The Problem of Problem Solving

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Teachers in Alberta are required to submit yearly professional growth plans to their school administration. Each goal undertaken by the teacher for that school year must align with a descriptor, set by Ministerial Order #016/97, of knowledge, skill or attribute required of teachers who possess a permanent teaching certificate in this province. One of the descriptors states, "Teachers are career-long learners."[1] My own career-long learning has focused on primary children's learning. It has taken me most of my thirty-year teaching career to work through the problem of problem solving.

The problem started much earlier than that. I was a reasonably strong student in elementary school, and though I never really had any difficulties with computation, I was quite anxious when we were assigned problems. I just never could read those problems and actually know what to do. When I faced a word problem such as

> At the school store Harry bought a textbook for which he paid 10 per cent less than the regular price of $1.40. How much did Harry pay for the book?

I found I was stumped. I couldn't make sense of the Physics 10 question that demanded to know how wide was the river based on the height of the tree that I could see on the other side. I managed to get my first degree without ever taking a university-level mathematics course.

As a young teacher of primary school children, I could pick and choose which problems I would ask my students to solve. If I thought the students would find it too difficult, I would just leave it out. That worked for the first twelve years of teaching, but then I was placed in a Grade 3 classroom. My task was to prepare these students for their first experience with the provincial achievement exam in mathematics—and it was *all word problems!*

I equated the mathematics test with a reading test. I believed that if my students could *read* the problem, they could *solve* the problem. We worked with a traditional textbook that contained lessons for practising beginning addition or subtraction that were followed with either words or pictures that gave the students the chance to apply the algorithm that they had just learned. These were often called story problems. If the lesson was about subtracting one-digit numbers from two-digit numbers, with regrouping, the problem might be

> Tyler had 23 hockey cards. He gave 6 to his younger brother. How many hockey cards does Tyler have now?

For the first two or three years in this teaching assignment I would create wall charts that I believed helped the students know which operation to apply by looking for the *hint words* in the actual word problem. If the question used the word *altogether*, the students knew that they were to add. If the question used the word *left*, the students knew that they were to subtract. The words were the key to the finding the correct answer. I was convinced that I had finally learned how to solve problems! But then a word problem like this came along:

> Joyce had 17 apples altogether. She had 8 red apples. How many were green?

It didn't fit the pattern. The students added according to the classroom chart, and although the hint word *altogether* was there in the question, the answer, 26, wasn't correct! I had a hard time explaining to the students why, for this problem, they had to subtract instead of add. They lost faith in my charts. They had a procedure but no understanding. "Students who memorize facts or procedures without understanding often are not sure when and how to use what they know, and such learning is often quite fragile" (Bransford, Brown and Cocking 1999). My students did not understand the procedure that I had created for them.

Not too long after, I tripped over the problem-solving strategies put forth by Polya. The four steps were logical. Paraphrased, they are (a) understand the problem, (b) devise a plan, (c) carry out the plan and (d) look back (Polya 1988). A new chart went up in the classroom. I used the words from my father's old math textbook (Banting, Banting and Brueckner 1936), which had been authorized by the ministers of education in Alberta and Manitoba.

I. What does the problem ask for?
II. What must be done to solve the problem if all the facts are not clear?
III. What is a reasonable answer?
IV. What checks should be used? (p 14)

The students were unsure of the second step. They did not have clear ideas of what they were meant to do without input from me, the teacher. I searched some more and began using *The Problem Solver 2* (Hoogeboom and Goodnow 1987), which was based on introducing a variety of strategies that the students could apply when solving a word or story problem. The strategies fit into Polya's second step, devising a plan: (a) act out or use objects, (b) make a picture or diagram, (c) use or make a table, (d) make an organized list, (e) guess and check, (f) use or look for a pattern, (g) work backwards, (h) use logical reasoning, (i) make it simpler, and (j) brainstorm (Hoogeboom and Goodnow 1987, viii). The binder of worksheets gave specific examples to work through with the students and a multitude of worksheets that allowed the students to practise the strategy that they had just been taught but had not really learned. This followed the traditional approach where

> the teacher demonstrates or leads a discussion on how to solve a sample problem. The aim is to clarify the steps in the procedure so that students will be able to execute the same procedure on their own. ... students practice using the procedure by solving problems similar to the sample problem. (Stigler and Hiebert 1997, 18)

At about this same time, elementary teachers in Alberta were being introduced to a new mathematics program of studies (Alberta Learning 1997). This new curriculum presented a shift from the manner in which mathematics had traditionally been taught. It suggested that the students would not be given prepared worksheets with algorithms to be completed silently and individually. Rather, it stated, "Problem solving, reasoning and connections are vital to increasing mathematical power and must be integrated throughout the program. A minimum of half the available time within all strands needs to be dedicated to activities related to these processes" (p 13). To help teachers recognize the areas where problem solving could be effectively added to the concepts being taught, a bold PS appeared after the actual specific outcome in the following manner: "6. Recognize, build, compare and order sets that contain 0 to 1000 elements. [PS, R, V]" (p 15).

This was the first time that I had been exposed to the idea that problem solving was an important component of mathematics. I had not understood that the role of problem solving was to actually develop the basic mathematical computational skills that I had been asking the students learn by rote or through the practice worksheets that I dutifully copied and distributed daily. I railed against the new textbooks that did not have practice pages before the introduction of a problem. I could not imagine how the students could solve a problem before they knew how to complete an algorithm correctly. Sadly, there was no one to help me understand this challenging change. My school district purchased the textbooks, but there was no assistance to help me change my own understanding. I was alone and I could rail against this change with indignation. What did *they* know about teaching kids? I had been successful and I did not need to change! "One cannot expect teachers to change their teaching practice simply because they have been told to" (Mewborn 2003, 49). I ignored the changes and for many years left those new textbooks, bindings uncracked, on the classroom shelves, using them only to press leaves in the fall.

It is now nearly fifteen years later. There are times when I marvel at how far I have come. Problem solving is not a problem for me or for my students any more because we no longer attempt textbook-generated word and story problems. The students no longer sit at their desks and work individually at worksheets with rows and rows of algorithms. The students and I are engaged in rich problem-solving activities. I enjoy mathematics, both the teaching and the learning, that occurs every day in the classroom. I see my students making sense of what they are doing and constructing their own knowledge. But how could this shift have taken place?

Just over five years ago, the schools were abuzz over another mathematics curriculum change, but this time it was different. Teachers were encouraged to attend professional development opportunities to help them understand the changes. I attended the first workshop at Barnett House, in Edmonton, led by a former teacher. Throughout the day, this teacher brought the curriculum changes into focus, explaining how they were intertwined and how the activities she shared could be done with all students. She espoused the same philosophy as Clements and Sarama (2009), who believe that "especially for younger children, mathematic topics should not be treated as isolated topics; rather, they should be connected to each other, often in the context of solving a significant problem or engaging in an interesting project" (p 207). The words of the new curriculum (Alberta Education 2007) began to have more meaning.

> Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type *How would you …?* or *How could you …?*, the problem solving approach is being modeled. … A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement … Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers. (p 6)

Tentatively, I took a chance and attempted an activity from the workshop. It meant that the students and I would be talking about the possibilities offered by the problem posed. It meant that we would work in groups, we would share our ideas and we would be willing to make mistakes. Luckily, it was April, so trust had already been established, and the students knew that I would not introduce an activity that they could not succeed at. Using wooden pattern blocks, the students were introduced to increasing patterns. A red square block was set down with two cream-coloured rhombus blocks placed on opposite sides to represent legs. This created a caterpillar-like creature. As each red square was added, two rhombi were added. After creating a creature with 4 squares (body) and 8 rhombi (legs), I challenged the students to use what they knew and predict the number of legs the creature would have when there were 6 body parts, 8 body parts and 10 body parts. With the students working in pairs, with manipulatives, the challenge began. As I walked about and listened to the students' conversations and attempts at a solution, I marvelled at the feeling in the air. It was electric! These students had never been so engaged. They didn't need me. They didn't want me. They wanted to work! I knew that I could never go back to the old way of teaching problem solving again.

> Once one accepts that the learner must herself actively explore mathematical concepts in order to build the necessary structures of understanding, it then follows that teaching mathematics must be reconceived as the provision of meaningful problems designed to encourage and facilitate the constructive process. In effect, the mathematics classroom becomes a problem-solving environment in which developing an approach to thinking about mathematical issues, including the ability to pose questions for oneself, and building the confidence necessary to approach new problems are valued more highly than memorizing algorithms and using them to get the right answers. (Schifter and Fosnot 1993, 9)

The challenge continues—not the challenge of *how to teach* using the new approaches required by the new mathematics curriculum, but *how to engage the students in meaningful investigations* and recognize opportunities to bring meaningful problems into the classroom. "[T]eaching is not just about starting with mathematically rich problems, even ones connected to what students are thinking. And it is also not just about listening to students and asking them to describe their thinking" (Franke, Kazemi and Battey 2007, 226). It is through these problems, which must be thought out, that new learning is encouraged. The problems must reach all of the students at the level at which they are currently constructing their own understanding of the mathematics being presented. The solutions must be their own. The problems must scaffold from students' prior knowledge and move into the next level of investigation. The problems must represent what is important about mathematics and illustrate real-world situations; they must engage and delight and offer opportunities for pondering, discussion, strategies, failure and success. They must allow a community of learners to work cooperatively and find solutions that are acceptable, not because the teacher says so, but because the community has looked for and found an acceptable explanation. "Classrooms need to be places where teachers and students are engaged in rigorous mathematics in ways that both parties learn" (Franke, Kazemi and Battey 2007, 228).

I think that next year, when I submit my professional growth plan, I might just write "Teachers are career-long learners." Period. Being able to learn is more powerful than having learned. It is the gift I wish to give my students.

Note

1. Teaching Quality Standard Applicable to the Provision of Basic Education in Alberta 1997 (Ministerial Order #016/97), 6

References

Alberta Education. 2007. Program of Studies. Mathematics Kindergarten to Grade 9. Available at http://education.alberta.ca/media/645594/kto9math.pdf (accessed September 20, 2011).

Alberta Learning. 1997. *Mathematics Kindergarten to Grade 6*. Available at http://education.alberta.ca/media/450754/elemmath.pdf (accessed September 20, 2011).

Banting, L, A D Banting and L J Brueckner. 1936. *Mathematics for Everyday Use: Book One*. Toronto, Ont: Winston.

Bransford, J D, A L Brown and R R Cocking, eds. 1999. *How People Learn: Brain, Mind, Experience, and School*. Washington, DC: National Academy Press. Cited in National Council of Teachers of Mathematics 2000, 20.

Clements, D H, and J Sarama. 2009. *Learning and Teaching Early Math: The Learning Trajectories Approach*. New York: Routledge.

Franke, M L, E Kazemi and D Battey. 2007. "Mathematics Teaching and Classroom Practice." In *Second Handbook of Research on Mathematics Teaching and Learning*, ed F Lester, 225–56. Reston, Va: National Council of Teachers of Mathematics.

Hoogeboom, S, and J Goodnow. 1987. *The Problem Solver 2: Activities for Learning Problem-Solving Strategies*. Mountainview, Calif: Creative Publications.

Mewborn, D S. 2003. "Teaching, Teachers' Knowledge, and Their Professional Development. In *A Research Companion to Principles and Standards for School Mathematics*, ed J Kirkpatrick, W G Martin and D Shifter, 45–52. Reston, Va: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, Va: NCTM.

Polya, G. 1988) *How to Solve It: A New Aspect of Mathematical Method*. 2nd ed. Princeton, NJ: Princeton University Press.

Schifter, D, and C T Fosnot. 1993. Introduction to *Reconstructing Mathematics Education: Stories of Teachers Meeting the Challenges of Reform*. New York: Teachers College Press.

Stigler, J W, and J Hiebert. 1997. "Understanding and Improving Classroom Mathematics Instruction: An Overview of the TIMSS Video Study." *Phi Delta Kappan* 79, no 1: 14–21.

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