Part I: Multiple-choice Problems

1. The number of integers between $\sqrt{13}-3$ and $\sqrt{32}-2$ is
(a) 1
(b) 2
(c) 3
(d) 4
2. For a letter weighing up to 20 grams, the postage is 3 cents per gram. The postage for each gram beyond 20 grams but up to 40 grams is 2 cents. The postage for each gram beyond 40 grams is 1 cent. The postage for a letter weighing 45 grams is
(a) $\$ 1.05$
(b) $\$ 1.10$
(c) $\$ 1.25$
(d) $\$ 1.35$
3. Eve claimed to have found $n$ integers whose sum is 0 and whose product is $n$. Ace said, "I do not think there is such an $n$." Bea said, "Yes, there is, and n must be odd." Cec said, "Actually, $n$ must be even." Dee said, "No, $n$ can be either odd or even." The one who was right was
(a) Ace
(b) Bea
(c) Cec
(d) Dee
4. The product of two numbers is equal to the product of two other numbers. Three of these four numbers are 3, 4 and 12. The number of possible values of the fourth number is
(a) 1
(b) 2
(c) 3
(d) 4
5. In the first round of World Cup soccer, there are four teams in each group. Every team plays each of the other three once, getting 3 points for a win, 1 point for a draw and 0 points for a loss. The top two teams in each group advance to the second round. The minimum number of points, which will guarantee advancement without getting involved in any tiebreakers, is
(a) 3
(b) 6
(C) 7
(d) 9
6. A merchant bought 3 pigs in a poke for $x$ dollars each, and later bought 2 more pigs in a poke for $y$ dollars each. When all 5 were sold at $\frac{x+y}{2}$ dollars each, the merchant lost money. The reason for this was
(a) $x<y$
(b) $x=y$
(c) $x>y$
(d) unrelated to the values of $x$ and $y$
7. A non-zero digit is added between two adjacent digits of 2004 to obtain a multiple of 7. The number of possible values of this digit is
(a) 1
(b) 2
(c) 3
(d) 4
8. The price of nuts in Walmart is $\frac{1}{3}$ that in Cocomart, and $\frac{7}{10}$ that in Doughmart. The price of nuts in Cocomart is some fraction of that in Doughmart. This fraction is
(a) $\frac{3}{7}$
(b) $\frac{10}{21}$
(c) $\frac{21}{10}$
d) $\frac{7}{3}$
9. A sequence of diagrams is constructed as follows. The first diagram is an equilateral triangle pointing up. Each subsequent diagram is obtained from the preceding one by dividing each equilateral triangle pointing up into four congruent equilateral triangles. The second and third diagrams are shown below. There are respectively 5 and 17 triangles in them, counting overlapping ones. The total number of triangles in the fifth diagram is
(a) 122
(b) 125
(c) 134
(d) 161

10. The diagram below shows a star polygon with seven sides. The total measure of its seven angles is
(a) 3600
(b) 5400
(c) 7200
(d) 9000

11. A video tape costs $\$ 10$ and sells at $\$ 16$ for a $60 \%$ profit. A video disc sells for a 40\% profit. If we sell one and a half times as many video tapes as video discs, the combined profit is $50 \%$. How much does a video disc cost?
12. In the expression $\frac{k^{2}}{k^{2}-10 k+50}$, a student put in $k=1,2,3 \ldots, 9$ What is the sum of the nine numbers so obtained?
13. What is the sum of all positive integers each equal to four times the sum of its digits?
14. Two birds in hand plus one bird in the bush are worth $\$ 50$. Three birds in hand plus two birds in the bush are worth $\$ 80$. How much are four birds in hand plus three birds in the bush worth?
15. The difference of two positive integers is 120. Their least common multiple is 105 times their greatest common divisor. What is their greatest common divisor?
16. One hundred students in a school line up in $n$ rows for a year-book picture, where $n$ > 1. Several students are in the first row. Each row starting with the second contains one student more than the preceding row. There are no students left over. What is the number of possible values of $n$ ?
17. Eddy picked a two-digit number, switched the two digits to get a new number, and subtracted the smaller of the two numbers from the larger one. The difference is the cube of Eddy's age. How old is Eddy?
18. The diagram below shows nine towns connected by twelve highways. There is a robber on each highway, and the number indicates the amount of money one must pay to travel along that highway. What is the lowest amount one has to pay in highway robbery in order to go from the town at the northwest corner to the town at the southeast corner?

19. In the quadrilateral $A B C D$, both $A B$ and $C D$ are perpendicular to $A D . A B=13, C D=8$ and $A D=12$. $F$ is a point on $B C$ such that $A F$ is perpendicular to $B C$. Find the length AF .
20. In triangle $A B C, \angle C A B=90^{\circ}$. D is a point on the extension of $B C$. The bisectors of $\angle A B C$ and $\angle A C D$ meet at $E$, as shown in the diagram below. What is the measure of $\angle \mathrm{BEC}$ ?

