delta-k

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To Become Wise to the World Around Us: Multiple Perspectives on Relating Indigenous Knowledges and Mathematics Education—*page 21*

MCATA Executive 2009/10

MCATA Mission Statement

Providing leadership to encourage the continuing enhancement of teaching, learning and understanding mathematics.

delta-k

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EDITORIAL

During the past year, I have been impressed with the resilient nature of my colleagues. I see teachers who, when faced with uncertainty and change, are able to respond in creative and generative ways. In this era of government restraint, we are responding in innovative and proactive ways to ensure the high quality of mathematics education in our province.

This issue of *delta-K* showcases the work of members of our mathematics education community. Two of our Mathematics Council (MCATA) executive members, Tancy Lazar and Chris Smith, were sponsored by Alberta Education to attend the Canadian Mathematics Education Forum (CMEF) in Vancouver last spring. Their report highlights their learning through participation in a working group on assessment practices. The article I co-authored with Liz Barrett, Narcisse Blood, Florence Glanfield, Lisa Lunney Borden, Theresa McDonnell, Cynthia Nicol and Harley Weston is based on the investigations of a CMEF working group on indigenous knowledges. These pieces emphasize math teachers' commitment to learning professionally within a broader community of Canadian teachers.

MCATA organized a fall symposium, held in October, to respond in a timely manner to the government's announcement of changes to the mathematics diploma exams. Darryl Smith, a long-time educator and supporter of MCATA, succinctly documents the symposium panel discussion. His report and the reports by mathematics education professor Elaine Simmt, recent high school graduate Ainslie Fowler, and teachers Donna Chanasyk and David Martin show teachers' dedication to real learning for our students and our willingness to take action when real learning is threatened.

The articles by Tim Sibbald, Joan Stevens, Stephanie Nash-Pearce, and Amanda Crampton and Paul Betts lead us in explorations of teaching a variety of mathematical concepts. Their lesson suggestions and experiences demonstrate the importance of sharing professional knowledge within our community.

Finally, the photographs from the 2009 MCATA conference, held in October, show teachers engaged in mathematical and pedagogical learning. Alberta's mathematics program of studies (Alberta Education 2007, 8) states that relationships are part of the nature of mathematics: "Mathematics is one way to describe interconnectedness in a holistic worldview. Mathematics is used to describe and explain relationships." Relationships are also part of the nature of professional learning. I invite you as mathematics teachers to become actively involved in our council. Attend the next MCATA conference (in October 2010), nominate a colleague for an award sponsored by the council or volunteer to serve on the executive. Seek out opportunities to build professional relationships.

Be connected. Be inspired. Find encouragement and support within the mathematics education community.

Reference

Alberta Education. 2007. *Mathematics Kindergarten to Grade 9.* Edmonton, Alta: Alberta Education. Also available at http://education.alberta.ca/media/645594/kto9math.pdf (accessed January 5, 2010).

Gladys Sterenberg

Meet Your MCATA Executive

Back row, left to right: Greg Forsyth, Elaine Simmt, Susan Ludwig, Carmen Wasylynuik, David Martin, Rod Lowry, Sharon Gach, Donna Chanasyk, Lisa Everitt, Lisa Hauk-Meeker, Robert Wong, Gladys Sterenberg

Front row, left to right: Tancy Lazar, Indy Lagu, Lori Weinberger, Chris Smith, Jennifer Baerg, Marj Farris

Not pictured: Daryl Chichak, Debbie Duvall, Marc Garneau, Carol Henderson, Christine Henzel, Martina Metz, Shauna Rebus

CMEF 2009: Participants' Report

Tancy Lazar and Chris Smith

A national conference for mathematics educators took place April 30–May 3, 2009, at Simon Fraser University in Vancouver. The Canadian Mathematics Education Forum (CMEF) 2009 focused on the ways in which resources and assessment define, inform and mould curriculum. Teachers at all levels worked collaboratively with representatives from school systems, school boards, colleges and universities, mathematics and statistics departments, faculties of education, ministries of education, parent groups, and business and industry to address the recurring theme of curriculum implementation in today's classrooms.

The forum began with keynote speakers and panel discussions that looked at the main obstacles students face in the mathematics classroom. These obstacles include

- disengagement from and fear of math,
- a lack of basic math skills,
- discomfort with mathematical language,
- calculator dependency,
- procedural dependency (step 1, step 2, . . .),
- an inability to interpret answers,
- an inability to communicate answers and
- assumptions that must be overcome.

We also looked at what we educators needed in the past and what we need now to meet the needs of our students and of our colleagues, particularly new teachers.

Then, the conference participants broke into a number of working groups to further discuss the obstacles faced by students and the needs of educators. We were part of the group called Rethinking Assessment, which considered the following questions:

- How does assessment affect the motivation in our class?
- How does the teacher know what the students know?
- How do the students know whether or not they know? And what they know?

A number of ideas came out of these discussions. However, one comment in particular stands out: "Our goal is not to teach; our goal is to help students learn."

By reflecting critically on this topic, teachers can change the whole dynamic of the classroom, not only in how they teach but also in how they assess. Assessment should be about developing a positive learning environment and setting up a path to help students develop their abilities. For a better understanding of where the Rethinking Assessment group was headed, think of a baseball card: the back of the card relays many statistics about a player and doesn't try to roll all of the numbers into one big category. Assessment can be looked at in the same way. With a number of different evaluations, students and educators can develop a better understanding of the learning taking place in the classroom.

Another key point made in our working group was the need for good mathematical problems and tasks that have an entry point for all students. It is through these problems that we can determine each student's current level of achievement. The problems should be open and rich enough to allow all students to demonstrate their understanding and to help educators see students' misconceptions.

So what can educators do? A climate in which growth can occur must be established. It was widely believed in our working group that too much focus is placed on changing a teacher's practice and not enough on changing perspectives. Sometimes colleagues pose the most resistance to new ideas, and this is where changing a perspective will help foster new ideas and techniques.

Our Rethinking Assessment working group put together a list of ideas and practices that we felt would benefit educators across the country:

- Focus on each student's success (learning) rather than on ranking students.
- Question everything.
- Students will rise to the level of their expectations.
- All students can learn.
- To know what students are thinking, we have to ask them.
- Assess to learn, not to control.
- Assessment means actively looking for understanding.
- Assessment is about the learning, not the teaching.

- Think of assessment as a relationship with students (coaching versus judging).
- We should evaluate what we value.
- Equity is not equality.
- Focus less on numbers; see assessment as a blend of the qualitative and the quantitative.
- Move away from one-size-fits-all to meeting kids where they are.
- Second chances are OK.
- Focus on differentiating assessments.
- Not everything you do has to be about the curriculum.
- Assessment is an inquiry process (not an event).
- Fairness doesn't mean sameness.
- Balance is important.

The conclusion of CMEF 2009 brought about questions about where to go next. CMEF will next be held sometime between 2012 and 2014, as there are other important national and international conferences to work around. Also, CMEF is looking toward creating a national organization similar to the National Council of Teachers of Mathematics (NCTM) in the United States. Several issues were brought up regarding this possible national organization, including bilingualism, organization, members, the mission statement and the need to get provincial teachers' associations involved. As for future topics at CMEF, curriculum and research were both brought up as potential starting points.

Photographic Memories: Conference 2009

Conference 2009: "Teaching Mathematics for Real Learning" was held October 23 and 24 at the River Cree Resort, in Enoch.

Conference 2009 Committee

Left to right: Susan Ludwig, Carmen Wasylynuik, David Martin, Rod Lowry, Elaine Simmt, Chris Smith, Donna Chanasyk

Award Recipients

- Dr Arthur Jorgensen Chair Award—Jennifer Baerg
- Alberta Mathematics Educator Award—Linda Arndt and Brenda MacDonald
- Friends of MCATA—Hank Reinbold and Didi Heer

Alberta Mathematics Educator Award winner Brenda MacDonald

Dr Arthur Jorgensen Chair Award winner Jennifer Baerg, with Art Jorgensen

2008/09 Executive Members

MCATA president Sharon Gach

Past president Elaine Manzer (left), secretary Donna Chanasyk (centre) and special projects director Lisa Hauk-Meeker (right)

Conference director Elaine Simmt

Daryl Chichak (membership director) and Pat Chichak (registration)

ATA president Carol Henderson (left) and Lisa Everitt (ATA staff advisor to MCATA), at the registration desk

Sharon Gach (left) and Elaine Manzer

Special projects director Lisa Hauk-Meeker (left) and newsletter editor Tancy Lazar, at the registration desk

Vice-president Susan Ludwig (left) and awards and grants director Carmen Wasylynuik, presenting the door prizes

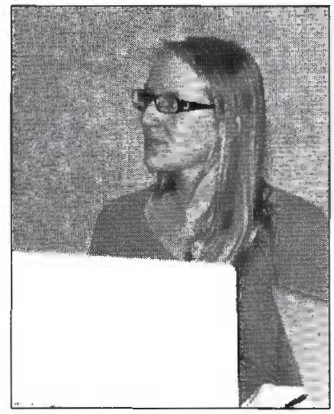
Keynote Speakers

Stephen Murgatroyd

Gerda deVries

Rina Zazkis

Participants






MCATA Fall Symposium Report

Darryl Smith

The MCATA fall symposium was held October 22 at the River Cree Resort, in Enoch. Minister of Education Dave Hancock weathered the storm, as members of the panel and the audience were unanimous in their responses to the questions up for discussion. The questions, which were inspired by the provincial government's recent decision to eliminate the written-response section of the mathematics and science diploma exams, were phrased as follows:

- What is the value and role of communication in mathematics literacy? What does communication in the mathematics classroom look like?
- What is the value of the mathematics diploma exams for Alberta students? In particular, what is the value of the written-response section of the mathematics diploma exams?

ATA president Carol Henderson served as moderator, and she did an excellent job of keeping the discussion open and to the point, tactfully and humorously enforcing time limits on speakers.

Minister Hancock began by saying that the removal of the written-response component was done in the context of planning for the future, and he questioned whether written-response questions help us to achieve outcomes and whether they are the best use of resources. The well-spoken minister candidly admitted that "assessment is a target," and it seems that the decision to eliminate the written-response component was based on what the government sees as a "high correlation" between the machine-scored and the written-response components. Numerous times throughout the panel discussion, he cited a correlation of 0.977 (which was given to him by his advisors, and no information was forthcoming on how the correlation had been calculated). He seemed to view the evaluation of Part A (written-response) questions as a duplication of effort and, of course, an unnecessary cost. "Classroom teachers are in the best position to do assessment," he said, and "removing Part A is in no way an indication that math/science literacy is unimportant." He went on to say that "there is not a lot of literacy in Part A"—a further attempt to justify the removal of the written-response section.

To his credit, Minister Hancock stated several times that he would talk to his advisors on many of these issues, and he seemed genuinely surprised that there was so much concern.

Panel member Carolyn Martin, a teacher, called the removal of the written-response section a travesty. Moderator Henderson suggested that if the written-response section is indeed removed, the weightings of the teacher mark and the diploma exam mark should be 80 per cent and 20 per cent, respectively, rather than the current 50-50 split. Indy Lagu, a professor of mathematics at Mount Royal University, declared that "communication is a necessary part of mathematics education," and he noted that he had not encountered even one multiple-choice question in pursuing his own mathematics education.

Sherry Bennett, of the Alberta Assessment Consortium, offered the opinion that the diploma exam is "high stakes" and implored the government to "not make decisions regarding examinations based on budget!" Both Lagu and Martin opined that machine-scored questions set students up for failure.

Elaine Simmt, a professor of secondary mathematics education at the University of Alberta, stated that "teaching mathematical communication ensures that the mathematically literate person can communicate mathematically." She noted that "society today is held together by mathematics." Simmt reported that the US-based National Council of Teachers of Mathematics (NCTM) holds that "all students should be able to organize their thinking mathematically," and she added that "nothing in the classroom is more fundamental than communication."

The people most affected by diploma exam decisions are, of course, the students, who were most ably represented by Ainslie Fowler, a recent high school graduate who is now studying English at the University of Alberta. Ainslie, in response to the first question, asked, "If communication is so important, why would it be done away with on the biggest exam of the year?" She observed that "written response is communication between student and teacher" and that "clear goals lead to an incentive to work hard." She was candid about her results on the Pure Mathematics 30 diploma exam. Unfortunately, she was one of the outliers with respect to Minister Hancock's stated correlation of 0.977: she scored 93 per cent on the written-response section, but about 20 percentage points lower on the machine-scored section. This is a clear example of panel member Donna Chanasyk's observation that "every question on a machine-scored exam is an all-or-nothing deal."

I congratulate Minister Hancock and all of the panel members for the professional manner in which they examined these questions. It remains to be seen what impact, if any, this frank and open discussion will ultimately have on the government's decision. We can only hope!

The Diploma Exam and the Role of Communication in Mathematics Literacy

Elaine Simmt

In October, I was a member of an MCATA panel responding to the decision of Alberta's minister of education to drop the written-response component from the diploma exams for Pure Mathematics 30 and Applied Mathematics 30. I speak as a mathematics education researcher who does classroom-based studies and as a professor in teacher education. My responses to the questions posed should be read with these roles in mind.

What Is the Value and Role of Communication in Mathematics Literacy?

From my perspective, communication in mathematics literacy plays at least two distinct roles relevant to the discussion at hand.

The first role is quite well understood by educators and the public: the mathematically literate person is able to communicate mathematically. More specifically, the mathematically literate person is able to interpret, translate and express his or her experience of a context mathematically, where and when appropriate.

The contexts for mathematics experiences are many and varied. Consumer, economic, health, industrial, scientific—any of these contexts can be understood mathematically. Take, for example, the following:

- The cost of heating a home
- The volume of oxyacetylene needed for a welding job
- The concentration of saline in an intravenous solution
- The loss of polar bear habitat
- The gross national product

Let's think for a moment about the possibilities for mathematical thinking and communication that emerge in the context of studying polar bear habitat. This context may not be immediately understood as having a significant mathematical dimension, given its scientific, economic, social and political aspects. However, studying the impact of the melting ice caps on the polar bear population requires all kinds of mathematics: measuring the surface area and volume of ice caps, correlating the average global temperature with the surface area, determining the average distances across water to ice floes, estimating seal populations, predicting bear population growth and so on. These are just a few examples of the questions within the broader study of polar bear habitat that require mathematical models to help us better understand.

Mathematical literacy, then, enables us to examine contexts and situations and to act on them in informed ways. Specifically, the ability to interpret, translate and express mathematically provides a frame through which we as members of communities can articulate an argument, relay information, justify a solution, explain a position and critique a practice—in other words, actively participate in and contribute to society.

The second role communication plays in mathematics literacy is less understood by the lay public. However, it is a concern of educators, and in the last few decades has been a major concern of mathematics education researchers. Indeed, this role of communication is highlighted in our program of studies.[1] This is the role that communication plays in thinking. Vygotsky (1934), a Russian psychologist, demonstrated how thinking and speech are intimately connected. Today, researchers have demonstrated the reciprocal relationship between communication and mathematical thinking (Sfard 2008).

The US-based National Council of Teachers of Mathematics (NCTM 2000) standards for school mathematics state that all students should be able to

> Organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; and use the language of mathematics to express mathematical ideas precisely.

In summary, mathematical communication is a form of thinking that enables us to interpret, translate and express our experiences. Communicating is intricately linked to reasoning, visualizing and problem solving—all of the higher-order processes we strive for in mathematics education. To actively participate in critical discussions on topics such as the loss of polar bear habitat or, dare I say, the development of the oil sands, students require extensive mathematical communication skills.

What Does Communication in the Mathematics Classroom Look Like?

Let me first say that the mathematics classroom is critical to the development of mathematical thinkers and communicators. It is in the public school classroom with knowledgeable others (teachers, mentors and peers) that young people encounter this powerful interpretive frame we call mathematics, a tool that enables them to participate in our local, national and international communities. It is in the classroom that they develop as active citizens.

The Western and Northern Canadian Protocol (WNCP 2008, 4) common curriculum framework for Grades 10–12 mathematics states,

> Teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding through:
>
> - taking risks
> - thinking and reflecting independently
> - sharing and communicating mathematical understanding
> - solving problems in individual and group projects
> - pursuing greater understanding of mathematics
> - appreciating the value of mathematics throughout history

It is obvious that all of these goals for the classroom hinge on communication. Hence, nothing in the mathematics classroom is more fundamental than communication.

So, what does communication in the mathematics classroom look like? There are others who are closer to the classroom than I am; however, I do make it a point to visit classrooms throughout the year, and my comments are based on those observations. I will take an example from pure mathematics. Think about this sequence: 1, 3, 5, 7, 9 . . . . How we communicate what this sequence is and what it means to us depends on how we think about it. We teach students to think about it by communicating its many interpretations. If asked to explain this sequence, we might say any of the following:

- It is a sequence of consecutive odd numbers.
- It is a sequence beginning with 1 and adding 2 for each consecutive term.
- It is an arithmetic sequence.
- It can be described by the formula $1 + (n - 1)2$, where 1 is the first term, n the number of terms in the sequence and 2 the common difference between terms.
- It is a linear function $\{(x, y) | y = 2(x - 1) + 1$ and $x \in N\}$.
- It is a linear function with slope +2 and y-intercept −1.

When the study of sequences involves a deep exploration of the mathematics—its connections, relationships and forms of representation—students have the opportunity to become powerful interpreters. Their mathematics grows deep, and they are able to study and comment on their experiences. Good math teaching involves rich mathematical communication. It teaches young people the value of mathematics and how to use it to become a strong interpreter, translator and communicator. So, then, if we know what good teaching and learning experiences look like, what does this say about the form of diploma exam questions?

What Is the Value of the Mathematics Diploma Exams for Alberta Students?

The mathematics diploma exams serve as exit exams to evaluate the achievement of the student learning outcomes specified in Alberta's programs of study for Grade 12 mathematics. The exam results are used by a number of parties and for a variety of purposes: postsecondary institutions, for admission requirements; scholarship committees, for ranking students; school administrators, for evaluating programs; teachers, for evaluating their instruction; and the ministry of education, for keeping school districts accountable for their spending.

As a university professor, I can speak to the role diploma exams play in the context in which I work. Specifically, they are used for student admission and scholarships in that they account for 50 per cent of a student's grade for courses that can be presented for admission. For this purpose and others, diploma exams are a valuable tool; hence, we are compelled to ensure that the exams reflect the program of studies legislated by the province and that they fairly assess a learner's understanding of that which is being evaluated.

What Is the Value of the Written-Response Section of the Mathematics Diploma Exams?

I will now look at Alberta's program of studies for mathematics to investigate the value of the written-response section of the mathematics diploma exams. Clearly, the value of the exams as a whole depends on their ability to fairly evaluate student learning of the major outcomes of school mathematics.

As stated in the program of studies (Alberta Education 2007, 2–3),

> The main goals of mathematics education are to prepare students to:
> - use mathematics confidently to solve problems
> - communicate and reason mathematically
> - appreciate and value mathematics
> - make connections between mathematics and its applications
> - commit themselves to lifelong learning
> - become mathematically literate adults, using mathematics to contribute to society.
>
> Students who have met these goals will:
> - gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
> - exhibit a positive attitude toward mathematics
> - engage and persevere in mathematical tasks and projects
> - contribute to mathematical discussions
> - take risks in performing mathematical tasks
> - exhibit curiosity.

Finally, the program of studies delineates seven process outcomes for mathematics: communication, connections, mental mathematics and estimation, problem solving, reasoning, technology, and visualization (p 4).

The value of the diploma exams for any of the users mentioned earlier depends on the extent to which the exams evaluate the achievement of the learning goals set out in the program of studies. To assess this, we would need to have the exams available to us for study. Unfortunately, they have not been publicly available for a number of years now, so we must simply trust the authorities when they tell us that the exams are a good tool and that they do what they are intended to do. This is a different issue but one that we must continually raise. Quite ironically, investigating the exams would entail the use of the critical mathematical literacy developed in public education.

Returning to the question of the value of the written-response questions in the diploma exams, there are a number of points to be made. Let me begin with a reminder of something that we have heard many times but that is often dismissed: "It is through our assessment that we communicate most clearly to students which activities and learning outcomes we value" (Clarke, Clarke and Lovitt 1990, 118). I do not want to focus on this statement here, but it cannot be ignored, so I will leave it to the teachers to consider. However, I can make some observations that others may not note—first from a broader perspective, and then in the context of Alberta high school mathematics.

Comparing our diploma exams with the exit exams of other countries, I note that national mathematics exams throughout the world (including the United Kingdom, New Zealand and China) use written response.[2] Indeed, students in those countries are asked to prove, to reason, to illustrate and to demonstrate their mathematical understanding through full written responses. Also, postsecondary math exams (at least in Alberta) are by and large written-response tests. Multiple-choice questions and other forms of forced response are not at all common at the postsecondary level.

With regard to the value of written-response questions for the student, there are a few points to be made. First of all, we need to recognize that students can make trivial errors, often as the result of exam anxiety or slips in concentration. In such cases, incorrect answers do not represent a lack of understanding of a concept. In multiple-choice and numerical-response exams, there is no room for such errors; the answer is simply marked wrong. Written-response questions, on the other hand, take into account these kinds of errors and partial marks can be awarded.

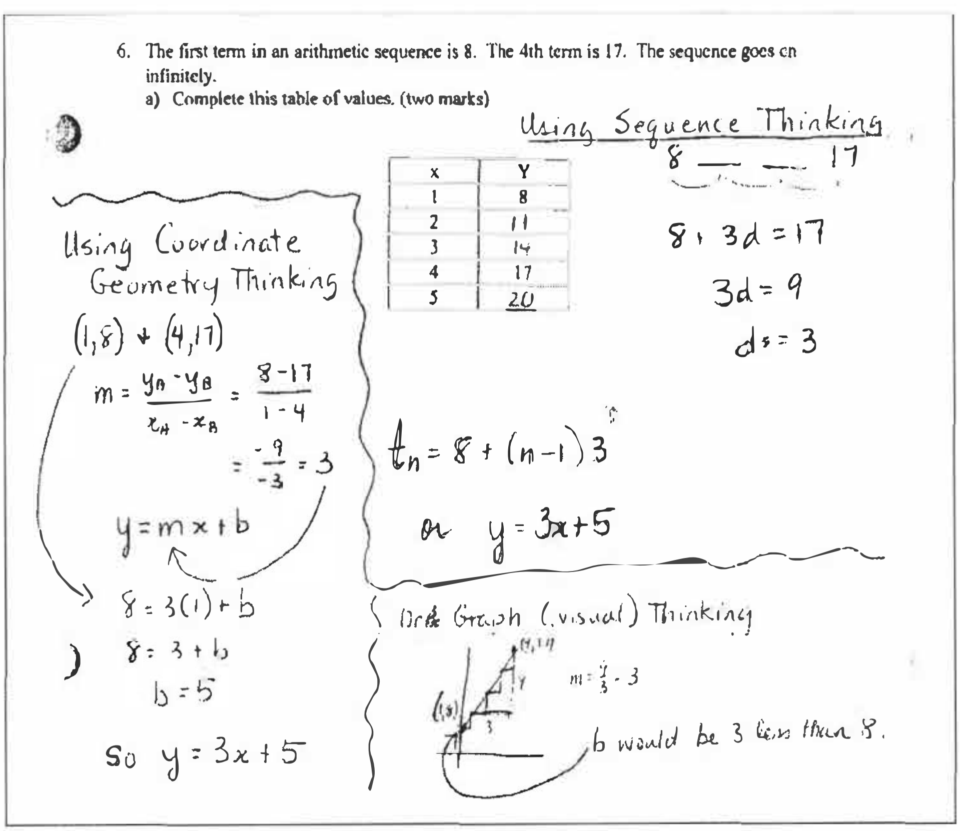
For example, a student computes a response to a trigonometry question and records his answer by filling in a bubble in the numerical-response section of the answer sheet. The answer is to be rounded to the nearest tenth so that a computer can mark it. The student hastily records the answer without rounding to the nearest tenth. His answer is now wrong, even though he *does* understand the trig concept being examined. If this item was supposed to measure the student's understanding, it has failed to do so. (One might argue that the item instead tested the student's ability to round.)

Further, the removal of written-response questions from the diploma exams is somewhat ironic. During their 12 years of mathematics education, students are taught to explain themselves, to think critically and creatively, to justify their positions and to reason mathematically. All of this education is perverted by an exam made up of purely forced-response questions. The task for students becomes one of reading carefully enough to figure out what the examiner wants and not thinking too hard, too creatively or too critically. Instead, students must reason in just the way the examiner has anticipated they will.

Consider Figure 1—an exam question from a high school math class. It is representative of a question on a final exam after the content of the whole course has been addressed. The work shown comes from the teacher's answer key. We see that the teacher has anticipated a number of paths to the correct answer. Given that this question is on a cumulative exam, it makes good sense that students may solve it using any one of a number of mathematical approaches: coordinate geometry, linear functions, sequences and series, algebraically, graphically, inductively, deductively and so on. Of particular note is how well the process outcomes for mathematics in Alberta are addressed by the written-response dimension of this question. Students can demonstrate what connections they have made, how they visualize the context, what mathematics they use to solve the problem, how they have reasoned through the problem and how well they can communicate their thinking.

In contrast, a numerical-response format for the same question might look like Figure 2. Note how the only evidence of the student's thinking is the four digits 1114, representing 11 and 14, the next two terms in the sequence. In this form, the student's answer can be read by a scanner. But what do we learn about this student's understanding? What process outcomes are addressed by this question when posed in this form? Can the student demonstrate an ability to make connections, visualize and communicate? What have we learned about this student's knowledge of sequences and series? Of what value is what we have learned? It is clear to me that there is much value in the written response, but it is less clear what value lies in the numerical response.

Figure 1
Written Response


6. The first term in an arithmetic sequence is 8. The 4th term is 17. The sequence goes on infinitely.
a) Complete this table of values. (two marks)

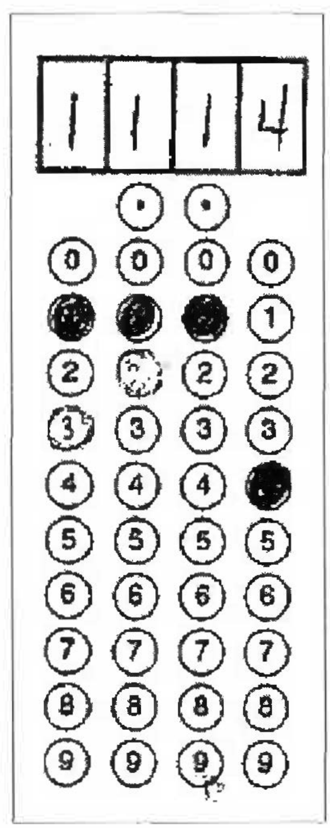
x	Y
1	8
2	11
3	14
4	17
5	20

Using Coordinate Geometry Thinking
(1,8) + (4,17)
$m = \frac{y_A - y_B}{x_A - x_B} = \frac{8-17}{1-4} = \frac{-9}{-3} = 3$
$y = mx + b$
$8 = 3(1) + b$
$8 = 3 + b$
$b = 5$
So $y = 3x + 5$

Using Sequence Thinking
8 __ __ 17
$8 + 3d = 17$
$3d = 9$
$d = 3$
$t_n = 8 + (n-1)3$
or $y = 3x + 5$

Or Graph (visual) Thinking
$m = \frac{9}{3} = 3$
b would be 3 less than 8.

Figure 2
Numerical Response



Let me conclude by suggesting that the primary value of the written-response section of the diploma exams has been to offer students a space where they can communicate their understanding of mathematics. It was a space where they played an active role in communicating (rather than passively selecting an answer expressed in someone else's voice). In the written-response section, they could be asked to examine a situation, interpret it, translate it and communicate their thinking. They could be asked to offer a critique or an evaluation of a situation. It was in those questions that they could demonstrate the higher-level thinking for which 12 years of schooling had prepared them.

The diploma exams must work in support of Alberta's program of studies for K–12 mathematics, which indicates that the process outcomes are every bit as important as the specific content outcomes. A multiple-choice exam is simply inadequate for thoroughly assessing 12 years of schooling. The purpose of all assessment should be to assess both *what* the learner understands and *how* the learner understands. I see no way to achieve this through a multiple-choice and numerical-response test.

The diploma exams should reflect our best practices in assessment and evaluation. Therefore, the decision to remove the written component from the mathematics exams must be reconsidered.

Notes

1. See, for example, the discussion of the seven process outcomes (Alberta Education 2007, 4–6).

2. Personal correspondence with A Watson, G Anthony and B Xu.

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Fall Symposium Report: A Student's Perspective

Ainslie Fowler

As a recent high school graduate, I appreciate that MCATA is seeking the student perspective on the role of communication in mathematics.

With regard to the first question—What is the value and role of communication in mathematics literacy?—I feel that the role of communication is to effectively relay concepts to the student and ensure that a relationship exists between student and teacher. The role of verbal communication in mathematics literacy is to bridge the gap between the language of math and the daily language of discourse.

However, the value of communication varies from teacher to teacher, and this variance can make or break a student's success in class. When teachers value communication highly, they adjust to different forms of communication to appeal to all kinds of students.

Communication in the classroom can take on these adjustments. Communicating the curriculum orally involves more than just daily lectures; it also involves the conversation between students and between students and teachers. Help sessions, student tutors and student presentations are all examples of oral communication that can increase student understanding of mathematics concepts. Visual communication also plays a key role in understanding. Written work (such as notes, daily quizzes to gauge growth or catch problems, and exams) is yet another vehicle for students to convey their status to the teacher.

These two forms of communication can't be successful unless a basic standard for communication is established before classroom work begins. This includes approachability, mutual respect and, more specific to the curriculum, goal-oriented tasks being made evident to students at all times. If students have clear goals tied to their assignments, their incentive to work will increase. Moreover, from the teacher's perspective, formative assessment (such as daily quizzes and small projects) communicates the class's status in a stress-free way. For the student, attending class and knowing the teacher should bring about greater academic achievement.

What is the value of the mathematics diploma exams for Alberta students? The diploma exams standardize testing for all Alberta students. Students are given a fair chance to show their knowledge—no matter where they live, what school they attend or who their teacher is. Overall, classroom work is targeted toward making success on the diploma exam seem like an attainable goal, and the exam represents a student's knowledge and effort over the year. Furthermore, diploma exams are an accumulation of all math units and concepts. Including more than one math concept in a question gives students an opportunity to demonstrate their ability to correlate units, to think outside the box (no memorizing of steps) and to show a holistic knowledge of the course.

For students, the written-response section of the mathematics diploma exams is paramount in proving one's complete understanding of concepts. If a student is prepared and has done well in the class, it is a nice payoff to be able to show his or her knowledge directly.

Multiple choice has its merits as a form of testing, but it is an indirect form, because the options can throw off even a prepared student. Multiple choice has a narrow scope in that it tests only the result, not one's understanding of the process. Multiple-choice questions are designed to be "hard" or "easy," and a rough estimate is made as to how many students will get each question wrong.

On the other hand, written response is a chance for students to clearly demonstrate their knowledge, from equation to answer. Showing one's work allows the marker to see one's logic, even if the final answer is wrong. If a student makes a simple arithmetical mistake, the answer will be wrong, but it will still be clear that the student understands the concept, through how he or she communicates the steps in solving the problem. With written response, students have more control over the answer.

A combination of multiple-choice questions and written work caters to every kind of test writer and learner. If one section of the diploma exams is eliminated, many students will suffer academically. (For example, I scored higher than 90 per cent on the written section and about 70 per cent on multiple choice.) Also, this balance reduces stress for students when writing the exam, because having two parts gives them a chance to redeem themselves if one part doesn't go well. Basing half of a student's final grade on 50 multiple-choice questions constitutes unbalanced assessment and will place immense stress on students.

Written response also prepares students for post-secondary math classes, where they will be expected to write out all their application questions and show every step. The calculator is a secondary device in written-response questions: students must show equations, show every step and even show the buttons they pressed on the calculator. The most important aspect of written response is how coherently the student can communicate the steps that have led to the answer. This occurs to a greater degree in post-secondary math courses, where calculators are often not allowed. Multiple choice can turn into a race to see who can type values into the calculator faster, as opposed to the combination of calculator use and logical explanation used in written response.

If communication is so important in the classroom and clearly yields success, then why would it be done away with in the biggest exam of the year?

Symposium Report

Donna Chanasyk and David Martin

What is the value and role of communication in mathematics literacy? What does communication in the mathematics classroom look like?

The objective of MCATA is to provide leadership to encourage the ongoing enhancement of teaching, learning and understanding mathematics. We define *mathematics literacy* as a student's ability to identify and relate to the role mathematics plays in society. Also, literacy involves creating and communicating conjectures, hypotheses and solutions to problems students may encounter in their current and future lives.

Communication is key. Many of us have had the experience of being lost in an unfamiliar city and asking a local for directions. The directions may be very poor, even if the person has driven that route many times. If asked to drive to the destination, the person could. But he or she has failed to communicate directions effectively. Similarly, students coming out of math classes need to know not only how to reach a solution but also how to communicate that solution effectively.

MCATA aims to offer teachers insights and teaching strategies intended to help students master mathematical communication. Simply achieving the correct answer is not sufficient. In a multiple-choice exam, every question is an all-or-nothing deal, and communication is no longer needed. Communication is present in the math classroom when students' creativity is challenged as they solve problems and search for the significance of key concepts. They defend their conclusions and solutions by explaining their reasoning in more than one way. Communication continues between students as they give each other feedback on their work. This communication gives all students an opportunity to improve and enhance their learning and understanding.

MCATA has been providing Alberta's math teachers with current research on mathematical communication, through its publications (papers, brochures, the newsletter and the journal). Alberta Education must also take a leadership role if mathematical literacy and communication are to become the norm in and out of the classroom.

What is the value of the mathematics diploma exams for Alberta students? In particular, what is the value of the written-response section of the mathematics diploma exams?

The primary purposes of student assessment are to facilitate student learning, to identify strengths and weaknesses, and to create a decision-making process for student progress. According to Alberta Education, the diploma exams have three main purposes: to certify the level of achievement, to ensure that provincewide standards are maintained, and to report individual and group results. The purposes of assessment and the purposes of the diploma exams do not seem to correspond at all. Large-scale assessment of groups of students is carried out to field test new ideas, create accountability and determine curriculum effectiveness. However, the inferences formed and reported are in reference to the performance of the group, not the individual student.

MCATA is opposed to standardized exams when an exam is not appropriate to the educational needs of the student and when the results are misused. The math diploma exams have become high-stakes exams for all students, constituting 50 per cent of their final mark. Even though teachers will continue to integrate mathematics communication into their practice, they may feel the need to spend considerable instructional time on teaching students how to read and answer multiple-choice and numerical-response questions. This is valuable time that could be spent on improving mathematical literacy.

Here in Alberta, we value what we test. The reality is that organizations such as the Fraser Institute evaluate schools based on diploma exam results. So do parents and school boards. Teachers must balance their desire to help students achieve real learning with the very real threat of scrutiny if diploma exam results are less than desirable.

While a few extra days of instruction in mathematics communication may be helpful, this will not alleviate the heavy pressure of an exam worth 50 per cent. Not every student is good at responding to multiple-choice and numeric-response questions, so the removal of the written-response component from the diploma exams will negatively impact some.

Teachers care about individual students and want them to strive to reach their potential. Also, young people are aware that their performance on the diploma exams will significantly affect their options after high school, and we cannot ignore their needs in this regard.

The numeric-response section may be labelled written response on the exam; however, in numeric response, students merely record their answers on the answer sheet and do not show their work. The answer is either right or wrong, and no partial marks are awarded. Thus, this is still an objective type of exam rather than a subjective one. In an objective test (multiple-choice, true-false or fill-in-the-box questions), there is only one correct response. Everything else is wrong. In a subjective test or written-response section, a student can earn partial credit even for an answer that is incorrect.

MCATA supports the new provincial math curriculum because we believe that it has benefits for students. These include "greater opportunity for conceptual understanding" and "course sequences . . . designed to prepare students for their future goals."[1] The first benefit allows students to go deeper into the ideas and concepts of mathematics, and thus allows for intensive understanding. Communication is the key to determining if conceptual understanding and learning have taken place. Written-response questions, therefore, play an enormous role in determining whether students have achieved the second benefit—being prepared for their future goals. Written-response questions allow students to demonstrate critical and creative thinking in the solving of mathematical problems.

MCATA requests that Alberta Education reinstate the written-response section of the diploma exams, using the previous exam format: multiple choice, numeric response and written response—in one exam, in one sitting.

Note

1. See http://education.alberta.ca/media/1089846/revisedhsmath.pdf (accessed January 8, 2010).

To Become Wise to the World Around Us: Multiple Perspectives on Relating Indigenous Knowledges and Mathematics Education

Gladys Sterenberg, Liz Barrett, Narcisse Blood, Florence Glanfield, Lisa Lunney Borden, Theresa McDonnell, Cynthia Nicol and Harley Weston

Significant curricular initiatives in mathematics have been undertaken across Canada to appropriately and respectfully consider indigenous knowledges and perspectives. For example, the Western and Northern Canadian Protocol (WNCP 2006, 3) common curriculum framework for K–9 mathematics now describes Aboriginal learners:

> Aboriginal students in northern and western Canada come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings including urban, rural and isolated communities. Teachers need to understand the diversity of cultures and experiences of students.
>
> Aboriginal students often have a whole-world view of the environment in which they live and learn best in a holistic way. This means that students look for connections in learning and learn best when mathematics is contextualized and not taught as discrete components.
>
> Aboriginal students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized.
>
> A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students.

These initiatives call on teacher educators, teachers, administrators and students to reconsider their received understandings of indigenousness. They challenge inherited conceptual frames derived from the lengthy colonial process of looking for mathematics in cultural activities and validating the activities as mathematical from a Euro-Western perspective. We wonder about the possibilities of enacting an Aboriginal perspective within our educational contexts.

In May 2009, a group of mathematicians and mathematics educators of non-Aboriginal and Aboriginal descent participated in a study group on indigenous knowledges at the Canadian Mathematics Education Forum (CMEF) in Vancouver. For three days, we engaged in authentic conversations (Clark 2001) as we investigated the following questions:

- What do indigenous knowledges offer for teaching and learning mathematics by both Aboriginal and non-Aboriginal students and teachers?
- What role does language play?
- What roles do place, community and culture play?

Narcisse Blood, who is recognized as an *Iitsitsskopa* (emplaced-for-a-reason, or elder), led our conversations and offered guidance and wisdom in our endeavour to better understand the relationship between indigenous knowledges and teaching mathematics.

This article continues these conversations as we draw on our educational experiences to consider the following question: What is the relationship between indigenous knowledges and mathematics education in our current research and teaching projects? We have organized the stories of our multiple perspectives into four sections: culturally responsive education, language and mathematics, learning mathematics from place, and relationships. Our hope is that you will be inspired to appropriately and respectfully consider indigenous knowledges and perspectives in your teaching and learning.

Culturally Responsive Education

> What has *culture* come to mean today? *Culture* came from anthropologists and the old country. The view back then was that any society other than ours is inferior. And anthropologists started saying, "No, that's not the case. As a matter of fact, some societies are every bit as complex as, if not more complex than, ours." That's where the term *culture* started from. But today it has changed. It has a different connotation, especially where I work and in the community. *Culture* has evolved into a definition of right. We're going to work with the universities, and they say, "Oh yeah, let's not leave out the cultural component." You know, as an afterthought. "Let's get Ryan and Narcisse in there to appease." It sounds good. Because when I was in high school, Indian Affairs came up with that same kind of thing: "We'll throw in culture. We're going to teach them how to bead." And they brought in Mrs Rosie Day Rider and the late Louise Cropped Ear, who knew what they were up to. In the classes, you'd be doing beadwork, and they'd be telling stories. It was not what Indian Affairs wanted—you know, "We'll throw a bead and feather in there to shut them up."
>
> *Narcisse Blood*

For the past few years, I have had the honour of working with researchers at the University of British Columbia, as well as teachers, students and community members in two rural communities and one urban community. In each community, we are exploring what it means to live culturally responsive mathematics education.

Our work is guided and informed by indigenous knowledges in multiple ways. First, our project brings together Aboriginal and non-Aboriginal scholars, school-based teachers and researchers, students, and community members to re-imagine mathematics teaching and learning. Second, our work is informed and guided by the framework of indigenous storywork developed by co-researcher Jo-ann Archibald (2008). Working with teachers, we are exploring "the seven principles related to using First Nations stories and storytelling for educational purposes . . .: respect, responsibility, reciprocity, reverence, holism, interrelatedness, and synergy" (p ix), in terms of our research and mathematics pedagogy.

We are exploring our awareness of these principles; how they could be used to connect mathematics, community, culture and indigenous knowledges; and what this could mean for improving students' mathematical experiences and emotions. For example, indigenous storywork as a methodology focuses our attention on the importance of building relationships that are respectful and mindful of community protocols, of deeply appreciating the stories and experiences shared, of giving back and paying forward our work so that future teachers and students can learn from our experiences, and of considering our multiple responsibilities in the research process. Experiences with the land and nature emphasize the connection of the inner passion and heart with the environment. For us, indigenous epistemologies can be characterized as being experiential, storied, relational, contextual and holistic.

Our goal is to transform mathematics education so that it serves the diverse interests and aspirations of all Aboriginal and non-Aboriginal students in our project communities and beyond. Our goal is actualized by the following processes:

- Embracing a holistic and interconnected view of mathematical knowledge
- Exploring practical approaches to using local and traditional knowledges as resources for mathematics pedagogy
- Using local cultural values to extend the possibilities for pedagogical practices, connecting with students and learning from students

Project teachers are exploring culturally responsive mathematics education in multiple ways. One way is by developing mathematics problems inspired by traditional legends. For example, teachers designed a lesson around an audio recording of "Raven Brings the Light," as told by community elders and youths. In this story, Raven brings light to the world after taking it from a series of nested bentwood boxes. The lesson, currently being piloted in classrooms, invites students to listen to the story and then explore the mathematics of building bentwood boxes out of paper.

A second way of exploring culturally responsive education is through developing digital storybooks.

Teachers and students have collected a series of images of the land and activities connected to the land. Together they have designed three mathematics photo books, which involved developing, adapting and writing mathematics problems inspired by the images.

A third way of exploring culturally responsive education has involved collaboration with the local museum. In this context, teachers are exploring the mathematics of the process of canoe building, from sapling to sea.

Teachers have been researching their own experiences of culturally responsive education in their classrooms, and they meet once every couple of months to share their ideas and stories.

Cynthia Nicol

During the past three years, I have had the opportunity to develop relationships with a number of Aboriginal elders and other leaders in the Aboriginal community in western Canada. I feel extremely privileged to have benefited from their sharing their knowledge.

Their knowledge has guided Judi MacDonald and me in the development of resources for teachers, students and parents in Aboriginal and non-Aboriginal communities. These resources, which are being written by education students at the University of Regina and the Gabriel Dumont Institute, have grown out of the audio and video capture of activities and conversations with members of the Aboriginal community. The use of audio and video material facilitates the inclusion of cultural context as an integral part of the resources, as well as making the content accessible to students in the early grades.

The following are three examples of events for which teaching resources are being prepared:

- Elder Glen Anaquod, from the Piapot First Nation, led students from Kitchener Community School through a teepee raising. This inner-city school in Regina has a high population of Aboriginal students. Through this activity, Elder Anaquod shared traditional Saulteaux teachings with students.
- Birchbark biting involves folding a paper-thin sheet of birch bark and putting a perforated design on it by biting. Rosella Carney, a Cree woman from La Ronge, Saskatchewan, is an artist in this art form. She is also a Cree-language instructor, and her interview includes a discussion of number words in Cree.
- Cassandra Opikokew was a student in the Indian Communication Arts program at the First Nations University of Canada and in the School of Journalism at the University of Regina, from which she graduated in 2009. In her interview, Cassandra talks about how she uses the skills she gained in both programs, as well as the importance of Aboriginal issues and voices in the media.

Harley Weston

Language and Mathematics

What we're talking about is a language that was built over thousands of years. So one of the barriers to learning is time—concepts of time, constructs of time. We work with the museums and some of the parks, like Writing-on-Stone Provincial Park. We have something similar about preserving sacred sites, but their concept of preserving is that it's in the past. Our being there, speaking the language and looking at the petroglyphs, takes it out of that model and brings it to the present. To get that across is quite difficult, but there's movement along that line. And, therefore, when we open a bundle, it's—if you want to put it this way—it's history happening right there. So the language is that it's evolved over thousands of years.

Narcisse Blood

I began my teaching career in the We'koqma'q First Nation, which is a Mi'kmaw community on Cape Breton Island in Nova Scotia. Mi'kmaw communities in Nova Scotia have a unique jurisdictional agreement with the Government of Canada that gives them control over their education system and collective bargaining power. These schools live within the tension between the desire to provide culturally responsive and language-rich programs for students and the legal requirement to offer provincially approved curricula.

Student disengagement from mathematics and science is a concern for many teachers in these schools, as they grapple with the tension between school-based mathematics and Mi'kmaw ways of reasoning about things seen as mathematical. Having taught secondary mathematics for 10 years, I have experienced this tension myself. It was this experience, and the related learning from community members, that brought me to my doctoral work. My goal was to work toward the development of culturally responsive mathematics curricula with participant schools; however, it soon became evident that a necessary first step would be to investigate the tensions between Mi'kmaw cultural ways of knowing and school-based mathematics.

My research took place in two schools and involved after-school discussions with teachers in the form of *mawikinutimatimk* (coming together to learn together).

This traditional community practice values the contributions of all participants and acknowledges that we each have something to teach and something to learn.

In our conversations, four key areas of concern emerged as themes:

- The need to learn from Mi'kmaw language
- The importance of attending to value differences between Mi'kmaw concepts of mathematics and school-based mathematics
- The importance of attending to ways of learning and knowing
- The significance of making ethnomathematical connections for students

Although interconnected and interdependent, each of these themes can be tied to the need to learn from Mi'kmaw language.

The important role of indigenous language in understanding mathematics was demonstrated by Denny (1981), who used a "learning from language" approach while working with a group of Inuit elders in northern Canada to explore mathematical words in the Inuktitut language. Rather than developing a curriculum and translating it into Inuktitut, they used the Inuktitut mathematical words to develop the curriculum and associated mathematics activities. More recently, Barton (2008) has shared his similar struggles in translating mathematics concepts into the Maori language. He argues that mathematics evolves with language and that

> A proper understanding of the link between language and mathematics may be the key to finally throwing off the shadow of imperialism and colonialisation that continues to haunt education for indigenous groups in a modern world of international languages and global curricula. (p 9)

This connection between language and the decolonization of education was certainly evident in my conversations with participants during my doctoral work.

The principal of one of the participant schools arrived at our session one day and told me that I should call my dissertation "Lost in Translation." She proceeded to talk about the difference between *Inuitasi* (our people's ways of thinking) and *aklasiweitasi* (anglophone ways of thinking). Many difficulties arise for children when their ways of thinking come into conflict with the teacher's ways of thinking. Something is lost in the translation of world views, ways of thinking and styles of communication. This principal was seeing conflicts arising daily as the students in her school struggled to find their way through a colonizing curriculum. She was searching for ways to resolve these challenges. The Mi'kmaw language holds the key to *Inuitasi*.

Three key pieces emerged in our discussion about the role language plays in understanding the link between indigenous knowledges and mathematics. First, there was an expressed need to include more Mi'kmaw language in the mathematics classroom. This group stressed the importance of reclaiming mathematical words and supporting Mi'kmaq-speaking teachers in developing a lexicon of words for use in their classes. Many participants shared their belief that using the Mi'kmaw language as much as possible could only benefit students. Several participants noted how much more responsive their students were when they were asked in Mi'kmaq to complete a task or do more work. One teacher commented on how her students often didn't understand what she meant when she asked "How many?" But, as she noted, "Say *tasikl* (how many—inanimate), and they get it."

Second, there is value in asking questions such as "What's the word for . . .?" or "Is there a word for . . .?" Through raising such questions, we began to gain new insight into the ways of thinking (*Inuitasi*) embedded in the language. It was interesting to discover that certain words that we assume to be understood by children in school-based mathematics perhaps do not even exist in the Mi'kmaw language. *Flat* is one such word. There is no equivalent word in Mi'kmaq, yet *flat* is commonly used in mathematics. When I asked elders in the community how they would describe the bottom of a basket (which is flat based on an anglophone world view), they said, "It is just the bottom of the basket. It's what lets the basket sit still." And when I asked them about a flat tire, I was told that the tire was "out of air." If *flat* has no equivalent word in Mi'kmaq, what happens when *flat* is used in the mathematics classroom, with the assumption that there is a shared understanding of its meaning?

Third, there is a sense of motion embedded in the Mi'kmaw language that is not apparent in school-based mathematics. In Mi'kmaq, shape and space words act as verbs and are dynamic. For example, to describe something that is straight, Mi'kmaq speakers might say *pektaqtek*, which means "from here to there, it pretty much goes straight." This dynamic nature of mathematical ideas comes into conflict with the tendency in school mathematics toward nominalization—turning actions and processes into nouns. Our group wondered what might happen if we drew on the verb-based discourse of Mi'kmaq rather than the noun-based discourse of school mathematics to engage our Mi'kmaw students in the learning of mathematics. Such "verbification" (using verb-based discourse) may provide Mi'kmaw students with increased understanding of and connection to mathematical concepts.[1]

While there is much more work to be done in exploring the connections between Aboriginal languages and indigenous knowledges for mathematics education, this area of investigation certainly seems to be important in supporting the development of culturally responsive mathematics education.

Lisa Lunney Borden

Something unique has happened in Mount Currie, British Columbia. Xit'olacw Community School is proud to be one of the oldest band-operated schools in Canada, and a group of dedicated elders and parents have worked tirelessly to ensure that it is one of the leading schools in First Nation language education. Reclaiming and protecting the language of the community is a priority. The school has one of the few immersion programs in the area, so students can learn and understand St'at'imcets, the Lil'wat language.

A teacher in the Clao7alcw (Raven's Next) program, Terri Williams, invited me into her classroom so that I could model some effective math strategies for teaching young children. It struck me that we should teach the same math program being used in other classes at the school, but translate it into the Lil'wat language. Everyone was excited by the idea, so we contacted the author of the math program, who was supportive of the idea and gave us permission to start translating the Grade 1 books.

This was the start of a language and math adventure. Lois Joseph, team leader of the Lil'wat7ul Culture Centre, used funding received from First Voices to bring elders together for intense discussions about what language would be best for describing mathematical concepts. New language had to be created to describe squares, rectangles, triangles and core patterns—a collaborative, living process of embracing and agreeing on new words that could and should be used.

What was interesting for me was watching the process unfold and observing the respect the elders showed toward each other throughout the discussions. This was and still is a mammoth team effort, and I wish to pay tribute to the team: Mary James, Laverne James, Priscilla Ritchie, Georgina Nelson, Veronica Bikadi, Jean Andrew, Vera Edmunds, Theresa Jones and Dixie Joe. Tanis Grandbois helped capture the data electronically, and Burt Williams is helping to edit the text. We thank Lois Joseph for all her support of the project, and we look forward to seeing other communities following Mount Currie's lead.

Liz Barrett

Nitawahsin-nanni: Learning Mathematics from Place

My colleague and I were asked to come and share at the university. But what stayed with me is, here are the buildings and there's a parking lot. And here are trees that go down to that environment. What struck me is the dissonance in terms of learning. What transformation will take place for the university to acknowledge the place and how tragic that can be. Because we're sitting up there overlooking the harbour, and you have all these ships and all these raw materials that are going to be shipped away somewhere. And how unsustainable that is.

Narcisse Blood

Alberta teachers are now required to infuse the curriculum with Aboriginal perspectives. Over the last two years, I have taken on this challenging but important endeavour in my middle school mathematics classes. I teach in Siksika, Alberta, one of the member nations of the Blackfoot Confederacy. I am of Cree and Irish Canadian descent.

In my classroom, I have observed that silent struggles with identity create the greatest barriers to success among my students. They long deeply to learn more of their proud history and accumulated knowledge, while also being profoundly involved and invested in a global world.

My experience has taught me that teaching from an Aboriginal perspective is best accomplished by focusing on the perspectives of the Siksika people. Because their knowledge has accumulated over the millennia in one place, it is fair to suggest that Blackfoot perspectives arose from their interaction with the land now labelled southern and central Alberta, eastern Saskatchewan and northern Montana. Their knowledge is rich in mathematics. I have come to see my job as a teacher as empowering Siksika students in recognizing their intuitive mathematical abilities. This is partly done by making time in the schedule for learning from place.

Learning from place continues to be a valid and meaningful method of interpreting and understanding the world, including mathematics. As most mathematicians know, mathematics can be found everywhere, and nature is no exception. Therefore, I decided not to rely solely on mathematics textbooks. Instead, we explore mathematics in the world, as well as in the textbook. The two complement each other well. For example, I taught part of both the shape and space units using the spokes, angles and circular geometry of the Majorville Medicine Wheel site. Students learned the concepts with help from the textbook; then, we explored those concepts through activities at the site and through the creation of models in the classroom.

By visiting sacred sites within the Blackfoot Confederacy and bringing some lessons outside, we soon realized that the land had become a very present third party. When taught outdoors, lessons don't follow the schedule a teacher has carefully planned. Suddenly other factors arise, such as weather, animals and other unexpected guests, and an abundance of student curiosity. This alternative method of teaching mathematics is challenging because it requires a plethora of planning, a field trip budget, an awareness of protocols, support from culturally knowledgeable staff and community members, flexible and supportive administration, and a willingness to relinquish some power and structure in teaching.

How each teacher meets the new goals set out by the province of Alberta will be varied and unique. For me, *infusion* is the wrong term. Teaching from an Aboriginal perspective is simply finding what is meaningful and relevant to students, and what honours the ancestors of the host territory in which teachers live and teach. It means teaching the curriculum and simultaneously addressing silent identity issues by revering the land and the people from which the students have come. This can be accomplished by continuing to find meaning in places and by inviting students to see the world mathematically and intuitively.

Theresa McDonnell

Working alongside my Aboriginal colleagues and friends, I am coming to understand my unique role as a person who can provide bridging experiences between teachers at federally and provincially funded schools. Perhaps this is best illustrated by a story of one such bridging experience.

In my role as a mathematics teacher educator, I have been invited to work in partnership with teachers in several First Nation communities. At one nation high school, we are investigating how learning from place might be enacted. To date, we have taken students on two field trips to sacred Blackfoot sites, including the Big Rock. Located west of Okotoks, Alberta, the Big Rock is the largest glacial erratic in the world and is part of a series of boulders stretching from Utah to Jasper. This site is significant to the Blackfoot community, and the splitting of the Big Rock as a result of the actions of Napi (a supernatural trickster of the Blackfoot peoples) is a familiar story.

On our field trip to the Big Rock, the elder who was with us began with an offering and stories of Napi. While we were there, elementary students from the Big Rock School, in Okotoks, arrived to investigate the meaning of their school's name. I knew one of the teachers, and once I found out why they were there, I suggested that her students might want to hear the Napi stories being told by the elder. Following protocol, I asked the elder if he could share the stories with them, and he was very excited about doing so. The elementary students were very respectful when listening to him. The principal of Big Rock School wrote an affirming piece about this experience, which was published in a local newspaper.

In this particular situation, my comfort with both federally and provincially funded school communities facilitated a bridging experience. For me, this was a profound experience, as I came to better understand the importance of creating opportunities for sharing knowledge between Aboriginal and non-Aboriginal communities and my possible role in this process.

For me, relating indigenous knowledges and mathematics education is focused on the act of listening. As a non-Aboriginal visitor, I am respectful of protocol, always mindful of my place within this place. Learning from place recognizes the intimate relationship indigenous people have with the land. In our visits to sacred places, the land has spoken to me as I have listened. Stories of place have grounded our visits, and our response to knowing the land has informed our acts. Carbaugh (1999) describes listening as "dwelling-in-place." He suggests that Blackfoot listening is a "highly reflective and revelatory mode of communication that can open one to the mysteries of unity between the physical and spiritual, to the relationships between natural and human forms, and to the intimate links between places and persons" (p 250). Through the stories told to us by cultural elders, I am learning to dwell-in-place.

Gladys Sterenberg

Relationship–Responsibility–Recursion

Really, what it is for my people, and I think all of us—and sometimes we forget—really, what it is all about is this relationship. Ultimately, that's who we are. It's relationship. If you want to learn about the Blackfoot, it's about relationship and relationship and relationship. About everything. Those that we see, and those that we don't see. So with that, I'm starting to get to know a lot of you. It's good to see you. And that's important. In our world, we don't take that for granted.

Narcisse Blood

"Florence, ohmigosh, this is FOIL! Why didn't anyone ever tell me that there was a relationship? I knew how to multiply, but I could never figure out FOIL!" These were the words of Cindy, a student in a mathematics education class for preservice elementary teachers, seven years ago. Cindy and her peers had been exploring the underlying concepts behind the multiplication of two two-digit numbers. In Cindy's work with base-10 blocks and the subsequent numerical representations, she came to realize that the multiplication of two two-digit numbers is the distributive property of multiplication, or FOIL (first, outside, inside, last). I now hold Cindy's words with me in my teaching and learning, and have held her words as I've come to know and acknowledge myself as an Aboriginal person in teaching mathematics.

When I consider Aboriginal perspectives in the mathematics classroom, I think about the importance of relationship. In my experiences as an Aboriginal person, I've come to understand the importance of acknowledging relationships in all aspects of life. For example, we have a relationship with the land (some call it Mother Earth) as we draw sustenance from it, and once we acknowledge our relationship with the land, we then have a responsibility to sustain that relationship and sustain the land. A second example is our relationships with our family: we draw sustenance from our family, and when we acknowledge that we draw sustenance from our relationship with family, then we also have a responsibility to sustain that relationship and sustain the family. Once we acknowledge the relationships we have with the land and with our family, and act on our responsibility as being a part of that relationship, then our relationship is now different from how it was. Hence, we are in a continuous cycle of acknowledging relationship, acting on our responsibility, and reliving or retelling the relationship (recursion).

What does this mean for a mathematics education class? Most of my teaching now involves teaching preservice teachers about what it means to teach mathematics. The planning work I do in my classes focuses on this notion of relationship–responsibility–recursion. In a classroom, there are multiple perspectives on relationship that a mathematics teacher holds in his or her practices. Teachers have relationships with mathematics; with their own experiences in learning mathematics; with their colleagues in the school; with the community in which the school resides; with their view about what it means to learn, to teach and, specifically, to teach and learn mathematics; with their family; with their students; and with themselves. I also believe that a teacher must acknowledge that within each of these relationships are other relationships that can be named. For example, within the relationship one has with mathematics, one might be aware that there are relationships within the content of mathematics itself.

My development as a mathematics teacher and a mathematics teacher educator, then, becomes a lifelong journey, as I will continue to come to know about the multiple relationships that exist within my classroom and will continue to have multiple responsibilities. In other words, as I attend to or notice (Mason 2002) the complexity and the responsibility of the relationships that exist in my classroom, my teaching continues to focus on sustaining relationships. My life as a teacher, then, is a series of recursive acts in sustaining relationships.

What might be an example? Suppose I am teaching about the multiplication of two binomials in a Grade 9 or 10 mathematics class. To focus on relationship, I might ask myself the following questions:

- How is it that I understand the multiplication of two binomials?
- In what way is the multiplication of two binomials related to the multiplication of two two-digit numbers?
- How can I invite my students to see that relationship?
- What were my experiences in learning about the multiplication of two binomials? How have those experiences informed my understanding?
- How does learning the multiplication of two binomials contribute to my students' deeper understanding of mathematical ideas?
- In what way will my actions as a teacher in teaching about the multiplication of two binomials sustain my relationship with my students? In what way will they sustain the relationship my students are developing with mathematics?
- In what way does how I see myself as a learner of mathematics influence the decisions I make as a teacher of mathematics and the tasks I ask my students to engage in while learning about the multiplication of two binomials?
- How do I share my understanding of the relationships within mathematics with my students in a way that sustains the relationship I have with them?
- How do I share my understanding of the relationship between the multiplication of two binomials and other areas of mathematics with my colleagues?
- How do I participate in the development of a shared understanding of these mathematical ideas within the community in which I teach?

In this way, mathematics teachers might begin to focus on the relationships between or the interconnectedness of our content, our lives as teachers, the lives of our students and the context in which we teach. Bopp et al (1988, 62) write that "the great lesson of the sacred circle is always that separate entities, when seen in light of the universe, are equal and necessary parts of the larger whole. It brings out the ancient teaching of the interconnectedness of all things."

Florence Glanfield

Conclusion

Colonization was an intentional act. If we try to convince our students that the only mathematics that exists and is worth studying has its roots in Euro-Western traditions, then we are engaged in such an intentional act. By honouring multiple perspectives on mathematical thinking and knowing, we can come to know mathematics and ourselves in a different way.

The WNCP's (2006) common curriculum framework for K–9 mathematics states that "the strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education." While we agree that token gestures of inclusion are inappropriate, we believe that relating indigenous knowledges and mathematics education is context-specific. Multiple perspectives are necessary and embraced. Moreover, any project must be initiated by the community.

At the end of our time together at CMEF, we were left with many questions: How do we teach in ways that are responsive to students and that challenge systemic inequities? How do we invite others to think with us in similar kinds of critical reflexive research? How do we engage in our own decolonizing? In this article, we have shared our stories and challenged others and ourselves with our questions. As we reflect on our experiences within this working group, we contemplate the next steps to be taken and invite others to walk alongside us as we attempt to live our lives differently.

Take our stories. They're yours. Do with them what you will. Use them in your planning. Tell them to other educators. Forget them. But don't say in the years to come that you would have lived your life differently if only you had heard our stories. You've heard them now.[2]

Notes

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1. See Lunney Borden (2009) for a more detailed discussion.

2. Adapted from the words of Thomas King (2003, 151):

Take it. It's yours. Do with it what you will. Tell it to your children. Turn it into a play. Forget it. But don't say in the years to come that you would have lived your life differently if only you had heard this story.

You've heard it now.

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Similar Triangles Versus Trigonometry

Tim Sibbald

Similar triangles are included in the shape and space strand of Alberta's Grade 9 mathematics curriculum. This provides a good footing for introducing trigonometric ratios in subsequent courses, because similar triangles familiarize students with the idea of using ratios of side lengths in a triangle. However, the premise of this article is that similar triangles provide a rich vein of mathematics when revisited from other parts of the math curriculum (in particular, Mathematics 20-1 and 30-1, and Pure Mathematics 20 and 30)—in effect, when viewed through the rear-view mirror.

This article looks at two distinct issues: (1) student approaches when using trigonometry to develop the equivalent ratios used in similar-triangle problems, and (2) the range of algebra questions that can be developed in similar-triangle questions.

Similarity by Trigonometry

The question of how similar-triangle ratios can be developed using trigonometry is fascinating—not because of the answer but, rather, because of the variety of ways the problem can be addressed. Thus, the focus here is not the answer; it is how students can explore and navigate this open-ended problem. (The answer will be provided simply to remove the focus from it. In a class, the answer would not be revealed, in the hopes that students would discover it themselves.) This topic is suited to the Mathematics 20-1 course, where the sine law is taught and students are familiar with trigonometric ratios.

A Simple Answer

Students usually perceive the problem as difficult and, in an effort to simplify it, avoid the sine law. The tendency is to start with right-angle trigonometry; however, this strategy complicates the problem, because one then needs a right angle. In fact, using the sine law is the simplest approach (using Figure 1 for clarity).

$$\frac{\sin(A)}{a}=\frac{\sin(B)}{b} \qquad \frac{\sin(A)}{c}=\frac{\sin(B)}{d}$$

$$\frac{\sin(A)}{\sin(B)}=\frac{a}{b} \qquad \frac{\sin(A)}{\sin(B)}=\frac{c}{d}$$

$$\frac{a}{b}=\frac{c}{d}$$

Why do students avoid this approach? They have difficulty accepting that they can derive useful conclusions from working with angles that have unknown values. In effect, mathematics teaching has led many students to believe that only the explicit information given to them is relevant in solving any given problem.

Use of the sine law could be encouraged by giving students a value for an angle. For instance, "I know that this problem could be done if angle A were equal to 10°." Students could then write expressions that involve that angle, and that might lead them toward the sine law. Also, the teacher could then ask students to consider what they could calculate for each triangle, a different question that would help consolidate the utility of the sine and cosine laws.

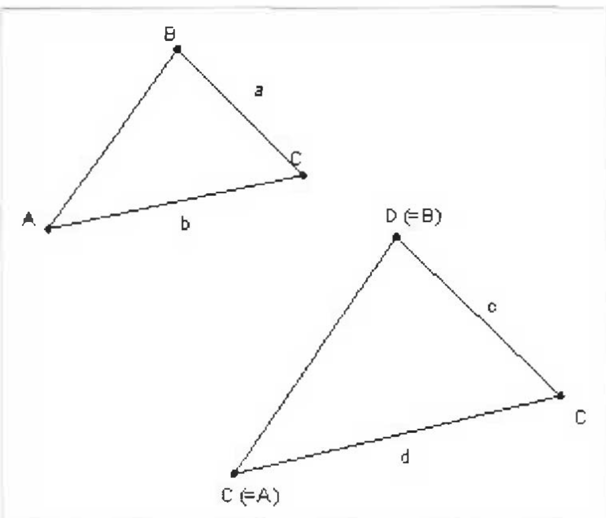
A Less Obvious Solution

Students try to keep the problem simple by using right-angle trigonometry, but that can only be done by constructing equivalent problems. The term *equivalent* here means that the given information is maintained and the information that is not specified can be changed in a manner that maintains the similarity.

One idea is to rotate a side about a point until a right angle is created; however, to keep the two triangles similar, the same degree of rotation must be

Figure 1

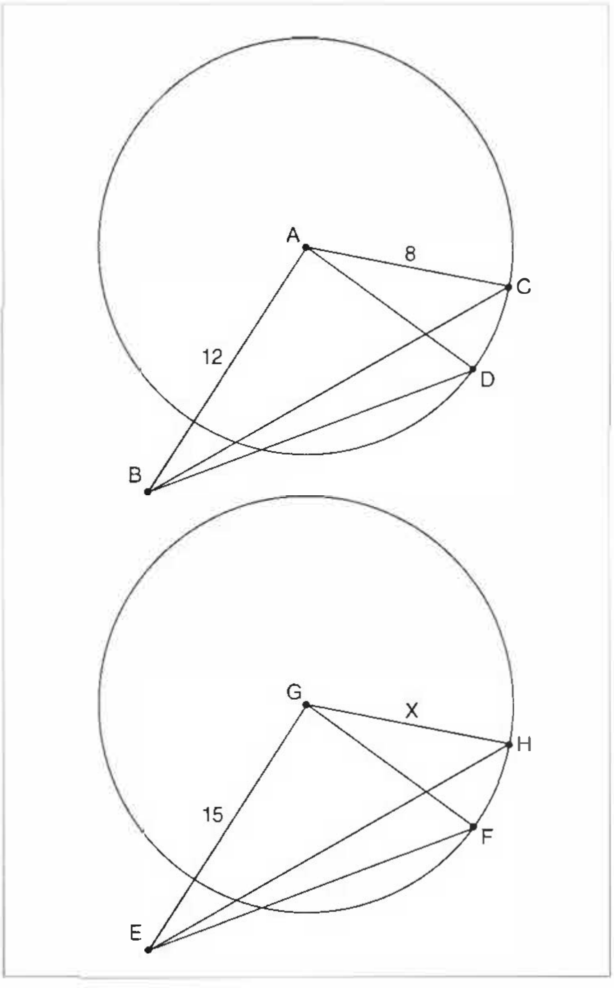
Similar Triangles



performed on the equivalent side around the equivalent point in the similar triangle (see Figure 2). The original question uses $\Delta BAC \sim \Delta EGH$, but $\Delta BAD \sim \Delta EGF$ by rotating both AC and GH to AD and GF, respectively. This process preserves the lengths given in the original question but allows $\angle BAD$ to be changed to any angle. There is a catch: changing $\angle BAD$ to 90° changes both of the other angles, $\angle ADB$ and $\angle DBA$, and that raises the question of how one can be sure that $\angle ADB$ is equal to $\angle GFE$. For students, this highlights the need to provide some details.

Figure 2

Making an Equivalent Similar-Triangle Question



When the transformation is done, and $\angle BAD = \angle EGF = 90°$, students can use right-angle trigonometry to solve for GF, which has the same length as GH and x.

$$\tan(\angle ABD) = \frac{8}{12} = \tan(\angle GEF) = \frac{x}{15}$$

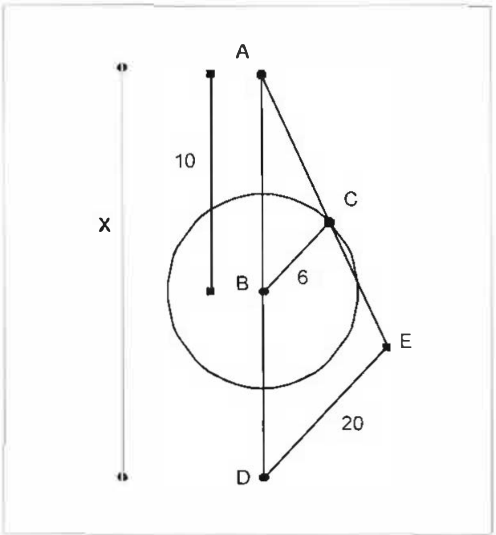
Transforming the question to an equivalent one that can be solved raises many doubts among students. This approach is not as straightforward as using the sine law, but it does arrive at a solution. Students have difficulty recognizing that changing the question is a legitimate approach, as long as the given information is preserved.

Another Approach

How can similar-triangle ratios be developed from trigonometry using the diagram in Figure 3?

Figure 3

Similar-Triangle Question



When I use this type of question, I present the problem without the circle. Discussion about how to solve this type of question leads to the addition of the circle, which promotes discussion about moving point C in a manner that allows $\angle ABC$ to be changed. This is analogous to what was done in the previous section. However, this approach has an added benefit: keeping DE parallel to BC is a simple way of articulating the condition for equivalence of questions.

Students sometimes do the unexpected, especially when they brainstorm in small groups. While several groups will follow up on the idea of changing $\angle ABC$ by moving point C, some groups will interpret this as rotating the line segment AE about point A. The latter approach leads to the idea that there is a maximum possible angle for A. It is then opportune to mention the concept of the tangent line and to point out that a tangent line, because of symmetry, has to be perpendicular to the radius line. Students then realize that they can move point C to the tangent point, keeping DE parallel to BC, and create a right angle at $\angle ACB$ (and the corresponding angle, $\angle AED$).

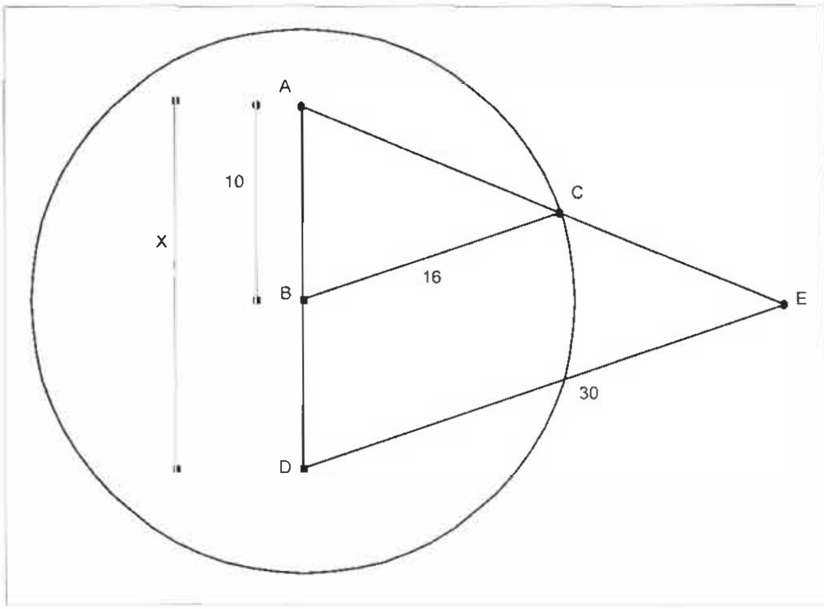
This approach works only if a tangent line can be created. In one class, my students realized that with C as a tangent point and $\angle ACB$ as 90°, they could use the sine law to determine the maximum value of angle A. They then argued that they could use the angle and sine ratio to determine the unknown length x. While I was impressed that they had come up with this solution, I felt obliged to point out that the model question (Figure 3) had BC shorter than AB, and that there could be a problem if BC were longer than AB (see Figure 4). Students then wrestled with the idea that moving point C would never make a tangent line and that there was no maximum value of angle A. In this case, their argument did not work.

Does one actually need the right angle? I suggest to students that the process shows that they can make angle A any value they choose, up to the maximum found in the earlier version of the question. If they can make angle A any value, can they solve the problem? For example, suppose angle A is 20°. Does this help solve the problem? Unfortunately, in my class, the students who were modifying $\angle ABC$ overheard this and argued that they could make $\angle ABC$ a specific value, allowing them to use the cosine law to find AC. This is an awkward, roundabout solution that can lead to x, but it has enough steps that the students did not get to the answer.

I was intrigued by my students' efforts and concluded that this type of question is invaluable. I had not anticipated the variety of approaches, and I had to think on my feet more than once. However, this article consolidates the major approaches students have used, helping teachers to better anticipate what may arise. It is particularly intriguing to observe what happens when the information given in the question is not sufficient. Few students recognize that they can impose new information, provided that it does not contradict the given information.

Figure 4

Modified Question with BC > AB



A Different Type of Question

The connection between trigonometry and similar triangles made me question if there are similar-triangle questions with pedagogical value that have been ignored because of the view that similar triangles are a stepping stone into trigonometry. This issue is exacerbated by a bias in teaching materials, where similar-triangle resources typically use numbers for side lengths (with three given and one unknown) and often include the same question formats.

In a moment of inspiration, I formed a similar-triangle question in which the unknown, x, appeared twice. See Figure 5.

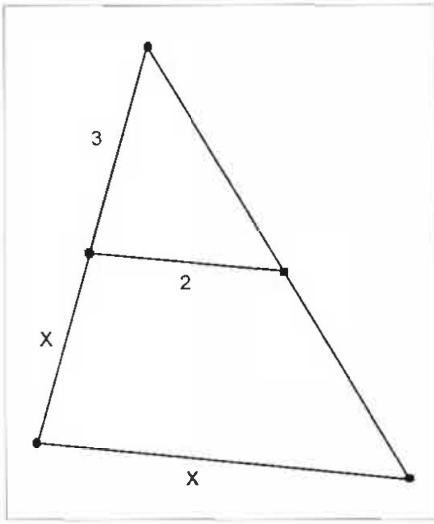
The problem is resolvable for 20-1 students as follows:

$$\frac{x+3}{3} = \frac{x}{2}$$
$$2(x+3) = 3x$$
$$2x + 6 = 3x$$
$$x = 6$$

This demonstrates a novel context for practising algebra.

Figure 5

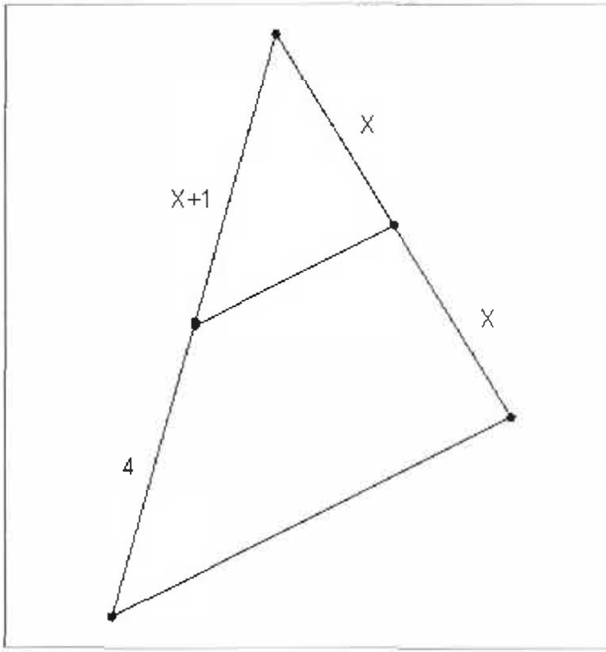
A Different Kind of Question?



Using other features of algebra is feasible. Consider the problem shown in Figure 6 and the following two methods of solving it.

Figure 6

Similar Triangle Requiring Algebra



The first method uses cancellation of the x values (since $x > 0$) on the left side of the equation:

$$\frac{2x}{x} = \frac{x+5}{x+1}$$
$$2 = \frac{x+5}{x+1}$$
$$2x + 2 = x + 5$$
$$x = 3$$

Another solution uses common factoring:

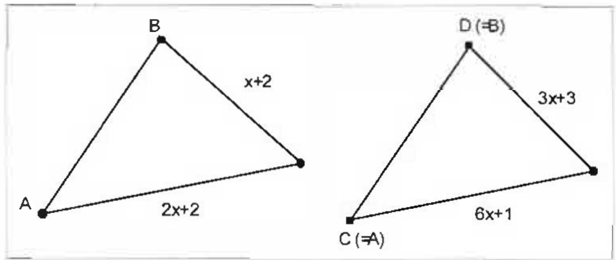
$$\frac{2x}{x} = \frac{x+5}{x+1}$$
$$2x(x+1) = x(x+5)$$
$$2x^2 + 2x = x^2 + 5x$$
$$x^2 = 3x$$
$$x(x-3) = 0$$

This could be used as a point of discussion for the domain; note that x cannot be 0 if this question is about a triangle.

It is also possible to have questions where quadratic terms arise in a manner in which they cancel. This assumes that students know how to multiply two binomials.

Figure 7

Example Requiring Algebra



Consider the example in Figure 7, which has the following solution.

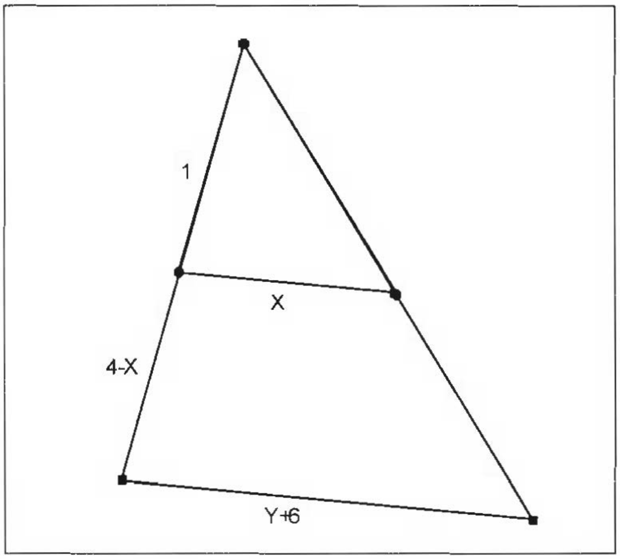
$$\frac{x+2}{3x+3} = \frac{2x+2}{6x+1}$$
$$6x^2 + 13x + 2 = 6x^2 + 12x + 6$$
$$13x + 2 = 12x + 6$$
$$x = 4$$

The general solution for this type of problem can be thought of geometrically as the intersection of two quadratic functions (as in the second line of the previous solution, considering each side to represent an independent quadratic). However, the general solution is not particularly insightful for designing questions, because it is beset by conditions representing the following requirements: all triangle sides must have positive lengths, and the problem must have a unique solution. There are cases with multiple solutions. Consider two equilateral triangles—one with all sides of length x, and the other with all sides of length $2x$. In this case, any positive value of x will suffice.

Last, similar triangles like this could be used to develop further types of questions, such as those requiring quadratic factoring with two unique positive solutions (in Figure 8, set $y = 0$ and solve for x). While the earlier questions have merit because of the connections to algebra and equation solving, casting problems of this type for factoring would be contrived. However, this type of problem, with the y in place, could be useful in higher mathematics classes to address domain and range issues. Consider the scenario in Figure 8, where the task is to determine the conditions for x and y so that the problem can be solved.

The relationship between x and y is quadratic ($y = -x^2 + 5x - 6$), and the restriction that side lengths must be positive provides restrictions for the domain and range. The domain is $0 < x < 4$, so that side lengths x and $4 - x$ are both positive. Over this domain the quadratic reaches a maximum at $x = 2.5$ and $y = 0.25$. The minimum value over the domain occurs as x approaches 0 and y approaches -6. So the range is $-6 < y < 0.25$. Note that this type of domain and range

Figure 8
Domain and Range Question



question can also be formed with linear functions, for instance by changing $4 - x$ to 4.

Concluding Remarks

This article highlights opportunities to use the concept of similar triangles for more than simply introducing trigonometry. The questions are designed for making connections between mathematical concepts that are often viewed as distinct. These connections lead to a deeper appreciation of the conceptual interconnections surrounding similar triangles.

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Circles Are Squares

Amanda Crampton and Paul Betts

We have been working together in a small rural school to improve our understanding using reform-based mathematics teaching strategies.

Last year, we tried a lesson in which we asked Grades 2 and 3 students to draw on 1 cm grid paper as many shapes as they could with an area of 16 cm^2, by counting the number of complete grid squares inside each shape. During this activity, one student became interested in trying to draw a circle with an area of 16 cm^2. Knowing that calculating or estimating the area of a circle is difficult, we were unsure of what to do with this sudden and unexpected interest.

Before long, most students were searching the classroom for objects with a circular face, trying to find one with an area close to 16 cm^2, and we observed significant mathematical thinking. For example, several students noted that small changes in the radius of the circle resulted in large changes in the area, which is an informal realization that area is a function of the square of a length (the relationship is quadratic, not linear).

After the lesson, we wondered whether we could create lessons to scaffold students' learning in order to help them acquire the skills to successfully determine the area of a circle. This year, we developed and implemented a series of inquiry-based lessons on area for Grades 2 and 3; the lessons were designed to reproduce the kinds of thinking we observed during the lesson described above. In this article, we describe those lessons.

Inquiry and Mathematics

Inquiry is a reform-based approach to teaching mathematics. It is premised on the idea that children can construct meaningful representations of mathematics, given a learning community that validates the thinking of all children (Cobb and Bauersfeld 1995).

We reject the idea that children must be told about mathematics. Rather, we wish to establish a learning environment in which students and the teacher work together to communicate ideas and use reasoning to jointly establish the validity of mathematical ideas. In order to foster individual and classroom mathematical engagement, mathematics educators often recommend the use of rich mathematical tasks. Such tasks are characterized as open-ended, with multiple possible entry points, trajectories and end points.[1]

Inquiry can be difficult to implement because of the tension between validating the thinking of all students and achieving the outcomes of the mathematics curriculum. The process for solving a rich mathematical task is not specified in advance; students must develop their own strategies, and teachers cannot control the activity (Smith, Bill and Hughes 2008). This lack of control means that teachers will make many decisions during an activity, trying to strike a balance between scaffolding student thinking and meeting broader mathematics teaching goals (such as curricular goals and the big ideas of mathematics).

The Mathematical Horizon

The mathematical horizon is a specific kind of broader mathematics teaching goal that teachers can attend to during inquiry lessons. Ball and Bass (2009, 5–6) define *horizon knowledge of mathematics* as "a sense of how the mathematics at play now [within a specific lesson] is related to larger mathematical ideas, structures, and principles." For example, knowing the relationship between area and radius for circles would be an example of horizon knowledge for lessons that explore finding the area of a circle by counting grid paper squares.

We agree with Ball and Bass (2009) that mathematics activities have greater potency when teachers attend to the mathematical horizon to inform their in-the-moment decision making. Thus, we have noted potential mathematical horizons for the lessons on area that we describe below. These should not be interpreted as the only mathematical horizons, teaching or curriculum goals, or learning goals that could guide teacher decision making.

Outline of Lessons

The overall mathematical horizon of this series of lessons is for students to develop a rich understanding of the meaning of area, including conservation of area and the informal precursors of area formulas.[2]

In this case, conservation of area is expressed by realizing that transforming squares (by moving or exchanging them, for example) does not change the area. We have found that many Grades 2 and 3 students need several experiences with conservation of area before it becomes meaningful. At this age, a child's apparent understanding of area conservation is not necessarily permanent and may be unstable.

We did not expect students to develop formulas for area. Rather, the intention was for students to experience the relationships evident in area formulas, and perhaps informally express those relationships. For example, for rectangles, area is a representation of multiplication. For circles, radius and area are not linearly related (rather, area is a function of the square of the radius).

The lessons outlined below are based on 40-minute periods. Each lesson begins on the learning carpet, a permanently gridded, 100-square floor carpet.[3] The focus is on exploring the notion of area, not formulas for calculating area, where the unit of area would be learning carpet squares.[4]

The mathematical horizon given for each lesson is intended to incrementally build toward the overall horizon described above. We also briefly describe the task and potential scaffolds to indicate some of the teacher decision making needed to ensure that the lessons remain inquiries, rather than devolving into traditional lessons where the teacher tells the students what to do. Possible scaffolds are questions that the teacher could ask students to restart their thinking when it appears to be stalled, when they are seeking teacher guidance, or when they appear to be frustrated with the task.

It should be expected that these lessons will play out differently when implemented, and teachers should be prepared to adapt in the moment.[5] We describe the lessons in further detail in a later section, where we also note specific events that occurred during our implementation.

Lesson 1

Mathematical Horizon
- Introducing area as covering

Task
- Make several swimming pools (rectangles) with 16 paper squares, so that the area of each pool is 16 squares.

Possible Scaffolds
- Should we remove squares or get extra squares?
- Do the various pool shapes still have the same area?

Materials
- Learning carpet
- 16 paper squares, plus extras (each paper square should be congruent to a square on the learning carpet)
- 1 cm grid paper (at least one sheet per student)

Lesson 2

Mathematical Horizon
- Experiencing area conservation

Task
- Make a triangle with an area of 16 paper squares.

Possible Scaffolds
- Is the shape really a triangle?
- Would it help to find area by cutting a paper square in half?
- What is the area of two of these half pieces of paper?

Materials
- Learning carpet
- 16 paper squares, plus extras
- Scissors
- 1 cm grid paper (at least one sheet per student)

Lesson 3

Mathematical Horizon
- Further experiencing area conservation

Task
- Make interesting shapes with an area of 16 paper squares.

Possible Scaffolds
- What about using pieces of paper of other sizes?
- Which pieces together make a square?

Materials
- Learning carpet
- 16 paper squares, plus extras
- Extra half pieces and precut pieces of various sizes (see Figures 3 and 4)
- "Paired Shapes" handouts (see the appendix for an example)
- Scissors

Lesson 4

Mathematical Horizon

- Consolidating area conservation
- Estimating area

Task

- Find the area of a shape bounded by string.

Possible Scaffolds

- When do pairs of pieces form a square?
- What about this little gap or overhang?

Materials

- Learning carpet
- 16 paper squares, plus extras
- Precut half pieces, one-quarter pieces, three-quarter pieces, one-sixth pieces, three-sixth pieces and five-sixth pieces
- Scissors
- String about 4 m long[6]

Lesson 5

Mathematical Horizon

- Estimating area

Task

- Find a circle with an area close to 16 cm^2.

Possible Scaffolds

- Is this an accurate circle?
- What about this gap or overlap?

Materials

- Learning carpet
- Plastic hoop
- Various objects with a circular base with a radius of up to 6 cm (mugs, plastic cups, cylinders, tape rolls, pop cans, bottle caps, Tupperware containers and lids, yogurt containers)[7]
- 1 cm grid paper

Detailed Lesson Descriptions

Lesson 1

Our first lesson was an introduction to area. On the learning carpet, we formed a rectangle with an area of 16 squares by placing 16 paper squares (each congruent to a learning carpet square) in two rows of eight (see Figure 1).

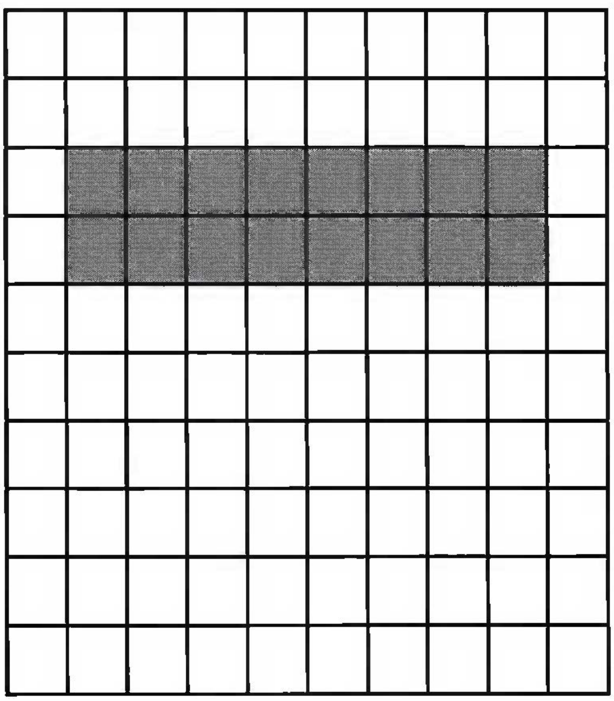
We asked the students, "If this rectangle is a pool, and we have to build a cover for the pool, how many squares big will that cover have to be?" After all the students had successfully counted the squares and mouthed the number to us,[8] we introduced the term *area*. For the rest of the lesson, we always used the terms *area* and *covering* to reinforce this new terminology.

We then began an inquiry into the core task of this lesson—making as many rectangular pools as possible with an area of 16 squares (using the 16 paper squares). Extra squares were in view of students, and we told them that they could have more squares or take squares away. One by one, students were selected to create new pools with the 16 paper squares, and with each new shape all students were asked to determine and then mouth the area.

As this was repeated several times, some of the students began to recognize that the area was unchanged, provided that no squares were added or removed. Other students continued to count squares, so we prompted them with the question, "Do we need any of these extra squares?" To provoke a deeper understanding, we said, "But it looks different. Are you sure it still has an area of 16 squares?" At first this question led students to recount the squares. However, later some students responded, "There are still the same amount of squares. I just moved them around," which indicated their recognition that the area was conserved.

To consolidate the lesson, we gave the students 1 cm grid paper (one piece for each student) and asked them to draw as many pool shapes as they could with an area of 16 squares (that is, square centimetres). We deliberately chose the word *shapes* rather than *rectangles* to allow for other possibilities, thereby maintaining an open-ended task, even though our intent was to consolidate the lesson.

Figure 1



Lesson 2

We began Lesson 2 by placing on the learning carpet a 4 × 4 rectangle with an area of 16 squares. We then asked students to determine and mouth the area. At this point, the students seemed comfortable with the term *area,* so we dropped the pool covering language. We then asked them for an explanation. They helped each other accurately count the number of squares. We repeated this task with one more block-like shape (a composite of two 2 × 4 rectangles in the shape of an L) to ensure an adequate review of area.

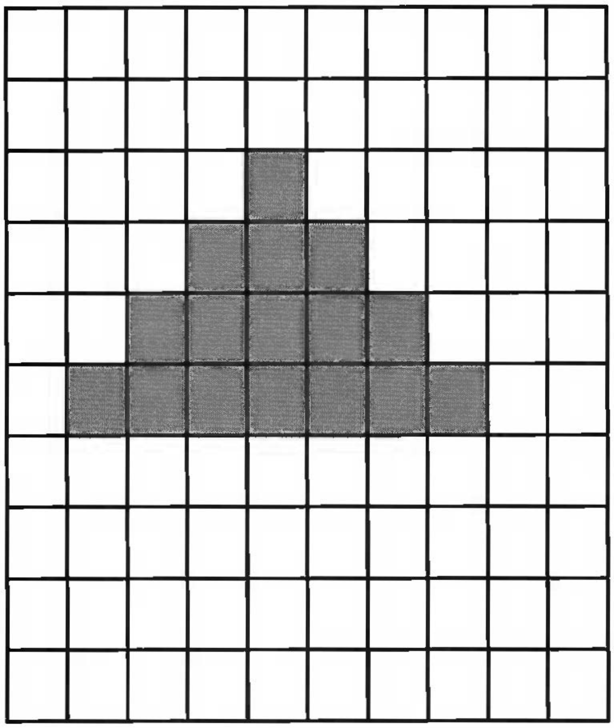
Then we began our next inquiry: "Using 16 paper squares, can we make a triangle with an area of 16 squares?" At first the students manipulated whole paper squares to create a stepped triangle-like shape (see Figure 2). We asked, "Is this really a triangle? Can we make the sides smooth?" Then, as hoped, one student suggested cutting a paper square. (It should be noted that throughout our questioning, scissors were placed near the learning carpet and were visible to students, but we did not refer to them.)

Upon cutting a paper square in half along its diagonal, the students placed both pieces back together on the learning carpet. When questioned about the area, they assured us that the shape still had an area of 16 squares. After moving the cut pieces into different locations, the students realized that they could cut more paper squares in half to make more of the "slanting part of a triangle."

As the students placed these half pieces in various locations, not all of them were assured that the area remained 16 squares. Those students counted pieces rather than whole squares (in other words, they counted a half square as one), and decided that the area was more than 16 squares. We resisted the urge to tell them what they were doing wrong. Instead, we acted unsure about what the area really was and asked, pointing to a half piece, "What is the area of one of these pieces?" We allowed students to move half pieces back together and then count whole squares to determine the area. Later we encouraged students to point to a pair that was counted as one. At this point, they discussed how to keep track of which pieces had already been counted, to avoid counting a piece more than once or not at all.

To consolidate, we asked students to draw on 1 cm grid paper at least two more shapes with an area of 16 cm^2—one that they had seen on the learning carpet and one new one. They used half squares in many of their new shapes.

Figure 2



Lesson 3

Lesson 3 began with a familiar shape on the learning carpet. The shape, which had an area of 16 squares and used half pieces, looked like a house (a 3 × 4 rectangle topped with a triangle made of four half pieces and two whole squares). The shape was familiar because it had been formed on the learning carpet in the previous lesson, and many students had also drawn it on the 1 cm grid paper.

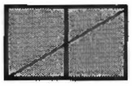
We reviewed what area is (that is, covering) and then calculated the area of the shape, remembering to match half pieces to create one full square. We motivated a new inquiry by stating, "This shape is boring. What other interesting shapes can we make?" Students took turns making new shapes, sometimes with help from others, and named them based on what they looked like (for example, rocket, tree and house). With each new shape, we determined its area, ensuring that it was still 16 squares.

Then, we again asked, "Can we make a more interesting shape?" The number of shapes that could be made with only whole and half squares was limited, so we were hoping that students would suggest using pieces of different sizes. Again, scissors were in view of students but were not referred to. Also in view were precut half pieces; one-quarter and three-quarter pieces; and one-sixth, three-sixth and five-sixth pieces.

To make one-quarter and three-quarter pieces, place two paper squares side by side, and cut from one corner to the opposite corner (see Figure 3). This produces four pieces—two small congruent pieces and two large congruent pieces. We will call these small and large pieces one-quarter pieces and three-quarter pieces, respectively.

Figure 3

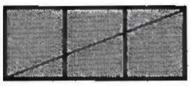
How to Cut One-Quarter and Three-Quarter Pieces



To make one-sixth, three-sixth and five-sixth pieces, place three paper squares side by side, and cut from one corner to the opposite corner (see Figure 4). This produces six pieces—two small congruent pieces, two medium congruent pieces and two large congruent pieces. We will call these small, medium and large pieces one-sixth, three-sixth and five-sixth pieces, respectively.[9]

Figure 4

How to Cut One-Sixth, Three-Sixth and Five-Sixth Pieces



Students started replacing full squares with two matching pieces that were not halves. At first, each time we replaced one full square with a new pair, we asked a student to create a new shape and then determine if the area was still 16 squares. This was repeated several times. Later, students wanted to try multiple replacements. We watched to make sure that they were pairing the pieces correctly (a pair is congruent to one whole square). For some students, the pairing was less obvious when the pairs were far apart, and they had to physically move the pieces together to confirm that the shape still had an area of 16 squares.

To consolidate their learning, students completed three "Paired Shapes" handouts (Appendix A is the first page). On these worksheets, students were asked to determine which shape in each pair had an area closest to 16 squares.

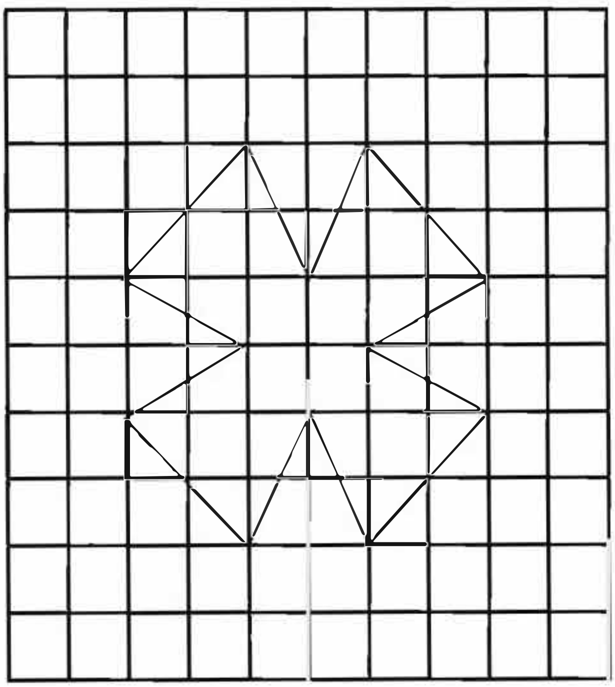
Lesson 4

Lesson 4 began with a 2 × 8 rectangle, formed on the learning carpet with paper squares and string along the perimeter. We asked the students to mouth the area of the rectangle. Then we removed the squares, leaving the string in place, and again asked the students to mouth the area. We repeated this review of area with two more shapes: an isosceles triangle with a base of 8 and a height of 4 (12 whole squares and 8 half pieces), and a right-angle triangle with a base of 4 and a height of 8 (12 whole squares, 4 one-quarter pieces and 4 three-quarter pieces). We reviewed these three basic shapes with an area of 16 squares to ensure that students were ready for the following task, which we prompted by asking, "What is the area of this four-leafed clover?" With string, we traced out the shape shown in Figure 5.[10]

Students worked together to fill in the shape, and we refrained from commenting except to encourage them to check their thinking. Finally, we began our inquiry: "Using the string, make any shape you want, provided that it is interesting in some way. The area does not have to be 16 squares. What do you think its area is?" As students tried to fill in their shapes with the available pieces (squares, halves, quarters and so on), they soon realized that there would be gaps smaller than any of the pieces. No students cut new shapes, which we would have allowed. Rather, they added more one-quarter pieces, trying to balance the remaining gaps (inside the shape) with the overhangs (the pieces going outside of the string).

After this activity, we were confident that most students were comfortable with area conservation and estimation of area. But we could have added another lesson similar to Lesson 4, to make sure that students were ready for Lesson 5.

Figure 5



Lesson 5

Our fifth and final lesson began with a plastic hoop on the learning carpet. We asked, "What do you think is the area of this circle?" Students used an approach similar to that used in Lesson 4 to cover the circle with paper pieces and estimate its area, by balancing gaps with overhangs. We were ready to repeat this review with a second plastic hoop if students did not appear ready for the subsequent inquiry.

We then challenged students to draw on 1 cm grid paper a circle with an area of 16 cm^2. Students were put into groups and given an assortment of objects with circular faces. The two main concerns that arose were related to drawing an accurate circle and accurately determining its area. For example, some students did not trace the circular face accurately, which called into question whether it was a candidate for having an area of 16 cm^2. At other times, students were not careful when matching gaps with overlaps when estimating the area. We saw both of these issues as opportunities rather than problems, as they gave students a chance to discuss the importance of accuracy in mathematics while also recognizing the need to estimate and improve estimates.

Final Thoughts and Considerations

The series of lessons above was triggered by an unexpected exploration of circles, which we initially did not know how to respond to. After implementing the lessons, we no longer wondered if exploring circles would be a waste of time. We had generated a series of inquiries leading to a productive exploration of the area of circles. Throughout, we attended to our mathematical horizon while still respecting the contributions and thinking of all students.

We have tried these activities several times (with some modifications and with different groups of children), and we have seen varying results. These differences can be navigated by teacher scaffolds that maintain student momentum for the inquiry while keeping the mathematical horizon in sight. For example, sometimes children will suggest cutting the shapes, while on other occasions a prompt from the teacher is needed ("What if we use scissors?"). The decision to cut leads to questions about how to cut (diagonally, lengthwise), which may require a teacher prompt such as "Can we cut to make two triangles?" Our rule of thumb is to be patient, building on what children do and say, rather than telling them what to do. We allow students to wind their way through an inquiry, and we provide a prompt only if they get stuck or if they are moving too far astray of our mathematical horizon.

Math classes are usually 40 minutes long. We have been surprised at how long students are willing to stay focused on these activities. We thought they would stay engaged for a much shorter time, leaving time for regular math class routines. On the contrary, each activity fills the entire math class, and sometimes goes over.

When we see student engagement waning, we wrap up the inquiry with a consolidation activity. This always occurs after significant mathematical thinking has been demonstrated by most students.

The greatest difficulty we have found is getting to all students so that they can share their thinking. To address this concern, we move back and forth from individual or small-group exploration to whole-class discussion. This teaching method provides just enough structure so that all students can participate and stay engaged, but is also flexible enough to take into account the diversity of thinking that can occur during an inquiry.

Why did we choose an area of 16 squares? Selecting this number was an inquiry in itself for us as teachers. After exploring various areas (12, 15 and 16), we decided that an area of 16 squares allows for generating many different shapes. For example, there are whole-number-dimension rectangles with an area of 16, as well as several triangles. Using an area of 12 or 15 would allow for only rectangular shapes, which would provide less opportunity to scaffold into half and other part pieces. That opportunity is critical to the success of the activities; it provides the openness needed for initiating and sustaining an inquiry, because there are more opportunities for mathematical thinking by students.

Beyond the specific decision to consider shapes with an area of 16 squares, planning the inquiries described above was much different from planning for a traditional, direct-instruction lesson. Our main goal was to explore area, rather than to target specific outcomes. It was only after the lessons that we recognized some of the specific outcomes and processes that had been met. This outcome openness fosters the flexibility needed for an inquiry to be successful. We also tried to anticipate several possible trajectories of student thinking, rather than creating a linear description of what should happen. This allowed us to stay flexible while also being ready with several possible scaffolds to keep the inquiry going. In a way, planning for an inquiry has in itself the elements of an inquiry.

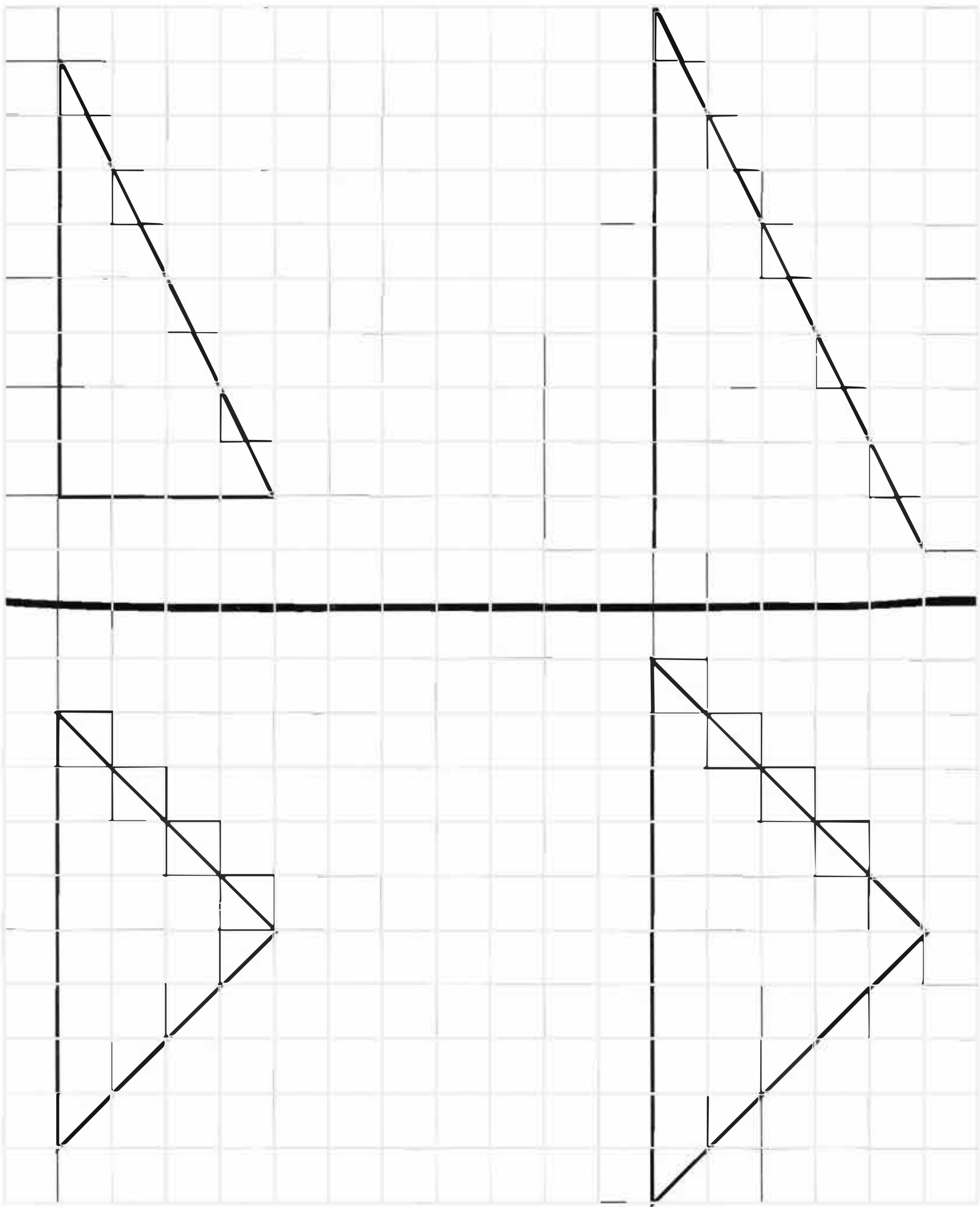
Unfortunately, it is impossible to anticipate everything that can happen. For example, we did not expect students to struggle with matching pairs to form a square: some students could not match pairs when the pieces were far apart, even though they could do so when the pieces were adjacent. We still are not sure why this would be the case. Further, we did not achieve all of our goals explicitly. For example, in Lesson 5, no students formally and explicitly noted that the area of a circle can change a lot when the radius changes only a little.

Nevertheless, we believe that significant mathematical thinking occurred during these inquiries, and we have laid an informal groundwork for lessons in later grades concerning area formulas.

Appendix A

Paired Shapes Handout

In pairs, which shape is closer to 16?



Notes

The research that informed the writing of this article was supported by a grant from the Imperial Oil Academy for the Learning of Mathematics, Science and Technology, a project of the Faculty of Education, University of Manitoba. Our thanks go to the school and the school division for supporting this work. We also appreciate the helpful comments from the anonymous reviewers. Special thanks go to the children who enthusiastically engaged with the lessons described in this article, which helped us learn about effectively implementing inquiry-based mathematics activities.

1. For two of many examples of rich mathematical tasks that promote inquiry, see Smith et al (2009) and Mooney (2008).

2. There are other, larger mathematical concepts that teachers could attend to during these lessons. For example, the idea of leaving no gaps or overlaps when covering an area is related to tessellation. In Lesson 4, connections between perimeter and area are possible.

3. The Learning Carpet can be ordered at www.thelearningcarpet.ca. Alternatively, use masking tape to create a 10 × 10 grid on a tile floor and cut paper squares of the same size as the tiles.

4. Throughout this article, unless otherwise noted, the unit for all areas is learning carpet squares (and is noted as square units or squares), and one unit of length is the side of a learning carpet square.

5. Smith, Bill and Hughes (2008) provide a framework for planning for inquiry lessons, which is premised on the need to anticipate possible student trajectories before implementing a lesson.

6. The string must be about 4 m long to ensure that there is enough string to surround various shapes with an area of 16 squares. For example, the 2 × 8 rectangle has an area of 16 squares and a perimeter equal to 20 sides of a square, but a square on a learning carpet has a side length of about 18 cm, so the rectangle's perimeter is 20 × 18 cm, or 360 cm. It is possible to make extremely irregular shapes with an area of 16 squares and with much larger perimeters, but with such shapes it is difficult to confirm that the area is 16 squares.

7. We wanted to make sure that circles of many sizes, with areas less than and greater than 16 cm^2, were available to students. On 1 cm grid paper, a circle with an area of 16 cm^2 will have a radius of approximately 2.26 cm.

8. Trevor Calkins (creator of the Power of Ten system for learning numeracy skills) recommends this questioning method to encourage all students to safely participate in whole-class discussions. When the teacher asks a question, rather than having students raise their hands or verbalize an answer, the teacher watches for each student to silently mouth the answer.

9. During the lesson, students did not use a consistent language to refer to these pieces. We chose fraction terminology for this article to avoid confusion and because the name of each piece is its area. This terminology was not used with students, as it would have confused them and detracted from the goals of the lesson.

10. We deliberately chose a shape with an area of 16 squares that hadn't arisen in previous lessons and that could be filled in only by using pieces of more than one size (in this case, half pieces, one-quarter pieces and three-quarter pieces).

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Math Class Enriched by a Little Bowl of Cereal

Joan Stevens

"Eat your cereal. It's good for you." This statement is especially true in the context of a math lesson that allows students to use real food to make change for a $10 bill.

In Alberta's K–6 mathematics curriculum (Alberta Learning 1997), an outcome for the Grade 3 measurement strand states that students will be able to "make purchases and change up to $10" (specific outcome 19, p 31). In an effort to address this outcome, I started by thinking about what it is that children love. The one thing I knew for sure was that they love to be fed, so I decided to bring to class several boxes of cereal (of at least two different kinds), as well as chocolate milk, bowls, paper cups and spoons. The students were going to purchase their cereal and milk.

As I was preparing, it became apparent to me that the lesson could be expanded into a review of mathematics strands learned earlier in the school year. I was excited at the prospect of being able to review estimating, measurement and addition.

I began the lesson by telling my students that they would have an opportunity to purchase a bowl of cereal and some chocolate milk using the $10 bills from our math money. When they saw what I had brought in, they eagerly awaited more information from me.

I then performed a demonstration of mass. I poured one type of cereal into a cup, and then poured an equal amount of another type of cereal into a different cup. We talked about what we thought the cereal would weigh in grams or kilograms. I asked the students if they thought the cereals would have equal masses. Some thought that they would, because there was the same amount of cereal in both cups.

I then put each cup of cereal on one side of a balance scale. The masses were not equal. Why was this? The students brainstormed ideas, and concluded that some cereals are heavier than others, even though there is the same amount. We talked about ingredients that would make a cereal heavier (such as raisins).

I then showed the students a 4 L container of chocolate milk. They were hooked. I asked them to estimate how many millilitres of milk a small paper cup would hold. From there, we determined how many millilitres of milk each student would get. The students came up with several strategies for determining how they would share the milk equally. Some students suggested that I put all the cups out and pour some milk in each, others used addition strategies and a few suggested using division to solve this problem. I did as the students asked and poured milk into each paper cup.

After this, I gave each student a menu, including prices (see Figure 1). The students then selected their items and paid the cashier. They had to verify their change before receiving their items. If their calculations were incorrect, they would eagerly go back to their desks and try again. When they had calculated the change correctly, they took their cereal and enjoyed it.

Figure 1

Menu	
Cereal	$2.25
Chocolate Milk	$1.00
Bowl	$0.50
Spoon	$0.25

At the end of the lesson, I had some cereal left. I decided to review chance and probability. On the board I wrote the phrases *impossible, likely, fifty-fifty* and *equal chance*. I had the students write their name on a piece of paper and then place it in a small container. I asked them what the probability was that their name would be drawn. When students' names were drawn, they were given a little more cereal and milk. We talked about chance and used the vocabulary on the board when talking about probability.

Then I asked the students to hand in the money they still had. We started with fives and skip counted while I kept tally on the board. Proceeding to the next denomination, we again skip counted and kept tally. We repeated this process until all the money had been collected and tallied on the board. Then, we subtracted the total from the total amount of money they had started with. Through this, they discovered how much they had paid for the snacks as a class.

This lesson started out as a lesson on making change from a $10 bill and expanded. In just one hour, I was able to review many other previously learned outcomes, and my students were engaged the whole time. I enjoyed this experience, as I saw how things unfolded and how making sense of math was possible for my students and me.

This lesson could be extended by allowing students to set up their own store and having them work through the whole process—shopping, selling and purchasing items they have priced.

Outcomes Addressed

The following outcomes come from Alberta's K–6 mathematics program of studies (Alberta Learning 1997).

Shape and Space (Measurement, Grade 3)

General Outcome

Estimate, measure and compare, using whole numbers and primarily standard units of measure.

Specific Outcomes

7. Select an appropriate object or nonstandard unit to measure capacity or volume of a container (estimation and mental mathematics, visualization).
9. Estimate, measure, record, compare and order the mass (weight) of objects, using standard units (g, kg) (estimation and mental mathematics, problem solving).
18. Estimate, count and record collections of coins and bills up to $10 (estimation and mental mathematics).
19. Make purchases and change up to $10 (problem solving).

Number (Number Concepts, Grade 3)

General Outcome

Develop a number sense for whole numbers 0 to 1000, and explore fractions (fifths and tenths).

Specific Outcome

1. Count by 2s, 5s, 10s and 100s to 1000, using random starting points (connections).

Reference

Alberta Learning. 1997. *Mathematics Kindergarten to Grade 6*. Edmonton, Alta: Alberta Learning. Also available at http://education.alberta.ca/media/450754/elemmath.pdf (accessed February 8, 2010).

Joan Stevens is a Grade 3 teacher at Westview Elementary Public School, in Fort McMurray. Joan loves teaching mathematics and is always looking for ways to differentiate her lessons so that all her students can contribute in a meaningful way. She hopes to instill in her students positive attitudes about their ability to do mathematics and to make sense of it.

Sharing and Grouping Cookies: Delicious Ways to Divide

Stephanie Nash-Pearce

Rationale

In this series of lessons, Grade 3 students will explore the concept of division and its relationship to equal sharing and equal grouping. This is an introduction to division, as division is not part of the curriculum prior to Grade 3. It is essential for students to understand the concepts of equal sharing and equal grouping, and to be able to apply the concepts to everyday situations. Whether it involves sharing food among friends or dividing fundraising money among classmates, division is an easy, logical way of ensuring that items are shared or grouped in equal parts or quantities.

Division occurs in contexts in which either the number of groups or the size of the groups is unknown. Equal sharing involves knowing the number of groups and finding the size of the groups. For example, if you know that there are 12 items and four children (the number of groups), then you can share the items equally to find out how many each child will get (the size of the groups). Equal grouping involves knowing the size of the groups and finding the number of groups. If you know that there are 12 items and these are put into containers holding three items (the size of the groups), then you can divide to find the number of containers needed (the number of groups).

Using children's literature enables students to take advantage of the material both during class and on their own time. Integrating literature incorporates language skills, reading, listening and writing into mathematics. Literature invites students to take an active role in their learning. The ideas presented in books can often be physically replicated by students, which means that drama can also be incorporated into the curriculum.

For example, *The Doorbell Rang*, by Pat Hutchins (1986), tells the story of a mother offering her children cookies that they must share. When the doorbell rings, more children arrive to share the cookies. This story can be re-enacted by children to bring math to life. *The Doorbell Rang* is an appropriate opener for a unit on division.

Outcomes and Processes

Alberta's new K–9 mathematics program of studies (Alberta Education 2007) indicates that Grade 3 students have had little experience with the terms *division, equal sharing* and *equal grouping*. While the words may be unfamiliar, understanding the concept is realistic at that level.

This lesson addresses Grade 3 specific outcome 12 (number strand):

> Demonstrate an understanding of division (limited to division related to multiplication facts up to 5 × 5) by:
>
> - representing and explaining division using equal sharing and equal grouping
> - creating and solving problems in context that involve equal sharing and equal grouping
> - modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
> - relating division to repeated subtraction
> - relating division to multiplication. (p 22)

The mathematical processes addressed in this lesson are as follows:

- *Connections.* By connecting language arts to math, students are able to see that literary pieces can teach math concepts. Having students divide themselves into equal groups provides them with a realistic experience.
- *Communication.* The teacher reads the book to students. The students suggest answers based on information provided in the book and discuss findings.
- *Problem solving.* Students develop their own problem-solving strategies as they respond to the question, "How many cookies will each person receive?" Students use this new knowledge of division to solve concrete examples in the classroom.
- *Reasoning.* Students are expected to justify and support their answers; therefore, they must explain how they determined particular answers.

- *Estimation.* Students can estimate the number of cookies each character in the book would receive, because this is their introduction to division.
- *Visualization.* The book provides images that relate to the math content. Since division is a new concept for them, students are exposed to a realistic scenario that uses the sharing of cookies to introduce division.

Lessons

Day 1

Introduction (15 minutes)

Teacher Activities

- Show students the cover of the book *The Doorbell Rang,* by Pat Hutchins.
- Ask students, "What do you think this book is about?"
- Read the book's back cover out loud: "Ma has made a dozen delicious cookies. It should be plenty for her two children. But then the doorbell rings and rings and rings." Ask students, "How have you changed your mind about what this book is about?"
- Ask students, "How many cookies are in a dozen?"
- Ask students, "Have you ever had to share your cookies before?"
- Read the book out loud. Each time another person is added to the scenario, ask students, "How many cookies will each person receive?" Students can use counters to demonstrate their understanding of division. They should respond before the answer on the next page of the book is revealed.
- After reading the book, pose the following questions:
  - "Did you like the book?"
  - "What was your favourite part?"
  - "When Grandma brings more cookies at the end of the book, do you think they will have to share those cookies too?"

Student Activities

- Respond to teacher-directed questions.
- Actively participate in the reading of the book.
- Form opinions about the story.

Development (15 minutes)

Teacher Activities

- Ask students the following questions:
  - "How do you think this story relates to math?"
  - "How did you share the cookies?" (Hint, if necessary: "Did you give them one at a time to each person?")
- Ask students, "Has anyone heard of the word *division* before? If so, what does it mean? Who can provide me with an example?"
- Encourage students to brainstorm ideas about division to form a class definition. Record their ideas on chart paper.

Student Activities

- Relate information in the story to math.
- Discuss the term *division* and pose meanings.

Day 2

Development, Continued (20 minutes)

Teacher Activities

- Review definition of *division.*
- Have students gather in the centre of the room. Tell them to form two equal groups. Ask them, "How many students are in each group?"
- Then, have each group split further into two equal groups. Ask students, "How many students are there in each group?" Split into two equal groups once again. Ask students, "How many ways can you make two equal groups?"
- Ask students, "If you have 12 cookies and you want to give three cookies to each child, how many children can you invite?"
- In groups of five, have students think of ways to divide the class. For example, 20 students could be divided into five groups of four students. Give each group a turn to divide the entire class into groups. Circulate as students perform this task. Select random groups and ask them, "How did you divide the groups?"

Student Activities

- Experience division by being broken up into groups.
- Form small groups and formulate strategies for dividing the entire class into smaller groups.

Closure (10 minutes)

Teacher Activities

- Continue the theme of division by having students return to their seats as follows:
  - "If you are wearing blue, return to your desk. Is there an even group of students remaining?"
  - "Those with brown hair, have a seat."
  - "Anyone with glasses, return to your desk. How many students remain? Has the class been divided into two equal halves?"

  Continue this process until all students are seated.
- Give students a chance to ask questions about division.

- Remind students that *The Doorbell Rang* will be available in the classroom for them to read in their spare time.

Student Activities

- Listen for cues to return to seats.
- Count those who remain standing to determine if the class is divided evenly.
- Ask questions about division.

Extensions

- Encourage students who have prior knowledge of division from outside of school to write what they know in their journals, in order to get a better idea of their level of prior knowledge.
- Present the following problem to students:

  A bag of candy sits on a table. If two kids share all the candy so that they both get the same number of pieces, there will be one candy left. If three kids share the same candy equally, there will be two candies left. If four kids share the candy equally, there will be three candies left. If five kids share the candy equally, there will be four candies left. If six kids share the candy equally, there will be five candies left. How many candies are in the bag? Is there more than one possibility?

Assessment

Use anecdotal records of student interaction to see if students are understanding the concept of division. This evaluation will serve as important information, as it is an indicator of students' previous knowledge.

References

Alberta Education. 2007. *Mathematics Kindergarten to Grade 9*. Edmonton, Alta: Alberta Education. Also available at http://education.alberta.ca/media/645594/kto9math.pdf (accessed February 8, 2010).

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Stephanie Nash-Pearce is a Grade 3 teacher at Good Shepherd School, a pre-kindergarten to Grade 8 school in Fort McMurray. She is interested in music and drama, and enjoys using fine arts in her teaching. She believes that teaching mathematics provides a new opportunity for learning every day, and she is proud to see the continued growth of her students.

Call for Manuscripts

Special Issue: Early Childhood Mathematics

delta-K invites submissions for a special issue on teaching and learning mathematics with young children (pre-kindergarten to Grade 3). Articles that provide classroom-tested activities and teaching strategies, offer insight into children's thinking and problem-solving strategies, address challenging classroom issues, or share findings from classroom-based research are encouraged.

The following topics are intended to provide guidance to authors. Manuscripts that address other issues and ideas are also welcome.

- Projects or problems that successfully prompt mathematical thinking and problem solving
- Classroom environments and teaching strategies that encourage and support risk taking, sense making, reasoning and mathematical habits of mind
- Activities that help young children connect mathematics to other curriculum areas (such as children's literature or science content) and real-world applications
- Innovative uses of technology to promote different forms of mathematical thinking
- Assessment strategies for beginning readers and writers
- Professional development that successfully supports and enhances early childhood educators' understanding of teaching and learning mathematics

Manuscripts may vary in length from brief reflections on relevant issues to longer articles that provide information on classroom projects and samples of children's work.

The deadline for submissions is **August 1, 2010.** Send manuscripts and inquiries to Lynn McGarvey (lynn.mcgarvey@ualberta.ca) or Gladys Sterenberg (gladyss@ualberta.ca).

Suggestions for Writers

1. *delta-K* is a refereed journal. Manuscripts submitted to *delta-K* should be original material. Articles currently under consideration by other journals will not be reviewed.
2. If a manuscript is accepted for publication, its author(s) will agree to transfer copyright to the Mathematics Council of the Alberta Teachers' Association for the republication, representation and distribution of the original and derivative material.
3. All manuscripts should be typewritten, double-spaced and properly referenced. All pages should be numbered.
4. The author's name and full address should be provided on a separate page. If an article has more than one author, the contact author must be clearly identified. Authors should avoid all other references that may reveal their identity to the reviewers.
5. All manuscripts should be submitted electronically, using Microsoft Word format.
6. Pictures or illustrations should be clearly labelled and placed where you want them to appear in the article. A caption and photo credit should accompany each photograph.
7. References should be formatted consistently using *The Chicago Manual of Style*'s author–date system.
8. If any student sample work is included, please provide a release letter from the student's parent/guardian allowing publication in the journal.
9. Articles are normally 8–10 pages in length.
10. Letters to the editor or reviews of curriculum materials are welcome.
11. Send manuscripts and inquiries to the editor: Gladys Sterenberg, 195 Sheep River Cove, Okotoks, AB T1S 2L4; e-mail gladyss@ualberta.ca.

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