Edmonton Junior High Mathematics Contest 2007

Multiple-Choice Problems

Problem 1

A sequence is simply a list of numbers in order. The sequence of odd integers is $1, 3, 5, 7, 9, \ldots$. If we add any number of consecutive odd numbers, always starting at 1, then the result will always be

A. an even number B. an odd number
C. a perfect square D. a perfect cube

Problem 2

You have an unlimited number of nickels (5 cents) and dimes (10 cents) from which you can use only nickels, only dimes or a combination of both to make a sum of 55 cents. The number of different combinations of coins that can be used is

A. 11 B. 6
C. 5 D. 4

Problem 3

In the real number system, if the sum of $a + 3.464\ 466\ 444\ 666\ldots$ is to be an integer, then a could have a value of

A. $0.535\ 533\ 555\ 333\ldots$ B. $1.646\ 644\ 666\ 444\ldots$

C. $2.202\ 200\ 222\ 000\ldots$ D. $3.313\ 311\ 333\ 111\ldots$

Problem 4

Consider any three consecutive integers *a*, *b* and *c*, where $a < b < c$ and $a > 1$. The expression that gives a correct relationship among *a*, *b* and *c* is

A. $ac = b^2 - 1$ B. $ac = b^2$
C. $2b = ac$ D. $c = a + b$

Problem 5

If $2x^2 + x + 3 + 2y = -4x^2 + 4x - 6$, then $y =$

A. $-12x^2 + 6x - 18$

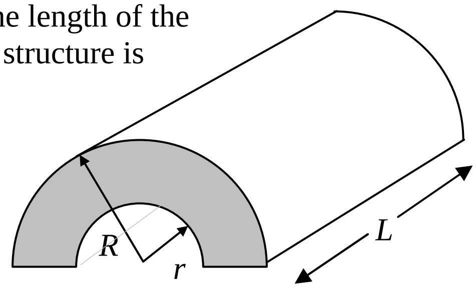
B. $-4x^2 + x - 6$

C. $-x^2 + 2.5x - 1.5$

D. $-3x^2 + 1.5x - 4.5$

Problem 6

An engineer designs a hollow reinforced concrete support structure in the shape of a semi-cylinder. If the inner radius is r, the outer radius is R and the length of the structure is L, then an expression for the volume of concrete in the structure is



A. $V = \frac{\pi}{2}\left(\frac{R}{r} - L\right)^2$

B. $V = \frac{\pi L}{2}(R - r)^2$

C. $V = \frac{\pi}{2}(LR - r)^2$

D. $V = \frac{\pi L}{2}(R - r)(R + r)$

Problem 7

A rectangle can be made longer and narrower without changing its area. For example, if the lengths of one pair of its sides are increased by 60%, then the lengths of its other pair of sides must be decreased by

A. 62.5%

B. 60.0%

C. 40.0%

D. 37.5%

Problem 8

The value of $\sqrt{3+\sqrt{8}}+\sqrt{3-\sqrt{8}}$ is equal to

A. 3

B. $2\sqrt{2}$

C. $2\sqrt{3}$

D. $\sqrt{6}$

Problem 9

Each student in a class of 25 students wrote 2 different tests. It is known that

- 18 students passed the first test.
- 22 students passed the second test.
- No students failed both tests.

The number of students who passed both tests is

A. 15 students

B. 10 students

C. 20 students

D. 40 students

Problem 10

If n is a whole number then the number of different values of n where $7n + 1$ is a multiple of $3n + 5$ is

A. 0

B. 1

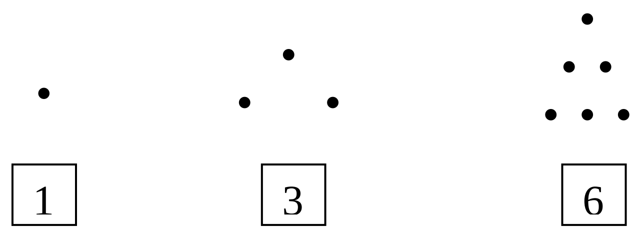
C. 2

D. infinite

Answers-Only Problems

Problem 1

Numbers such as 1, 3 and 6 are sometimes referred to as triangular numbers, because the value of the number can be represented by a triangular shape as shown below.



The sum of the first 10 triangular numbers is __________ .

Problem 2

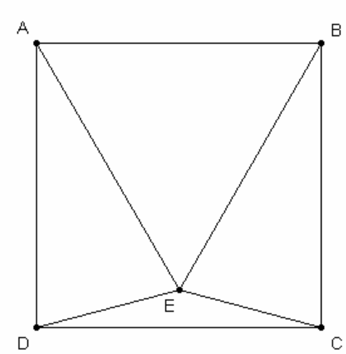
John and Sam both leave point A at the same time, heading in exactly opposite directions. If John walks at 4 km/h and Sam walks at 3.5 km/h, then the number of minutes it takes for them to be 2.5 km apart is __________ .

Problem 3

A set of 5 numbers has an average of 13. If a 6^{th} number is included, then the average is 23. The value of the 6^{th} number is __________ .

Problem 4

In the diagram to the right, quadrilateral ABCD is a square, and ΔABE is an equilateral triangle. The measure in degrees of $\angle ECD$ is __________ .

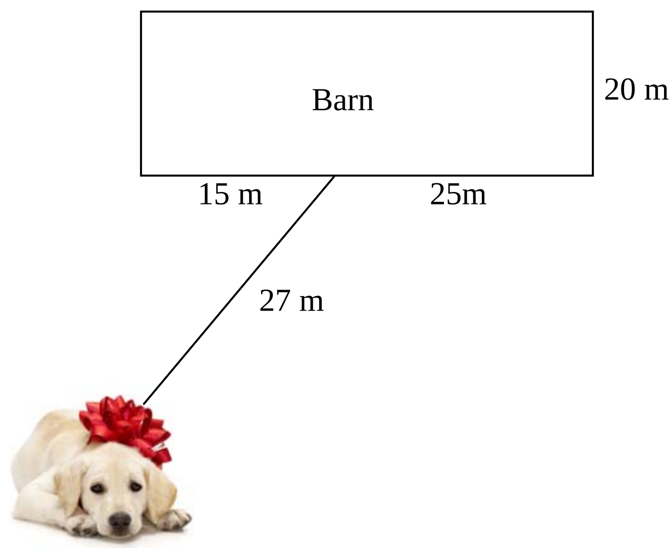


Problem 5

Each integer from 1 to 100 inclusive is written on an identical card, one number per card. The cards are placed into a box and mixed thoroughly. If a single card is drawn at random, then the probability that the number on the card is divisible by either 3 or 5, expressed as a decimal to the nearest hundredth, is __________ .

Problem 6

A farmer fastens the end of his dog leash to the edge of his barn at a point that is 15 m from one corner and 25 m from another corner of the barn, as illustrated in the diagram below. The barn is 20 m wide and the leash is 27 m long. The area of the region where the dog is able to reach while leashed to the wall, to the nearest whole square metre, is __________ .

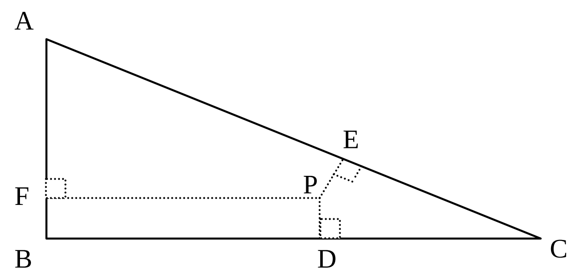


Problem 7

Consider the irrational number $0.454\ 554\ 555\ 455\ 554\ldots$. The total number of 5s that occur in the number before the digit 4 appears for the 100^{th} time is __________ .

Problem 8

A point P, is inside a triangle ABC, where $AB = 7$, $BC = 24$ and $CA = 25$.
If $PD = 2$, and $PE = 2$, then the length of PF to the nearest whole number is ___________ .



Problem 9

When 80, 97 and 158 are divided by a certain even positive integer, the sum of the three remainders is 39. This even positive integer is ___________ .

Problem 10

Let a, b, and c be non-zero numbers such that $a+b+c=0$. The value of
$a\left(\frac{1}{b}+\frac{1}{c}\right)+b\left(\frac{1}{c}+\frac{1}{a}\right)+c\left(\frac{1}{a}+\frac{1}{b}\right)$ is ___________ .