Print ID #: _____

School Name:_____   Student Name:_____

(Print: First Name &   Last Name)

2011 Edmonton Junior High Math Contest

Multiple Choice
(PRINT neatly using CAPITAL letters)

Numeric Response

Part A: (4 pts each)	Part B: (6 pts each)	Part C: (8 pts each)

Part A
1. E
2. B
3. E
4. C
5. B

Part B
6. D
7. B
8. D
9. D
10. B

Part C
11.   1
12.   1005
13.   15
14.   28
15.   77 777 779 779

Instructions:

1. Grid paper, scrap paper, and non-programmable calculators **ARE** permitted. You may write on the booklet.

2. Programmable calculators, cell phones, and wireless devices **ARE** **NOT** allowed.

3. To avoid others from seeing your answers, **DON'T** print your answers **TOO** **LARGE**, and be sure your answers are **HIDDEN FROM VIEW** at all times.

4. Each **CORRECT ANSWER** in:
   - Part A is worth 4 points,
   - Part B is worth 6 points,
   - Part C is worth 8 points.

5. Each **BLANK** in:
   - Part A is worth 1 point,
   - Part B is worth 2 points,
   - Part C is worth 0 points.

6. Each **INCORRECT ANSWER** is worth 0 points.

7. You have 60 minutes of writing time.

8. When done, carefully **REMOVE** and **HAND IN** only this **COVER PAGE**.

MARKER USE ONLY

Part A: _____ $\times\, 4 +$ _____ $\times\, 1\ =$ _____
       (# Correct)   *(# Blank)*

Part B: _____ $\times\, 6 +$ _____ $\times\, 2\ =$ _____
       (# Correct)   *(# Blank)*

Part C: _____ $\times\, 8$ $=$ _____
       (# Correct)

                                 Total: = _____

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Part A: Multiple Choice. Place the letter that corresponds to the correct answer on the blank provided. Each correct answer is worth 4 points. Each unanswered question is worth 2 points, up to a maximum of 3 blanks in parts A and B combined.

1. The square root of half Mitchell's age in years is half the sum of the first 3 prime numbers. What is Mitchell's age in years?

    A) 9

    B) 10

    C) 18

    D) 25

    E) 50 ←

Let a = Mitchell's age

$$\sqrt{\frac{a}{2}} = \frac{2+3+5}{2}$$

$$\sqrt{\frac{a}{2}} = 5$$

$$\frac{a}{2} = 25$$

a = 50

Mitchell is 50 years old.

2. A bag contains red, yellow, and green gumdrops. Of the total, $\frac{1}{4}$ are red, $\frac{1}{3}$ are yellow, and the remaining 70 gumdrops are green. How many gumdrops are in the bag?

    A) 120

    B) 168←

    C) 192

    D) 204

    E) 210

Let x = # of gumdrops

$$\frac{1}{4}x + \frac{1}{3}x + 70 = x$$

$$70 = \frac{5}{12}x$$

$$168 = x$$

The bag contained 168 gumdrops.

3. Kendra has a basket containing 4 types of fruit. She has 3 times as many bananas as apples. There are 4 more pears than bananas, and 2 less lemons than apples. What is the least number of pieces of fruit that could be in Kendra's basket?

      A) 10

      B) 14

      C) 18

      D) 20

      E) 26 ←

x = # of apples

$3x$ = # of bananas

$3x + 4$ = # of pears

$x - 2$ = # of lemons

$x + 3x + 3x + 4 + x - 2 =$ _____

Since there is at least one of each type of fruit, in order for there to be one lemon, x must be 3. By substitution, this will yield 26 fruits.Therefore, the answer E) 26, x= 3, which yields a positive integer for each type of fruit, is the answer.

4. In the figure shown, at the right, the radius of each circle is 3 cm. The centres of the circles represent the vertices of a square. What is the area of the closed shaded region, to the nearest square centimetre?

A) 4

B) 6

C) 8 ←

D) 16

E) 18
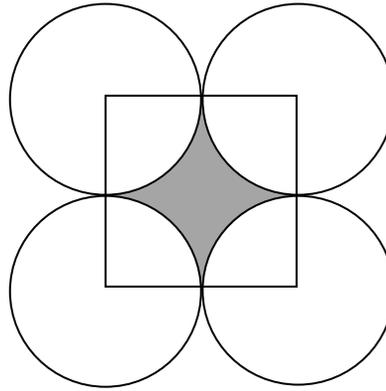
The area of the square − 4(one-fourth of a circle) = the shaded region.
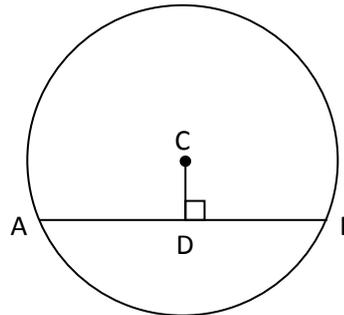
$$s^2 - \pi r^2 =$$

$$6^2 - \pi 3^2 =$$

7.72

To the nearest square centimetre the answers is 8.

5. In the figure shown at the right, the length of segment AB = 16 cm and the length of segment CD = 6 cm. What is the radius of Circle C, to the nearest centimetre?

A) 8

B) 10 ←

C) 14

D) 17

E) 20

Draw in the radius from A to C. A right triangle is formed. Use the Pythagorean property to find the radius, r.

$$r = \sqrt{6^2 + 8^2}$$

$$r = \sqrt{100}$$

r = 10

The radius is 10 cm.

Part B: Multiple Choice. Place the letter that corresponds to the correct answer on the blank provided. Each correct answer is worth 6 points. Each unanswered question is worth 2 points, up to a maximum of 3 blanks in parts A and B combined.

6. What is the sum of the first 63 terms of the following sequence?

1, -2, 3, -4, 5, 1, -2, 3, -4, 5, 1, -2, 3, -4, 5, 1, -2, 3, -4, 5, ….

    A) 34

    B) 36

    C) 37

    D) 38 ←

    E) 40

The pattern 1, -2, 3, -4, 5 occurs 12 times in the first 63 terms.

$1 + (-2) + 3 + (-4) + 5 = 3$.

The 61st term is 1, the 62nd term is -2, and the 63rd term is 3.

$(12)(3) + 1 + (-2) + 3 = 38$.

The sum is 38.

7. The first 13 terms of a number pattern are shown below, What is the 15th term?

1, 1, 2, 2, 4, 6, 3, 9, 12, 4, 16, 20, 5, …

    A) 25

    B) 30 ←

    C) 35

    D) 36

    E) 38

The following table shows a pattern to the sequence of numbers:

Starting number	Square the starting number	Starting with 1, add the next consecutive integer to the previous number.	Divide the previous number by the same number that was added in

			the previous step
1	$1^2 = 1$	$1 + 1 = 2$	$2 \div 1 = 2$
2	$2^2 = 4$	$4 + 2 = 6$	$6 \div 2 = 3$
3	$3^2 = 9$	$9 + 3 = 12$	$12 \div 3 = 4$
4	$4^2 = 16$	$16 + 4 = 20$	$20 \div 4 = 5$
5	$5^2 = 25$	**$25 + 5 = 30$** **(This is the 15$^{\text{th}}$ term.)**	$30 \div 5 = 6$

Or

There are three separate patterns here.

1 1 2 2 4 6 3 9 12 4 16 20 5

Red is 1, 2, 3, 4, 5 consecutive numbers

Green is 1, 4, 9, 16 square numbers

Blue is 2, 6, 12, 20 … we can say it's +4, +6, +8, + 10 in between terms. This would give $20 + 10 = 30$. Or a more elegant solution would be $n(n+1) = 5(6)$ where n is the term number

8. A large cube with an edge of 8 cm is made from the least possible number of centicubes. Although the object looks solid, it is hollow inside. How many centicubes are needed to make the object?

     A) 96

     B) 169

     C) 216

     D) 296 ←

     E) 384

If the large cube was solid it would be made up of 8^3 centicubes. The hollow portion inside has a volume of 6^3 cm^3. So, the numbers of cubes needed is $8^3 - 6^3 = 296$.
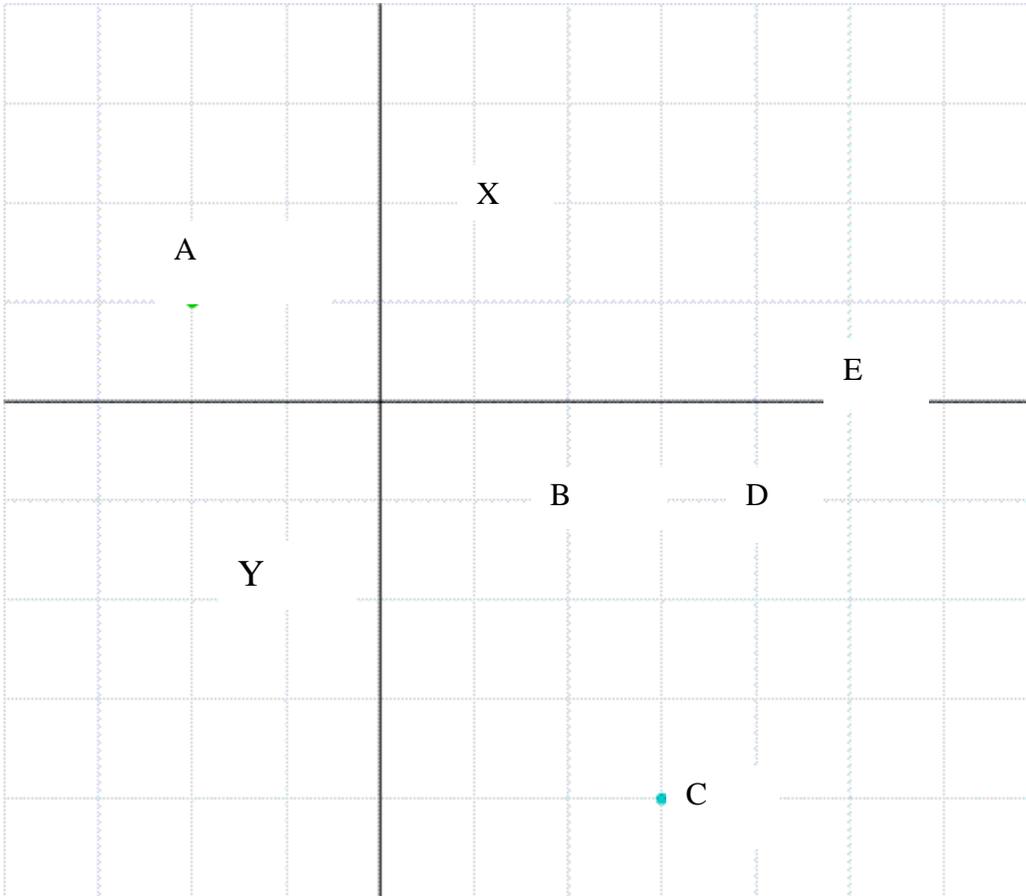
9. If (1, 2) and (-1, -2) are two vertices of a square, which of the following points could not be another vertex of the square?

     A) (-2, 1)

     B) (2, -1)

     C) (3, -4)

D) (4, -1) ←

E) (5, 0)

Label point (1, 2) with X. Label point (-1, -2) with Y. Label the points in the answer with their corresponding letter, A, B, and so on. Plot and label the points.

Use the origin as the turn center, rotate point X 90 ° three times, and the images of X will be at points B, Y, and A. Use the point (2, -1) as the turn center, rotate point X 90° three times, and the images of X will be at point E, C, and Y. Therefore, only point D (4, -1) could not be another vertex of a square.

10. Let ABCD be a quadrilateral with AB parallel to CD and CD = 2AB. Let E be a point on CD so that AE is parallel to BC.
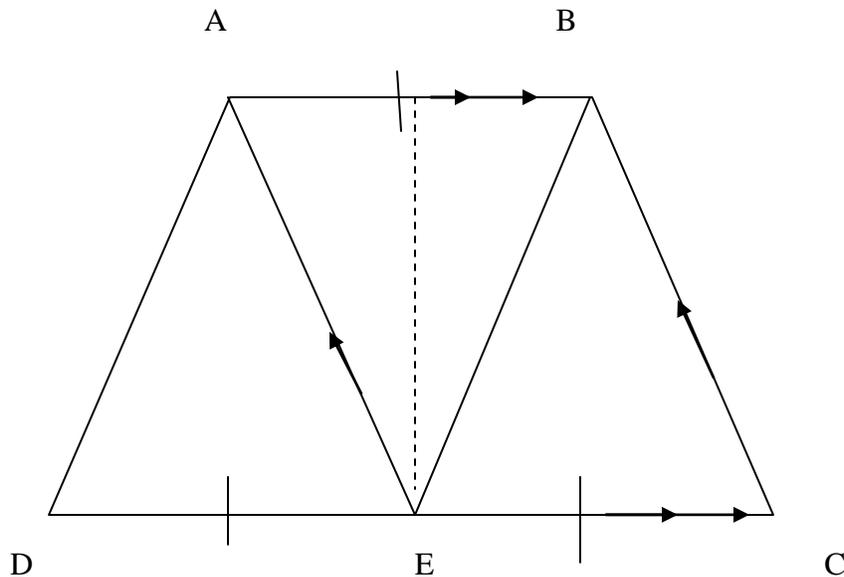
Find the ratio of the areas of ADE to ABCD.

A) 1 : 4

B) 1 : 3 ←

C) 1: 2

D) 2: 3

E) 3: 4

Since AB is parallel to CD and AE is parallel to BC, quadrilateral ABCE is a parallelogram. Therefore, side AB is equal to segment EC. Since CD = 2AB, CE = ED. Since AB is parallel to CD, all three triangles have the same height. Therefore, the three triangles are congruent and are equal in area. The ratio of the areas of triangle ADE to quadrilateral ABCD is 1 : 3.

Part C: Numeric Response. Place the correct answer on the blank provided. Each correct answer is worth 8 points. Each unanswered question is worth 0 points.

11. Find all natural numbers $n \geq 1$ for which $n(n-1)(n+1) + 3$ is prime.

n, n-1, and n+1 represent 3 consecutive integers. Therefore, $n(n - 1)(n + 1)$ is a multiple of 3. The sum of a multiple of 3 and 3 cannot be prime. Therefore, $n(n - 1)(n + 1)$ must be equal to zero. By the zero property, three cases occur: $n = 0$, or $n - 1 = 0$, or $n + 1 = 0$. The first and third cases yield numbers that do not satisfy the condition that $n \geq 1$, therefore, the case, $n - 1 = 0$, is the only one that works. There is only one answer, and that is **n = 1.**

12. The numbers between 1 and 2011 are written on a piece of paper. Logan circles the even numbers with red circles and Miranda circles the multiples of 5 with blue circles. How many numbers are circled with only one color?

Logan circle 1005 numbers, Miranda circled 402 numbers. The only numbers which are circled by two colors are the multiples of 10; thus there are 201 numbers circled both with red and blue colors. There are 1005 – 201 = 804 numbers circled only with red, and 402 – 201 = 201 numbers circled only with blue. In total there are 804 + 201 = **1005 numbers circled with only one color.**

13. The table below shows the integers from 1 to 25 in a 5 by 5 array. Choose five of the numbers, with one in each row and one in each column, such that the smallest of the five chosen numbers is a large as possible. What is the largest possible value for this number?

11	17	25	19	16
24	10	13	15	3
12	5	14	2	18
23	4	1	8	22
6	20	7	21	9

At the most four of the five numbers chosen can be from those on the border of the array. That means that at least one number among the nine at the centre can be chosen. The largest number there is 15. Therefore, the smallest of the chosen numbers cannot be greater than 15. By choosing 15, 25, 18, 23, and 20, there are five numbers with no two in the same row or column, and **the smallest of them is 15.**

14. In the diagram below, triangle ABC has a right angle at A and AB = AC. A circle passing through A cuts AC at E, AB at F and CD at D, with AE = AF. If the measure of angle CAD is 17°, what is the measure of angle ACF?
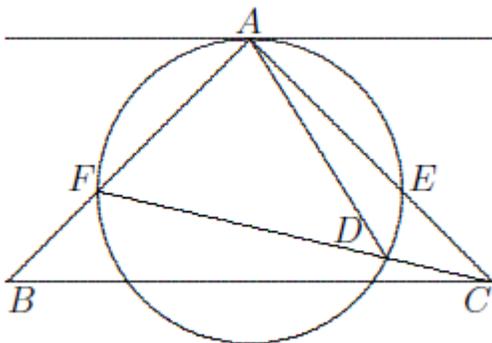
Angle A is a common angle to both triangle AFE and ABC. Angle AFE = angle ABC = angle AEF = angle ACB = 45°. Thus, triangle AFE is similar to triangle ABC.

17° = Angle CAD = angle EAD = angle EFD as the two angles are subtended by the same arc DE.

FE is parallel to BC. Angle EFD = angle EFC = angle FCB because they are alternate interior angles.

Angle ACF = angle ACB - angle FCB = 45° – 17° = 28°.

The measure of angle ACF is 28°.

15. What is the smallest positive integer which is divisible by both 7 and 9, each digit is 7 or 9, and there is at least one 7 and at least one 9?

To be divisible by 9, the sum of the digits must also be divisible by 9. Since 7 and 9 have no common factors, the number of copies of 7 must be a multiple of 9 as well. Since there is at least one 7, there are at least nine of them. As for divisibility by 7, all the digits 7 may be replaced by 0 and digits 9 by 2, so we are looking for the smallest positive multiple of 7 whose digits are 0 and 2. This is 2002. Putting everything together, **the number we want is 77777779779.**