Helping Grade 1 Students Understand the Equals Sign: A Difficult but Not Impossible Task

Laurent Theis

Context

Difficulty in developing a proper understanding of the equals sign is a widespread, though often unrecognized, phenomenon among lower-level elementary school students. When I discussed my research on Grade 1 students' understanding of the equals sign with elementary school teachers, most of them were surprised that the equals sign could pose such difficulty for their students. I encouraged those teachers to ask several of their students about the meaning of the equals sign. The teachers were surprised to find that most students saw the equals sign not as an indicator of a relationship but, rather, as an indication to perform an operation or write an answer.

The aim of my research was to describe the development of Grade 1 students' understanding of the equals sign. To observe this development, I chose to involve students in a constructivist teaching experiment in which they worked on the meaning of the equals sign in number sentences involving addition.

At the beginning of my research, I conducted a pretest involving 11 Grade 1 students in an urban district. In six to nine half-hour individual sessions, I then taught to students the equals sign as an indicator of a relationship. Each session was videotaped and then transcribed, allowing me to analyze the students' reasoning. Approximately 10 days after the last lesson, I conducted a posttest, which required the students to answer questions similar to those in the pretest.

In this article, I will describe how Melissa, who was initially assessed by her teacher as being an average performer in mathematics, managed to develop an accurate understanding of the equals sign. I will also highlight the difficulties of two other participants, Mathieu and Caroline, in acquiring this new understanding.

Misunderstanding of the Equals Sign as an Operator

During the pretest, the students were asked to assess whether various number sentences were correct and to complete some equations. They were asked to justify their answers. When they stated that a number sentence was not correct, I asked them to change it to make it correct. They also had to explain the meaning of the equals sign. At this stage, my aim was to gain access to the children's understanding; I did not yet try to make the children think differently about the equals sign.

From the beginning of the pretest interview, Melissa showed an ambiguous understanding of the equals sign. To gain access to her initial understanding, I asked her to tell me whether the number sentence $4 + 5 = 9$ was correct or incorrect. Her answer led me to believe that Melissa saw the equals sign as an indicator of a relationship:

TEACHER [*showing* $4 + 5 = 9$]. Can you tell me whether this number sentence is correct or incorrect?

MELISSA. It is correct.

TEACHER. Why do you think it is correct?

MELISSA. Because we do lots of these additions in school.

TEACHER [*pointing to the equals sign*]. Can you tell me what this sign means?

MELISSA. It says that when it is equal, it also means "the same thing."

However, when I used the number sentence 7 = 3 + 4, Melissa's answers showed that she accepted only number sentences in which the equals sign preceded the last number.

TEACHER [*showing 7 = 3 + 4*]. Can you tell me whether this number sentence is correct or incorrect?

MELISSA. It is the wrong way round. In the last example, the plus sign was here [*indicating placement after the first number*], and now it is here [*indicating placement before the last number*].

TEACHER. Do you think that we can write a number sentence like this?

MELISSA. I'm not sure we can. I don't think so.

TEACHER. How should this number sentence be so that it is correct?

MELISSA. We should put the plus sign here [*indicating placement after the first number*] and the equals sign here [*indicating placement before the sum*]. It would be 4 plus 3 equals 7.

Melissa had used a strategy of reading backward.

Her answers when she had to assess other number sentences showed that she accepted only $a + b = c$ number sentences.

TEACHER [*showing 3 + 4 = 6 + 1*]. Can you tell me whether this number sentence is correct or incorrect?

MELISSA. It is wrong.

TEACHER. Why do you think it is wrong?

MELISSA. Because it is not equal here [*showing the position of the equals sign*].

TEACHER. Why can't you put the equals sign here?

MELISSA. Because, when you add up, you have to .... There is something wrong here, because 4 plus 3 equals 7, not 1.

This example shows that it was important to Melissa that the equals sign was followed immediately by the sum of the numbers preceding it; she would not accept that the equals sign was followed by another sum.

Similar difficulties appeared when Melissa was asked to complete a number sentence. When I asked her to add the correct number to the number sentence 6 + 2 = __ + 3, Melissa stated that 8 must be the missing number.

TEACHER. Why do you think that 8 is the missing number?

MELISSA: Because 6 plus 2 equals 8. It is 6 plus 1, and 1 again, which adds up to 8.

This understanding of the equals sign as an operator (which must be followed by the answer to a question—the operation that precedes it) was not isolated to Melissa. All the students I interviewed displayed, in various degrees, a similar understanding of the equals sign.

The research literature confirms that this understanding of the equals sign is very common (Carpenter and Levi 2000; Sáenz-Ludlow and Walgamuth 1998). While this conception of the equals sign allows students to solve number sentences like 2 + 5 = __, significant difficulties arise when they have to assess number sentences like 8 = 7 + 1 or complete equations like __ = 2 + 3. Students are often unable to find the correct answer, or they choose to read the number sentence backward. That strategy, while efficient for number sentences involving addition, will cause significant problems with number sentences containing subtraction. Because subtraction is not commutative, backward reading will lead to a wrong answer.

Difficulties also arise when students are asked to complete equations like 3 + 5 = __ + 2 (Falkner, Levi and Carpenter 1999; Sáenz-Ludlow and Walgamuth 1998; Shoecraft 1989). A common error of students who view the equals sign as an operator would be to designate 8 (the sum of the numbers preceding the equals sign) as the unknown number. This type of error is not only common with Grade 1 students but also frequently found with older students. In a research study involving 752 elementary school students, the success rate for both Grade 1 and Grade 6 students who were asked to complete 8 + 4 = __ + 5 was below 10 per cent (Falkner, Levi and Carpenter 1999).

The conception of the equals sign as an operator also plays an important role in learning algebra. As Bodin and Capponi (1996) point out, this conception has been clearly identified as a main obstacle in the transition from arithmetic to algebraic thinking.

Which Classroom Strategies Allow Children to Develop a More Accurate Understanding of the Equals Sign?

In the past, several researchers have tried to find strategies that would allow students to develop a better understanding of the equals sign. Their results have been mixed. For instance, 30 years ago, Denmark, Barco and Voran (1976) proposed a balance model that Grade 1 students could use to illustrate various number sentences. However, their research was inconclusive, and they decided that Grade 1 students are simply too young to conceive of the equals sign as an indicator of a relationship.

However, the failure of Denmark, Barco and Voran's (1976) strategy can be explained by the type of number sentence a balance model can illustrate. Generally, a number sentence can be represented in at least two ways. I will use the number sentence $2 + 5 = 7$ to explain the differences between the types of representation.

In one situation, someone has two marbles in the left hand and five marbles in the right hand—seven marbles altogether. In this situation, the seven marbles do not exist independently from the two marbles and the five marbles; they represent the sum of the two subgroups. Later, I will refer to this situation as an inclusive representation.

In another situation, someone has two green marbles and five red marbles, and another person has seven marbles. Both people have the same number of marbles. I will refer to this situation as a comparative representation, because in the solid representation, the seven marbles are not physically the same as the two marbles and the five marbles.

A balance model can be used to illustrate only a comparative representation, because the seven marbles are distinct from the two marbles and the five marbles when the two sides of the balance are stable. However, students' first experiences with addition and number sentences refer much more often to an inclusive situation, where they are attempting to find the sum of the two addends. Denmark, Barco and Voran's (1976) use of a comparative representation, which does not correspond to students' previous experiences with addition, could explain, at least partly, why the exclusive use of a balance model did not help the Grade 1 children in their study to understand the equals sign as an indicator of a relationship.

More recently, Carpenter, Franke and Levi (2003) experimented with different methods of challenging students' conceptions of the equals sign. They found that using true-false number sentences was an effective way to change students' misconceptions. In their classroom throughout the year, they repeatedly engaged Grade 1 students in discussions about true-false number sentences by presenting a list of number sentences, some true and some false. Subgroups of students were organized for discussion purposes. The classroom discussions focused on the children's justifications for considering a number sentence to be true or false. The researchers also encouraged students to use words that expressed the equality relation more directly. For instance, the statement "8 is the same amount as 5 plus 3" gives a clearer description of the underlying relationship than does "8 equals 5 plus 3." This approach allowed most Grade 1 students to use the equals sign appropriately by the end of the year (Carpenter and Levi 2000). However, the researchers point out that the students' understanding was fragile; therefore, it is important to work on understanding of the equals sign regularly.

A Sequence of Activities to Help Children Develop a Better Understanding of the Equals Sign

Several principles guided the development of a sequence of activities presented to the children in my teaching experiment.

First, my interventions during the sequence consisted mainly of questioning the children's strategies. The only information I explicitly gave them was an explanation, during the first activity of the sequence, of the equals sign as a symbol that says there is the same amount on both sides.[1] I referred to this explanation several times during the sequence, but I intentionally did not show the students specific strategies that could influence the way they resolved the situations.

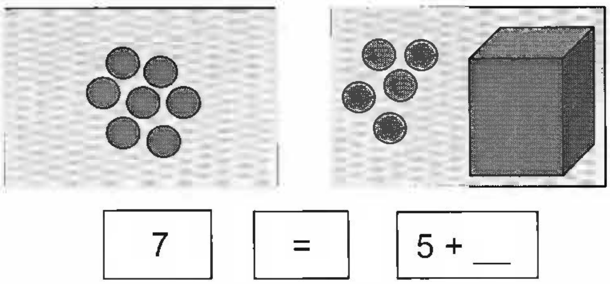
Second, the children were asked to work on two types of tasks throughout the teaching experiment. The first type of task asked them to determine whether a given number sentence was true or false. When they detected an error in a number sentence, students were asked to modify the sentence in such a way as to make it correct. In the second type of task, the children were asked to complete various equations.

Third, I used various types of equations and number sentences. At the beginning of my sequence of activities, we worked exclusively on $a + b = c$ and $a = b + c$ number sentences. Later, I introduced the $a + b = c + d$ structure, which is more difficult for children to understand.

Fourth, to help establish a link between mathematical symbolization and concrete representation, I introduced each number sentence or equation coupled with solid objects at the beginning of the work on each type of question. Those concrete representations were gradually withdrawn later in the sequence, with the aim of facilitating students' ability to work on number sentences and equations solely with mathematical symbols.

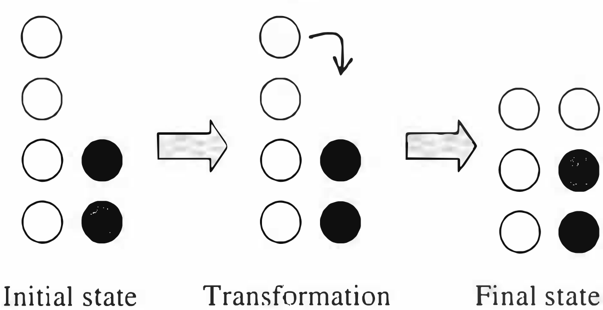
Fifth, the unknown numbers in the various equations were represented in two ways. In one situation, children had to add as many objects as necessary to a transparent plastic bag to make both collections equal. In another situation, the unknown number was represented by a nontransparent box that contained the correct number of objects. The children were told that the same number of objects were present on both sides and then asked to calculate how many objects were in the box (see Figure 1). This situation was more difficult for the students to work on, because they could not see or manipulate the solid objects that represented the unknown.

Figure 1



Finally, throughout the sequence, I alternated comparative and inclusive representations. Previously, I had mapped out the differences between the types of representation in $a + b = c$ number sentences. The same differences could apply to $a + b = c + d$ number sentences. For instance, a comparative representation of $4 + 2 = 3 + 3$ implies the presence of four red and two black marbles in one hand, and three black and three red marbles in the other hand. An inclusive representation of $4 + 2 = 3 + 3$ implies the transfer of one marble from the collection of four marbles to the collection of two marbles, as illustrated in Figure 2.

Figure 2



Study Results

Accurate Understanding for Some Students

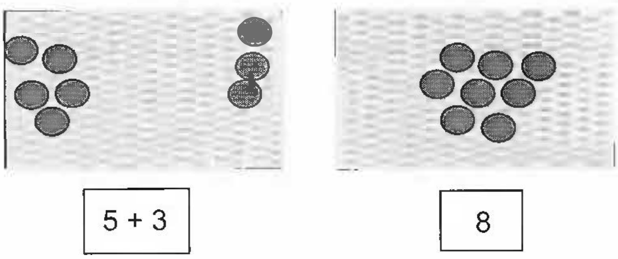
Melissa, like many other participants, made significant progress in understanding the equals sign during the teaching experiment. At the end of the study, she was able to correctly complete $a + b = \_\_ + d$ number sentences. Nevertheless, understanding the equals sign as an indicator of a relationship was a challenge for her. In this section, I will explore Melissa's difficulty accepting a new meaning of the equals sign at the beginning of the study. I will then describe my perceptions of her understanding of the equals sign during the posttest, which shows that she made significant progress during the teaching experiment.

Accepting a New Meaning of the Equals Sign

The aim of the first activity of the sequence was to introduce the children to the meaning of the equals sign as an indication of the same quantity on both sides.

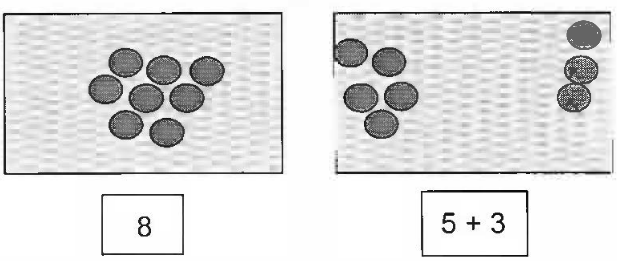
I first presented to the students a comparative representation of the number sentence $5 + 3 = 8$ (see Figure 3).

Figure 3



The students then had to discover whether the same quantities were represented in both situations, and whether the equals sign could be used between 5 + 3 and 8. The situations were then inverted, to get the number sentence $8 = 5 + 3$, as shown in Figure 4.

Figure 4



The children were then asked whether there was still the same number of counters on both sides and whether it was appropriate to use the equals sign between 8 and 5 + 3. At first, Melissa was not sure if she should use the equals sign in this situation.

TEACHER. Do you think that you can use the equals sign now?

MELISSA. I think so.

TEACHER. Can you read the number sentence aloud?

MELISSA. Can I read this way? [*She indicates a progression from the right to the left.*]

TEACHER. You can do it the way you think it should be done.

MELISSA. You have to turn this around, put the 5 plus 3 here [*indicating placement before the equals sign*] and the 8 here [*indicating placement after the equals sign*].

At that point, I explained to Melissa that the equals sign means that there is the same quantity on both sides of the sign, but Melissa was unwilling to adopt this new meaning: "I don't think that it is the way you just explained it. If you have 8 here [before the equals sign], it doesn't work. We haven't learned the equals sign, but I think it means ... I don't remember what it means." Her strong resistance to changing her conception of the equals sign was a first indicator of her difficulty accepting the equals sign as an indicator of a relationship.

Accurate Understanding at the End of the Sequence

After seven half-hour sessions of working with Melissa on the described tasks related to the meaning of the equals sign, I conducted a posttest interview aimed at illustrating her understanding. During the posttest, Melissa's answers clearly showed that she now understood the equals sign as an indicator of the same quantity on both sides of the sign. For instance, when I asked her to assess whether the number sentence 4 + 2 = 6 + 1 was correct, her answer revealed that she had made significant progress in her understanding of the equals sign:

TEACHER [*showing 4 + 2 = 6 + 1*]. Can you tell me whether this number sentence is right or wrong?
MELISSA. Do I have to say whether it is the same thing?
TEACHER. I want to know whether the number sentence is right or wrong.[2]
MELISSA. It is wrong, because 4 plus 2 equals 6, and there is an equals sign that tells us that it is the same thing, but after the equals sign, it is 6 plus 1, which equals 7, not 6.
TEACHER. How could you modify this number sentence to make it a correct one?
MELISSA. [*She replaces the 1 with a 0.*]

During the posttest, Melissa was also able to complete number sentences correctly, without having to use a concrete representation.

TEACHER [*showing 7 + 1 = __ + 2*]. Can you tell me what number you have to write in the box to make this a correct number sentence?
MELISSA [*after thinking awhile*]. It must be 6.
TEACHER. Why do you think it should be 6?
MELISSA. Because I thought 8 minus 2, which equals 6.
TEACHER. Can you read me the number sentence now?
MELISSA. 7 plus 1 equals 6 plus 2.

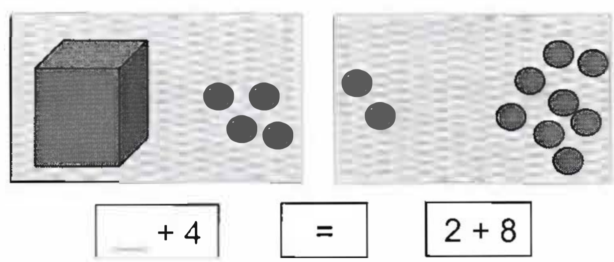
More Difficult Progress for Other Students

Although it seems that Melissa was able to develop a more coherent understanding of the equals sign as an indicator of a relationship, progress was more difficult for other children. This section describes some of the difficulties Mathieu and Caroline experienced during our research.

Mathieu was initially perceived by the teacher as one of the strongest students in the class. During the pretest, his understanding of the equals sign was similar to Melissa's. By the end of the study he was, like Melissa, able to correctly assess $a + b = c + d$ number sentences and to complete $a + b = \_\_ + d$ number sentences. However, as the following examples illustrate, Mathieu's conception of the equals sign was much more fragile than Melissa's.

Throughout the teaching experiment, Mathieu showed a tendency to return to his conception of the equals sign as an operator, even if in other situations he considered it an indicator of a relationship. For example, during the last session Mathieu had to complete the number sentence __ + 4 = 2 + 8. This number sentence is illustrated by Figure 5, in which the nontransparent box contains six objects.

Figure 5



Mathieu had to determine how many objects were in the box if the use of the equals sign between __ + 4 and 2 + 8 was to be possible.

TEACHER. If we can use the equals sign here, how many objects should be in the box?
MATHIEU. There must be 4, because 4 plus 4 equals 8.
TEACHER. Can you read the number sentence?
MATHIEU. 4 plus 4 equals 2 plus 8.

Mathieu had transformed the number sentence into an $a + b = c$ structure, which allowed him to think, once again, in terms of a question–answer pattern.

Mathieu also showed a tendency, especially during the early sessions, to read certain number sentences backward. For instance, when asked to complete the number sentence 7 = 2 + __, he first thought that 9 was the missing number. Then, reading backward seemed to be an adequate strategy for him. "Can I read in the other direction [from right to left], too?" When I asked him about the reasons for this change of direction, he referred explicitly to an understanding of the equals sign as an operator: "You always put the operation first and the result after."

During the posttest, even if Mathieu was generally able to use the equals sign as an indicator of a relationship, he referred to the equals sign as an operator on one occasion, at the beginning of the posttest, when he insisted on reading the number sentence 8 = 4 + 4 backward.

TEACHER [*showing 8 = 4 + 4*]. Can you tell me whether this number sentence is right or wrong?
MATHIEU. It is correct, because 4 plus 4 equals 8.
TEACHER. Can you read the number sentence?
MATHIEU. 8 plus ... no, this doesn't work. But you always have to begin on the side where the window is. 4 plus 4 equals 8.
TEACHER. Why do you read this way?
MATHIEU. You always read this way. You always start on the side where the window is.

In the room where I conducted the posttest, the window was to Mathieu's right, whereas the window was to his left in the classroom. This change was sufficient to encourage Mathieu to read backward, indicating the fragility of his conception of the equals sign as an indicator of a relationship.

Caroline, another student on whom I conducted an in-depth analysis, was perceived by her teacher as having major difficulties in school, particularly in mathematics. During the pretest interview, she displayed an understanding of the equals sign similar to that of Melissa and Mathieu. However, unlike the other two students, Caroline had major difficulties during the teaching experiment in developing a coherent understanding of the equals sign as an indicator of a relationship. She returned to her understanding of the equals sign as an operator on numerous occasions and was often unable to manipulate the concrete representation appropriately. At the beginning of the posttest interview, she seemed convinced that the equals sign is used as an operator.

TEACHER [*showing 8 = 4 + 4*]. Can you tell me whether this number sentence is right or wrong?
CAROLINE. It is wrong.
TEACHER. Why do you think it is wrong?
CAROLINE. It is the wrong way around.
TEACHER. How do you think it should be?
CAROLINE. These [*indicating 4 + 4*] should be at the beginning, then the equals sign, and finally the answer.

However, later in the interview she seemed to remember that the equals sign is an indicator of a relationship.

TEACHER [*showing 4 + 2 = 6 + 1*]. Do you think that this number sentence is right or wrong?
CAROLINE. It is correct.
TEACHER. Why do you think it is correct?
CAROLINE. No, it is wrong, because it is not the same. The additions are not the same. In the first one, it is 4 plus 2, and 6 plus 1 equals 7.
TEACHER. So, do you think that this number sentence is right or wrong?
CAROLINE. It is wrong, because there is not the same amount on both sides.

From this moment on in the posttest, Caroline's answers were coherent with an understanding of the equals sign as an indicator of a relationship. However, the fact that she considered the equals sign as an operator at the beginning of the posttest interview clearly indicates that her understanding of the equals sign was fragile.

Discussion

Several conclusions may be drawn from my research.

First, if the equals sign is not taught explicitly, children will likely develop a conception of the equals sign as an operator. In the class in which I conducted my research, the equals sign had not been explicitly investigated by the teacher, and all the students initially believed that they had to write an answer after the equals sign. This finding is consistent with other research on the understanding of the equals sign: several other researchers have confirmed children's common conception of the equals sign as an operator. Furthermore, this conception is held not only by Grade 1 students but also by much older students. Carpenter, Franke and Levi (2003) support the necessity of explicitly teaching the equals sign as an indicator of a relationship, because developing an understanding of the equals sign is not simply a process of maturation but, rather, must be addressed more directly.

Also, it seems realistic to allow students to change their conception of the equals sign under certain conditions. This idea is consistent with recent research (Carpenter and Levi 2000; Falkner, Levi and Carpenter 1999), which suggests that even Grade 1 students can develop a flexible understanding of the equals sign.

Even if it is possible to influence a change in students' conceptions of the equals sign, students are often reluctant to change and will try to stick with their initial conception. I have described strategies the participants in my research used, which can also be found in the literature. For example, Sáenz-Ludlow and Walgamuth (1998, 185) found that it is difficult to make children understand that the equals sign is an indicator of a relationship: "The dialogues and the arithmetical tasks on equality indicate these children's intellectual commitment, logical coherence and persistence to defend their thinking unless they were convinced otherwise." Carpenter, Franke and Levi (2003, 12) also point out the difficulty of changing students' conceptions of the equals sign:

> Children may cling tenaciously to the conceptions they have formed about how the equals sign should be used, and simply explaining the correct use of the symbol is not sufficient to convince most children to abandon their prior conceptions and adopt the accepted use of the equals sign.

Carpenter and Levi (2000) also observed that students' new understanding of the equals sign was not stable. After the researchers had investigated the equals sign with Grade 1 students, many of the children returned to their initial conceptions of the sign several months after the end of the teaching sessions. Therefore, Carpenter, Franke and Levi (2003) recommend a continuation of the use of nonconventional number sentences throughout the year. Sáenz-Ludlow and Walgamuth (1998) emphasize that developing an adequate understanding of the equals sign is a long-term process. It is therefore important to adopt a long-term approach in the classroom and to have children work repeatedly on the meaning of the equals sign.

Considering Grade 1 students' difficulties with the equals sign, one might wonder whether the introduction of the equals sign should be delayed. Several arguments support this proposition. As Dougherty (2004, 29) mentions, an early introduction of the equals sign means that teachers will need to undo childrens' misconceptions later: "In order for older children to solve equations with meaning, we have to first undo their idea about the equals sign before any approach to solving an equation makes sense."

However, is postponing the introduction of the equals sign a viable solution? Sáenz-Ludlow and Walgamuth's (1998) research seems to indicate that even when children are confronted later with the equals sign, they have difficulties understanding the symbol as an indicator of a relationship. In their research, the equals sign was taught to Grade 3 students who had not had to use the equals sign in previous tasks. During Grades 1 and 2, they had seen additions and subtractions written horizontally, without the equals sign. However, even those students tended to conceive of the equals sign as an operator. The learning of this symbol as an indicator of a relationship is therefore also a significant cognitive obstacle, and postponing the introduction of the equals sign is not sufficient to help students better understand its meaning. In this context, Dougherty (2004, 29) recommends helping children work on the equality relationship at a concrete level in measurement situations before switching to a numerical level:

> Showing that students can solve equations with different methods at an earlier age is encouraging. If they are capable of using these methods, even after coming from a strictly numerical perspective in their early beginnings in mathematics, what would be possible if students started with a focus on the structure of mathematics within a measurement context?

Furthermore, it is important to remember that children have probably already developed a conception of the equals sign, even before starting school. Many children's books, especially those that deal with counting, use the equals sign, so most children have already encountered the symbol.

There seems to be no easy solution in addressing young children's misunderstanding of the equals sign. However, the important thing to remember is that if the equals sign is not taught explicitly, children will develop a conception of this symbol as an operator—a conception that will be difficult to deconstruct later. On the other hand, the constant use of appropriate classroom activities that promote an accurate understanding of the equals sign seems to help even young children develop a better understanding of the symbol and the underlying relationships.

Notes

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1. Because the definition of the equals sign is a convention, it is impossible for children to simply discover its meaning, especially because they are already convinced that the equals sign is always followed by an answer to a question preceding it.

2. Here, I chose to repeat the question I had asked Melissa rather than answer her question. I did not want to influence her answer or give her hints about the meaning of the equals sign.

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Laurent Theis has been an assistant professor at Université de Sherbrooke in Quebec since 2003. In 2004 he completed his PhD thesis on Grade 1 students' understanding of the equals sign. His current research interests are the development of elementary students' algebraic reasoning and at-risk students' problem-solving abilities. As a member of the Sherbrooke Centre for Research in Youth, Science Teaching and Learning (CRYSTAL-Sherbrooke), he is also working on an inservice teacher training project that promotes the implementation of problem-solving approaches in early secondary school.