Print ID # ____________ School Name ____________________________________

Student Name ____(**Print** First, Last) $\overline{}$ 100

2016 Edmonton Junior High Math Contest SOLUTION

Part A: Multiple Choice		Part B (short answer)		Part C(short answer)	
1.	E	8.	1 OR 100%	14.	$\frac{1}{6}$
2.	C	9.	24	15.	17
3.	B	10.	2	16.	0
4.	D	11.	11.4	17.	20
5.	C	12.	96	18.	518
6.	D	13.	3	19.	1344
7.	A				

Part A: __________× 4 + ________ × 2 = ___________ Blank answers ≤ 3.
Correct blank
Part B: __________× 5 = __________ MARKER ONLY
Correct
Part C: __________× 7 = __________
Correct
Total: = __________

Instructions:

1. Calculator, grid paper and scrap paper are permitted. You may write on the booklet.
2. Programmable calculators and cell phones are not allowed.
3. Each correct answer in Part A is worth 4 points, each correct answer in Part B is worth 5 points, and each correct answer in Part C is worth 7 points. In Part A each blank is worth 2 points each up to a maximum of 3 blanks.
4. Each incorrect answer is worth 0 points.
5. Unanswered questions in Parts B and C are worth 0 points.
6. You have 60 minutes of writing time.
7. All participants (grade 7 to 9) in the same school MUST write at the same time.
8. When done, carefully REMOVE and HAND IN this TOP page. You may keep the contest.
9. DO NOT discuss or post any answers on social media.

GOOD LUCK!

Edmonton Junior High Math Contest 2016

Part A: Multiple Choice: Each correct answer is worth 4 points. Each unanswered question is worth 2 points to a maximum of 3 unanswered questions.

1. On a contest with 30 questions:

   - Each correct answer is awarded 6 points;
   - Each incorrectly answered question is penalized 2 points;
   - Each unanswered question is penalized 1 point.

   Abby answered 70% of the questions. The ratio of correct answers to incorrect answers to unanswered question is 5:2:3. How many points did she receive?

   A. 45 B. 126 C. 63 D. 81 E. 69

Solution:

Abby answered 0.70 x 30 = 21 questions. With the ratio 5:2:3, it translates to 15 questions correct, 6 questions incorrect, 9 questions unanswered. The total points earned is 15(6) – 6(2) – 9(1) = 69.

2. Working at a constant rate, if Ben can cut a metal pipe into 5 pieces in 30 minutes, how many minutes would it take to cut a similar pipe into 15 pieces?

   A. 210 B. 170 C. 105 D. 90 E. 65

Solution:

5 pieces requires 4 cuts (in 30 min). Since 15 pieces requires 14 cuts, then:

Minutes : Cuts = 30 : 4 = m : 14. Therefore m = $\frac{30(14)}{4}$ = 105 min.

3. Each of the first 6 prime numbers is written on the 6 different faces of a red cube. Each of the first 6 composite numbers is written on the 6 different faces of a blue cube. The two cubes are tossed/rolled once. What is the probability that the sum of the numbers rolled is 13?

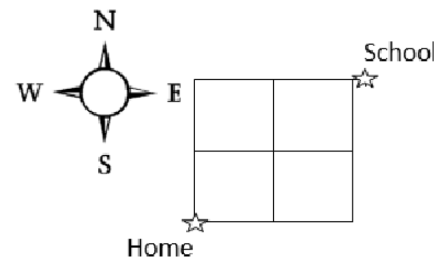
   A. $\frac{1}{18}$ B. $\frac{1}{12}$ C. $\frac{1}{9}$ D. $\frac{1}{6}$ E. $\frac{1}{3}$

Solution:

Red = {2, 3, 5, 7, 11, 13} Blue = {4, 6, 8, 9, 10, 12} Favourable outcomes = {(3,10), (5,8), (7,6)}

P(Sum = 13) = $\frac{3}{6(6)}$ = $\frac{1}{12}$

4. The distance from home to school is a total of 4 blocks. Jane must stay on the pathways and walk either east or north, as shown in the diagram.



How many ways are there in total for Jane to walk from home to school and back again, if she must return using a different path?

A. 5   B. 6   C. 11   D. 30   E. 36

Solution:

There are a total of 6 different pathways from Home to School.

Therefore, there are 6 - 1 = 5 different pathways back.
Therefore, the total different pathways for the return trip = 6(5) = 30.

5. Two rectangles with integer dimensions each have an area of 216 cm^2. The length of the first rectangle is 30 cm greater than that of the second rectangle. However, the width of the first rectangle is 5 cm less than that of the second rectangle. What is the difference in the perimeters, in centimetres, of the two rectangles?

A. 25   B. 35   C. 50   D. 60   E. 70

Solution:

Possible Dimensions:

L	216	108	72	54	36	27	24	18
W	1	2	3	4	6	8	9	12

The only two possibilities for the rectangles' dimensions with lengths differing by 30 and widths differing by 5 are:
Rectangle #1: 24 x 9 (with perimeter= 66 cm) and Rectangle #2: 4 x 54 (with perimeter= 116 cm).
Therefore, the difference in their perimeters = 116 – 66 = 50 cm

Alternate solution:
Let dimensions of Rectangle #2 be: Length = L; and Width = W. Therefore: Perimeter = 2L + 2W.
Therefore, dimensions of Rectangle #1 are:
Length = (L + 30); and Width = (W – 5). Therefore: P = 2(L + 30) + 2(W – 5) = 2L + 2W + 50
Therefore, the Difference in perimeters is: (2L + 2W + 50) – (2L + 2W) = 50 cm.

6. A family agreed to share the total cost of buying a $2268 (including GST) gaming computer.

- Dad paid for $\frac{1}{6}$ of the total cost.
- Then the oldest daughter, Kylee, paid for $\frac{1}{5}$ of the remaining cost.
- Then the son, Shawn, paid for $\frac{1}{4}$ of the remaining cost.

- Then the middle daughter, Erin, paid for $\frac{1}{3}$ of the remaining cost.
- Then the youngest daughter, Kassidy, paid for $\frac{1}{2}$ of the remaining cost.

If Mom paid for the remaining amount, then approximately how much more did the children pay compared to the amount paid by the parents?

A. \$300 B. \$450 C. \$600 D. \$750 E. \$900

Solution:

Dad's Share = $\frac{2268}{6}$ = \$378 Kylee's Share = $\frac{2268-378}{5}$ = \$378 Shawn's Share = $\frac{1890-378}{4}$ = \$378

Erin's Share = $\frac{1512-378}{3}$ = \$378 Kassidy's Share = $\frac{1134-378}{2}$ = \$378 Mom's Share = \$378

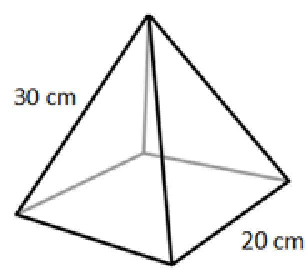
Note: Each person paid \$378! Since there were 4 children and 2 parents:

The 4 children paid a total of $(4 - 2)(378) = 2(378) =$ \$756 more than what the parents paid together.

Therefore, the closest answer is \$750.

7. The net of the square pyramid shown at the right would consist of 4 congruent isosceles triangles (each with sides measuring 30 cm, 30 cm, 20 cm) and one square base.

What is the total surface area, to the nearest whole cm^2, of the pyramid?



A. 1531 B. 1585 C. 1600 D. 1665 E. 2800

Solution:

Using the Pythagorean Theorem, the altitude of each isosceles triangle = $\sqrt{900-100} = \sqrt{800}$ cm.

Therefore, SA = 4 triangles + Square base = $\frac{4(\sqrt{800})(20)}{2} + (20)(20) = 40(\sqrt{800}) + 400 = 1531.37 \approx 1531$

Part B: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 5 points.

8. Two different digits from 1 to 9 are chosen randomly to form a 2-digit number. By reversing the order of the digits, a second 2-digit number is formed. What is the probability that the sum of these two numbers will be a multiple of the sum of the digits?

Solution:

Let the units digit be U, and the tens digit be T. Therefore the value of the 2-digit number = 10T + U.
By reversing the digits, the value of that 2-digit number = 10U + T.
Therefore, the sum of the two 2-digit numbers = (10T + U) + (10U + T) = 11U + 11T = 11(U + T).
Therefore, the sum must be eleven times the sum of the two digits. Therefore, the Probability = 100%.

9. How many positive proper fractions in lowest terms have a denominator of 90?

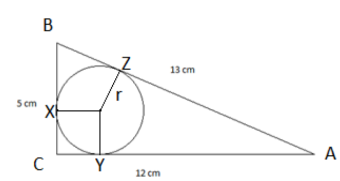
Solution:

The numerator of a basic fraction (with a denominator of 90) must be either 1, or any prime number less than 90 that is not a factor of 90. (The prime factors of 90 are: 2, 3, and 5. The quantity of possible numerators can be determined without listing all of them; just eliminate all multiples of these three prime numbers, as follows:

- To begin with, there are 90 numbers from 1 to 90, inclusive.
- Of these 90 numbers, eliminate all multiples of 2. That leaves 45 remaining possibilities (all are odd).
- Now eliminate all odd multiples of 3: (3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, 75, 81, 87). There are 15 of them. Therefore, the remaining number of possible numerators = 45 – 15 = 30 numbers.
- Now eliminate all remaining odd multiples of 5: (5, 25, 35, 55, 65, 85). There are only 6 of them. Therefore, the remaining number of possible numerators = 30 – 6 = 24 numbers!
- If you would like the list of all 24 possible numerators, they are: {1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89}.

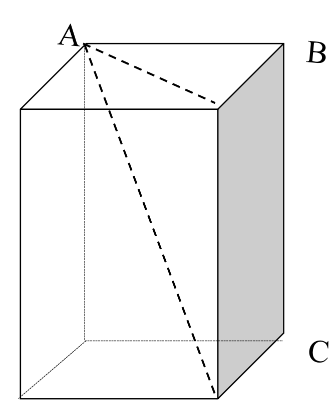
10. A circle is inscribed in a right triangle as shown. What is the radius, in cm, of this circle?

Solution:



AY = AZ = 12-r
BX = BZ = 5-r
AB = AZ + BZ
13 = (12 – r) + (5 – r)
2r = 12 + 5 – 13
Therefore, r = 2 cm

11. A carpenter stores his nails in a metal box in the shape of a rectangular prism with a square base that measures 5 cm by 5 cm by 9 cm. To the nearest **tenths** of a cm, what is the length of the longest nail that can be stored inside the box?



Solution:

The longest nail is represented by AC.

AB = $\sqrt{25+25} = \sqrt{50}$ cm.

AC = $\sqrt{50+81} = \sqrt{131}$ = 11.4 cm

12. What is the difference between the sum of all multiples of 3 less than 50 and the sum of all multiples of 4 less than 50?

Solution:

$$(3 + 6 + 9 + \ldots + 48) - (4 + 8 + 12 + \ldots + 48)$$
$$= 8(51) - 6(52) = 408 - 312 = 96$$

13. How many 9s are in the product 999 999 999 x 20 162 016?

Solution:

(1000000000 – 1) x 20162016 = 20162016000000000 – 20162016 = 20162015979837983.
Therefore, three 9's.

Part C: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 7 points.

14. Three people are coloring the same piece of $8\frac{1}{2}\ by\ 14$ paper.

Abby starts on the left side and painted $\frac{1}{2}$ of the paper red.
Ben starts on the right side and painted $\frac{3}{4}$ of the entire paper green.
Cathy starts in the middle and painted $\frac{1}{3}$ of the entire paper, evenly on either side of the centre line, using blue color.

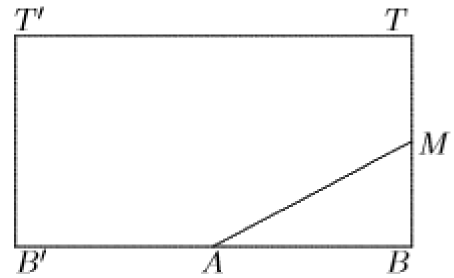
What fraction of the paper has all three color painted on it?

Solution:
The least common multiple of 2, 4 and 3 is twelve. We could divide the paper into twelve equal sections and label each color accordingly.

Red	Red	Red	Red	Red	Red						
			Green	Green	Green	Green	Green	Green	Green	Green	Green
				Blue	Blue	Blue	Blue				

From the diagram we could see that two of the vertical sections contain all three color, $\frac{2}{12} = \frac{1}{6}$.

15. $\overline{AB}$ is a diameter of the base of a cylinder and T is point on the opposite base of the cylinder directly above B. M is the midpoint of $\overline{TB}$. If $\overline{MB}$ = 8 and $\overline{AB}$ = $\frac{30}{\pi}$, what is the shortest distance between A and M along the curved surface of the cylinder?



Solution: Cut the cylinder along $\overline{TB}$ and unrolled it into the rectangle TBB'T'. Then A is the midpoint of $\overline{BB'}$. Since the radius of the base of the cylinder is $\frac{15}{\pi}$, $\overline{BB'}$ =2 $\pi\left(\frac{15}{\pi}\right)$ = 30 so that $\overline{AB}$ = 15 in the rectangle. The shortest path between A and M along the curved surface of the cylinder becomes the segment AM in the rectangle. By Pythagoras' Theorem, AM = $\sqrt{\overline{MB}^2 + \overline{AB}^2}$= 17.

16. The fraction $\frac{1}{7}$,when expressed as a repeating decimal, is equal to 0.142857142857 ...

Note that 1 is in the tenths place, 4 is in the hundredths place and so on. Let m be the 20th digit to the right of the decimal point and n be the 104th digit to the right of the decimal point, find n – m.

Solution

Any one of the digits (1, 4, 2, 8, 5 and 7) will reappear six places later. Note that 20 + 6(14) = 20 + 84 = 104. This means the two digits are the same; hence, n – m = 0.

17. There were 11 baskets of Easter eggs, containing 14, 15, 19, 20, 22, 23, 24, 26, 27, 34 and 40 eggs respectively. John and Mary took all but one basket, each getting several baskets. John had twice as many eggs as Mary at this point. Mary then gave one of her baskets to John and now John had three times as many eggs as Mary. Which basket, indicated by the number of eggs above, did Mary give to John?

Solution

There are 264 Easter eggs in total. Firstly, John had twice as many eggs as Mary. This means the total number of eggs between them is divisible by 3. After the exchange, John had three times as many eggs as Mary. This also means the total number of eggs between them is divisible by 4.

Let J, M be the number of Easter eggs that John and Mary have. L be the number of Easter eggs left behind. Since (J + M) + L = 264, we have both (J + M) and 264 divisible by 3 and 4, it follows that the basket not taken is also a multiple of 3 and of 4. The only basket that is divisible by 3 and 4 is 24.

Between John and Mary, they share 264 – 24 = 240 Easter eggs. At start, John would have 160 eggs and Mary 80 eggs. After the exchange, John would have 180 eggs and Mary 60 eggs. Therefore, Mary gave away the basket that contains 20 eggs.

18. How many copies of the digit 0 are there among the digits of the first 2016 positive integers?

Solution:

Among the units digits, the copies of 0s are in 10, 20, ..., 2010, so that there are 201 of them. Among the tens digits, the copies of 0s are in 100 to 109, 200 to 209, ..., 2000 to 2009, so that there are 200 of them. Among the hundreds digits, the copies of 0s are in 1000 to 1099 and 2000 to 2016, so that there are 117 of them. The total is 201+ 200 + 117 = 518.

19. E and F are the respective midpoints of the sides $\overline{AB}$ and $\overline{BC}$ of a rectangle ABCD of area 2016. G is the point of intersection of $\overline{AF}$ and $\overline{CE}$. What is the area of the quadrilateral AGCD?

Solution:

Denote the area of the polygon P by [P]. Then [ABCD] = 2016. Let [GEB] = x and [GFB] = y. Since AE = EB and BF = FC, [AGE] = x and [CGF] = y.

Now $2x + y = [ABF] = \frac{1}{2}[ABC] = [CBE] = x + 2y$.

Hence x = y. Since [ABC] = 1008, x = y = 168, so that [AGCD] = [ABCD] − 2x − 2y = 1344