# The Diploma Exam and the Role of Communication in Mathematics Literacy

*Elaine Simmt*

In October, I was a member of an MCATA panel responding to the decision of Alberta's minister of education to drop the written-response component from the diploma exams for Pure Mathematics 30 and Applied Mathematics 30. I speak as a mathematics education researcher who does classroom-based studies and as a professor in teacher education. My responses to the questions posed should be read with these roles in mind.

## What Is the Value and Role of Communication in Mathematics Literacy?

From my perspective, communication in mathematics literacy plays at least two distinct roles relevant to the discussion at hand.

The first role is quite well understood by educators and the public: the mathematically literate person is able to communicate mathematically. More specifically, the mathematically literate person is able to interpret, translate and express his or her experience of a context mathematically, where and when appropriate.

The contexts for mathematics experiences are many and varied. Consumer, economic, health, industrial, scientific—any of these contexts can be understood mathematically. Take, for example, the following:

- The cost of heating a home
- The volume of oxyacetylene needed for a welding job
- The concentration of saline in an intravenous solution
- The loss of polar bear habitat
- The gross national product

Let's think for a moment about the possibilities for mathematical thinking and communication that emerge in the context of studying polar bear habitat. This context may not be immediately understood as having a significant mathematical dimension, given its scientific, economic, social and political aspects. However, studying the impact of the melting ice caps on the polar bear population requires all kinds of mathematics: measuring the surface area and volume of ice caps, correlating the average global temperature with the surface area, determining the average distances across water to ice floes, estimating seal populations, predicting bear population growth and so on. These are just a few examples of the questions within the broader study of polar bear habitat that require mathematical models to help us better understand.

Mathematical literacy, then, enables us to examine contexts and situations and to act on them in informed ways. Specifically, the ability to interpret, translate and express mathematically provides a frame through which we as members of communities can articulate an argument, relay information, justify a solution, explain a position and critique a practice—in other words, actively participate in and contribute to society.

The second role communication plays in mathematics literacy is less understood by the lay public. However, it is a concern of educators, and in the last few decades has been a major concern of mathematics education researchers. Indeed, this role of communication is highlighted in our program of studies.[1] This is the role that communication plays in thinking. Vygotsky (1934), a Russian psychologist, demonstrated how thinking and speech are intimately connected. Today, researchers have demonstrated the reciprocal relationship between communication and mathematical thinking (Sfard 2008).

The US-based National Council of Teachers of Mathematics (NCTM 2000) standards for school mathematics state that all students should be able to

> Organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; and use the language of mathematics to express mathematical ideas precisely.

In summary, mathematical communication is a form of thinking that enables us to interpret, translate and express our experiences. Communicating is intricately linked to reasoning, visualizing and problem solving—all of the higher-order processes we strive for in mathematics education. To actively participate in critical discussions on topics such as the loss of polar bear habitat or, dare I say, the development of the oil sands, students require extensive mathematical communication skills.

## What Does Communication in the Mathematics Classroom Look Like?

Let me first say that the mathematics classroom is critical to the development of mathematical thinkers and communicators. It is in the public school classroom with knowledgeable others (teachers, mentors and peers) that young people encounter this powerful interpretive frame we call mathematics, a tool that enables them to participate in our local, national and international communities. It is in the classroom that they develop as active citizens.

The Western and Northern Canadian Protocol (WNCP 2008, 4) common curriculum framework for Grades 10–12 mathematics states,

> Teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding through:
>
> - taking risks
> - thinking and reflecting independently
> - sharing and communicating mathematical understanding
> - solving problems in individual and group projects
> - pursuing greater understanding of mathematics
> - appreciating the value of mathematics throughout history

It is obvious that all of these goals for the classroom hinge on communication. Hence, nothing in the mathematics classroom is more fundamental than communication.

So, what does communication in the mathematics classroom look like? There are others who are closer to the classroom than I am; however, I do make it a point to visit classrooms throughout the year, and my comments are based on those observations. I will take an example from pure mathematics. Think about this sequence: 1, 3, 5, 7, 9 . . . . How we communicate what this sequence is and what it means to us depends on how we think about it. We teach students to think about it by communicating its many interpretations. If asked to explain this sequence, we might say any of the following:

- It is a sequence of consecutive odd numbers.
- It is a sequence beginning with 1 and adding 2 for each consecutive term.
- It is an arithmetic sequence.
- It can be described by the formula $1 + (n - 1)2$, where 1 is the first term, $n$ the number of terms in the sequence and 2 the common difference between terms.
- It is a linear function $\{(x, y)|y = 2(x - 1) + 1$ and $x \in N\}$.
- It is a linear function with slope +2 and $y$-intercept $-1$.

When the study of sequences involves a deep exploration of the mathematics—its connections, relationships and forms of representation—students have the opportunity to become powerful interpreters. Their mathematics grows deep, and they are able to study and comment on their experiences. Good math teaching involves rich mathematical communication. It teaches young people the value of mathematics and how to use it to become a strong interpreter, translator and communicator. So, then, if we know what good teaching and learning experiences look like, what does this say about the form of diploma exam questions?

## What Is the Value of the Mathematics Diploma Exams for Alberta Students?

The mathematics diploma exams serve as exit exams to evaluate the achievement of the student learning outcomes specified in Alberta's programs of study for Grade 12 mathematics. The exam results are used by a number of parties and for a variety of purposes: postsecondary institutions, for admission requirements; scholarship committees, for ranking students; school administrators, for evaluating programs; teachers, for evaluating their instruction; and the ministry of education, for keeping school districts accountable for their spending.

As a university professor, I can speak to the role diploma exams play in the context in which I work. Specifically, they are used for student admission and scholarships in that they account for 50 per cent of a student's grade for courses that can be presented for admission. For this purpose and others, diploma exams are a valuable tool; hence, we are compelled to ensure that the exams reflect the program of studies legislated by the province and that they fairly assess a learner's understanding of that which is being evaluated.

## What Is the Value of the Written-Response Section of the Mathematics Diploma Exams?

I will now look at Alberta's program of studies for mathematics to investigate the value of the written-response section of the mathematics diploma exams. Clearly, the value of the exams as a whole depends on their ability to fairly evaluate student learning of the major outcomes of school mathematics.

As stated in the program of studies (Alberta Education 2007, 2–3),

> The main goals of mathematics education are to prepare students to:
>
> - use mathematics confidently to solve problems
> - communicate and reason mathematically
> - appreciate and value mathematics
> - make connections between mathematics and its applications
> - commit themselves to lifelong learning
> - become mathematically literate adults, using mathematics to contribute to society.
>
> Students who have met these goals will:
>
> - gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
> - exhibit a positive attitude toward mathematics
> - engage and persevere in mathematical tasks and projects
> - contribute to mathematical discussions
> - take risks in performing mathematical tasks
> - exhibit curiosity.

Finally, the program of studies delineates seven process outcomes for mathematics: communication, connections, mental mathematics and estimation, problem solving, reasoning, technology, and visualization (p 4).

The value of the diploma exams for any of the users mentioned earlier depends on the extent to which the exams evaluate the achievement of the learning goals set out in the program of studies. To assess this, we would need to have the exams available to us for study. Unfortunately, they have not been publicly available for a number of years now, so we must simply trust the authorities when they tell us that the exams are a good tool and that they do what they are intended to do. This is a different issue but one that we must continually raise. Quite ironically, investigating the exams would entail the use of the critical mathematical literacy developed in public education.

Returning to the question of the value of the written-response questions in the diploma exams, there are a number of points to be made. Let me begin with a reminder of something that we have heard many times but that is often dismissed: "It is through our assessment that we communicate most clearly to students which activities and learning outcomes we value" (Clarke, Clarke and Lovitt 1990, 118). I do not want to focus on this statement here, but it cannot be ignored, so I will leave it to the teachers to consider. However, I can make some observations that others may not note—first from a broader perspective, and then in the context of Alberta high school mathematics.

Comparing our diploma exams with the exit exams of other countries, I note that national mathematics exams throughout the world (including the United Kingdom, New Zealand and China) use written response.[2] Indeed, students in those countries are asked to prove, to reason, to illustrate and to demonstrate their mathematical understanding through full written responses. Also, postsecondary math exams (at least in Alberta) are by and large written-response tests. Multiple-choice questions and other forms of forced response are not at all common at the postsecondary level.

With regard to the value of written-response questions for the student, there are a few points to be made. First of all, we need to recognize that students can make trivial errors, often as the result of exam anxiety or slips in concentration. In such cases, incorrect answers do not represent a lack of understanding of a concept. In multiple-choice and numerical-response exams, there is no room for such errors; the answer is simply marked wrong. Written-response questions, on the other hand, take into account these kinds of errors and partial marks can be awarded.

For example, a student computes a response to a trigonometry question and records his answer by filling in a bubble in the numerical-response section of the answer sheet. The answer is to be rounded to the nearest tenth so that a computer can mark it. The student hastily records the answer without rounding to the nearest tenth. His answer is now wrong, even though he *does* understand the trig concept being examined. If this item was supposed to measure the student's understanding, it has failed to do so. (One might argue that the item instead tested the student's ability to round.)

Further, the removal of written-response questions from the diploma exams is somewhat ironic. During their 12 years of mathematics education, students are taught to explain themselves, to think critically and creatively, to justify their positions and to reason mathematically. All of this education is perverted by an exam made up of purely forced-response questions. The task for students becomes one of reading carefully enough to figure out what the examiner wants and not thinking too hard, too creatively or too critically. Instead, students must reason in just the way the examiner has anticipated they will.

Consider Figure 1—an exam question from a high school math class. It is representative of a question on a final exam after the content of the whole course has been addressed. The work shown comes from the teacher's answer key. We see that the teacher has anticipated a number of paths to the correct answer. Given that this question is on a cumulative exam, it makes good sense that students may solve it using any one of a number of mathematical approaches: coordinate geometry, linear functions, sequences and series, algebraically, graphically, inductively, deductively and so on. Of particular note is how well the process outcomes for mathematics in Alberta are addressed by the written-response dimension of this question. Students can demonstrate what connections they have made, how they visualize the context, what mathematics they use to solve the problem, how they have reasoned through the problem and how well they can communicate their thinking.

In contrast, a numerical-response format for the same question might look like Figure 2. Note how the only evidence of the student's thinking is the four digits 1114, representing 11 and 14, the next two terms in the sequence. In this form, the student's answer can be read by a scanner. But what do we learn about this student's understanding? What process outcomes are addressed by this question when posed in this form? Can the student demonstrate an ability to make connections, visualize and communicate? What have we learned about this student's knowledge of sequences and series? Of what value is what we have learned? It is clear to me that there is much value in the written response, but it is less clear what value lies in the numerical response.

**Figure 1**

*Written Response*

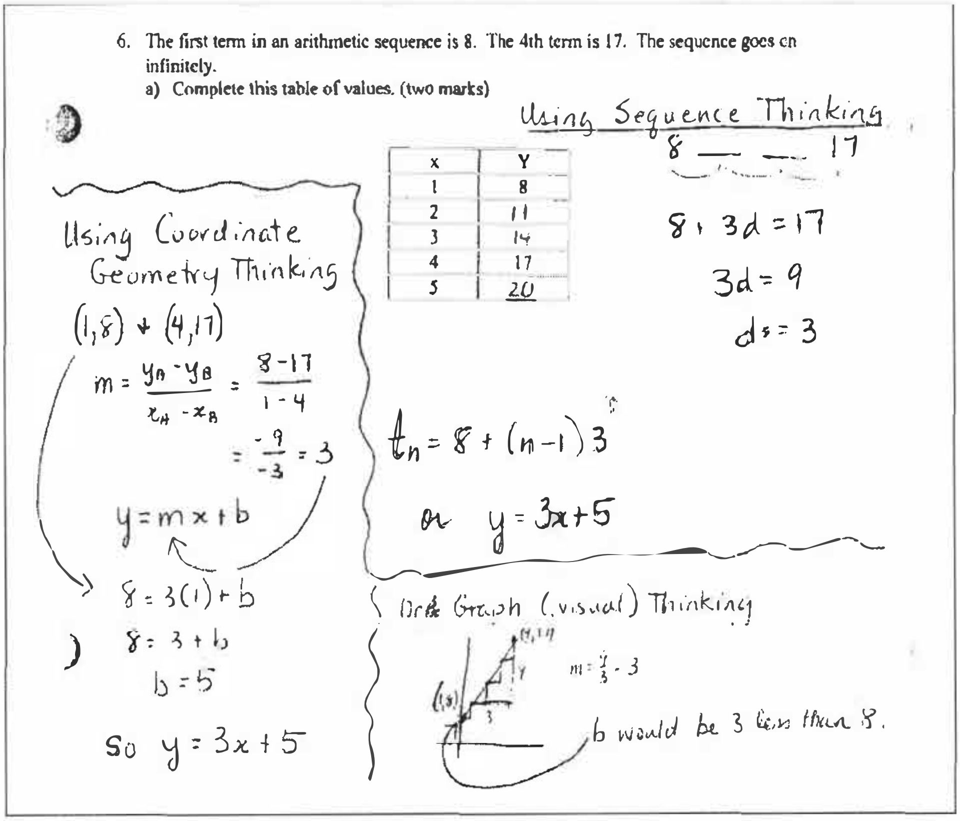
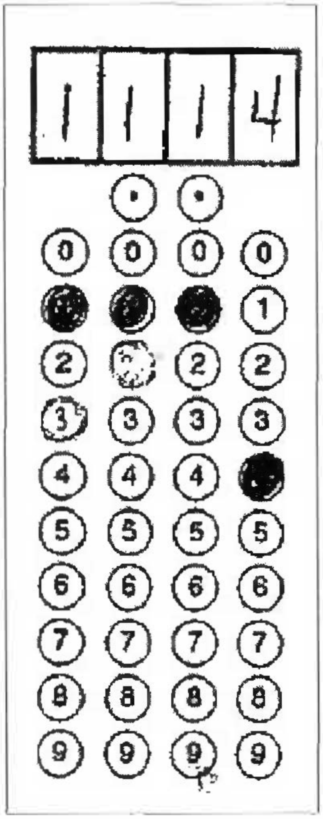



**Figure 2**

*Numerical Response*



Let me conclude by suggesting that the primary value of the written-response section of the diploma exams has been to offer students a space where they can communicate their understanding of mathematics. It was a space where they played an active role in communicating (rather than passively selecting an answer expressed in someone else's voice). In the written-response section, they could be asked to examine a situation, interpret it, translate it and communicate their thinking. They could be asked to offer a critique or an evaluation of a situation. It was in those questions that they could demonstrate the higher-level thinking for which 12 years of schooling had prepared them.

The diploma exams must work in support of Alberta's program of studies for K–12 mathematics, which indicates that the process outcomes are every bit as important as the specific content outcomes. A multiple-choice exam is simply inadequate for thoroughly assessing 12 years of schooling. The purpose of all assessment should be to assess both *what* the learner understands and *how* the learner understands. I see no way to achieve this through a multiple-choice and numerical-response test.

The diploma exams should reflect our best practices in assessment and evaluation. Therefore, the decision to remove the written component from the mathematics exams must be reconsidered.

## Notes

1. See, for example, the discussion of the seven process outcomes (Alberta Education 2007, 4–6).

2. Personal correspondence with A Watson, G Anthony and B Xu.

## References

Alberta Education. 2007. *Mathematics Kindergarten to Grade 9.* Edmonton, Alta: Alberta Education. Also available at http://education.alberta.ca/media/645594/kto9math.pdf (accessed January 6, 2010).

Clarke, D J, D M Clarke and C J Lovitt. 1990. "Changes in Mathematics Teaching Call for Assessment Alternatives." In *Teaching and Learning Mathematics in the 1990s,* 1990 Yearbook of the National Council of Teachers of Mathematics (NCTM), ed T J Cooney and C R Hirsch, 118–29. Reston, Va: NCTM.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics.* Reston, Va: NCTM.

Sfard, A. 2008. *Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing.* New York: Cambridge University Press.

Vygotsky, L S. 1934. *Thought and Language.* Ed and trans E Hanfmann and G Vakar. Cambridge, Mass: MIT Press, 1962.

Western and Northern Canadian Protocol (WNCP). 2008. *The Common Curriculum Framework for Grades 10–12 Mathematics.* Also available at www.wncp.ca/media/38771/math10to12.pdf (accessed January 6, 2010).