Print ID # ____________

$\overline{\quad}$
100

School Name ______________________ Student Name ______________________________
(Print First, Last)

2013 Edmonton Junior High Math Contest Answers and Solutions

Part A: Multiple Choice

1. C
2. C
3. A
4. B
5. D

Part B (short answer) Part C(short answer)

Part B	Part C
6. $\frac{3}{2}$ **or 1.5**	**15. 4**
7. 7	**16. 10**
8. $\frac{1}{4}$	**17. 24**
9. 28	**18. 987 654 320**
10. 15	**19. 2 & 17**
11. 30%	
12. 64	
13. 34	
14. 28 & 29	

Part A: __________× 4 + ________ × 2 = __________ Blank answers ≤ 3.
Correct blank
Part B: __________× 5 = _________ MARKER ONLY
Correct
Part C: __________× 7 = __________
Correct
Total: = __________

Instructions:

1. Calculator, grid paper and scrap paper are permitted. You may write on the booklet.
2. Programmable calculators and cell phones are not allowed.
3. Each correct answer in Part A is worth 4 points, each correct answer in Part B is worth 5 points, and each correct answer in Part C is worth 7 points. In Part A each blank is worth 2 points each up to a maximum of 3 blanks.
4. Each incorrect answer is worth 0 points.

5. Unanswered questions in Parts B and C are worth 0 points.
6. You have 60 minutes of writing time.
7. When done, carefully REMOVE and HAND IN only page 1.

Edmonton Junior High Math Contest 2013

Place your answers on the answer sheet provided.

Part A: Multiple Choice: Each correct answer is worth 4 points. Each unanswered question is worth 2 points to a maximum of 3 unanswered questions.

1. If a stack of 5 dimes has a height of 6 mm, then what would be the value, in dollars, of a 1.5 m high stack of dimes?

A) \$1.25
B) \$12.50
C) \$125.00
D) \$125.50
E) \$1250.00

Five dimes have a height of 6 mm, therefore,
5d = 6
d = 1.2,
Therefore, 1 dime has a height of 1.2 mm.
1.5 m = 1500 mm
1500 ÷ 1.2 = 1250 dimes
1250 × 0.1 = 125
The value of 1250 dimes is \$125.00
The answer is C.

2. There are about 7.06 billion people in the world, and there are about 35 million people in Canada. What percent of the world population is in Canada?

A) 0.005 %
B) 0.05 %
C) 0.5%
D) 5.0 %
E) 5.5 %

35 000 000 ÷ 7 060 000 000 = 0.00495
0.00495 ≈ 0.5 %
The answer is C

3. A large soup pot is in the shape of a right circular cylinder, and it has no lid. When filled to the top, it can hold 9.42 L of soup. The height of the pot is 30 cm. Approximately how many square centimeters of metal are needed to make the pot? Round the answer to the nearest whole cm^2. (1 L = 1000 cm^3, use 3.14 for all your calculation)

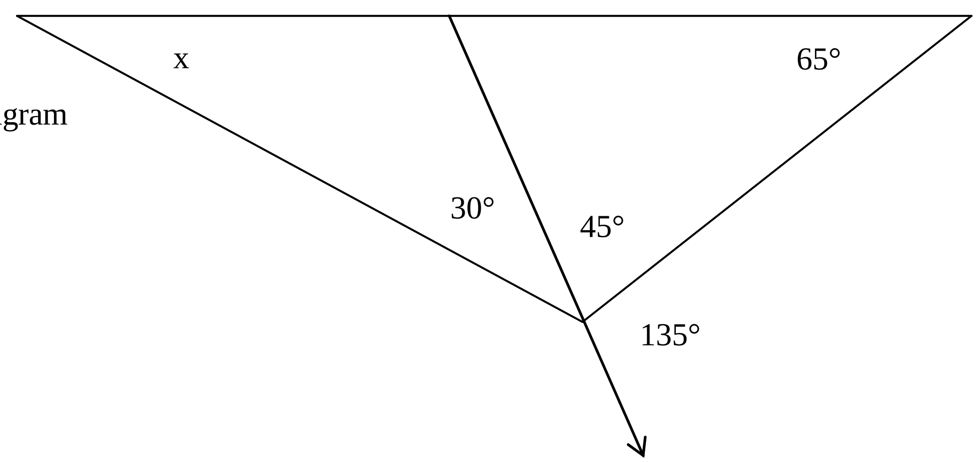
A) 2198
B) 2218
C) 2838
D) 3010
E) 3140

9.42 L = 9420 cm^3

$9420 = \pi r^2 h$
$9420 \div (30\pi) = r^2$
r = 10 cm

Find the surface area of the bottom and the lateral side.
$SA = \pi r^2 + 2\pi rh$
$SA = \pi(10)^2 + 2\pi(10)(30)$
$SA = 100\pi + 600\pi = 700\pi = 2198\ cm^2$. The answer is A.

4. Without a protractor, determine the number of degrees for *x*. Note: the diagram is NOT drawn to scale.



A) 30
B) 40
C) 45
D) 60
E) 65

The angle marked as 135° forms a supplementary pair with a 45° angle. The missing angle is 180 – (65 + 45 + 30) or 40°.
The answer is B.

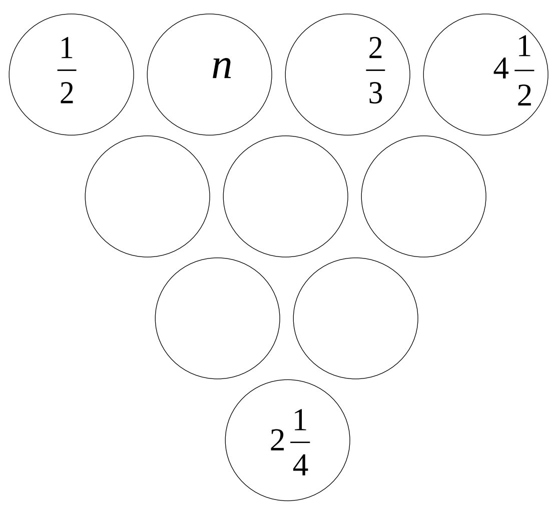
5. Robert wanted to buy Mandy a gold bracelet while it was on sale for $160 off the regular price. He planned to pay it off with 2 equal monthly payments of $340. Instead, it went on sale for only $75 off the regular price, and he paid for it with 5 equal monthly payments. How much was each of his monthly payments? (Assume that there is no interest nor GST.)

A) $89
B) $136
C) $151
D) $153
E) $168

The regular price of the bracelet is: 160 + 2(340) or $840.
The sale price with the $75 discount is $840 - $75 or $765.
765 ÷ 5 = 153. The monthly payment is $153.
The answer is D.

Part B: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 5 points.

6. The number in each circle is the product of the 2 numbers above it. What is the value of *n*?



Start with the first two numbers in the first row.

$$\frac{1}{2} \bullet n = \frac{n}{2}$$

The value of the left circle in the second row is $\frac{n}{2}$**.**

Next find the product of n and $\frac{2}{3}$**.**

n $\bullet$ $\frac{2}{3}$ = $\frac{2n}{3}$

The value of the middle circle in the second row is $\frac{2n}{3}$.

Find the product of the last two numbers in the first row.

$\frac{2}{3} \bullet \frac{9}{2}$ **= 3**

The value of the last circle in the second row is 3.

Find the product of $\frac{n}{2}$ and $\frac{2n}{3}$.

$\frac{n}{2} \bullet \frac{2n}{3} = \frac{n^2}{3}$

The value of the left circle in the third row is $\frac{n^2}{3}$.

Find the product of $\frac{2n}{3}$ and 3.

$\frac{2n}{3} \bullet$ **3 = 2n**

The value of the right circle in the third row is 2n.

Find the product of $\frac{n^2}{3}$ and 2n.

$\frac{n^2}{3} \bullet$ **2n =** $\frac{2n^3}{3}$

$\frac{2n^3}{3} = \frac{9}{4}$

Solve for n.

n = $\frac{3}{2}$

The answer is $\frac{3}{2}$ of 1.5.

7. The sum of 8 consecutive odd integers is –32. By how much does the median exceed the minimum number?

Let x = first odd #
x + 2 = second odd #
x + 4 = third odd #
x + 6 = fourth odd #
x + 8 = fifth odd #

$x + 10$ = sixth odd #
$x + 12$ = seventh odd #
$x + 14$ = eighth odd #
$8x + 56 = -32$
$x = -11$
The eight consecutive odd numbers are: –11, –9, –7, –5, –3, 1, 1, 3.
The median is $(-5 + -3) \div 2$ or –4.
The median exceed the minimum value of –11 by $-4 - (-11) = 7$.
The answer is 7.

Alternate solution:

Let the eight consecutive odd numbers be $n - 8$, $n - 6$, $n - 4$, $n - 2$, n, $n + 2$, $n + 4$, $n + 6$. The median is $(n + n - 2)/2 = n - 1$.

The difference is $(n - 1) - (n - 8) = 7$.

8. What fraction of the numbers from 1 to 100, inclusive, is prime? Express your answer in lowest terms.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

There are 25 prime numbers between 1 and 100: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

$$\frac{25}{100} = \frac{1}{4}$$

The answer is $\frac{1}{4}$.

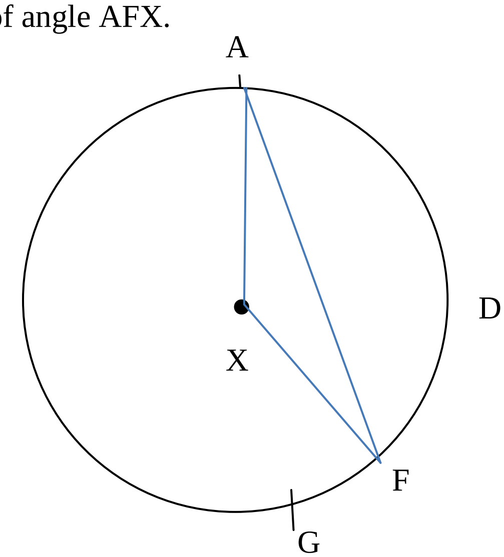
9. The three dimensions in centimetres (length, width and height) of a right rectangular prism are all natural numbers. The volume of the prism is 770 cm^3. What is the least possible sum that the three numbers can have?

The following table lists some of the possible dimensions and the sum of the dimensions:

Length	Width	Height	Sum
1	77	10	88
2	5	77	84
7	11	10	28
5	7	22	34
5	14	11	30
2	11	35	48
2	7	55	64

The least sum is 28. The answer is 28.

10. Twelve points are equally spaced on a circle with centre X. Points are labeled sequentially clockwise around the circle using the letters A to L. To the nearest degree, and without the use of a protractor, calculate the measure of angle AFX.



Since there are twelve points spaced equally on the circle, all twelve arcs are equal. Each arc has a central angle of 360° ÷ 12 or 30°. ∠ AXF subtends five of these arcs and has a measure of 30° x 5 or 150°. ΔAXF is an isosceles triangles, therefore ∠ AFX = (180 – 150)/2 = 15°.

11. Kylee has a set of 5 cards numbered from 1 to 5. Kassidy has a set of 10 cards numbered from 1 to 10. If they each pick one card from their deck at random, what is the probability that the product of the 2 chosen numbers is odd? Write your answer as a percent.

The sample space consists of 50 ordered pairs. Fifteen of these: (1,1), (1,3), (1,5), (1,7), (1,9), (3,1), (3,3), (3,5), (3,7), (3,9), (5,1), (5,3), (5,5),(5,7), (5,9) have an odd product. The probability is 15 ÷ 50 or 30%.

12. A 3-digit number has the following properties. The hundreds digit is a composite number, the tens digit is a prime number, and the units digit is greater than 2 but less than or equal to 6. How many such 3-digit numbers are there in total?

The hundred's digit could be: 4, 6, 8, 9.
The ten's digit could be: 2, 3, 5, 7.
The one's digit could be: 3, 4, 5, 6.
There are 4 × 4 × 4 or 64 possible 3-digit numbers.
The answer is 64.

13. Svitlana takes $1\frac{1}{2}$ h to cycle to her friend's house if she averages 340 m/min. How many minutes should it take her to make the same trip if she travels at an average speed of 54 km/h in her car? Express the answer rounded to the nearest whole number of minutes.

Find the distance traveled.
d = rt
d = (340 m/min)(90 min)
d = 30 600 m or 30.6 km

54km/hr = 0.9 km/ min

Find the time for the rate of 54 km/hr

t = $\frac{d}{r}$

t = $\frac{30.6km}{0.9km/\min}$

t = 34 min
The answer is 34.

Alternate solution:

Find the distance traveled.
d = rt
d = (340 m/min)(90 min)
d = 30 600 m or 30.6 km

Find the time for the rate of 54 km/hr

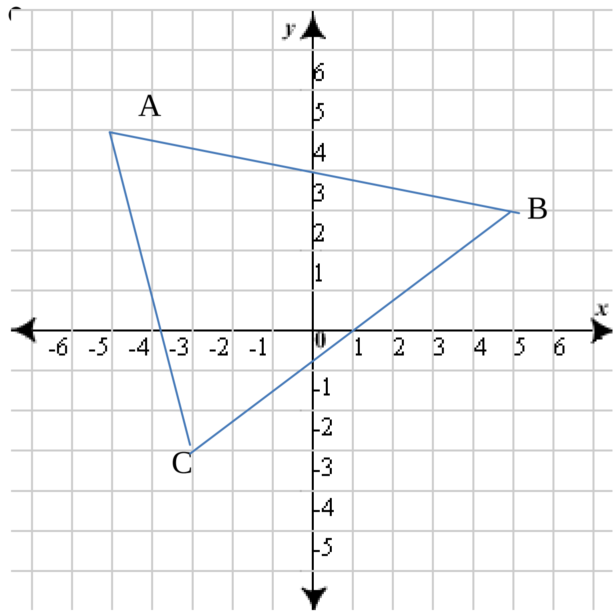
t = $\frac{d}{r}$

t = $\frac{30.6}{54}$

t = 0.56…. hour
t = 34 min
The answer is 34.

14. Points A(-5, 5), B(5,3) and C(-3,-3) are vertices of a triangle. The perimeter of ΔABC is between which two whole numbers?

The distance between AC is $\sqrt{8^2 + 2^2} \approx 8.246$
The distance between AB is $\sqrt{10^2 + 2^2} \approx 10.198$
The distance between BC is $\sqrt{6^2 + 8^2} = 10$
Perimeter ≈ 8.246 + 10.198 + 10 ≈ 28.444, which is between 28 and 29.



Part C: Short Answer: Place the answer in the blank provided on the answer sheet. Each correct answer is worth 7 points.

15. The digits: A, B, C, D, E, F, G, H, and I, not necessarily all different digits, are arranged in a 3 by 3 configuration. The first two rows, ABC and DEF, are three-digit prime numbers. The third row GHI and the first column ADG are three-digit cubes. The last two columns BEH and CFI are three-digit squares. What is the value of digit E?

This is the configuration:
A B C
D E F
G H I

There are five 3-digit cubes: 125, 216, 343, 512, 729.
ADG and GHI are cubes. If ADG is 125 then GHI is 512. If ADG is 512, then GHI is 216. No other combinations will work.

A 3-digit square number cannot end in 2, so eliminate ADG: 125 and GHI: 512. Therefore, the value for ADG is 512 and the value for GHI is 216.
5 B C
1 E F
2 1 6

There are four 3-digit square numbers that end in 1: 121, 361, 441, 841. There are four 3-digit square numbers that end in 6: 196, 256, 576, 676.

If 5BC is a prime number, it cannot end in 2, 5 or 6, this gives CFI = 196

5 B 1
1 E 9
2 1 6

If BE1 = 121, then 1E9 = 129. However, 129 is not prime.
If BE1 = 361, then 5B1 = 531. However, 531 is not prime either.
That leaves 441 or 841 for BE1. Both gives E = 4.

Therefore the value of digit E is 4.

Also note 841 won't work as this would give 5B1 = 581 which is not prime either.

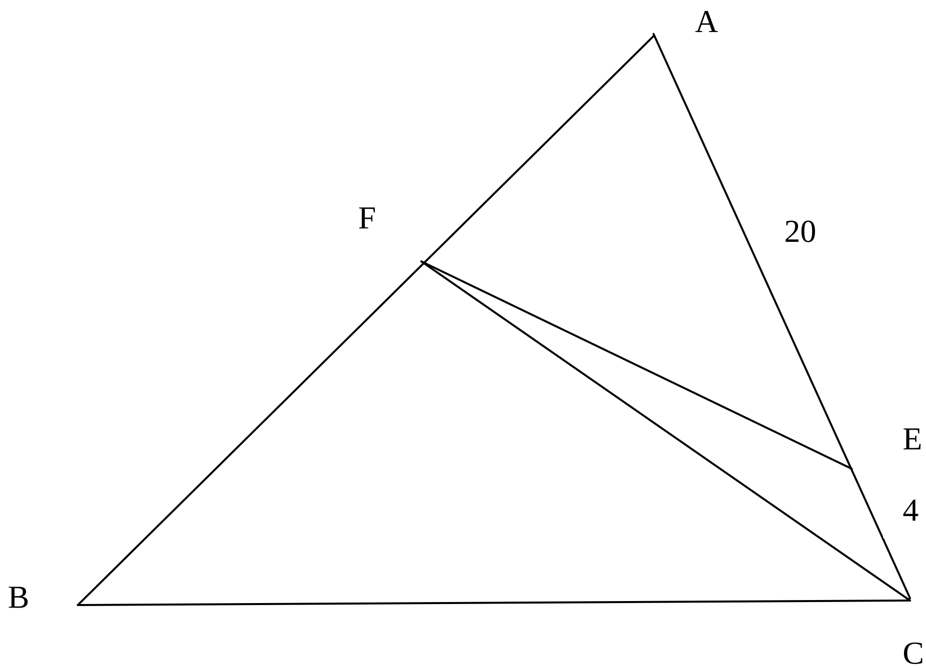
16. In triangle ABC, AB = 25 and CA = 24. E is a point on CA and F is a point on AB such that EF cuts ABC into two regions of equal areas. If CE = 4, what is the length of BF?

Connect CF. Let *x* represent the area of triangle CEF. Using CE and EA as the bases, the two triangles CEF and AFE have the same height. Since the area of triangle CEF = *x*, then area of triangle AFE = 5*x*. The area of triangle BFC = 5x - x = 4x. The area of triangle ACF = *x* + 5*x* = 6*x*.

Let m = length of BF and (25 – m) = length of AF.
Using FB and FA as the bases, the two triangles FCB and FAC have the same height, H.

Area of ΔFCB = $\frac{m(H)}{2} = 4x$
Area of ΔFAC = $\frac{(25-m)(H)}{2} = 6x$

Rewriting both equations in terms of H, we have $\frac{8x}{m} = \frac{12x}{25-m}$**. Solve for m, we have m = 10.**
The answer is 10.



17. How many numbers between 100 and 1,000,000 have all digits the same and are divisible by 3?

The number can have either 3, 4, 5, or 6 digits.
If the number has 3 digits, it has the form *aaa*, with $1 \leq a \leq 9$. The sum of the digits is 3*a* which is always divisible by 3. There are 9 three digit numbers that satisfy this condition.

If the number has 4 digits, it has the form *aaaa*, with $1 \leq a \leq 9$. The sum of the digits is 4*a* which is divisible by 3 only when *a* is 3, 6, or 9. There are 3 four digit numbers that satisfy this condition.

If the number has 5 digits, it has the form *aaaaa*, $1 \leq a \leq 9$. The sum of the digits is 5*a* which is divisible by 3 only when *a* is 3, 6, or 9. There are 3 five digit numbers that satisfy this condition.

If the number has 6 digits, it has the form *aaaaaa*, with $1 \leq a \leq 9$. The sum of the digits is 6*a* which is always divisible by 3. There are 9 six digit numbers that satisfy this condition.
In total there are 9 + 3 + 3 + 9 = 24 such numbers.
The answer is 24.

18. What is the largest number whose digits are all different and the number is NOT divisible by 9?

Since the number has distinct digits, it has at most 10 digits. If the number has 10 digits, then its digits must be exactly 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in some order. But then

the sum of the digits is 45, and the number is divisible by 9. Thus the number cannot have 10 digits.

If the number has 9 digits, then one of the 10 digits must be missing. The sum of the digits then is 45 – (the missing digit). In order for this number not to be divisible by 9, the missing digit can be anything except 0 or 9.

Since we are looking for the largest, 9-digit number, the missing digit must be as small as possible, therefore, it must be 1.

This shows that our number has exactly the digits: 0, 2, 3, 4, 5, 6, 7, 8, 9. Since the largest number is wanted, the digits must be decreasing. Therefore, the number is 987 654 320.
The answer is 987 654 320.

19. There exits two prime numbers: p and q, such that $2p + 3q = 99$. The sum of p and q is also the product of 2 other prime numbers: m and n. Find m and n.

$2p + 3q = 99$
$2p = 99 - 3q$
$2p = 3(33 - q)$
p must be divisible by 3. Since p is prime, $p = 3$
Substitute $p = 3$ into the equation.
$2(3) + 3q = 99$
Solving for q, $q = 31$
The sum of p and q is 34.
34 can be factored only 2 ways: $1 \times 34 = 34$ and $2 \times 17 = 34$.
The numbers 1 and 34 are not prime, but the numbers 2 and 17 are prime.
Therefore m and n have the values of 2 and 17.
The answers are 2 and 17.