# Irritating Things

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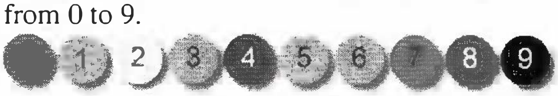
Beading with numbers can be an exciting way to engage children's mathematical thinking and develop their proficiency in addition. In this paper, we will explore a hands-on problem, "Irritating Things" (Galileo Educational Network Association [GENA] 2009), which combines colour coding with integers from 0 to 9, to create colourful beaded bracelets. In solving this problem, students will use logical thinking and addition to translate the numbers into beads. The patterns and relationships in this problem can be represented and described with words, tables, numbers and bracelets. The multiple representations will help students organize and communicate their ideas. By working with multiple representations, students will gain flexibility in their thinking and develop their proficiency in addition. The problem has multiple solutions; therefore, students can experience different ways to solve a problem. Working with problems with multiple solutions and representations gives students opportunities to discuss and learn each other's problem-solving strategies and solutions. This might help them gain deeper understanding of addition, pattern and their own reasoning ability. "Irritating Things" easily lends itself to differentiated instruction, giving all students opportunities for challenges and success. While the problem has considerable depth and can extend into computer programming, coding theory and discrete mathematics at a university level, the following activities are targeted to Grades 4 to 6.

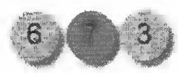
## "Irritating Things" Problem

There are 10 beads of different colours, numbered from 0 to 9.




1. Pick a first and second bead. They can be the same number, or not. For example, pick bead # 6 and bead #7.


2. To get the third bead, add the numbers on the first and second beads. If the sum is more than 9, just use the last (ones) digit of the sum. Adding 6 + 7 equals 13. Using the rules, drop the first digit and use 3.




3. To get the next bead, add the numbers on the last two beads used, and use only the ones digit.


4. Keep going until the first and second beads repeat, in that order.
5. Tie them in a loop to make a bracelet. (Don't use the last two beads, since they ust repeat the first two beads.)

## Instructional Strategies

### Day 1: Creating a Colour-Coded Addition Key

Before students begin beading, we suggest that students make their own colour addition chart to help them with the beading addition. In the top row, start with the + sign in the uppermost left-hand corner. Sequentially, place the numbers 0 through 9 in each column of the top row. In the left-most column, sequentially place the numbers 0 through 9. Then fill in the table with the accurate operations and colour.

Developing this colour-coded addition key should take one 45-minute class. It is important that all the students work with the same colour codes—for example, a red bead should always be 0. If everyone has the same colour code, finding addition errors will be much easier. Also, in order that patterns in the bracelets be recognized, all beads have to use the same number representation. See the appendix for a template that can be used in class.

**Figure 1: Example of a colour-coded addition key**

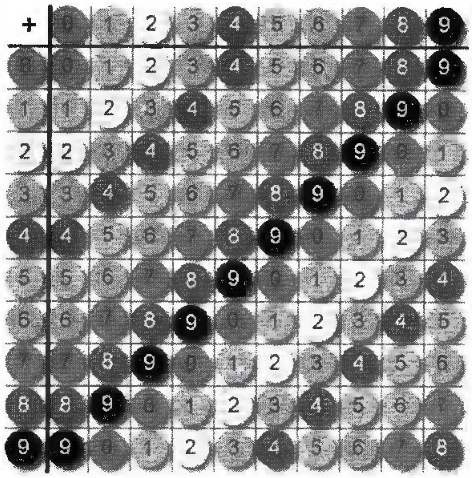
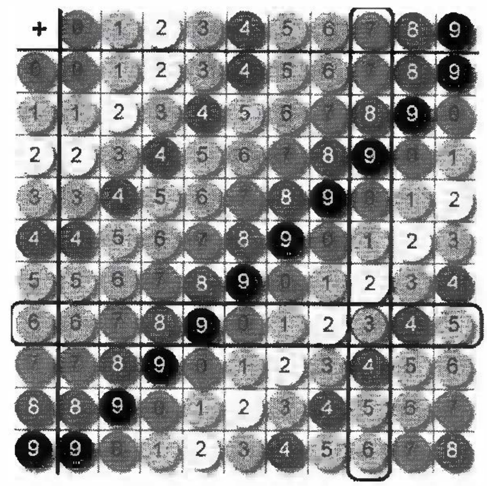


**Figure 2: Example of adding 7 + 6 on the colour-coded addition key**



## Day 2: Introducing the Problem

***Materials needed:***

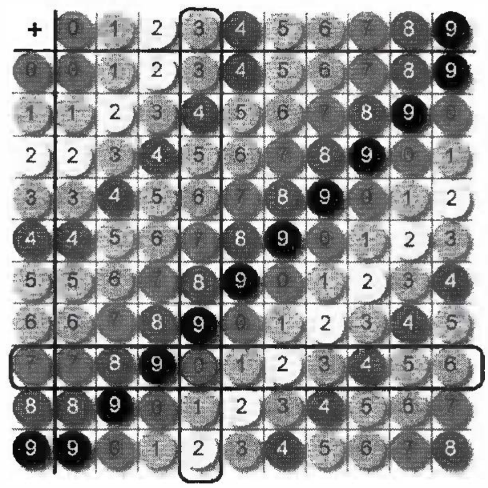
- Numerous beads with at least 10 different colours. Each group will need approximately 150 different-coloured beads. Inexpensive opaque pony beads from craft stores work well.
- Thick cording or shoelaces to thread beads—five strings per pair of students.

Demonstrate to the students how to begin to create their bracelet. Pick two beads, for example, . . Add the numbers on the beads together: + = 13. There are no beads that represent 13, so there is one more step to find the next bead. The rule is to drop off the first digit. When the first digit is removed from 13, the next bead is . Reinforce how to find the next bead with the colour-coded addition key. A SmartBoard is useful for demonstrations with the colour-coded addition key. Circle the row beside the 6 and the column under the 7. The number 3 is the bead that is where the 6 row and 7 column intersect. Students can follow their colour-coded addition key with their fingers.

Next, string the third bead on the shoelace .

To find the fourth bead, add the last two beads together: + = 10. There is no bead that represents 10, so when the first digit is dropped, a remains. Demonstrate how to find the fourth bead with the colour-coded addition key. Circle the row beside the 7 and the column under the 3. The number 0 is the bead where the 7 row and the 3 column intersect.

**Figure 3: Example of adding 7 + 3 with the colour-coded addition key**



String the fourth bead on the shoelace.

Encourage students to work in pairs, checking their work as they put each new bead on the bracelet. Pairing students together encourages collaboration and makes the problem easier to tackle. Persuade each student to be responsible for the accuracy of the addition. Plan for the bracelet making to take a couple of classes to complete.

## Day 4 or 5: Finding the Patterns in Class Discussion

Find bracelets that are the same size. Have students compare their bracelets. Lead them to discover that bracelets of the same size are actually the same bracelet. Starting with any two beads on the bracelet eventually results in the same bracelet. For instance, if a student picked 4 and 2, the bracelet would be the same bracelet with the same length if they picked 8 and 4.

Another interesting pattern is that when the total lengths of all the bracelets are added: 1 + 3 + 4 + 12 + 20 + 60 = 100. 100 equals the total number of beads in the rows and columns of the colour coded addition key. 100 equals 10 rows times 10 columns for multiplication—the number of different ways beads can pair together.

## Solutions to "Irritating Things"

There are exactly six different bracelets, with the numbers of pairs of beads of 1, 3, 4, 12, 20 and 60. Expect many errors in addition. If the students have created a bracelet with a total number of pairs beads that is not equal to 1, 3, 4, 12, 20 or 60, you can be certain that an addition error has been made. Have the students find their error. If available, a few parent volunteers could help students find the error. Once an error has been found, remove all the beads past the error. In our experience, students express dismay when the error is found and they have to pick up from the mistake. However, they very quickly go back to beading again.

One idea for differentiating is to encourage a group with slower addition skills to start with one of the smaller bracelets. Pick out the beads for 9 and [illegible] so that students will find a bracelet that is 12 beads long, or pick out the beads 2 and 8 for a pair to find a bracelet that is 20 beads long. They will find success more quickly with a smaller bracelet.

When students find their bracelet, there are several more to be found. The students with quicker addition skills can find more bracelets. Ask them to find a different-sized bracelet. With differentiation, the entire class can be engaged.

## Mapping to the Program of Studies

"Irritating Things" addresses the mathematical processes discussed in the front matter of the mathematics program of studies (Alberta Education 2007). "Irritating Things" connects addition to pattern; encourages the development of fluency with addition, visualization of addition and pattern; and develops mathematical reasoning through problem solving. The colour-coded addition key is a guide that can correspond to developing strategies for mental addition.

Students solve this problem primarily by using whole-number addition to discover a repeating pattern. Throughout the problem solving, students are translating whole numbers to colour-coded beads. The chart provides another representation to facilitate the translation. The representation of whole-number addition symbolic beads and a pictorial chart address many specific outcomes in the program of studies. The following figure outlines the specific outcomes by grade.

## Algebraic Extension

The steps for addition and dropping the 10s column for addition provide an excellent exploration for algebra and the distributed property. The following explanation is beyond the elementary levels of the activities, but might be useful for further understanding of the problem.

This is an algebraic explanation of the solution. Consider the first bead as $a$ and the second as $b$. The first bead can be represented as $1a + 0b$, and the second as $0a + 1b$. When you add them together—$1a + 0b + 0a + 1b$, you get the third bead $1a + 1b$. The sequence of subsequent beads follows:

$1a + 0b$
$0a + 1b$
$1a + 1b$
$1a + 2b$
$2a + 3b$
$3a + 5b$
$5a + 8b$

Solution:

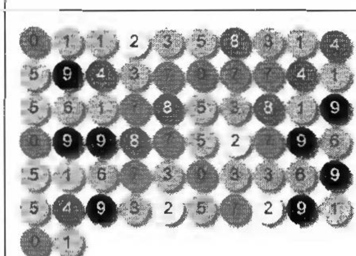
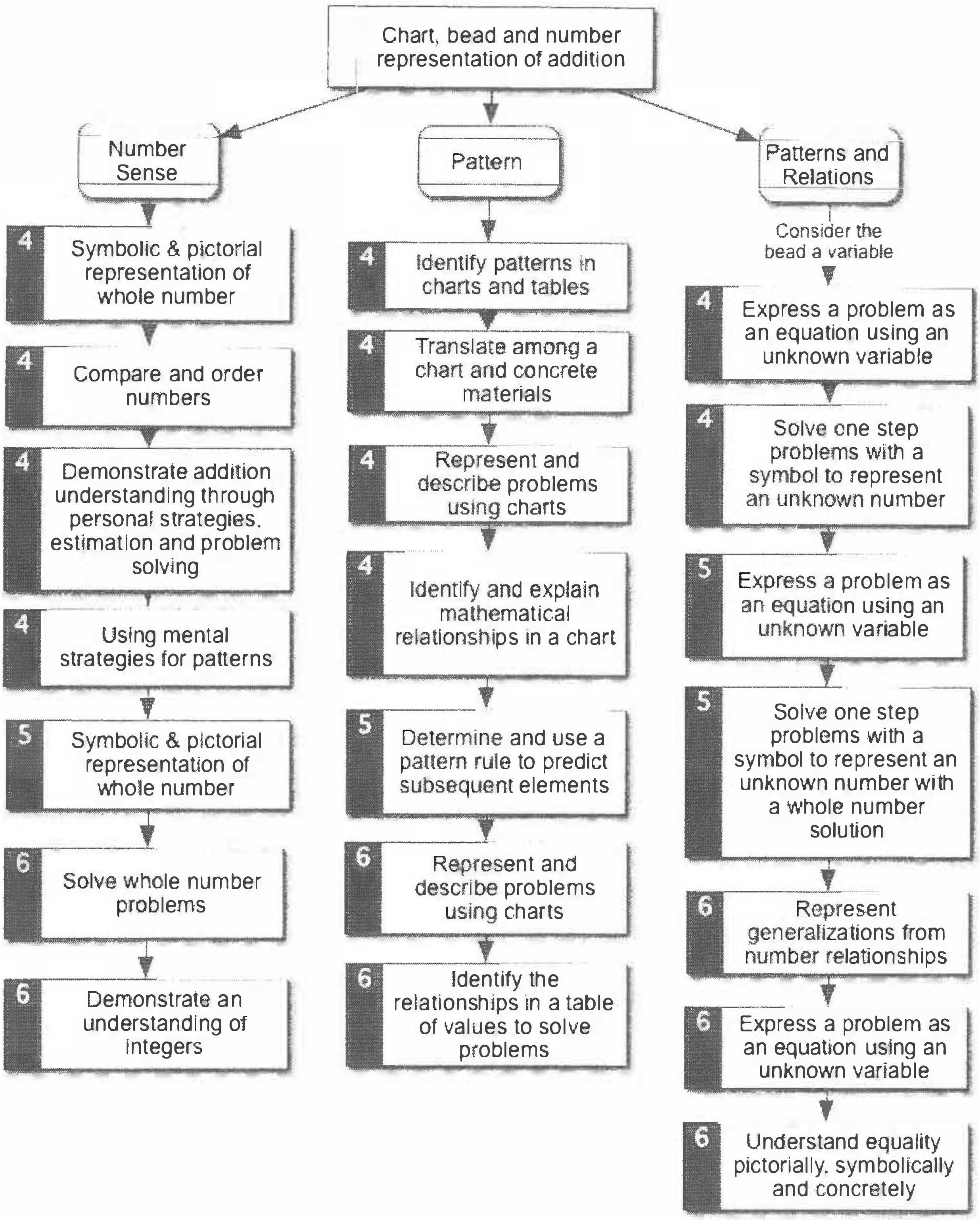
| # of pairs | 1 | 3 | 4 | 12 | 20 | 60 |
|---|---|---|---|---|---|---|
| Sequence | 0 | 0-5-5 | 4-2-6-8 | 2-1-3-4-7-1-8-9-7-6-3-9 | 2-2-4-6-0-6-6-2-8-0-8-8-6-4-0-4-4-8-2-0 |  |

**Figure 4: Specific outcome curriculum mapping for Grades 4 to 6 (Alberta Education 2007)**



The next bead requires the special condition of dropping the 10 digit. We need to apply the distributed formula to see how this works. The next bead in the sequence is $8a + 13b$. Using the distributive formula for $b$, we get $8a + (10 + 3)b = 8a + 10b + 3b$. However, the rule means that $10b$ is equal to zero. Thus, the next bead is $8a + 3b$. The sequence continues as follows:

$8a + 3b$
$3a + 1b$
$1a + 4b$
$4a + 5b$
$5a + 9b$
$9a + 4b$
$4a + 3b$
$3a + 7b$
$7a + 0b$
$0a + 7b$
$7a + 7b$
...

Continuing with this algebraic sequence would eventually lead to 60 pairs of beads before the sequence repeated: the maximum size of the bracelet. This is one algebraic proof of the maximum size. Notice that if $a$ or $b$ equals 0, then the bracelet will be the maximum bracelet size. If $a$ and $b$ both equal 0, then the minimum bracelet is found. What bracelets are found with other values of $a$ or $b$?

## Conclusion

"Irritating Things" provides an exciting opportunity to explore addition and pattern with problem solving. Creating a colour-coded addition key reinforces the correspondence of number and addition. Building the bracelets provides a non-pencil-and-paper method for developing proficiency with addition. Finding that each bracelet of the same size is the same bracelet is an exciting discovery of pattern. Importantly, this set of activities is rigorous, demanding and fun.

"Irritating Things" is a rich problem that can be explored in different ways. For instance, what happens when a different number of beads is used? How about 8 beads? How about 12 beads? Are there more bracelets? Or are there fewer? How long is the longest bracelet? How long is the shortest? Using different numbers of beads takes students into operations with a non-10 base. Most of all, have fun exploring this investigation into addition and pattern.

## Bibliography


Alberta Education. 2007. *Mathematics Kindergarten to Grade 9*. Edmonton, Alta: Alberta Education. Available at http://education.alberta.ca/media/645594/kto9math.pdf (accessed September 22, 2011).

Galileo Educational Network Association (GENA). 2009. "Irritating Things." Available at www.galileo.org/math/puzzles/IrritatingThings.htm (accessed September 22, 2011).

Papert, S. 2002. "Hard Fun." Available at www.papert.org/articles/HardFun.html (accessed September 22, 2011).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## Appendix: Colour-Coded Addition Key Template

| + | ⓪ | ① | ② | ③ | ④ | ⑤ | ⑥ | ⑦ | ⑧ | ⑨ |
|---|---|---|---|---|---|---|---|---|---|---|
| ⓪ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| ① | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| ② | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| ③ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| ④ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| ⑤ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| ⑥ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| ⑦ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| ⑧ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| ⑨ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |