Assessing Young Children's Attention to Pattern and Structure

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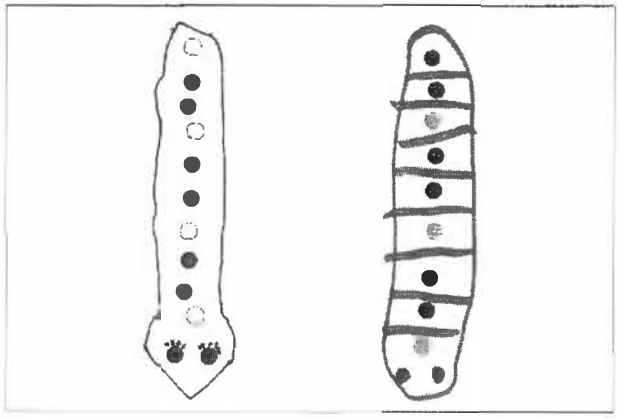
> A mathematician, like a painter or a poet, is a maker of patterns.
> —G H Hardy

Mathematics is sometimes described as "the science of patterns" (Devlin 1994; Steen 1988, 611). As Steen (1990) wrote, "Mathematics is an exploratory science that seeks to understand every kind of pattern—patterns that occur in nature, patterns invented by the human mind, and even patterns created by other patterns" (p 8). Across North America, mathematics curricula in the early years emphasize the importance of pattern as a way for children to make connections to the world around them and as the foundation for algebraic thinking (NCTM 2000). From pre-K to Grade 2, children are expected to recognize, identify, duplicate, extend and translate simple sequential patterns using a variety of attributes including sounds, actions, colours, shapes, objects and numbers. Early childhood classroom walls are often adorned with a variety of colour- and shape-patterning products. However, these products often don't reveal the range of mathematical reasoning that takes place when the patterns are made. For example, examine the patterns in Figure 1 created by Jun and Mason, both age 6. Both children have created a similar repeating pattern successfully and independently, but their reasoning about patterning is very different.

Figure 1:
Jun's (left) and Mason's (right) repeating patterns



Jun described her pattern as "yellow-blue-blue-yellow-blue-blue-yellow ..." and pointed to each dot on her snake. When asked to describe her pattern, she said, "There are two blues between the yellows." And when asked how many dots made up her snake, she pointed and counted, "1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and then 11, 12 for eyes." Jun has met many of the expectations for repeating patterns, and we might assume that she knows repeating patterns well, but the curriculum expectations do not provide a clear indication of what teachers should be looking for in children's descriptions and do not help teachers recognize the link to algebraic thinking that underlies patterning activities. Jun's interpretation of the pattern as "two blues between the yellows" makes it difficult for her to see the structure of the pattern as a whole.

Mason's response provides a contrast in experience and reasoning about patterns. As Mason was making his pattern (before the lines were drawn), he was asked to describe his pattern he said, "It's a red-red-green pattern. That's the core. Do you want me to circle the core?"

"No, that's okay. Just keep making the pattern for your snake."

"I could change it by putting a green dot at the beginning [tail] and make it a green-red-red-green pattern ... No, wait. It would just be a green-red-red pattern, but I'm just going to keep it [as red-red-green]." He finished putting down his dots and I asked, "You used the word core. How many times does the core repeat?"

"Three."

"Do you know how many dots you used for your pattern?"

"Uh ... nine."

"Oh [expecting him to count]! How did you get that?"

"Well, I know that six and three is nine, so it was easy."

"Where did the six come from?"

"Two of these [two units of the core] are six and one more is another three. So nine."

Mason's description of his pattern, his identification of the pattern core, his flexibility in counting the core units as a group of three dots and then using that information to determine the number of dots

altogether provide a solid basis for later understanding of multiplication, algebraic expressions and functional relationships.

This paper provides an example of a repeating patterns assessment task that can be used with children aged 4 to 8. The task and variations of it reveal children's reasoning about patterns. Four types of reasoning are shown to orient teachers' attention during the patterning process and also provide guidance for instruction. Although the content of the task uses repeating colour patterns—which are the simplest form of pattern and attribute—the task may easily be adapted for other repeating patterns (for example, border, hopscotch) with a variety of visual attributes (for example, shapes, objects).

Repeating Patterns Assessment Task

(adapted from Papic and Mulligan 2007)

Materials and Preparation

- Connecting cubes in six colours: Create a two-colour ABB tower (for example, yellow-green-green) (see Figure 2) and a collection of individual cubes in the same two colours, plus a third colour used as a distracter (for example, black). Create a second two-colour ABB tower in different colours (for example, orange-blue-blue) and a collection of individual cubes in the same two colours, plus a third colour (for example, white).
- Strips of legal size paper cut in half (that is, 5.5" × 14")
- Coloured dot stickers in three or four colours
- Markers

Set-Up

Working with pairs of students, give each child an ABB tower and coloured blocks (see Figure 2).

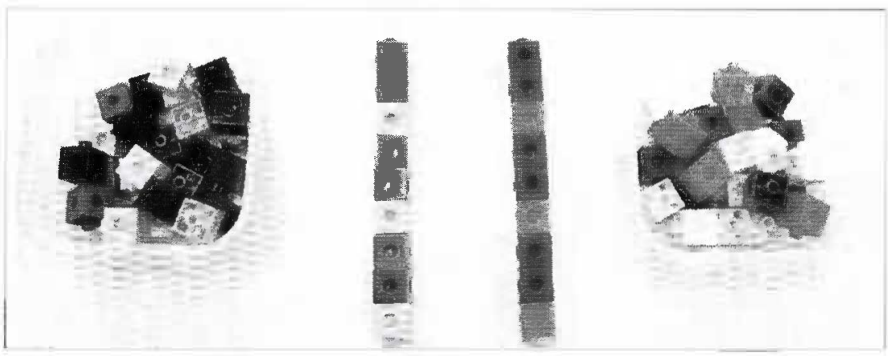
Assessment Task

The following questions represent many of the outcomes for patterns in the early grades including identify, describe, copy, extend, compare and create patterns.

1. **Identify/describe:** Give each child a premade ABB tower and ask, "Is it a pattern? Why do you think it is (or isn't)? Describe the pattern." For kindergarten, ask, "What is the part that repeats?" For Grades 1 and 2, ask, "What is the pattern core?"
2. **Reproduce:** Give each child a set of individual cubes (two of the correct colour and a third colour) and say, "Make a tower exactly the same as this one." Depending on the children's previous experiences, either leave the tower on the table for them to make comparisons (preschool to Grade 1) or show the tower for five seconds and then hide it (Grades 1 to 3). If they have difficulty, show it again for a few seconds. If they still have trouble, leave the tower out to be copied.
3. **Extend:** "Can you add more blocks to your tower? What would come next on the tower? How do you know that block comes next?"
4. **Compare:** Have the pairs of children compare their towers with each other and ask, "How are the two towers the same? How are they different?"
5. **Create:** Remove the blocks and give each child coloured circle stickers and a strip of paper. "Make your own pattern with coloured stickers."
6. **Identify/describe:** "Did you make a pattern? How do you know? Describe your pattern. What is the pattern core? How many times does the core unit repeat?" It may be helpful to have the child circle the core units with a marker.

This assessment task may be modified for a variety of materials and for the experiences of the children being assessed. The general goal of the assessment task is to understand the children's reasoning about patterns. Not every question needs to be asked, and

Figure 2: ABB Towers



modifications may be made depending on the child's responses. The next section provides a range of children's patterning strategies, from preschool to Grade 2, in response to aspects of the assessment task.

Children's Attention to Pattern and Structure

The assessment task is not a measure of understanding, but an indicator of how children perceive patterns and what strategies they use when working with patterns. The information gathered is intended to inform instruction. In this section, four types of responses are provided based on working with children from age 4 to 7. The range of responses is not intended to be developmental—that is, children will not necessarily go through each phase. In fact, children will attend to patterns differently, depending on the attribute. For example, children are often very successful with patterning tasks that focus on colour patterns, but they might have difficulty when patterns focus on shape, sound or other attributes. Differences in children's responses, such as those seen with Jun and Mason at the beginning of the paper, are due primarily to previous experiences and instructional orientation.

1. Inattention to pattern and structure

When asked, "What is a pattern?" Abed (age 4) did not have a definition or description. Not being able to define a pattern is not necessarily an indicator of understanding, so the assessment continued, and Abed was asked to make a copy of the orange-blue-blue tower he was given. Although I tried to encourage him to build the same tower, he either did not understand or was not interested. He was eager to build another tower, but he did so by randomly putting the blocks together (see Figure 3). When it got too long and started breaking apart, he began adding blocks to the original tower. The circle around the blocks in Figure 3 shows the original tower that remained intact. Abed appeared very motivated to build with the blocks, but he did not attend to the pattern as he did so.

2. Direct comparison strategy

Sophie (age 5) was given the yellow-green-green tower and was asked, "Is it a pattern?" She responded, "Yes," and described it as "yellow-green-green-yellow-green-green-yellow-green-green" as she pointed to each block in the tower.

"How do you know it is a pattern?" She responded, "Because it has yellow and green and they keep going."

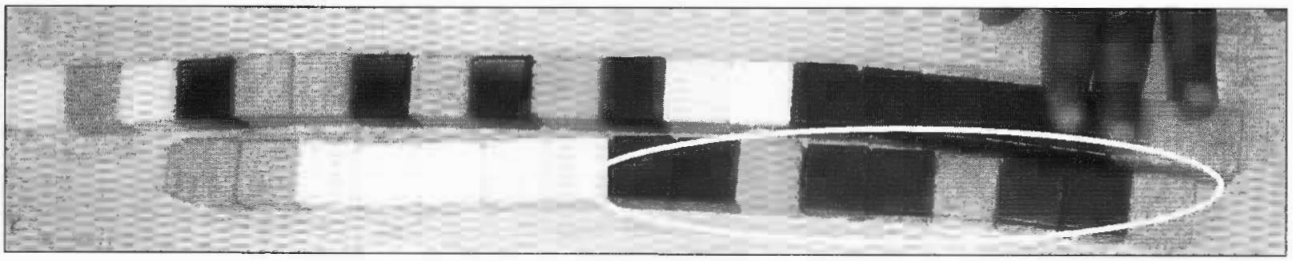
"Do you know what the core is?" She shrugged her shoulders.

"Here are some more blocks. I want you to make a tower exactly like this one, okay?" Sophie got a yellow block and then a green one and put them together. After this initial building she lined up her tower with the premade one to determine which colour would go on next. Her completed tower was identical to the original, but to examine her process more closely I created a revised task.

On a strip of paper I used yellow and green stickers to make a yellow-green-green pattern and asked her to make a copy. This time she also tried to use a direct comparison strategy by placing a finger on the original pattern at the left and putting a matching sticker on her pattern on the right. Once she had placed a sticker, she looked back to the original tower and found a dot just above the height of the sticker she had just placed to find the next sticker in line. Since there were longer gaps in her sticker tower, she missed one of the green stickers in the middle of the pattern.

Children who use a direct comparison strategy will often be able to successfully copy patterns when objects fit together; however, they have more difficulty when they are asked to copy a pattern with stickers or stamps or by drawing, because the spaces between elements can vary, and it is more difficult to line patterns up to make a direct comparison. Children using this strategy may say that the original and copy (like that in Figure 4) are the same by looking at it. It is only when they read the pattern and hear the verbal pattern breaking down that they are able to correct the pattern. For example, when Sophie read

Figure 3: Inattention to pattern and structure



her pattern, "Yellow-green-green-yellow-green-yellow," she heard the error and said, "Oh! I made a mistake."

3. Recursion strategy

Hua (age 6) described his tower as orange-blue-blue-orange-blue-blue-orange-blue-blue and said that it would keep going. He was able to copy and extend the pattern with blocks fairly easily. As he was building and extending the tower I asked, "How do you know which colour comes next?" He had just put an orange on and confidently said, "Blue comes next."

I asked, "And then what?"

"It's another blue."

"Then what?"

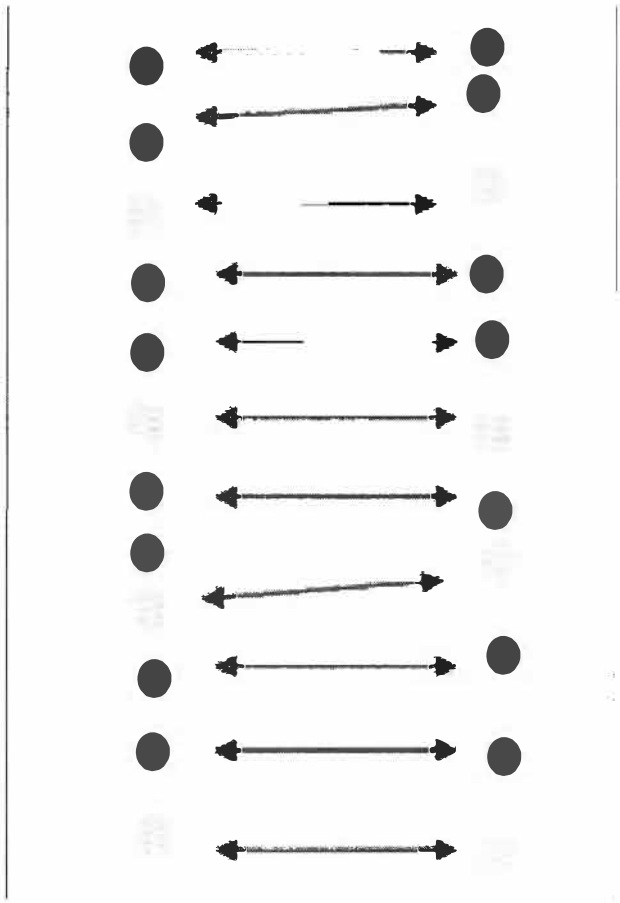
"Orange."

"So how do you know what comes next?"

"I look at this one [pointing to the last block put on] and then put the next one on."

Hua's response suggests that he is using a recursive strategy to build the tower. He knows what comes next by looking at the last block that was put on.

Figure 4: Original pattern (left) and Sophie's direct comparison process (right)



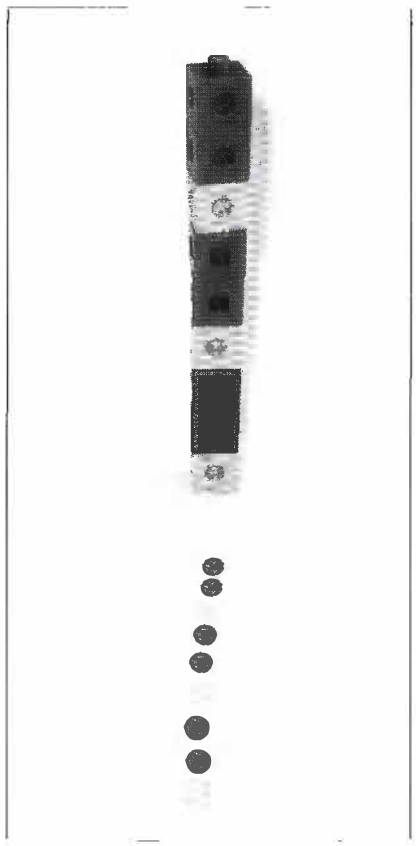
Children are often able to produce the expected pattern using a recursion strategy, but it is in the making and extending of patterns that the recursion process appears. A recursive strategy is used frequently by children (and adults), but it becomes less effective with more challenging patterns when the number of elements in the core unit gets longer, when the materials used are less familiar and when the shift is made to number patterns. For example, in the number pattern 4, 7, 10, 13, a child might use a recursive strategy of plus 3 to determine that 16 comes next. However, the only way the child can determine, for example, the tenth number in the pattern using a recursive strategy is by adding 3 until the tenth number is reached.[1]

4. Core unit strategy

I showed Reagan (age 7) the yellow-green-green tower for three or four seconds and then put it behind my back and asked her to make the same pattern with stickers. She quickly and easily placed the stickers on the page.

"Wow! That was fast. How did you know how to build the tower?"

Figure 5: Reagan's copy of tower using pattern unit strategy



Reagan said, "I remembered yellow-green-green [core unit] and there were three of them [units]."

If a child can see the pattern core, she doesn't have to remember every single block. Reagan showed that she needed to remember only the core unit and how many units there were. Looking for a core unit allowed Reagan to look for a relationship between the number of times the core unit is repeated and the number of elements in the core. Reagan demonstrated flexibility in being able to count with units other than one. A core unit strategy is also directly related to identifying a relationship in a function. A functional approach allows a person to determine any number of elements in a pattern without having to know all of the numbers in the sequence.

Conclusion

Human beings are naturally inclined to make sense of their environment by searching for patterns in images, objects and events. While early patterning activities might produce pretty pictures for classroom walls, supporting young children's understanding of patterns provides an excellent starting place for mathematical thinking. This paper provides an example of an assessment task, but the questions asked during the task are also important for daily instruction in patterns:

- Is it a pattern? Why do you think so?
- How are the two patterns the same? How are they different?
- What is the pattern core? How many times does it repeat?

Instruction needs to draw children's attention to what is and what is not a pattern, finding similarities and differences in patterns and the structure of patterns by attending to the pattern core. Our assessment of children also needs to shift, from the patterning products that children produce to the reasoning and strategies they use in the process of copying, extending, comparing and creating patterns. Without a shift in our instruction and assessment, many children will continue to be successful in the outcomes related to patterns by focusing primarily on the repeating elements in a pattern (for example, red-green-red-green), but an understanding of patterns requires attention to the core unit that repeats (for example, red-green repeated three times). Understanding units and flexibly counting and comparing units is essential in many topics in mathematics, including place value, measurement, fractions, multiplication and unit circles in geometry. Patterns are at the heart of mathematics and mathematical thinking. Early childhood educators have the opportunity to help children see mathematics as the science of patterns, rather than just as exercises in counting and computation.

Note

1. The more efficient alternative is to determine a functional relationship. In the example of 4, 7, 10, 13, the function rule is "times 3 plus 1." The tenth number would be 10 x 3 + 1 = 31.

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