Print ID # ___________ School Name ___________________________________

Student Name ____(**Print** First, Last) ____ /100

2018 Edmonton Junior High Math Contest SOLUTION

Part A: Multiple Choice		Part B (short answer)		Part C (short answer)	
1.	C	8.	24	14.	3
2.	B	9.	4	15.	35
3.	C	10.	$\sqrt{2}$ or 1.414	16.	45
4.	E	11.	7	17.	6
5.	A	12.	$\sqrt{7}$ *or 2.646*	18.	110
6.	E	13.	18	19.	8
7.	B				

Part A: _________× 4 + _______ × 2 = __________ Blank answers ≤ 3.
Correct blank
Part B: _________× 5 = _________ MARKER
Correct ONLY
Part C: _________× 7 = _________
Correct
Total: = _________

Instructions:

1. Calculator, grid paper and scrap paper are permitted. You may write on the booklet.
2. Programmable calculators and cell phones are not allowed.
3. Each correct answer in Part A is worth 4 points, each correct answer in Part B is worth 5 points, and each correct answer in Part C is worth 7 points. In Part A each blank is worth 2 points each up to a maximum of 3 blanks.
4. Each incorrect answer is worth 0 points.
5. Unanswered questions in Parts B and C are worth 0 points.
6. You have 60 minutes of writing time.
7. All participants (grade 7 to 9) in the same school MUST write at the same time.
8. When done, carefully REMOVE and HAND IN this TOP page. You may keep the contest.
9. DO NOT discuss or post any answers on social media.

GOOD LUCK!

Part A: **Multiple Choice:** Each correct answer is worth 4 points. Each unanswered question is worth 2 points to a maximum of 3 unanswered questions.

1. In a triangle, all angles are integer values. The measure of the smallest angle is 20°. What is the measure of the largest possible angle?

A) 80° B) 89° C) 139° D) 140° E) 160°

Solution

The two remaining angles have a sum of 160. Since the measurements of the angles are integers, the next smallest angle should be 21°. Therefore, the largest angle would be 180 – 20 –21 or 139°.

2. The original price of an item is reduced by 20% and the new price is further reduced by 20%. The final sale price is the same as a single reduction of what percentage of the original price?

A) 30% B) 36% C) 40% D) 60% E) 64%

Solution

If the item was priced at \$100, a 20% reduction, would result in a reduced price of $100 \times 0.8 =$ \$80. A 20% reduction of the reduced price would be $80 \times 0.8 =$ \$64. The twice reduced price is 64% of the original, or $100 - 64 =$ a single reduction of 36% of the original.

3. 16, 24, 36, … are the first three terms of a **geometric** sequence. With the exception of the first term, each term thereafter is the product of the previous term and a constant.

30, 45, 60, … are the first three terms of an **arithmetic** sequence. With the exception of the first term, each term thereafter is the sum of the previous term and a constant.

If both sequences are to continue, for which n does the n^{th} term of the geometric sequence {16, 24, 36, ...} first become larger than the n^{th} term of the arithmetic sequence {30,45,60, ...}

A) 4 B) 5 C) 6 D) 7 E) 8

Solution

Each term of the geometric sequence is multiplied by the constant 1.5.
Each term of the arithmetic sequence is added by the constant 15.
We can simply list out the next few terms of each sequence.

Geometric sequence	16	24	36	54	81	121.5
Arithmetic sequence	30	45	60	75	90	105

We can see the geometric sequence is larger starting from the 6th term.

4. Solve for x

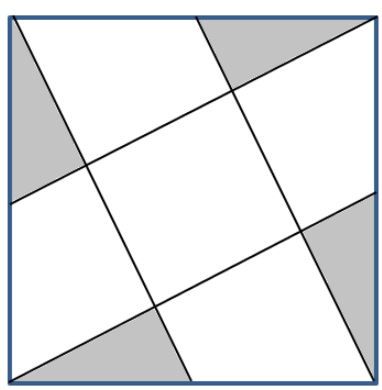
$$\frac{x}{2018} + \frac{2017(2019)}{2018} - 2019 = -1$$

A) –2018 B) 2018 C) 0 D) –1 E) 1

Solution

X = $(2018)^2$ – (2017)(2019) = 1

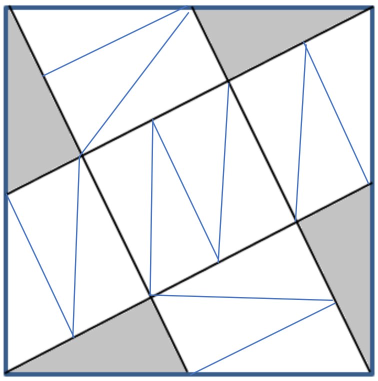
5. Midpoints of each side are connected to one of the vertices of a square. What percent of the square is shaded?



A) 20% B) 25% C) 30% D) 35% E) 15%

Solution

The diagram can be subdivided into congruent triangles as follows



The entire square is made up of 20 congruent triangles. The shaded part has 4 pieces out of 20. This means the shaded part covers 20% of the square.

6. In the sequence of numbers: 1, 3, 2, –1, ... each term after the first two is equal to the term preceding it minus the term preceding that, $t_n = t_{n-1} - t_{n-2}$.
What is the sum of the first one hundred terms of the sequence?

A) 0 B) –1 C) 21 D) 16 E) 5

Solution

Writing out the sequence, we have 1, 3, 2, –1, –3, 2, 5, 3, –2, –5, –3, 2, 5, 3, –2, –5 ,–3, 2, 5, 3, –2, –5 ...

After the first four numbers in the sequence, there is a repetition of these six terms:
–3, 2, 5, 3, –2, –5. The remaining 96 terms are grouped six terms at a time of which each group has a sum of 0.

The sum is $1 + 3 + 2 + -1 + \left(\frac{96}{6}\right)(0) = 5$

The sum of the first 100 terms is 5.

7. A woman, her brother, her son and her daughter (all relations by birth) are chess players. The worst player's twin (who is one of the four players) and the best player are of opposite sex. The worst player and the best player are the same age. Who is the worst player?

A) woman B) son C) brother D) daughter E) insufficient information to determine

Solution

If the son is the worst player, the daughter must be his twin. The best player must then be the brother. This is consistent with the given information, since the brother and the son could be the same age. The assumption that any of the other players is worst leads to a contradiction.

If the woman is the worst player, her brother must be her twin and her daughter must be the best player. But the woman and her daughter cannot be the same age.

If the brother is the worst player, the woman must be his twin. The best player is then the son. But the woman and her son cannot be the same age, and the woman's twin, her brother, cannot be the same age as the son.

If the daughter is the worst player, the son must be the daughter's twin. The best player must then be the woman. But the woman and her daughter cannot be the same age.

Part B: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 5 points.

8. A large cube is assembled together with 125 smaller identical cubes. This large cube is placed on a table top and spray painted on five of the faces. After drying, the large cube is disassembled with all the smaller cubes placed in a bag. How many cubes have exactly 2 faces painted?

Solution

Of the twelve edges, only 8 edges have cubes that have exactly 2 faces painted. Each edge has 3 such cubes giving a total of 8 x 3 = 24 cubes.

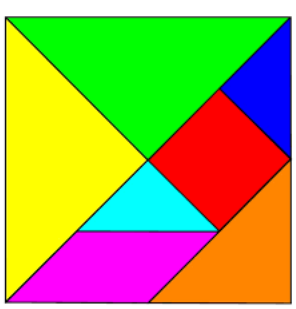
9. Juice cans are sold in a pack of 6 or 12. There are 12 448 cans in the warehouse waiting to be put into different packs. If the cans are placed in any combination of 6 and 12 packs, how many cans would be left over?

Solution

Let n represent the number of 6 packs and let m represent the number of 12 packs.

Then $6n + 12m = 12\ 448$
$6(n + 2m) = 12\ 448$
$12\ 448 \div 6 = 2074$ with a remainder of 4.
4 cans would not fit into 6 or 12 packs.

10. Below is a picture of an ancient Chinese puzzle, called a tangram. The seven pieces are arranged to form a large square. If the edge of the large square is one unit, find the perimeter of the smaller square in the picture. Leave the answer in simplest radical form.



Solution

All the triangular pieces (small, medium and large sizes) are isosceles right triangles. This means they are all similar.

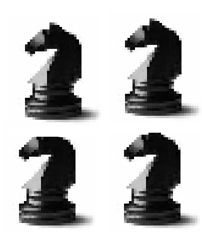
The largest triangle measures $\frac{\sqrt{2}}{2}$ by $\frac{\sqrt{2}}{2}$ by 1.

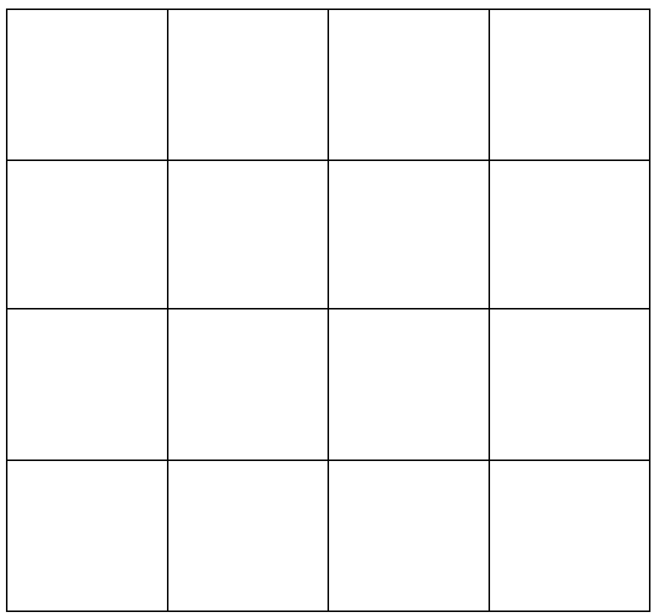
The smallest triangle is a reduction with scale factor of ½, it measures $\frac{\sqrt{2}}{4}$ by $\frac{\sqrt{2}}{4}$ by $\frac{1}{2}$.

The side of the small square is the same as one of the leg of the small isosceles triangle, $\frac{\sqrt{2}}{4}$.

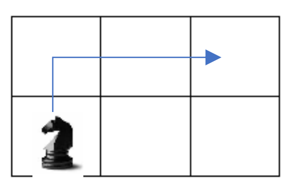
The perimeter of the small square is $4\left(\frac{\sqrt{2}}{4}\right) = \sqrt{2}$

11. Sally places 4 knights on a 4 by 4 board.



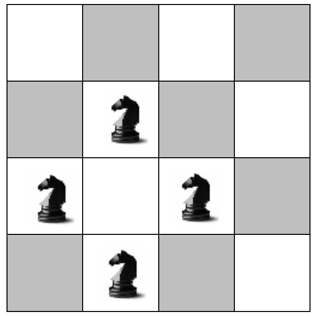


Knights can only attack a square that is 2 by 3 away.

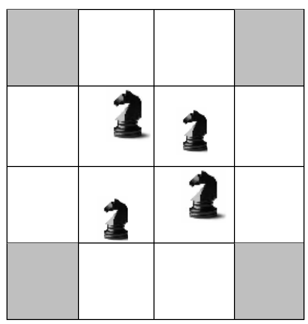


If any square is attacked by at least two different knights, Sally colours the square black. What is the maximum number of black squares possible?

Solution

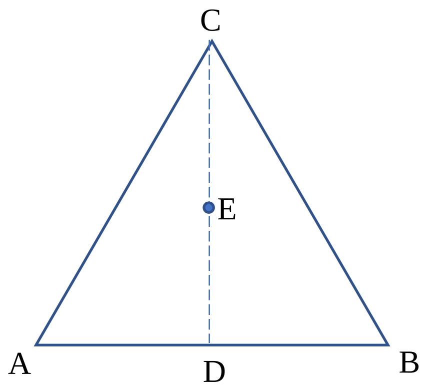


Where knights will doubly attack all the blackened squares. In total there are 7 black squares. To prove this is the maximum, we note that any knight in one of the centre squares will attack 4 squares, and any knight on an edge will attack 3 squares. Note that the above configuration maximizes the number of black squares given that 2 knights are on an edge and 2 knights are in the centre. We only have 4 knights, so the only way for Sally to colour 8 squares is if all knights are attacking 4 squares, or in other words, that they are all in the centre.



Clearly this does not result in 8 blackened squares.

12. ABC is an equilateral triangle with a height of $2\sqrt{3}$. E is the midpoint of the altitude CD. Find distance $\overline{AE}$. Leave answer in radical form.



Solution

ΔABC is an equilateral with interior angles at 60° each. ΔCBD is a 30°–60°–90° triangle with side lengths in the ratio of 1–$\sqrt{3}$–2. Since $\overline{CD}$ is $2\sqrt{3}$, we must have $\overline{CB} = \overline{CA} = \overline{AB}$ = 4.

It follows that

$$(\overline{AE})^2 = (\overline{AD})^2 + (\overline{DE})^2$$

$$\overline{AE} = \sqrt{(2)^2 + \left(\sqrt{3}\right)^2} = \sqrt{7}$$

13. In triangle *ABC*, ∡*CAB*=108° and *AB*=*AC*. The bisector of ∡*ABC* meets *CA* at *E*, and the perpendicular to *BE* at *E* meets *BC* at *D*. Determine ∡*ADE*.

Solution

Extend BA and DE to meet at F. By symmetry, BED and BEF are congruent right triangles.

Now ∡EFB=∡EDB=90°−∡EBC=72° while ∡EAF=180°−∡CAB=72° also. Hence EA=EF=ED.

Note that we have ∡BEA=180°−∡CAB−∡ABC=54° and ∡DEA=∡BED+∡BEA=144°.

It follows that $\measuredangle ADE = \frac{1}{2}(180° - \measuredangle DEA) = 18°$.

Part C: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 7 points.

14. Each of the digits 0 to 9 is written on a card. You can select any number of cards from the group. The digit(s) can then be rearranged in any order to form a number. What is the minimum number of cards from which a multiple of 3 with up to three digits can always be formed?

Solution

If we have only two cards, the numbers on them may be 1 and 4, and none of 1, 4, 11, 14 or 41 is a multiple of 3. Hence two cards are not enough. We claim that three cards are enough. If any of 0, 3, 6 or 9 is on one of them, we have a one–digit multiple of 3. Suppose this is not the case. If we have all of 1, 4 and 7 or all of 2, 5 and 8, we have a three–digit multiple of 3 since 1+4+7=12 and 2+5+8=15 are multiples of 3. If this is also not the case, we take one number from each triple. Their sum will be a multiple of 3, and they will form a two–digit multiple of 3.

15. A class of 31 students invite some students from another school to the Valentine dinner. There are 19 tables at which one or two students may sit. If each boy has exchanged Valentine cards with exactly three girls and each girl with exactly two boys, how many students have attended the dinner?

Solution

The ratio of boys to girls is 2:3. Hence the total number of students is a multiple of 5. It is greater than 31 and less than $19 \times 2 = 38$. Hence it must be 35.

16. How many different five–digit numbers have the property that if one digit is removed from it, reading from left to right, the number 2018 will be obtained?

Solution

If the deleted digit is in the first place before 2, we have exactly 9 choices for adding a digit because we cannot add 0. It may appear that we have 10 choices if the deleted digit is in the second place between 2 and 0. However, 22018 has already been counted. So we have only 9 choices. Suppose the deleted digit is in the third place between 0 and 1. We still have only 9 choices since we cannot choose 20018. Hence the total number of choices is $9 \times 5 = 45$.

17. In the school badminton club, there are 18 boys and 18 girls, with 12 in each of Grades 7, 8 and 9. The school wants to enter as many mixed–double teams as possible in the City Championship. The partners in each pair must be from the same grade. What is the minimum number of mixed–double pairs the school can enter?

Solution

Suppose all Grade 7 students are boys while all Grade 8 students are girls, the school can enter exactly 6 Grade 9 teams. Suppose the number of teams that can be entered is less than 6. Then there are more than 24 students with no partners. Such students from each grade must be of the same gender. By symmetry, we may assume that all students without partners from Grades 7 or 8 are boys and all students without partners from Grade 9 are girls. Since the total number of boys is equal to the total number of girls, we have more than 12 girls with no partners, and they are all in Grade 9. This is a contradiction since there are only 12 students in Grade 9.

18. Consider a non–negative number boring if it is made of only the same digits, and cool if it is made of only distinct digits. Single digits i.e. 1, 4, 3 etc. are only cool. Note that digits with repeats such as 21330 are neither cool nor boring. What is the smallest positive integer greater than or equal to 11 that cannot be represented as the sum of a boring and a cool number?

Solution

We claim the smallest positive integer is 110 . Note that the difference between 110 and any two–digit boring number will also be a boring number, so there are no cool numbers that can sum with a boring number to obtain 110.

We now prove this is the minimum. All two–digit boring numbers are the sum of the given boring number and 0. We know all two–digit boring numbers are separated 10 numbers, which correspond to adding the cool numbers 1, 2, ..., 10, thus any two–digit number is the sum of a boring number and a cool number less than or equal to ten. For three–digit numbers, note that we only need to consider numbers greater than 99 + 10 = 109, of which there are none that are also less than 110, thus the minimum is proved.

19. In triangle *BAD*, *BD*=3, *AD*=4 and *AB*=5. *C* is the point on the extension of *BD* such that *DC*=1. *PQRS* is a rectangle with *P* and *S* on *BC*, *Q* on *AB* and *R* on *AC*. Determine the maximum perimeter of *PQRS*.

Solution

Since $\frac{BD}{DA} = \frac{BP}{PQ} = \frac{3}{4}$ and $\frac{CD}{DA} = \frac{CS}{SR} = \frac{1}{4}$

Note that PQ = SR.

Hence BP+CS = PQ($\frac{3}{4} + \frac{1}{4}$) = PQ.

It follows that the perimeter of PQRS is equal to 2(PQ+PS) = 2(BP+CS+PS) = 2BC =2(3+1) = 8.

Since the value is constant, the maximum value is also 8.