Ready, SET, Play: Learning Elementary School Mathematics Through an Attribute-Based Game

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The use of engaging activities such as games in elementary mathematics classrooms can help children form a positive disposition towards mathematics and provide meaningful learning opportunities. Games have been recommended as a way for students to develop an understanding of mathematical ideas before they move toward abstractions (Diénès 1971). Ernest (1986) identified three educational uses of games in mathematics class: gaining skill-based fluency, developing conceptual understanding and refining problem-solving approaches. Analysis of students' learning through varied game contexts has identified benefits such as (1) the context of mathematics class allows for children to readily mathematize authentic contexts originating outside the classroom (Linchevski and Williams 1999); (2) small-group settings in games, with peers and a teacher, support mathematical conversations in which learning occurs (Polaki 2002); and (3) children demonstrate improvement in attitude and motivation for learning mathematics (Lopez-Morteo and Lopez 2007).

Commercial card and board games have an important presence in many homes of the students in our classrooms. We identify games as "commercial" when they are marketed to the general public and are easily purchased through bookstores or toy stores. While often these games are used for enjoyment, we realized that many games have an educational element that can be employed in elementary mathematics classes. Different from instructional games—games created to intentionally learn or practice specific mathematics skills—commercial games can be seen as authentic contexts in which to experience mathematical ideas. Through our interactions with elementary school students in several classrooms, we explored the mathematical experiences available through commercial games.

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In this article, we offer a sense of the richness of mathematical learning possible through incorporating a card game. We use the game SET as a specific example, first describing the rules and explaining the context of our work. Students' mathematical thinking embedded in their game play offers a strong case for incorporating SET as an opportunity to learn in mathematics class. We end by identifying curricular connections and offering some ideas for differentiating to engage all students in experiencing mathematical learning through games.

Try Making a Set!

We explored the use of the game SET in an elementary school mathematics club. The commercial card game is a visual discrimination game that students can learn quickly and that allows students to explore identifying attributes and sorting in a captivating manner. SET is a game for one or more players, and takes approximately 15 minutes to play. Players aim to make a set that consists of three cards. Twelve cards are placed face up in a rectangular arrangement on a surface for all players to see, as depicted in Figure 1.

Cards are made up of figures with four varying attributes: shape, colour, quantity and shading.

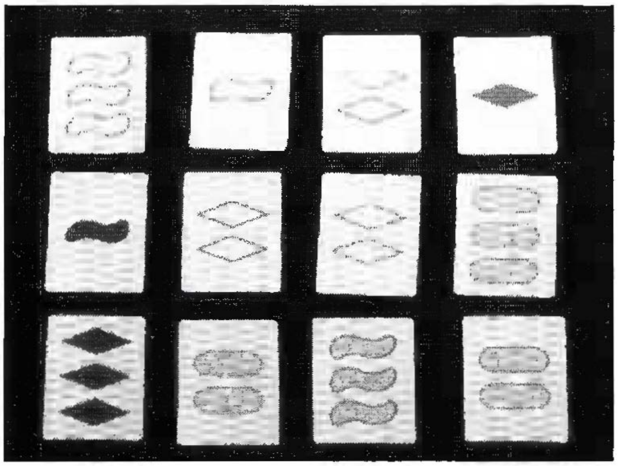

Figure 1: *The game SET*

In order to make a set, a player must identify 3 cards on which the individual attributes are either all the same or all different across the 3 cards. Once a player identifies a set, the player must say "set" out loud and remove the cards from the table; the other players verify that the set is correct. If the player forms a valid set, the player keeps the cards; otherwise, the cards are returned to the table. Figure 2 shows examples of sets. During game play, cards are replaced from the deck so that 12 cards remain at all times on the table until the cards run out. Game play is over once all cards have been used. Players count the number of sets they have collected during the game. Each set counts as a point and the player with the most points wins.

SET can be purchased inexpensively at many stores, making the game easily obtainable for teachers to add to their classroom materials to be used as part of a mathematics lesson or station. We found that students appreciated having a "real" game to play in class. Within limited budgets, it is possible to construct a homemade version of the game because it relies only on cards as playing materials. As a result of searching online we also discovered that the game is available for free through several Internet sites, including

- www.setgame.com/set/daily_puzzle,
- http://smart-games.org/en/set/start/ and
- www.lsrhs.net/faculty/seth/Puzzles/set/set.html.

SET could also be used in the classroom by app for iPad and iPod touch. We invite readers to give one of these online versions a try before continuing.

Context

To explore students' mathematical thinking within the game of SET we attended an elementary school math club over a three-month period. The math club met weekly during lunch hour as an extracurricular activity. Grades 4 to 6 students were invited to participate, with an average of more than 30 students attending each week. In the small elementary school that was the setting for the research project, almost half the Grades 4 to 6 students attended at times over the three months.

In math club, students would eat their lunch while a teacher introduced a new game. Often instructional videos were used to highlight game rules, such as this one for the game SET: www.youtube.com/watch?v=bMhJmrJVP4Q. If no new game was introduced that week, students would be asked to share some strategies they developed from the previous week. After the introduction the students selected who they wanted to play with and which game they wanted to play out of seven possible games: Farkle, SET, Othello, Gobblet Gobblers, CirKis, Equilibrio and Quartex. Students had the option to play in pairs or individually against their opponent(s). After the students had chosen their game, the rest of math club time was given to open exploration and playing the selected game.

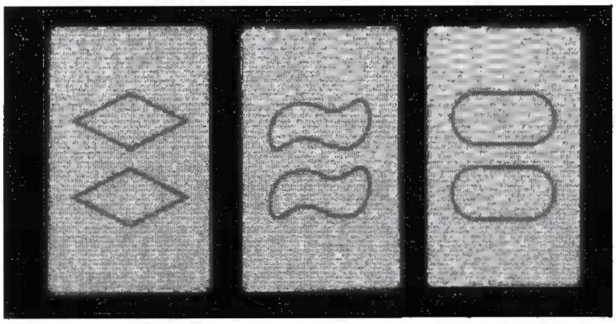

Figure 2a: *Example 1 of a set*

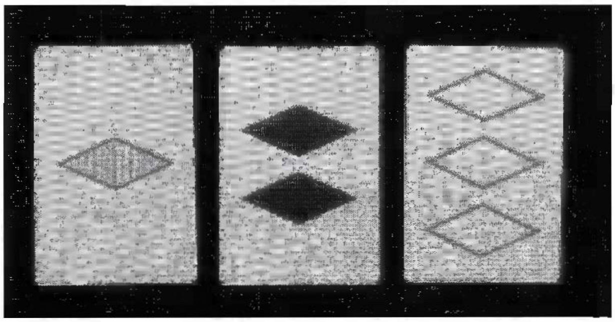

Figure 2b: *Example 2 of a set*

Two teachers, two parent volunteers and a researcher were present to play with the students and to pose questions to prompt mathematical thinking. Several times the students were invited to fill in record sheets to help them identify some of their mathematical thinking and to share some of their personal strategies. A couple of examples of reflective prompts include "How do you look for sets?" and "When you can't see a set right away, what do you do?" The students who took part in our research project also participated in informal interviews in pairs to further demonstrate their mathematical thinking and share their thoughts on the games in math club. The examples of students' thinking in this article come from both their interviews and their record sheets.

Students' Mathematical Thinking Through SET

We highlight students' learning through three particular Grades 4 and 5 students: Nicole, Zahra and Rimira (pseudonyms are used for all students). We selected these students because they saw themselves as experts and had developed facility with SET. In general, the students in math club felt they were a game expert if "you have a good chance of winning and a lot of strategies," as one student explained to the group. It became apparent that to only have one strategy to rely on in playing wasn't seen as effective by students, but a variety of strategies allowed their play to respond to each game situation, often leading to a win.

The students who played SET, however, identified that there is more to being an expert than simply winning the game. Zahra, who was very quick to identify sets, attributed her exceptionality at SET to having "good looking." In this brief explanation, Zahra demonstrates a capability to notice her own mathematical

They grew as mathematical learners.

thinking, indicating that visualizing was an important process for her as she built sets systematically.

As we observed students playing SET, explored their record sheets and interviewed the students, we were pleased with the mathematical thinking and learning occurring. We are excited to share some of the students' thinking as examples of the possibility for rich mathematical learning embedded in an engaging commercial game like SET. In particular, students developed a range of strategies as they tried to improve, they engaged in important mathematical processes and refined their use, and they grew as mathematical learners.

Learning to Sort by Using Personal Strategies

When we asked students about what made the games used in math club mathematical in nature, a common response was similar to Nicole's—the games "teach us lots of new strategies." While personal strategies are often used in the program of studies in conjunction with arithmetic computations (Alberta Education 2014), the development of strategies is

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important more generally in all mathematical learning because it indicates that students are making procedures and approaches that are meaningful to them. Related to "relational understanding" (Skemp 2006)—where students not only know what to do to craft a solution to a mathematical question, but they also know the reasons for why they carried out a particular procedure—students develop the *why* in creating their own personal strategies.

Many personal strategies emerged as students learned the game and moved toward being expert players. Their personal strategies relied on discriminating among attributes, classifying groups of cards, and developing and applying sorting rules. When students were prompted to explain how they searched for a set, students demonstrated a range of ways to proceed systematically that worked best for them.

Some students used the cards as physical objects to identify sets. They would sort cards that had potential in forming a set and narrow down related attributes. For example, Zahra preferred to physically pick up or point at cards to help her keep track of the cards she was sorting to make a set. Figure 3 shows Zahra beginning a set by selecting two cards with the same shape (ovals), same shading (solid), different colours (green and red), and different quantities (one and two). The importance of using manipulatives in mathematics class has been well documented previously (eg, Boggan, Harper and Whitmire 2010; Moyer 2001; Sowell 1989). By using the SET cards in a concrete fashion, students like Zahra were able to keep track of their possible sets.

Figure 3: *Zahra manipulating the cards*

While some students who are more competitive might not want to point to cards to reveal possible starting places for making a set, we noticed that the physical grouping of cards did not inhibit Zahra from winning many of her matches. In fact, Bonnie (the supervising teacher for math club) exclaimed that Zahra "was amazing at SET!" She followed up with Zahra by posing the question, "How do you see these [sets] so fast?" For us, this was an important question because it pointed to the process of visualizing that students began to apply as they moved beyond using the cards as manipulatives.

Another common strategy students developed was to begin with a partial set. Seen as an act of "specializing" (Mason, Burton and Stacey 2010), this refers to using specific examples or narrowing the problem into a smaller problem to be solved. For students playing SET, that often meant quickly identifying two cards that constituted part of a set. In the picture above, Zahra portrays this approach. Rimira expressed this as her most frequent strategy. She would "look for two that are pairs and then I try to see if there is something that can go with that and if not I just move on to a different two." Students using this strategy would move across the table with a pair of cards as a point of comparison, having established a sorting rule that could then be applied to the other cards on the table. Generating the sorting rules created an efficient way of discriminating, supporting growth in pattern noticing.

In another form of specializing, students would also systematically explore the 12 cards laid out on the table. A powerful approach to thinking mathematically is to be systematic in exploring the situation, where "success is more likely if the specializing is done systematically" (Mason, Burton and Stacey 2010, 5). The most systematic approach the students demonstrated was to isolate one attribute on the SET cards at a time.

During math club, Nicole explained her strategy to us as "I look for one colour, and then I look at another colour and then the other colour." Beginning with the attribute of colour, Nicole used the attribute to direct how she looked at the rest of the cards on the table. Rather than sorting by four attributes, specializing allowed Nicole to narrow down the possibilities and systematically keep track of the leading attribute. If this did not result in forming a set, Nicole would be able to justify why and move to isolating another attribute confidently.

Bonnie also described Zahra's systematic approach as "she would pick a characteristic and look for that particular one. But that would lead her to another one. … It's like she was following a trail of them." When we asked Zahra in an interview about her expertise, she also confirmed that she liked to focus on "the detail on things" to help her make a match aiding her development of focusing on attributes. Nicole and Zahra's personal strategies established a very systematic way to play the game, which allowed them to develop a logical argument for quickly sorting cards by attributes.

Through the development and use of personal strategies, students began to make statements about broader game play. For instance, Nicole found that it was easiest to find sets where all of the cards are all the same in each of the individual attributes. This built on her personal strategy of attending to one attribute at a time. One generalization Zahra made was that the hardest sets to find were ones where the attributes were all different. In this case, she would have to look at every card rather than applying sorting rules created from pairs of cards. The process of generalizing is so important that it has been described as "the life-blood

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of mathematics" (Mason, Burton and Stacey 2010, 8). Rather than specific strategies, the generalizations that students formed articulated a global approach to sorting situations grounded in noticing patterns across many instances of playing SET.

Developing Mathematical Processes

The mathematical processes that appear in the program of studies (Alberta Education 2014), such as communication, visualization, reasoning and problem solving, are also important elements in the mathematics classroom that are integrated across all learning opportunities for students. As the students collaborated during game play, they were continually engaged in developing the mathematical processes as "shared mathematics promotes the development of problem solving, reasoning and communication skills" (Alberta Education 2010, 217). In particular, we noticed growth in the students' mathematical processes of communication, visualization and reasoning.

Communication

Communication in an elementary mathematics classroom can be characterized as "a way of clarifying students' thinking and understanding" and "a way of revealing their thinking, their reasoning, and what they know and do not know" (Greenes and Schulman 1996, 160). Students were talking with their peers frequently while they were playing SET. As they identified a set, they defended the composition by verbalizing to their opponents the validity of the set. Peers were appropriately skeptical so that no player would claim an undeserved set. Platz (2004) emphasizes that "there is a need for children to not only sort and classify objects but for them to communicate their thinking as to how they sorted or classify the set of objects provided to them" (p 90).

As students were learning SET, they often discussed the attributes of the shapes on the cards with their peers in order to assess whether they had identified a set. This helped establish a common way of grouping attributes—for instance, the patterns filling the shapes were referred to as "shading," "pattern" or "detail"—providing a rationale for developing mathematical terminology for mathematics class. As students became experts, they communicated their thinking very clearly to others.

Through game play and the rule that a player must show the other players the identified set, the students became fluent at communicating the components of their set. For example, Nicole justified why three cards were a set by explaining, "Same shape, same colour and same detail. One, two, three." Although brief, Nicole's statements were complete enough to explain her rationale and satisfy her opponent. In addition to the specificity of identifying a set, we have already demonstrated above that students communicated their strategies on how to locate a set that moves toward expressing generalizations valued in mathematics.

Visualization

Visualization is a mathematical process that is rooted in action—the act of visualizing, which grows out of manipulating objects and creating (pictorial) representations. As students learn mathematical ideas, we can understand the early development as "image making" and "image having" to develop mental images of mathematical ideas (Pirie and Kieren 1994). Visualizations are recognized as

> physical objects (i.e., illustrations, computer-generated displays); mental objects pictured in the mind (i.e. mental schemes, mental imagery, mental constructions, mental representations); or cognitive processes (i.e., cognitive functions in visual perception, manipulation and transformation of visual representation by the mind, concrete to abstract modes of thinking, and picturing facts). (Macnab, Phillips and Norris 2012, 104)

In the case of SET, students were using the cards (illustrations on physical objects) to scaffold the ability to see possible groups of cards mentally as they transformed the arrangement of cards.

SET is advertised as a game of "visual perception" (www.setgame.com), which could occasion moments for students to improve in visualizing. Zahra mentioned on several occasions that she depended on her visualization to aid her in finding a set. For instance, when asked why SET belongs in math class Zahra acknowledged that the game "gives your eye a little workout." By saying this, Zahra recognizes the importance of visualizing in the game and its place in mathematics class. To her, visualizing during a mathematics activity is a fundamental component of becoming mathematically literate. Students used the mathematical process of visualization to sort and organize objects and data, recognize same and different, form mental images, and focus on attributes. Students carry with them the mental images they create to learn related mathematical ideas in meaningful ways.

Reasoning

Mathematical reasoning is viewed as one of the most important mathematical processes and "involves exploring the mathematics at hand; generating, implementing, and evaluating conjectures; as well as justifying our thinking and actions as we engage in mathematics" (Thom 2011, 234). Students playing SET were exploring the mathematical actions of discriminating among attributes, sorting by attributes and generating rules for classification. They created conjectures as to which cards would belong in a set and identified a possible set to be assessed by opponents. The need to justify was inherent in the game and valued by students as they played. Even the students noticed that the game "gets your mind going" and "gives your brain a workout," becoming aware that they needed to be cognitively active to succeed in a mathematical game like SET.

Deductive reasoning structures began to emerge as students made if–then statements as they were playing. If students had difficulty finding a set, rather than getting frustrated they persevered and talked among their opponents, making statements such as, "If there was this one here [gesturing], then the set could have gone through." Rather than imposing this important mathematical structure, students used if–then statements in an authentic and meaningful way. If–then statements also mark in the students' reasoning that they are developing relationships among the attributes and classifications, and "students who understand such relationships are reasoning at higher levels" (Fox 2000, 573). We were excited to hear this way of generating conjectures because of the foundational experiences in reasoning that can lead to later facility with constructing proofs. As students worked together during game play, they were continually engaged in refining the mathematical processes integral to *doing* mathematics.

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Growing a Positive Disposition

Each week, students eagerly picked games at the beginning of math club time. They showed enthusiasm as they developed winning strategies and defended their plays to their peers. In describing the development of mathematical proficiency, researchers have identified a productive disposition as one of five components necessary for students' success, defining it as a "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (National Research Council 2001, 116). We see a willingness to engage in mathematical tasks, such as thinking mathematically during SET, as an indication of a productive disposition. It incorporates and moves beyond a positive attitude so that students "appreciate and value mathematics" (Alberta Education 2014)—a goal within the Alberta mathematics curriculum. We noticed that the students who played SET were engaged, exhibited perseverance and were confident.

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We asked the students why they like playing SET, as they often returned to the game week after week. Nicole explained that the game is "simple and fun." While it may not be the primary reason to implement the game into your mathematics class, Nicole's statement is worth considering. Nicole demonstrated in her play that she was often successful at finding sets, and connected her efficacy with the straightforward nature of the play. Rimira echoed Nicole's sentiment when she identified that the game could be used "for people who have trouble at math, they can have an easier way to learn math." Both students had confidence in their ability to play a mathematical game. Students can feel the most successful when they believe that they can accomplish something and can be more engaged and motivated to learn when the activity they are doing is enjoyable.

Out of a positive disposition often comes a willingness to persevere in problem-solving situations. We found this to be the case in playing SET on a number of occasions. Zahra identified several times where "there is no matches or anything," making it impossible to make a set within a group of 12 cards laid out on the table. When this occurred, she eagerly explained how no set could be formed. This required perseverance by systematically eliminating all possibilities. Additionally, notice one of Zahra's final reflections on her learning through the game, shown in Figure 4 overleaf. Like Zahra, students often found the last round most challenging as cards on the table diminished, but also took great pride in identifying the last set of the game, regardless of how long it took. Rather than causing frustration, coming across challenging groups of cards provided opportunities for

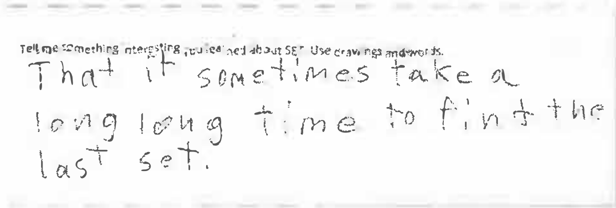

Figure 4: *Zahra's final reflection*

students to develop and use different strategies while discussing with their peers whether a set existed.

We also noticed students' confidence as they sustained engagement in playing SET, seeing themselves as growing mathematical learners. While the game was relatively quick to learn how to play, students enjoyed the variation of each group of 12 cards being different and the competition of being first to identify a set. Nicole explained that she would "just play [the game] a lot" and chose it frequently during math club. Her choice to play often provided the opportunity to develop a range of strategies, refining older strategies and testing newer ones. Even outside of math club, Rimira mentioned that she could "play SET on the Internet." While Rimira did not consider mathematics to be her favourite subject in school, Rimira's choice to play a mathematically oriented game outside of mathematics class speaks to her evolving sense of her capability in mathematical tasks. These three students demonstrate, on behalf of their peers, how incorporating commercial games with mathematical ideas can foster a shift in perspective about mathematics.

Curriculum Connections

The qualities of mathematical thinking and engagement that the students demonstrated are important reasons to incorporate SET in elementary school mathematics classrooms. We are mindful that in addition to the broad goals for students and mathematical processes that are to be incorporated in all aspects of learning mathematics in school, it is beneficial to connect mathematical tasks with specific learning outcomes.

Students in the early grades of elementary school focus on identifying attributes in objects, sorting by a predetermined rule (comparing), identifying a rule used to sort objects (pattern noticing), and creating and expressing rules for sorting (generalizing). The Alberta program of studies (Alberta Education 2014) includes the following specific learning outcomes:

- Kindergarten: Sort a set of objects based on a single attribute, and explain the sorting rule.
- Grade 1: Sort objects, using one attribute, and explain the sorting rule.
- Grade 2: Sort a set of objects, using two attributes, and explain the sorting rule.
- Grade 3: Sort objects or numbers, using one or more than one attribute. (p 60)

While SET consists of more attributes than listed above, we believe that extending the trajectory in this group of learning outcomes into later elementary grades is beneficial for encouraging growth of complexity in distinguishing attributes and patterning.

Experiences of thinking mathematically in these ways supports students' emerging understanding of complex mathematical ideas in later grades.

Building on these foundational experiences, "sorting, classifying, and ordering facilitate work with patterns, geometric shapes, and data" (National Council of Teachers of Mathematics 2000, 91). In other words, experiences of thinking mathematically in these ways supports students' emerging understanding of complex mathematical ideas in later grades. Additionally, while students classify, sort and categorize information they are also learning about relationships between the objects that they are organizing. Through further analysis of the Alberta program of studies (Alberta Education 2014), we noticed this learning trajectory through multiple content substrands in elementary school.

- Patterns: Students' understanding of *same* and *different* through identifying attributes supports pattern building and pattern noticing, while creating rules leads directly to making generalizations in pattern expressing. These are important algebraic skills.
- 3-D Objects and 2-D Shapes: Students' abilities to notice attributes to describe characteristics of shapes and classify to construct properties of geometric shapes lead to developing geometric relationships and engaging in deductive reasoning.
- Data Analysis: Students' experiences in classifying inform the way students analyze data through categorizing based on attributes; sorting in different ways highlights the interpretive nature of working with data, which in turn shapes their representation of data.

Identifying attributes, sorting into groups and classifying with rules becomes important in learning how to generalize, work with geometrical shapes, analyze

Figure 5: *Matching Madness*

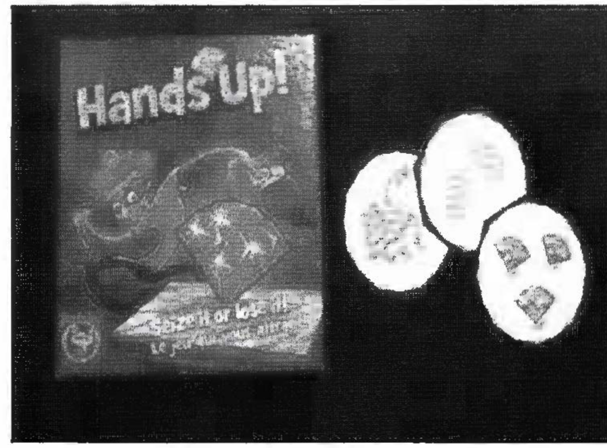

Figure 6: *Hands Up!*

data, classify information, use deductive and inductive reasoning and think systematically during problem solving (Maherally 2014). The breadth of mathematical learning that relies on the experiences students have while playing SET could support their subsequent success in learning mathematics.

Differentiating with Attribute Games

One of the encouraging aspects of incorporating commercial games into mathematical learning is that students are engaged and excited. For us, the success of *all* students is important and so we also explored ways to differentiate with attribute games. Differentiating within the game of SET is possible: for instance, instead of racing to be the first to find a set, students could be encouraged to take turns. SET has also been used in high school and college contexts to support mathematical reasoning (Quinn, Koca Jr and Weening 1999).

Beyond differentiating within the game, we located several different attribute-based games that could be used within the same classroom to address students' differing needs or across grade levels within a school. Below, we offer an explanation of how these games differ from the original SET game with some images. The different options are presented from least difficult to most challenging.

SET Junior is a double-sided board game variation of SET, in which the figures on tiles use only three attributes (shading is excluded). On one side of the board, players are limited to direct matching tiles in their hand to the preprinted SET figures on the board, akin to direct correspondence. Game play is similar to SET on the second side of the board, with some features making the game easier: (1) only 10 tiles are displayed, limiting the number of comparisons to be made; and (2) examples of sets are depicted around the board, supporting players in identifying the range of sets allowed. Both versions take about 10 minutes to play.

Matching Madness is a frog-themed game with four different attributes and a shorter playing time of about 10 minutes (Figure 5). Three differences make the game easier for children to play: (1) a greater variation in the formation of the shapes makes it easier to spot similarities; (2) players look for similarities only, rather than similarities and differences; (3) matches are made only between the top card in the discard pile and the cards in a player's hand, limiting the number of comparisons; and (4) a die directs the attention of players to focus on only one attribute in each round.

Hands Up! captures the imagination of children in its robbers-and-jewels theme (Figure 6). The game is limited to three attributes, but maintains a challenging aspect in identifying similarities or differences across each attribute. As a result of cards being constantly added to the table from players' hands, the game becomes more challenging than Matching Madness. Hands Up! adds some special cards that modify rules when played, making sets easier to make by narrowing down the choice of sets to scaffold game play. This provides momentary breaks in the intensity to find sets and allows for a more balanced play among opponents.

SET Cubed is a Scrabble-inspired game composed of dice that are rolled and then placed on a board resembling that of Scrabble (Figure 7, overleaf). This version is the most sophisticated of the attribute games because it challenges players not only to make sets with three dice but also to build new sets with

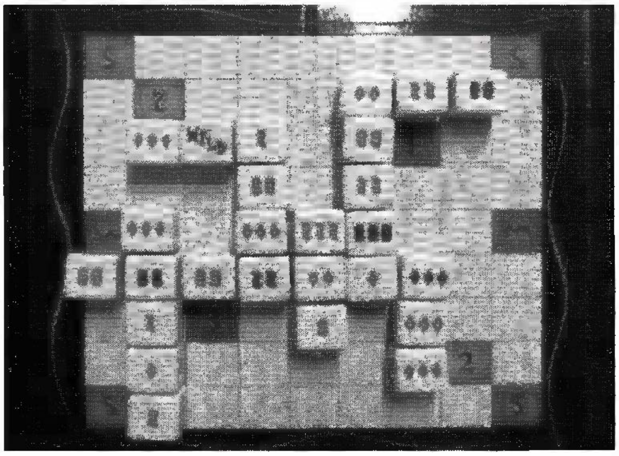

Figure 7: *SET Cubed*

the dice that are placed on the board throughout the game. Similar to SET Junior, SET Cubed does not include the shading attribute in the game; therefore, sets are made using only three attributes. The scoring is a bit more complex in SET Cubed, making it important for players to play strategically while placing their dice on the playing board.

Extending the Inquiry

One phenomenon we did not explore in this article was how students extended their exploration of the game beyond what was structured by the games. For instance, some students began to analyze the quantities and types of cards that comprised the deck. In incorporating SET into a classroom, we invite readers to attend to how their students' curiosity and interest provokes further inquiry. We imagine many productive moments of mathematical thinking!

We hope that sharing the students' engagement and mathematical thinking through the game of SET encourages teachers, parents and other educators to give the game a try. What other personal strategies are possible for children to develop? Are there other mathematical processes that arise as important in the game play? In what ways are children willing to re-engage in learning mathematics and share this with their parents? We invite you to consider how playing SET with your students may provide rich, foundational experiences for meaningful learning in mathematics class.

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