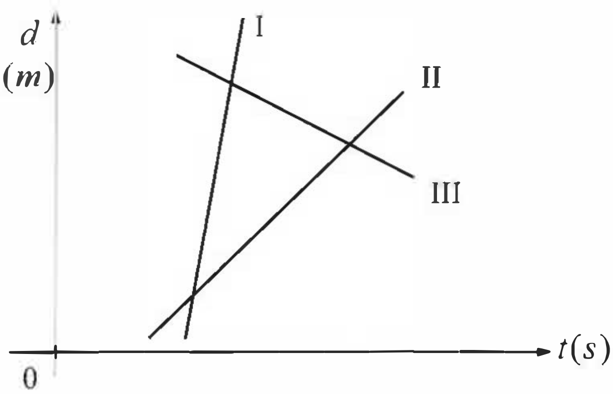
# Three Speedy Whales Solutions

*Editor's note: This problem appeared in the December 2010 issue of* delta-K. *The author's solution and teachers' solutions are presented below.*

Once upon a time three whales swam along a north–south shoreline in the ocean near the island. Children at the island lighthouse observed the whales' motion and graphed their coordinates. They noticed that the intersections of three position-time graphs form an isosceles triangle (see Figure 1 below). If the first whale swam at 7 m/s [N] and the second at 1 m/s [N], with what velocity did the third whale swim?

**Figure 1**




## Author's Solution

### *Gregory V Akulov*

Certainly, there are several different ways to solve this problem. I believe that the shortest, most beautiful solution uses the slope of the angle bisector formula.

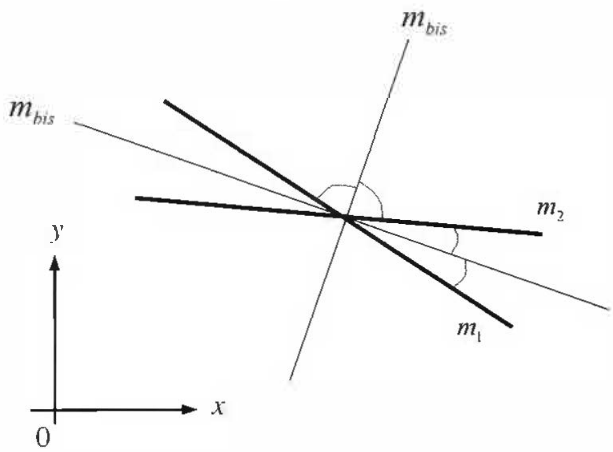
Since the triangle is isosceles, then graph III is parallel to the one of two angle bisectors between lines I and II. Noticing that the slope is equal to velocity, and using a formula for the slope of the angle bisector for $m_1 = v_1 = 7$ m/s and $m_2 = v_2 = 1$ m/s, we get that the third whale swam at

$$v_3 = m_{bis} = \frac{8}{-6-\sqrt{8^2+6^2}} = -0.5 \text{ m/s, or } 0.5 \text{ m/s[S]}.$$

If $m_1$ and $m_2$ are the slopes of two intersecting lines, and $m_{bis}$ is the slope of the line that bisects angle between them (as shown on Figure 2), then

$$m_{bis} = \frac{a}{b \pm \sqrt{a^2+b^2}}, \text{ where } a = m_1 + m_2,\ b = 1 - m_1 m_2 .$$

**Figure 2**




This relationship was found and first published in Canada in 2009.

## Teachers' Solution

### *Darryl Smith*

Percy Zalasky and I collaborated on this problem, and we both began by writing multiple equations that were of little use. The key to doing the problem is to realize that the slope of any position–time graph gives the velocity, so conversely, if we know the velocity, we can determine the slope of the corresponding line. Since the velocity associated with line I is 7 and the velocity associated with line II is 1, we can determine the angle at the vertex of the isosceles triangle by using $tan^{-1}(7) - tan^{-1}(1) = 36.9$ *degrees*.

The rest of the solution follows from there, and we get an answer of about 0.5 m/s south.

The solution is fairly elegant, and even a Grade 10 class could do this, albeit perhaps with a bit of guidance.