School Name ______________________________ Teacher Name________ __________ /100

Student Name ___________________ ______________________(**Print** First, Last) Grade ____

2019 Edmonton Junior High Math Contest

Part A: Multiple Choice		Part B (short answer)		Part C (short answer)	
1.	D	8.	3	14.	4
2.	A	9.	899	15.	11
3.	B	10.	17	16.	21
4.	D	11.	15, 24	17.	5
5.	A	12.	14	18.	$\frac{1}{512}$
6.	E	13.	203616	19.	8
7.	E				

Part A: _________× 4 + _______ × 2 = __________ Blank answers ≤ 3.
Correct blank
Part B: _________× 5 = _________ MARKER ONLY
Correct
Part C: _________× 7 = _________
Correct
Total: = _________

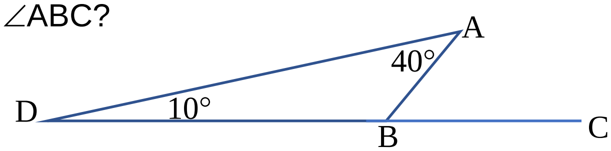
Instructions:

1. Calculator, grid paper and scrap paper are permitted. You may write on the booklet.
2. Programmable calculators and cell phones are not allowed.
3. Each correct answer in Part A is worth 4 points, each correct answer in Part B is worth 5 points, and each correct answer in Part C is worth 7 points. In Part A each blank is worth 2 points each up to a maximum of 3 blanks.
4. Each incorrect answer is worth 0 points.
5. Unanswered questions in Parts B and C are worth 0 points.
6. You have 60 minutes of writing time. It is recommended that you enter the answers on the Google form after the contest. This allows you the full 60 minutes of contest writing time.
7. All participants (grade 7 to 9) in the same school MUST write at the same time.
8. DO NOT discuss or post any answers on social media.

GOOD LUCK!

Part A: **Multiple Choice:** Each correct answer is worth 4 points. Each unanswered question is worth 2 points to a maximum of 3 unanswered questions.

1. What is the measure of $\angle ABC$?



A) 30° B) 40° C) 45° D) 50° E) 60°

Solution:
$\angle DBA = 180° – (40° + 10°) = 130°$.
$\angle ABC = 180° – \angle DBA = 180° – 130° = 50°$. Answer: D

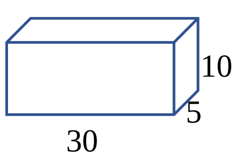
$\angle ABC$ is also known as the exterior angle to $\angle DBA$. It has a measure of the sum of the other two angles in the triangle. $\angle ABC = \angle ADB + \angle DAB$.

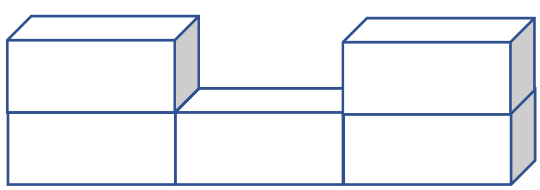
2. $4^{3n} = 16.\ What\ is\ 4^{6n}?$

A) 256 B) 32 C) 16 D) 8 E) 1

Solution:
$4^{3n} = 16$
$(4^{3n})^2 = 4^{6n}$
$16^2 = 4^{6n}$
$256 = 4^{6n}$ Answer: A

3. The solid on the left shows the measurement of one brick. Find the surface area of the solid on the right without the bottom. u^2= units square





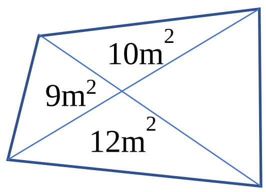
A) 3450 u^2 B) 3750 u^2 C) 4200 u^2 D) 3900 u^2 E) 2250 u^2

Solution:

There are 5 front, 5 back, 3 right, 3 left and 1 top view in total.
This gives (2)(5)(30 x 10) + (2)(3)(5 x 10) + (3)(30 x 5) = 3000 + 300 + 450 = 3750 u^2.

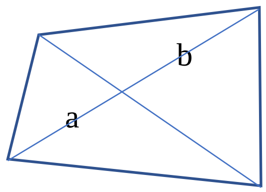
Answer: B

4. Find the area of the blank region rounded to tenths.



A) 14.5 m^2 B) 11.5 m^2 C) 7.5 m^2 D) 13.3 m^2 E) 10.8 m^2

Solution:



$\frac{ah_1}{2} = 9, \frac{bh_1}{2} = 10$ this gives a:b = 9:10 or b = $\frac{10}{9}a$

Similarly, $\frac{ah_2}{2} = 12, \frac{bh_2}{2} = \frac{10}{9}\left(\frac{ah_2}{2}\right) = \frac{10}{9}(12) = 13.3$ Answer: D

5. If $\frac{9^{4x-1}}{27} = \frac{3^{2x+1}}{81^{0.5-x}}$, what is the value of x?

A) 2 B) 1.5 C) 3 D) – 1.5 E) – 0.5

Solution:

Rewriting the powers using 3 as a base, we have

$$\frac{3^{8x-2}}{3^3} = \frac{3^{2x+1}}{3^{2-4x}}$$

8x – 2 – 3 = (2x + 1) – (2 – 4x)
8x – 5 = 6x – 1
2x = 4
X = 2 Answer: A

6. 2x – 3(4y + 4) – ((3x – (4x – (3y – 4z)) – 4(2x – 5y + 6z))) = ax + by + cz + d.
What is the value of a + b + c + d?

A) 16 B) 9 C) 3 D) – 9 E) – 8

Solution:
2x – 3(4y + 4) – ((3x – (4x – (3y – 4z)) – 4(2x – 5y + 6z)))
= 2x – 12y – 12 – ((3x – (4x – 3y + 4z)) – 8x + 20y – 24z)
= 2x – 12y – 12 – (3x – 4x + 3y – 4z – 8x +20y – 24z)
= 2x – 12y – 12 – (– 9x + 23y – 28z)
= 2x – 12y – 12 + 9x – 23y + 28z

= 11x – 35y + 28z – 12

11 – 35 + 28 – 12 = 39 – 47 = – 8 Answer: E

7. How many integer solutions are there if $-20 \leq x < 10$ and $-30 < \frac{5x-4}{2} \leq 30$?

A) 24 B) 22 C) 23 D) 19 E) 21

Solution:
$-60 < 5x - 4 \leq 60$
$-56 < 5x \leq 64$
$-11.2 < x \leq 12.8$
Combining $-20 \leq x < 10$ and $-11.2 < x \leq 12.8$, we have $-11 \leq x \leq 9$. This gives 21 integer solutions. Answer: E

Part B: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 5 points.

8. A blue marble weighs 100 grams, and a red marble weighs 500 g. Michelle weighs all her marbles and she discovers that the average weight of her marbles is 200 g. The number of blue marbles is how many times that of the number of red marbles?

Solution:
Lets b, r be the number of blue and red marbles respectively. Then, their average weight is

$$\frac{100b + 500r}{b + r} = 200$$

100b + 500r = 200b + 200r
300r = 100b
3r = b

This means that Michelle has 3 times blue marbles as red marbles. Answer: 3

9. Find the rightmost three digits of $1 + 11 + 111 + 1111 + \cdots + \underbrace{11111}_{2019\ digits}$

Solution:
In the ones place, we have 2019 1’s. In the tens place, we have 2018 1’s. In the hundreds place, we have 2017 1’s.
This gives 2019(1) + 2018(10) + 2017(100) =2019 + 20180 + 201700 = …899. The rightmost three digits are 899. Answer: 899

10. There are two whole numbers such that the square of their sum is 68 more than the square of their difference. What is the product of the two whole numbers?

Solution:
Let a be the larger number and b be the smaller number.
Then $(a + b)^2 = (a - b)^2 + 68$
$a^2 + 2ab + b^2 = a^2 - 2ab + b^2 + 68$
$4ab = 68$
$ab = 17$
Therefore, their product is 17. Answer: 17

11. Find all two-digit numbers with the property that the number is three times the product of its digits.

Solution:
Let the two-digit number be 10a + b.
We have
$10a + b = 3ab$
$10a = 3ab - b$
$10a = b(3a - 1)$
$$\frac{10a}{3a-1} = b$$
Since both a and b are integers with $a > 0$ and $0<b<9$, we only have two such numbers when a = 1 and b = 5 or a = 2 and b = 4. The numbers are 15 and 24. Answer: 15, 24

12. Given a sequence of 20 positive integers, it is known that the sum of any 3 consecutive terms in the sequence is even. Find the maximum number of odd integers that can exist in this sequence.

Solution:
Grouping 3 integers at a time, one of them must be an even number. With 20 integers, this gives 6 even numbers; thus, a maximum of 20 – 6 = 14 odd integers.

110 110 110 110 110 110 11 is one such example. Answer: 14

13. For some real positive number n, let $\lfloor n \rfloor$ be the greatest integer less than or equal to n. For example, $\left\lfloor \frac{1}{4} \right\rfloor = 0,\ \left\lfloor \frac{8}{7} \right\rfloor = 1,\ and\ \lfloor 3 \rfloor = 3$. Evaluate
$$\left\lfloor \frac{1}{5} \right\rfloor + \left\lfloor \frac{3}{5} \right\rfloor + \left\lfloor \frac{5}{5} \right\rfloor + \cdots + \left\lfloor \frac{2019}{5} \right\rfloor$$

Solution

After evaluating a few of the expressions, we notice a pattern for the answers of individual terms as

0 + 0 + 1 + 1 + 1 + 2 + 2 + 3 + 3 + 3 + … + 402 + 402 + 403 + 403 + 403

We now need to sum the odd and even series separately before adding the two totals

0 + 0 + 2 + 2 + … + 402 + 402 = $2\left((402)\left(\frac{\frac{402-0}{2}+1}{2}\right)\right) = 81204$

1+1+1+3+3+3+ … 403+403+403 = $3\left((403+1)\left(\frac{\frac{403-1}{2}+1}{2}\right)\right) = 122412$

Sum = 81204+122412 = 203616. Answer: 203616

Part C: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 7 points.

14. The tens digit of a two-digit prime number is smaller than its units digit, and the number obtained by interchanging the two digits is also a prime number. How many such numbers are there?

Solution:

Both digits must be among 1, 3, 7 and 9. Thus there are six candidates, namely, 13, 17, 19, 37, 39 and 79. We must eliminate 39 which is composite. We must also eliminate 19 because 91 is composite. On the other hand, 31, 71, 73 and 97 are all prime. Hence there are four such numbers. Answer: 4.

15. Three boys play badminton. Two of them play in the first game. In each subsequent game, the waiting boy replaces the loser of the preceding game. When they stop, Caleb has played 10 games and Ryan 21 games. How many games has Evan played?

Solution:

Ryan can only play Caleb 10 times. Since he cannot play Caleb two games in a row, he must have played Evan 11 times. Hence Evan has played 11 games. Answer: 11.

16. The students in a class are divided into groups of size m ≤ 5 to work on mathematics projects, and also into groups of size n ≤ 5 to work on nature projects. For each student, the pair (m, n) is different. At most how many students are in this class?

Solution

There are 25 different values of (m, n), namely, (1,1), (1,2), …, (5,5). The class can have at most 21 students. It can have that many students, with (m, n) ≠ (2,3), (3,2), (3,3) or (4,4). Answer: 21.

Below is a chart denoting which student belongs to which ordered pair.

A = (1,1)	B = (1, 2)	C = (1, 3)	D = (1, 4)	E = (1, 5)
F = (2, 1)	G = (2, 2)	H = (2, 4)	I = (2, 5)	J = (3, 1)
K = (3, 4)	L = (3, 5)	M = (4, 1)	N = (4, 2)	O = (4, 3)
P = (4, 5)	Q = (5, 1)	R = (5, 2)	S = (5, 3)	T = (5, 4)
U = (5, 5)				

Here is a diagram showing who is in which group.

Math					Nature				
Group of 1	Group of 2	Group of 3	Group of 4	Group of 5	Group of 1	Group of 2	Group of 3	Group of 4	Group of 5
A B C D E	FG, HI	JKL	MNOP	QRSTU	A F J M Q	BG, NR	COS	DKHT	EILPU

17. D, E and F are points on the sides BC, CA and AB, respectively, of triangle ABC. AD, BE and CF meet at G. If AG=GD, BG=2GE and CG=xGF, determine x.

Solution:
Triangles ABC and GBC have the same base. Hence their areas are proportional to their altitudes on BC. Since AD=AG+GD=2GD, the area of GBC is 1/2 that of ABC. Since BE=BG+GE=3GE, the area of GCA is 1/3 that of ABC. Hence the area of GAB is 1−1/2−1/3=1/6 that of ABC. It follows that 6GF=CF=CG+GF, so that CG=5GF. Answer: 5.

18. Initially, there are one 1 and nine 0s. In each move, we can replace any two of the numbers by their average. What is the minimal value of the number obtained after nine moves?

Solution:

Combining a nonzero number with a 0 reduces the nonzero number to one half its value. Combining two 0s simply eliminates one of them. Since we have only one nonzero number at any time, we should always combine it with a 0. Hence the minimum value of the final number is $\frac{1}{2^9} = \frac{1}{512}$ Answer: $\frac{1}{512}$

19. A shop sells two kinds of items, hamburgers and hot dogs, each costs an integral number of dollars. Alice spends 43 dollars for 7 items. Michelle spends 42 dollars for 9 items. The number of hamburgers they buy differ by at least 2. Christopher spends 41 dollars in this shop. How many items does he buy?

Solution:

Let a hamburger cost x dollars and a hot dog y dollars. Since 7 items can cost more than 9 items, x ≠ y. By symmetry, we may assume that x ≥ y. Since 7 hamburgers cost more than 43 dollars, x ≥ 7. Since 9 hot dogs cost less than 42 dollars, y ≤ 4.

Let Alice and Michelle buy m and n hamburgers respectively. Then we have $mx + (7 - m)y = 43$ and $nx + (9 - n)y = 42$. Subtraction yields $(x - y)(m - n) = 2y + 1$. It follows that $x - y$ divides $2y + 1$. Note that $x - y \geq 3$ while $2y + 1 \leq 9$. Since $m - n \geq 2$, we must have $x - y < 2y + 1$. The only possibility is $x - y = 3$ and $2y + 1 = 9$, so that x = 7 and y = 4. Thus, Alice buys 5 hamburgers and 2 hot dogs while Michelle buys 2 hamburgers and 7 hot dogs. Christopher may spend 7, 14, 21, 28 or 35 dollars on hamburgers, leaving 34, 27, 20, 13 or 6, respectively. The only multiple of 4 is 20. Hence Christopher buys 3 hamburgers and 5 hot dogs for 8 items. Answer: 8.

Alt. solution:

Another approach is to use a table of values to organize the relationship between cost of a hot dog (d) and the cost of a hamburger (H). Assuming hamburgers cost more than hot dogs, we may also have Michelle bought more hot dogs as she also has 9 items that costs less than 7 items. Each trial keeping in mind Alice would have bought 2 or more hamburgers than Michelle

Alice 7 items, $43	Michelle 9 items, $42	Verification by using an equation
3H + 4d	H + 8d + 1	3H + 4d = H + 8d + 1 2H = 4d + 1, not possible as left side is even and right side is odd
4H + 3d	H + 8d + 1	4H + 3d = H + 8d + 1 3H = 5d + 1 When d = 1, H = 2 but 2 + 8(1) + 1 ≠ 42
5H + 2d	H + 8d + 1	5H + 2d = H + 8d + 1 4H = 6d + 1, not possible as left side is even and right side is odd

6H + 1d	H + 8d + 1	6H + d = H + 8d + 1 5H = 7d + 1, When d = 2, H = 3 but 3 + 8(2) + 1 ≠ 42
4H + 3d	2H + 7d + 1	4H + 3d = 2H + 7d + 1 2H = 4d + 1, not possible as left side is even and right side is odd
5H + 2d	2H + 7d + 1	5H + 2d = 2H + 7d + 1 3H = 5d + 1, When d = 1, H = 2 but 2 + 7(1) + 1 ≠ 42 When d = 4, H = 7 and 2(7) + 7(4) + 1 = 14+28 = \$42 5(7) + 2(4) = 35 + 8 = \$43

This gives each hot dog is \$4 and a hamburger is \$7.
With \$41, we have an equation 41 = 4(5) + 7(3), or 5 + 3 = 8 items. Answer = 8