Print ID # _____   School Name _____KEY_____

Student Name _____(Print First, Last) $\overline{100}$

2015 Edmonton Junior High Math Contest

Part A: Multiple Choice		Part B (short answer)		Part C(short answer)	
1.	C	8.	90	14.	46
2.	B	9.	82	15.	152
3.	A	10.	166	16.	35
4.	E	11.	(1,3)	17.	0, OR NONE
5.	A	12.	118	18.	2, 4, 6, 6 Or a = 6, b = 2, c = 4, d = 6
6.	E	13.	98	19.	145
7.	C				

Part A: _____× 4 + _____ × 2 = _____   Blank answers ≤ 3.
         Correct   blank
Part B: _____× 5   = _____   MARKER
         Correct   ONLY
Part C: _____× 7   = _____
         Correct
Total: = _____

Instructions:

1. Calculator, grid paper and scrap paper are permitted. You may write on the booklet.
2. Programmable calculators and cell phones are not allowed.
3. Each correct answer in Part A is worth 4 points, each correct answer in Part B is worth 5 points, and each correct answer in Part C is worth 7 points. In Part A each blank is worth 2 points each up to a maximum of 3 blanks.
4. Each incorrect answer is worth 0 points.
5. Unanswered questions in Parts B and C are worth 0 points.
6. You have 60 minutes of writing time.
7. All participants (grade 7 to 9) in the same school MUST write at the same time.
8. When done, carefully REMOVE and HAND IN this TOP page. You may keep the contest.
9. DO NOT discuss or post any answers on social media.

GOOD LUCK!

Part A: Multiple Choice: Each correct answer is worth 4 points. Each unanswered question is worth 2 points to a maximum of 3 unanswered questions.

1. A sequence of 12 whole numbers is formed by doubling the preceding number. If the tenth number in the sequence is 3072, then what is the median of the set of numbers?

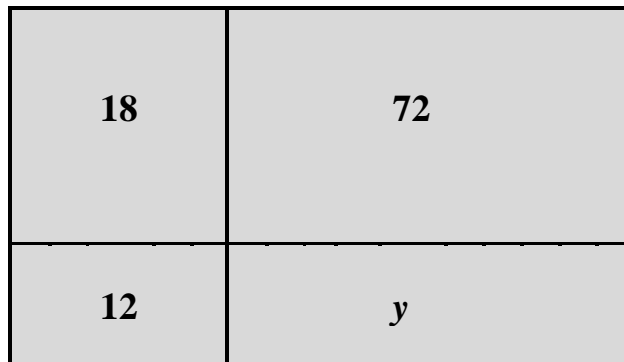   A. 192   B. 256   C. 288   D. 310   E. 144

    Answer: C (288)

    $10^{th} = 3072$; $9^{th} = 1536$; $8^{th} = 768$; $7^{th} = 384$; $6^{th}= 192$.

    Median = $(6^{th} + 7^{th}) / 2 = (192 + 384) / 2 = 288$

2. A *large rectangle* is divided into 4 sub-rectangles with areas of 12, 18, 72, and *y* square units, as shown below: NOTE: Not drawn to scale!
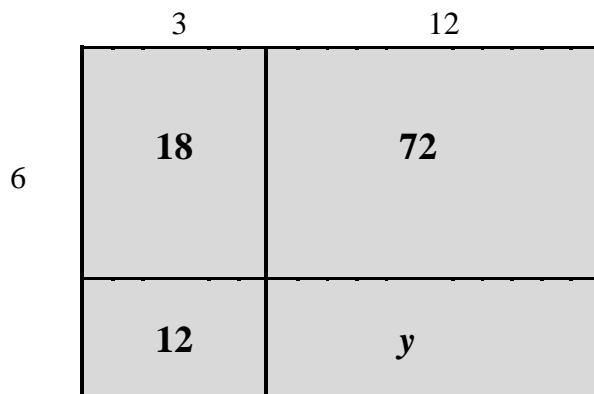
    What is the area, in square units, of the bottom-right *sub-rectangle*?

    A.   40   B. 48   C. 56   D. 62   E. 72

    Answer: B (48)
    Using common factors, the dimensions of the sub-rectangles are:

    $y = (4)(12) = 48$ sq. units

3. If n% of 6 kilometer is 150 meter, then what is 6% of n kilometer?

A. 0.15 km   B. 0.3 km   C. 3 km   D. 9 km   E. 15 km

Answer: A (0.15 km)

$$\frac{n}{100} = \frac{150}{6000} \qquad n = 2.5 \qquad \text{Answer} = (0.06)(2.5) = 0.15$$

4. If 6 machines can assemble 9 toys in 4 minutes, then without stopping, how many machines would be needed to assemble 8100 toys in 1 full day?
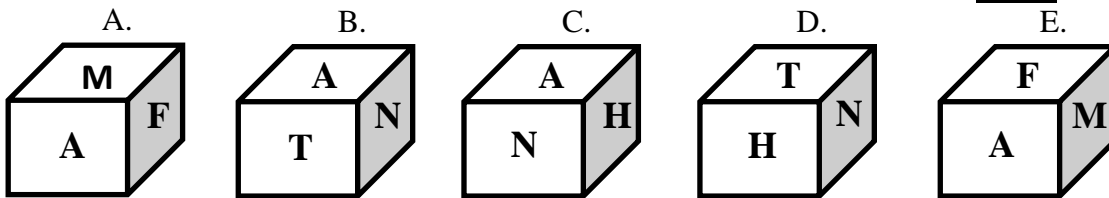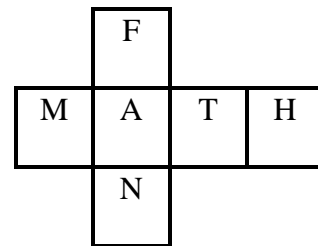
A. 6   B. 8   C. 12   D. 14   E. 15

Answer: E (15 machines)

Number of Machines	Number of Toys	Number of Minutes
6	9	4
	$(\times 15)$	$(\times 15)$
6	135	$60 = 1h$
	$(\times 24)$	$(\times 24)$
6	3240	$1440 = 1$ day
$(\times 2.5)$	$(\times 2.5)$	
Answer $= 15$	8100	$1440 = 1$ day

5. The net of a cube is shown at the right.

One of the figures below shows the correct cube. Which is it?

(Please ignore the direction of the letters.)

```
      +---+
      | F |
  +---+---+---+---+
  | M | A | T | H |
  +---+---+---+---+
      | N |
      +---+
```

A.   B.   C.   D.   E.

Answer: A

Take the top face and front face as correct. In A, the right face should show F, which it does. In B, the right face should show F. In C, the right face should show T. In D, the right face should show F. In E, the right face should show T. Since only A passes the right face test and one of the figures is correct, it must be A.

6. The altitude of a triangle is half the side of a square. If the area of the triangle is three times that of the area of the square, then the ratio of the base of the triangle to the side of the square is _____.

   A. 2:1   B. 4:1   C. 6:1   D. 8:1   E. 12:1

   Answer: E (12:1)

   Let Side length of square $= s$.   Therefore: Area $_{(square)} = s^2$

   Let Base length of triangle $= b$ Given: Altitude of triangle $= \frac{s}{2}$ Area $_{(triangle)} = b \left(\frac{s}{2}\right)\left(\frac{1}{2}\right) = \frac{bs}{4}$

   Since $\frac{bs}{4} = 3\, s^2$ It follows $b = 12\, s$ Therefore $b:s = 12:1$

7. If you skip-count backwards by 5s starting from 83 and not going below 0, then how many of those whole numbers will be multiples of 3?

   A. 4   B. 5   C. 6   D. 7   E. 8

   Answer: C (6 numbers)

   We can step back 16 times before going below 0, so that 17 numbers are counted. The first multiple of 3 counted is 78 and the last one is 3. Thus we are skip-counting backwards by 15s starting from 78 and not going below 0. We can step back 5 times, yielding 6 numbers.

Part B: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 5 points.

8. A pizza consists of the following:

   - 2 different toppings: fresh tomato, green peppers, ham, mushrooms, pepperoni, and pineapple.
   - 1 of 2 types of tomato sauce: mild and tangy.
   - 1 of 3 types of crust: thick, thin, and stuffed with cheese.

   How many different pizzas are there?

   Answer: 90

   The first topping can be chosen in 6 ways, and the second 5. Since the order is immaterial, there are 15 ways of choosing the toppings. Hence the number of different pizzas is the product of 15, 2 and 3, or 90.

9. Given:

- The lengths, in cm, of the sides of a right triangle are positive integers.
- One arm of the triangle is 9 cm shorter than the hypotenuse.
- The hypotenuse of the right triangle is equal to the radius of a circle.
- Rounded to the nearest whole number, the area of the circle is 2642 cm^2.
- The length and width of a rectangle are equal to the two arms of the right triangle.

What is the perimeter, in cm, of the rectangle?

Answer: 82 cm

When 2642 is divided by π, the answer is 841 to the nearest integer. Hence the length of the hypotenuse is 29 cm, and one arm has length 20 cm. From Pythagoras' Theorem, the other arm has length 21 cm, so that the perimeter of the rectangle is 2(20+21) = 82 cm.

10. A flower garden is in the shape of a regular polygon. Each of its interior angles measures 120º. The perimeter of the garden is 48 m. What is the area, in m^2, of the garden, rounded to nearest integer?

Answer: 166 m^2

The exterior angle of the regular polygon is 60º. Since the sum of all exterior angles is 360º, the polygon is a regular hexagon. Each side has length 8 m. The area, in m2, of an equilateral triangle of side length 8 m is the square root of 3 times ¼ of the square of 8. Six times is value is closest to 166.

11. The vertices of a quadrilateral located on a coordinate plane are: A(-6,4), B(2,10), C(8,2), D(0,-4).
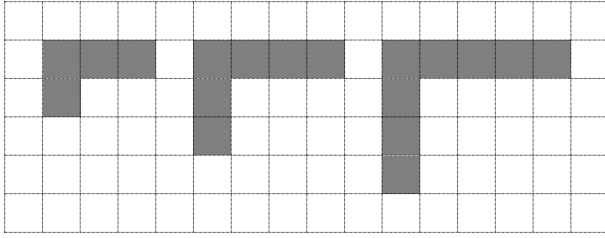
What is the ordered pair that represents the location of point P such that P is equidistant to the vertices of the quadrilateral?

Answer: (1,3)

The x-coordinate $= \dfrac{-6+8}{2}$ or $\dfrac{2+0}{2} = 1$

The y-coordinate $= \dfrac{4+2}{2}$ or $\dfrac{-4+10}{2} = 3$

12. If the pattern of shapes shown below in the diagram continues, what is the perimeter of the 28ᵗʰ shape, in units of length?

Answer: 118 units

The perimeter of the first shape is 10. In forming the next figure, an extra square is added to each arm of the preceding figure, thereby increasing the perimeter by 4. It follows that the perimeter of the 28th shape is 10 plus 4 times 27, or 118.

13. From the diagram below, what is the area of the shaded region, rounded to the nearest square unit?
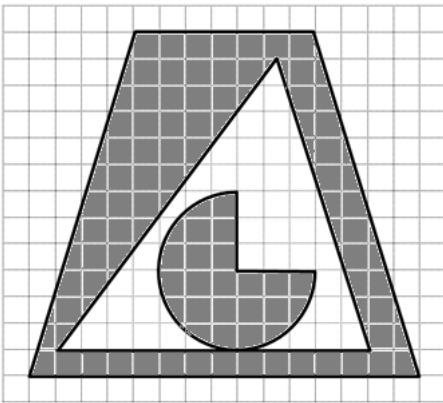
Answer: 98 square units

The outer trapezoid has height 13, top base 7 and bottom base 15. Hence its area is 143. The inner triangle has height 11 and base 12. Hence its area is 66. The difference is 77. To this we add the area of ¾ of a circle of radius 3. Rounding off to the nearest integer, we obtain 98.

Part C: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 7 points.

14. A 31-year old father has two children. All 3 people share the same birthday. The father's present age is equal to the sum of the children's ages in 8 years. What is the sum of the present ages of the 3 people?

Answer: 46

In 8 years, the sum of the children's age is 31. Hence the current sum is 15 and the desired total is 46.

15. Find a number consisting of three different digits such that if we write it after 523, we obtain a six-digit number which is divisible by each of 7, 8 and 9.

Answer: 152

523 000 / (7 × 8 × 9) ≈ 1037.69

Then: (1038) × (7 × 8 × 9) = 523 152

Therefore: The Three different digits are: 1, 5, and 2

Alternate solution:

The least common multiple of 7, 8 and 9 is 504 and 523-504=19. When 19000 is divided by 504, the remainder is 352. Since 504-352=152, we can add 152 after 523 to get a multiple of 504. Adding 504 to 152 yields 656, which has two identical digits. Adding 404 to 656 yields a four-digit number. Hence 152 is the unique answer.

16. With the engine on, a steam boat takes five days to go down the river from A to B, but seven days to go up the river from B to A. Find the number of days the steam boat will take to drift down the river from A to B with the engine off. The speed of the steam boat in still water and the speed of the current of the river are both constant.

Answer: 35 days

Let the speed of the boat be b and the speed of the current be c. Then $b + c = \frac{1}{5}$ and $b - c = \frac{1}{7}$.

Hence $2c = \frac{1}{5} - \frac{1}{7} = \frac{2}{35}$ so that $c = \frac{1}{35}$. The boat will take 35 days to drift.

17. How many values of n such that $1 \le n \le 2015$ and when we multiply n by (n + 2015) we get a power of 3?

Answer: 0

In order for this to happen, both n and (n + 2105) must be multiples of 3. Since 2015 is not a multiple of 3, the number of such n is 0.

18. Given:

a, b, c, d are four whole numbers,
a + b + c = 12,
a + b + d = 14,
a + c + d = 16,
b + c + d is equal to one of the previous sum 12, 14 or 16.

What are the values of these four numbers?

Answer: a = 6, b = 2, c = 4, d = 6

If we add up the four equations, the total must be a multiple of 3 since each of a, b, c and d appears 3 times. As 12+14+16 = 42 is a multiple of 3, so must b + c + d. This means the repeated sum is 12.

Now a = d = c + 2 and c = b + 2. Hence a = d = 6, b = 2 and c = 4.

19. Find the largest three-digit number which is equal to the sum of the factorials of its digits. The factorial of a positive integer is the product of all the numbers from 1 up to and including that positive integer. For example, 3! reads 3 factorial = 3 x 2 x 1 = 6.

Answer: 145

The factorial of a positive integer n is denoted by $n!$. Now 7! = 5040 is too large. Hence none of the digits can be 7, 8 or 9. Also 6! = 720, none of the digits can be 6 either.

Since 4! = 24, at least one of the digits must be 5. We cannot have three 5s since 555 6= 5!+5!+5!.

If we have two 5s, then the first digit must be 2, but 255 ≠ 2! + 5! + 5!.

With only one 5, the first digit must be 1, and 5!+1! = 121. The middle digit cannot be 5. Hence the last digit is 5, and the middle digit must be 4. Thus the only such number, and therefore the largest such number, is 145.