are some activities the DEHR committee undertakes:

- Studying, advising and making recommendations on policies that reflect respect for diversity, equity and human rights
- Offering annual Inclusive Learning Communities Grants (up to $2,000) to support activities that support inclusion
- Producing *Just in Time*, an electronic newsletter that can be found at www.teachers.ab.ca; Teaching in Alberta; Diversity, Equity and Human Rights.
- Providing and creating print and web-based teacher resources
- Creating a list of presenters on DEHR topics
- Supporting the Association instructor workshops on diversity

Specialist councils are uniquely situated to learn about diversity issues directly from teachers in the field who see how diversity issues play out in subject areas. Specialist council members are encouraged to share the challenges they may be facing in terms of diversity in their own classrooms and to incorporate these discussions into specialist council activities, publications and conferences.

Diversity, equity and human rights affect the work of all members. What are you doing to make a difference?

Further information about the work of the DEHR committee can be found on the Association's website at www.teachers.ab.ca under Teaching in Alberta, Diversity, Equity and Human Rights.

Alternatively, contact Andrea Berg, executive staff officer, Professional Development, at andrea.berg@ata.ab.ca for more information.

MCATA Executive 2011/12

MCATA Mission Statement

Providing leadership to encourage the continuing enhancement of teaching, learning and understanding mathematics.

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