Print ID # ___________ School Name ________________________________

Student Name ____(Print First, Last) ___ 100

2014 Edmonton Junior High Math Contest – ANSWER KEY

Part A: Multiple Choice

1.	C
2.	B
3.	A
4.	A
5.	E

Part B (short answer)

6.	10
7.	68
8.	T
9.	116
10.	74
11.	139
12.	2:3
13.	10
14.	9

Part C (short answer)

15.	9079
16.	21
17.	816
18.	24
19.	80

PRINT NEATLY

Part A: _________ × 4 + _______ × 2 = ___________ Blank answers ≤ 3.
Correct blank

Part B: _________ × 5 = _________ MARKER ONLY
Correct

Part C: _________ × 7 = _________
Correct

Total: = _________

Instructions:

1. Calculator, grid paper and scrap paper are permitted. You may write on the booklet.
2. Programmable calculators and cell phones are not allowed.
3. Each correct answer in Part A is worth 4 points, each correct answer in Part B is worth 5 points, and each correct answer in Part C is worth 7 points. In Part A each blank is worth 2 points each up to a maximum of 3 blanks.
4. Each incorrect answer is worth 0 points.
5. Unanswered questions in Parts B and C are worth 0 points.
6. You have 60 minutes of writing time.
7. When done, carefully REMOVE and HAND IN this TOP page. You may keep the contest. GOOD LUCK!

Edmonton Junior High Math Contest 2014 SOLUTION

Part A: Multiple Choice: Each correct answer is worth 4 points. Each unanswered question is worth 2 points to a maximum of 3 unanswered questions.

1. Which of these numbers is greater than its reciprocal?

A. $-1.\overline{5}$ B. 0.995 $\boxed{\text{C.}\quad -99.9\%}$ D. $0.\overline{3}$ E. $\frac{2}{5}$

Solution:

A = $-\frac{14}{9} < \frac{-9}{14}$ B = $\frac{199}{200} < \frac{200}{199}$ C = $\frac{-999}{1000} > \frac{-1000}{999}$ D = $\frac{1}{3} < 3$ E = $\frac{2}{5} < \frac{5}{2}$

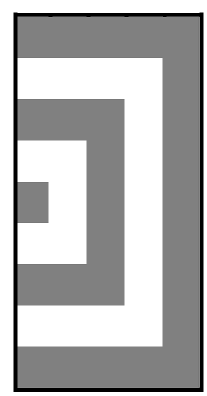
2. What number is doubled when $\frac{3}{4}$ of it is subtracted from 99?

A. 32 $\boxed{\text{B.}\quad 36}$ C. 40 D. 44 E. 52

Solution: Let n be the number.

$$2n = 99 - \frac{3}{4}(n)$$
$$\frac{11n}{4} = 99$$
$$n = 36$$

3. A target is made of dark and white strips of equal width as shown at the right.



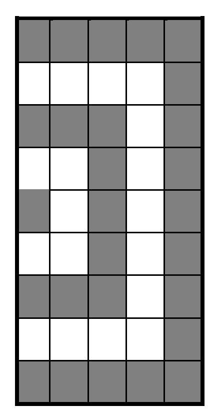
If a dart is thrown and lands randomly inside the target, what is the probability that it will land on white?

$\boxed{\text{A.}\quad \frac{2}{5}}$ D. $\frac{1}{2}$

B. $\frac{3}{8}$ E. $\frac{1}{3}$

C. $\frac{4}{9}$

Solution:



The shape can be divided into 45 individual squares. The white squares are $\frac{18}{45} = \frac{2}{5}$ of the entire target.

4. How many 2-digit whole numbers less than 40 are divisible by the product of its digits?

$\boxed{\text{A. } 5}$ B. 4 C. 3 D. 2 E. More than 5

Solution: There are exactly five of them: 11, 12, 15, 24 and 36.

5. A florist has 72 roses, 90 tulips and 60 daffodils, and uses all of them to make as many identical bouquets as possible. How many flowers does the florist put in each bouquet?

A. 6 B. 18 C. 24 D. 29 $\boxed{\text{E. } 37}$

Solution: The GCF(72, 90, 60) = 6. This gives 6 bouquets with 12 roses, 15 tulips and 10 daffodils; a total of 37 flowers in each of the bouquet.

Part B: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 5 points.

6. A rectangle has an area of 48 cm^2, and a perimeter of 28 cm. What is the length of the rectangle's diagonal, rounded to the nearest whole centimeter?

Solution: Knowing that L(W) = 48 and L+W = 14, we have L = 8 and W = 6. The diagonal is $\sqrt{8^2 + 6^2} = 10\ cm$

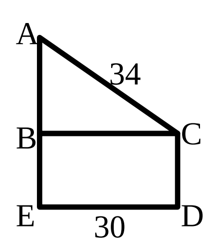
7. When a 2-digit number is multiplied by the sum of its digits, the product is 952. What is the 2-digit number?

Solution: The prime factorization of 952 is $2 \times 2 \times 2 \times 7 \times 17$. Two-digit divisors are 14, 17, 28, 34, 56 and 68. Checking all cases, we have 952=68(6+8).

8. Twenty-six people are seated in a circle, and are lettered alphabetically from A to Z. Beginning with Person A, and proceeding in a clockwise direction, each alternate person leaves the circle. What is the letter of the last person to leave?

Solution: After the first round, BDFHJLNPRTVXZ are left, and the next to go is B. After the second round, DHLPTX, and the next to go is H. After the third round, DLT are left and the next to go is L. After the fourth round, DT are left and the next to go is D. After the fifth round, only T is left.

9. In the rectangle BCDE, BC=30 cm. A is on the extension of EB, and AC=34 cm. The area of triangle ABC is 30 cm^2 less than half of the area of BCDE. What is the perimeter of the quadrilateral ACDE?



Solution: In the rectangle BCDE, BC=30 cm. A is on the extension of EB, and AC=34 cm. By Pythagoras' Theorem, AB=16 cm and the area of triangle ABC is 240 cm^2. Hence the area of BCDE is 2(240+30) or 540 cm^2, so that CD=18 cm. The perimeter of the trapezoid ACDE is 34+18+30+18+16=116 cm.

10. The age of a tortoise is 52 years more than the combined age of two elephants. In 10 years, the tortoise will be twice as old as the two elephants combined. How old is the tortoise now?

Solution: Suppose the tortoise is x years old and the two elephants together are y years old. Then $x - y = 52$. In ten years' time; $x + 10 = 2(y + 20)$. Hence $y + 52 = 2y + 30$ so that $y = 22$ and $x = 74$.

11. The angle bisectors of the two acute angles of obtuse triangle, ΔXYZ, intersect at Point W. The measure of $\angle Z$ is 98°. What is the measure, in degrees, of $\angle XWY$?

Solution: In degrees, $\angle X$ plus $\angle Y$ is 82, $\angle WXY + \angle WYX$ is 41 and $\angle XWY$ is 139.

12. Maria purchased a number of peaches and apples. The mean mass of the peaches is 170 g. The mean mass of the apples is 140 g. The mean mass of all the fruit is 152 g. What is the ratio of the number of peaches to apples purchased?

Solution: Let p = number of peaches and a = number of apples

This gives $\frac{170p+140a}{p+a} = 152$ or $18p = 12a$. Thus the ratio of p:a = 2:3

13. Two sides of a scalene acute triangle measure 12 cm and 13 cm. If the length of the third side is also an integer, then how many lengths are possible for the third side to be?

Solution:

Let ABC be the triangle where AC = 13cm and BC = 12cm.

When AB = 5cm, we have a right angle triangle at $\angle ABC$.

When $\angle ACB = 90°$, AB = 17.69.

We now have 5 < third side < 17.69 so that it is an acute triangle. This gives 12 possible lengths for the third side from 6 to 17 cm. In order to be scalene, we need to eliminate both 12 and 13 from the list; hence, there are a total of 10 possible lengths for the third side.

14. What is the largest n such that n^n is an n-digit number?

Solution:

A quick check would reveal that 10^{10} would give a total of 11 digits. In fact when n is greater than 10, the resulting power will always have more than n digits. Thus the greatest n = 9 giving $9^9 = 387420489$ (9 digits)

Part C: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 7 points.

15. Consider the 2014 digit number consists of 2013 nines followed by 1 one.

$$\overbrace{99\ldots99}^{2013}1$$

The smallest factor is 1 and the largest factor is the number itself. Let M be the second smallest factor and N be the second largest factor. What is the sum of the digits of M and N?

Solution:

Firstly, we know the number is not divisible by 3 as it divides into all the 9's except the last digit of 1.

As for 7, it will divide into six 9's evenly. The longest string of 9's would be 2010 digits. This leaves 9991 which 7 do not divide evenly.

Eleven divides evenly into a pair of 9's. However, eleven does not divide into the last two digits of 91.

The number 13 will go into six 9's evenly. Similar to 7, it does not divide evenly into 9991.

Seventeen divides evenly into a string of sixteen 9's. Leaving thirteen 9's followed by a 1, which 17 does not divide evenly.

Nineteen divides evenly into a string of eighteen 9's. Leaving fifteen 9's followed by a 1, which 19 also does not divide evenly.

Similarly twenty three divides evenly into a string of twenty two 9's. Leaving eleven 9's followed by a 1, which 23 does not divide evenly.

The next prime number to try is 29. Like the previous quotients, 29 divides evenly into a number made up of a string of twenty eight 9's. The quotient is 0344827586206896551724137931. The sum of its digits is 126. This also means there are 71 sets of this number giving a total of 1988 digits of 9's. We still have a number with 25 digits of 9's followed by a 1. A total of 26 digits left. Fortunately, 29 multiply by the quotient less the last two digits of "31" results in exactly 25 digits of 9's followed by a 1. This concludes that 29 divide evenly into the original number.

To recap, the second smallest factor is 29. The sum of the digits is 2 + 9 = 11.

The second largest factor is a number of the form

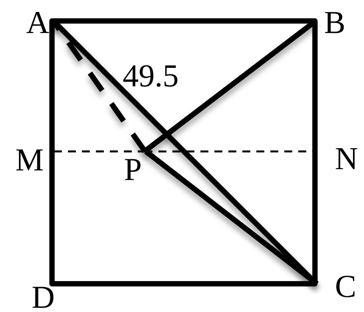
$$\overbrace{0344827586206896551724137931}^{71\ sets}03448275862068965517241379$$

The sum of the digits is 126 x 72 – 4 = 9068.

Therefore the total sum of the digits of M and N is 11 + 9068 = 9079.

16. *ABCD* is a square with *AC* = 49.5 cm. *P* is a point inside *ABCD* such that *PB* = *PC*, and the area of triangle *PCB* is one third of the area of *ABCD*. What is the length, in cm, of *PA*? Round your answer off to the nearest integer.

Solution:



Let L be the length of one side of the square.

Using Pythagorean property, we have

$2L^2 = 49.5^2$. This gives $L^2 = 1225.125$, L = 35cm

As well, knowing that 3 times the area of ΔPCB is equal to L^2, we have

$3\left(\frac{\overline{PN}\times 35}{2}\right) = 35^2$, or $\overline{PN} = 23\frac{1}{3}$ and $\overline{PM} = 11\frac{2}{3}$

It follows that $\overline{PA} = \sqrt{17.5^2 + 11\frac{2}{3}^2} = 21\ cm$

17. A three-digit number is equal to 17 times the product of its digits, and the hundreds digit is 1 more than the sum of the other two digits. Find all such three-digit numbers.

Solution:
Let a, b, c be the three digits not necessarily different. As well, we should only consider product that is less than 999 ÷ 17 = 59. Since we have the hundreds digit 1 more than the sum of the other two digits, we could use the following table to sort out the three digits.

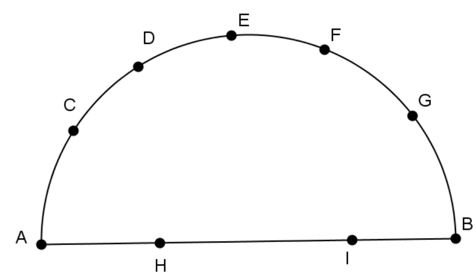
Original number	a	b	c	Product abc
	9	1	7	63
	9	2	6	108
	9	3	5	135
	9	4	4	144
17(48) = 816	8	1	6	48
	8	2	5	80
	8	3	4	96
17(35) = 595	7	1	5	35
17(56) = 952	7	2	4	56
	7	3	3	63
17(24) = 408	6	1	4	24
17(36) = 612	6	2	3	36
17(15) = 255	5	1	3	15
17(20) = 340	5	2	2	20
17(8) =136	4	1	2	8
	3	1	1	3

Therefore, only one such number exists and it is 816.
Alternate solution:
The number is divisible by any of its digits. Using its hundreds digit, the quotient is greater than 100 and less than 111. It is also a multiple of 17, so that it has to be 102. Now 102 = 17 x 6. So the last two digits are 1 and 6 or 2 and 3. It is easy to check that 861, 632 and 623 are not multiples of 17 but 816 is.

18. A magazine receives 32 articles, of length 1, 2, ..., 32 pages respectively. The first article starts on page 1 and all other articles starts on the page after the preceding article. The articles may be arranged in any order. What is the maximum number of articles that can start on an odd-numbered page?

Solution:
Put all 16 articles of even length first, so that they all start on odd-numbered pages. Of the other 16, half of them will start on odd-numbered pages, for a total of 24. This cannot be higher because an article of odd length changes the parity of the starting page number. This parity must change at least 15 times, so that at least 8 articles must start on even-numbered pages.

19. The diagram shows nine points as shown. How many triangles are there whose vertices are chosen from the nine points?



Solution:
There are six ways to choose two points from the straight line and each pair can form a triangle with each of the point on the curve, this gives 6 x 5 = 30 triangles.
There are ten ways to choose two points from the curve and each pair can form a triangle with each of the point on the straight line, this gives 10 x 4 = 40 triangles.
Lastly, all three vertices can be chosen from the curve alone. There are ten ways to do so. In total, there are 30 + 40 + 10 = 80 triangles.