2010 Edmonton Junior High Math Contest

Part I: Multiple Choice (PRINT neatly, use CAPITAL letters, 4 points each)

Part II: Numeric Response(PRINT small but legibly, 6 points each)

1. C	2. A	11. 60°	12. 11
3. B	4. B	13. 20	14. 168
5. E	6. E	15. 0.037	16. 3540
7. D	8. B	17. 10.9	18. 0.60
9. C	10. A	19. 23	20. 11.4

Part I: _____ x 4 + _____ x 2 = _____   (be sure blanks ≤ 3)

       Correct   Blank

MARKER ONLY

Part II: _____ x 6 = _____

       Correct

Total = _____   (enter the total score above)

Instructions:

1. Calculator, grid paper and scrap paper are permitted. You may write on the booklet.
2. Programmable calculators and cell phones are not allowed.
3. Don't write your answers too LARGE to avoid others seeing your answers. COVER your answers at all times.
4. All fractions must be proper and reduced to lowest terms.
5. Each correct answer is worth 4 points for Multiple Choice and 6 points for Numeric Response.
6. Each incorrect answer is worth 0 points.
7. Each unanswered question in Part I is worth 2 points up to a maximum of 6 points.
8. Unanswered questions in Part II are worth 0 points.
9. You have 90 minutes of writing time.
10. When done, carefully REMOVE and HAND IN only page 1.

Multiple Choice Questions

1. The side lengths of a rectangle are whole numbers and the perimeter is 202 cm. What is the largest area possible?

    A) 1000 cm^2

    B) 2050 cm^2

    C) 2550 cm^2

    D) 3050 cm^2

    E) none of these

Solution: C) 2550 cm^2

The largest area is obtained when the rectangle's shape is closest to a square. Since $202 \div 4 = 50.5$, and the sides must be whole numbers, the shape closest to a square would have dimensions of 50 and 51. The area is $50(51) = 2550$. Thus, the answer is C) 2550 cm^2.

2. x is a positive number with the property that $x^2 + \dfrac{1}{x^2} = 23$. The value of $x + \dfrac{1}{x}$ is

    A) 5

    B) 7

    C) $\sqrt{23}$

    D) $\sqrt{46}$

    E) none of these

Solution: A) 5

The square of $x + \dfrac{1}{x}$ is

$$(x + \frac{1}{x})^2 = x^2 + 2x(\frac{1}{x}) + \frac{1}{x^2}$$

$$= x^2 + \frac{1}{x^2} + 2$$

Replace $x^2 + \dfrac{1}{x^2}$ with 23

$= 23 + 2$

$= 25$

Since, $(x + \dfrac{1}{x})^2 = 25$, the square root of $(x + \dfrac{1}{x})^2$ is $x + \dfrac{1}{x}$ and $x + \dfrac{1}{x} = \sqrt{25}$ and $\sqrt{25} = 5$. This is answer A) .

3. How many whole numbers, *n,* can be found so that $\dfrac{2}{7} < \dfrac{n}{11} < \dfrac{2}{3}$?

    A) 3

    B) 4

    C) 5

    D) 6

    E) none of these

Solution is B) 4

Multiply $\dfrac{2}{7} < \dfrac{n}{11} < \dfrac{2}{3}$ by 11.

$11 \, (\dfrac{2}{7} < \dfrac{n}{11} < \dfrac{2}{3})$ which is equivalent to

$\dfrac{22}{7} < n < \dfrac{22}{3}$ which is equivalent to

$3.14 < n < 7.33$

Therefore, there are four whole numbers: 4, 5, 6, and 7. Which is answer B)

4. If p, $p + 10$, and $p + 14$ are all prime numbers, how many possible values are there of *p*?

    A) 0

    B) 1

    C) 2

    D) 3

E) infinitely many

5. In how many ways can two A's, two B's and two C's be arranged in a row so that no two adjacent letters are the same?

A) 120

B) 90

C) 60

D) 45

E) 30

6. Given: $3^x = 9^{y-2}$ and $2^{x-3} = 8^y$, find the value of xy.

A) -126

B) -60

C) 0

D) 60

E) 126

Given: $3^x = 9^{y-2}$, rewrite 9^{y-2} as base 3

$3^x = 3^{2(y-2)}$

Therefore, $x = 2y - 4$

Given: $2^{x-3} = 8^y$, rewrite 8^y as base 2

$2^{x-3} = 2^{(3)y}$

Therefore, $x - 3 = 3y$,

Since, $x = 2y - 4$, replace x with $2y - 4$ in the equation $x - 3 = 3y$

$x - 3 = 3y$

$2y - 4 - 3 = 3y$

Solve for y:

$y = -7$

Substitute $y = -7$ into the equation: $x = 2y - 4$

$x = 2y - 4$

$x = 2(-7) - 4$

$x = -18$

Therefore, $xy = (-18)(-7)$

$xy = 126$ which is answer E)

7. If *x* represent the units digit in the product of the first 100 prime numbers, and *y* represents the sum of the units digits of the first 10 multiples of 3, find $x + y$.

    A) 5

    B) 15

    C) 18

    D) 45

    E) 48

The solution is D) 45.

In the list of the first 100 prime numbers, 2 and 5 appear. Therefore the unit's digit of the product will be a 0. The first 10 multiples of 3 are: 3, 6, 9, 12, 15, 18, 21, 24, 27, and 30. The sum of these numbers is 45. So, x = 0 and y = 45. x + y = 0 + 45

Therefore, the sum is 45, which is answer D).

8. Twenty-four teens, washed one-third of the cars lined up for a charity car wash in 4 h. Then more teens joined the car-washing team, and the remaining cars were washed in another 6 hours. How many teens joined the team?

    A) 4

    B) 8

    C) 16

    D) 32

    E) 64

Solution: The answer is B) 8.

Let C represent the number of cars lined up to be washed.

Use the formula: Work = Rate × Time

$\dfrac{1}{3}C = 24(4)$ where 24 (the number of workers) represents the Rate and 4 represents the Time

Solve for C.

C = 288.

Therefore, there were 288 cars lined up to be washed.

Let N represent the number of workers joining after 4 hours.

Two-thirds of the cars remain to be washed by 24 + N workers in 6 hours. Substitute into the work formula.

W = R × T

$\dfrac{2}{3}C = (24 + N)6$

Replace C with 288 and solve for N.

$\dfrac{2}{3}(288) = (24 + N)6$

$192 = 144 + 6n$

$48 = 8N$

$8 = N$

$N = 8$, which is answer B).

9. Kristoff earned $10.00 each day for a week shoveling walks. On the eighth day he earned $1.75 more than his average earnings for all 8 days. How much did Kristoff make on the eighth day

    A) $11.00

    B) $11.75

    C) $12.00

    D) $13.75

    E) $15.00

Solution is: C) $12.00

The earnings for 7 days at $10.00/day is $70. Add this to *a,* which represents the average of all eight days plus the $1.75 increase. Divide by 8 and set equal to *a*.

$$\frac{70 + a + 1.75}{8} = a$$

71.75 + a = 8a

71.75 = 7a

10.25 = a

Add $1.75 to $10.25 to find his earnings on the 8th day.

$1.75 + $10.25 = $12.00, which is answer C).

10. $\dfrac{2}{f}$ and $\dfrac{5}{2f}$ represent positive fractions. Which expression represents the sum of the two fractions that are evenly spaced between $\dfrac{2}{f}$ and $\dfrac{5}{2f}$?

    A) $\dfrac{9}{2f}$

B) $\dfrac{10}{f}$

C) $\dfrac{1}{6f}$

D) $\dfrac{55}{12f}$

E) $\dfrac{25}{6f}$

Numeric Response

11. ABCD is a trapezoid, with side AB parallel to side CD. The sides: AB, BC and DA are equal and are of length 2 cm. Side CD has length of 4 cm. The measure of angle ADC is _____.
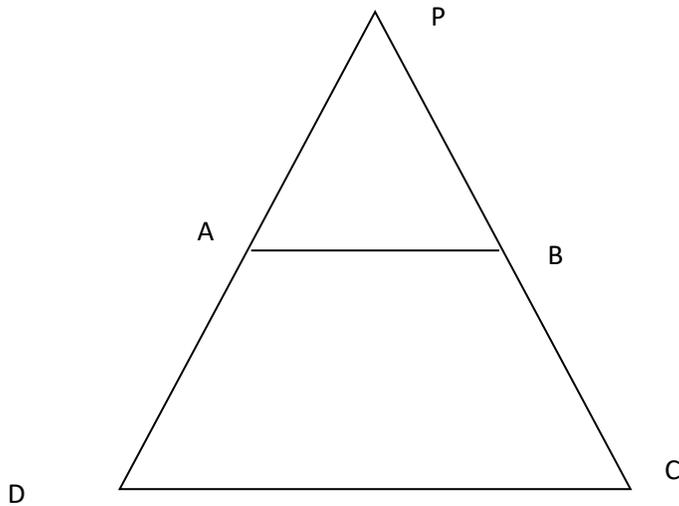
Let P be the point of intersection of AD and BC. Since AB is parallel to CD and AB is half of CD, AB is the midline of triangle PCD. Therefore, A is the mid-point of PD and B is the mid-point of PC. And PC = PD = CD = 4 cm. Therefore, triangle PCD is equilateral, and angle ADC has a measure of 60°.

12. The product of the digits of the number 176 is 42. How many other three-digit numbers have 42 as the product of their digits?

The solution is 11.

A product of 42 can be formed two ways using three whole numbers:

$1 \times 6 \times 7$ or $2 \times 3 \times 7$

Rearranging these numbers, the following three-digit numbers can be formed:

167, 176, 671, 617, 761, 716, 237, 273, 327, 372, 723, and 732. Since, the number 176 was given, there

are 11 other ways.

13. Alyssa said to Bryan, "If you give me half of your money, I will have just enough to buy that horse." Bryan replied, "If instead you give me two-thirds of your money, I will have just enough to buy that same horse." Neither gave, and instead spent all their money buying pigs, each of which cost the same. If Alyssa bought 30 pigs, how many did Bryan buy?

The solution is 20.

Alyssa's money plus half of Bryan's was equal to Bryan's money plus two-thirds of Alyssa's. Therefore, one third of Alyssa's money was equal to half of Bryan's. Since Alyssa bought 30 pigs, she

bought 10 pigs with one third of her money. And, Bryan bought 10 pigs with half of his money. Or he bought 20 pigs total.

14. The length and area of a rectangle can be expressed as x^2y and x^3y^2, respectively, where x and y are natural numbers. If the area is 1323 m^2, what is the perimeter of the rectangle?

The solution is 168 m.

Since the area is 1323 m^2 , and x and y are natural numbers, 1323 must be written as the product of a perfect square number and a perfect cube number. The prime factorization of 1323 is $3 \times 3 \times 3 \times 7 \times 7$. Therefore, x = 3 and y = 7.

Area of a rectangle: A = lw

$$x^3y^2 = (x^2y)w$$

$$\frac{x^3y^2}{x^2y} = w$$

This simplifies to: xy = w

The perimeter of a rectangle is 2l + 2w

P = 2l + 2w

P = 2(x^2y) + 2xy, replace x with 3 and y with 7.

P = 2($3^2$7) + 2(3)(7)

P = 168.

The perimeter of the rectangle is 168 m.

15. The sum of three positive integers is 9. What is the least possible product of their reciprocals? Express your answer to the nearest thousandth.

The solution is 0.037.

| Possible Sums	Product of Reciprocals
1 + 1 + 7	$1 \times 1 \times \frac{1}{7} = \frac{1}{7}$
1 + 2 + 6	$1 \times \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
1 + 3 + 5	$1 \times \frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$
1 + 4 + 4	$1 \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

$2 + 2 + 5$	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{5} = \frac{1}{20}$
$2 + 3 + 4$	$\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{24}$
$3 + 3 + 3$	$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$

The smallest product $= \frac{1}{27} = 0.037$

16. A large rectangular field is subdivided into 4 congruent rectangular fields as shown at the right. The area of the large rectangular field is 3468 m². Find the area of a square field that has the same perimeter as the large field before it was subdivided. Express your answer to the nearest whole square metre.
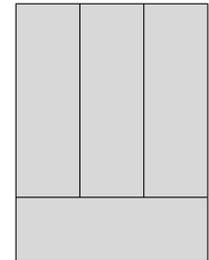
The solution is 3540 m².

Since the rectangle is divided into 4 congruent rectangles, the original rectangle has a length to width ratio of 4 to 3. So, the perimeter of the original rectangle can be expressed as 14x and the area as $12x^2$.

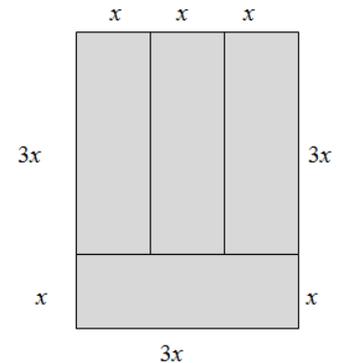Therefore, $12x^2 = 3468$. and x = 17.

The perimeter of the original rectangle is: $2(4x) + 2(3x)$

$$P = 2(4)(17) + 2(3)(17)$$

$$P = 238 \text{ m}$$

Therefore, a square with perimeter 238 m would have side length of $238 \div 4$ or 59.5 m.

The area of the square is 59.5^2 or 3540.25 m². Rounded to the nearest whole metre squared: 3540 m²

17. The height of a cone is increased by 10% and the radius is decreased by 10%. By what percent, expressed to the nearest tenth of a percent, will the volume of the cone change? (Volume of cone $= \dfrac{\pi R^2 H}{3}$, where R is the radius and H is the height)

The solution is 10.9%.

Let the R = 10 units and the H = 10 units.

F   A
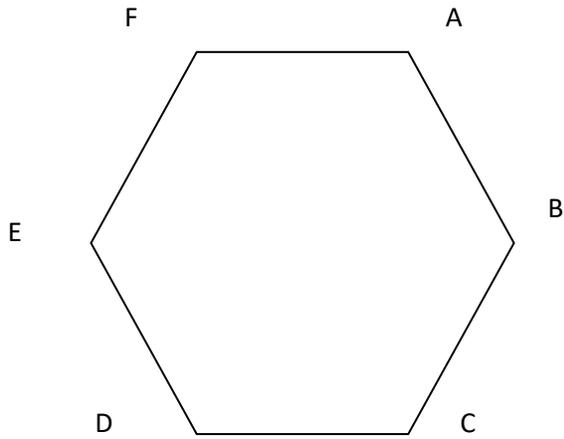
Substitute into the formula: $V = \dfrac{\pi R^2 H}{3}$

$V =$

$\dfrac{\pi 10^2 10}{3}$

E

B

$V =$

$\dfrac{1000\pi}{3}$

D   C

The altered cone has a height increase of 10%, so the new H is 11. The altered cone has a radius decrease of 10%, so the new R is 9. Substitute into the volume formulas.

$$V = \frac{\pi R^2 H}{3}$$

$$V = \frac{\pi 9^2 11}{3}$$

$$V = \frac{891\pi}{3}$$

This is a decrease in volume of $\dfrac{1000\pi}{3}$ - $\dfrac{891\pi}{3}$ or $\dfrac{109\pi}{3}$ which is 10.9 % of the original volume.

18. If you randomly select 3 vertices of a regular hexagon, and then connect them, what is the probability that a right triangle will be formed? Express your answer as a decimal, to the nearest hundredth.

The solution is 0.60

There are 20 triangles that can be formed: ABC, ABD, ABE, ACE, ADC, ADE, AFC, AFD, BCD, BCE, BCF, BDF, CDE, CDF, DEF, EDB, EFA, EFB, FAB, FEC. Of these, 12 are right triangles: ABD, ABE, AFC, AFD, ADE, ADC, BCF, BCE, CDF, EDB, EFB, FEC. Therefore, the probability of selecting a right triangle is $\dfrac{12}{20}$ or 0.60

19. Kylee is a cheerleader and has a drawer that contains four colours of poms. Ninety are gold, 70 are green, 50 are blue, and 40 are red. She randomly pulls out poms, one at a time, without looking at the colours. What is the fewest number of poms that Kylee must remove to be certain that she has at least 10 pairs of matching poms? The solution is 23.

20. Travis left home, traveled 12 km west to the grocery store, then 6 km south to the post office, then 2 km west to the bank, then x km north to the library. The shortest distance from the library to Travis's home is 15 km. What is the farthest distance that the library can be from the bank? Express your answer to the nearest tenth of a kilometer.

Library

Grocery   Home

Library
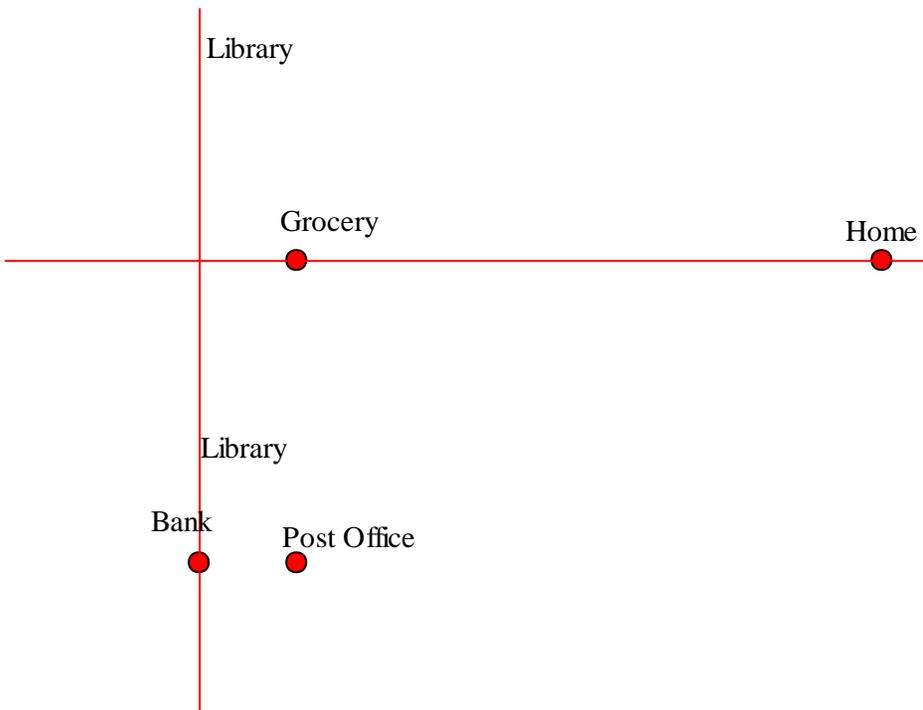
Bank   Post Office

Suppose home is at the point (17, 8) and the grocery is 12 units west (left) at the point (5, 8). Then the post office would be 6 units to the south (down) at the point (5, 2). The bank would be 2 units west (left) at the point (3, 2). The library is located on the vertical line passing through the point (3, 2). Form a right triangle with hypotenuse of 15 km which is the shortest distance from the library to Travis's home. One leg is the horizontal distance from home (17, 8) to the vertical line with x-coordinate of 3, which has a length of 14. Find the other leg using the Pythagorean property.
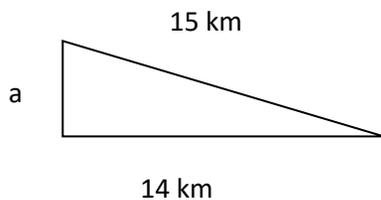
$a^2 = c^2 - b^2$

$a^2 = 15^2 - 14^2$

$a^2 = 225 - 196$

$a^2 = 29$

$a = \sqrt{29}$

15 km

a

14 km

a = 5.385

The library could either be below the horizontal line joining home at (17, 8) to the vertical line with x-coordinate of 3 or it could be above the line. If it is below the horizontal line the distance would be 6 - 5.4 = 0.6 km. If above the horizontal line the distance would be 6 + 5.4 = 11.4 km. Since the farthest distance was requested, the answer is 6 + 5.4 or 11.4 km.