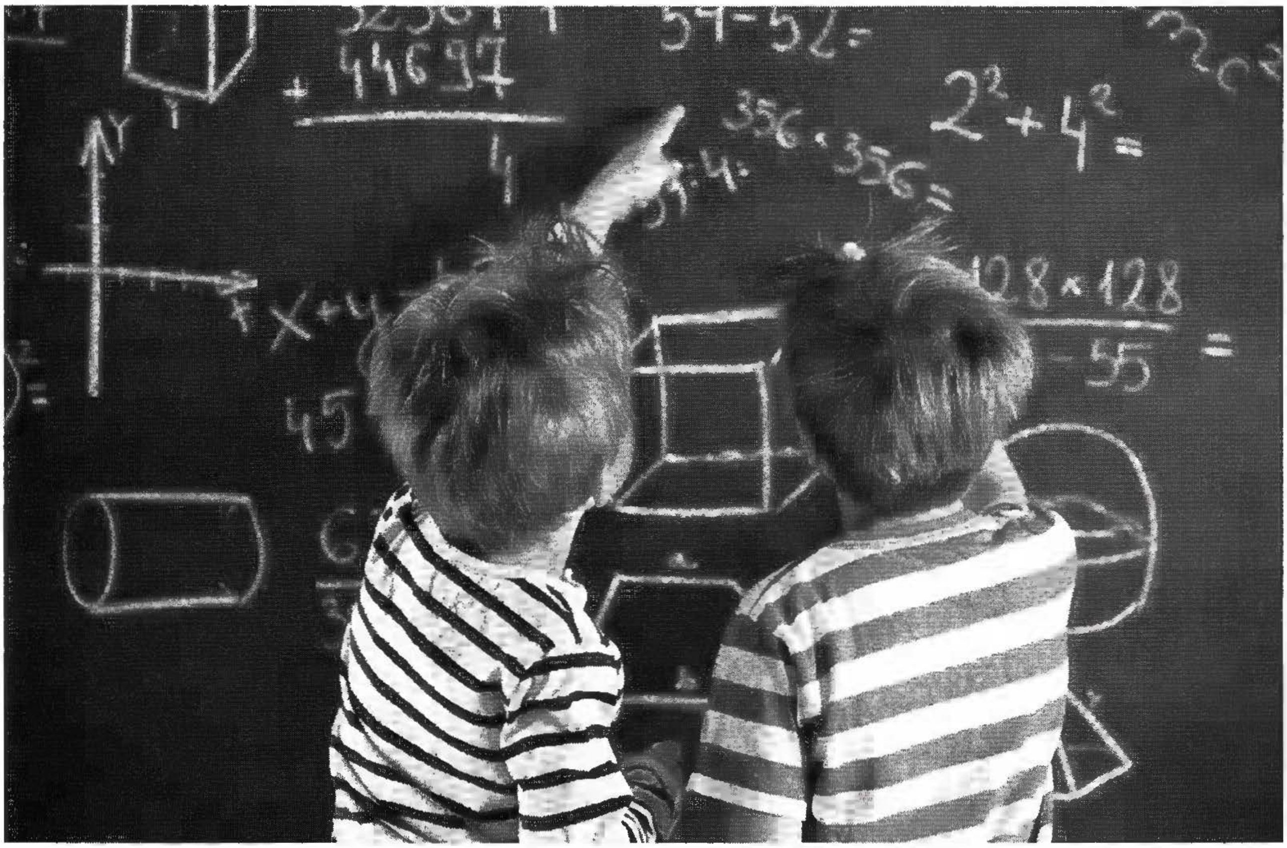
Automaticity: Building the Foundation for Mathematical Mastery

Ronald Vanden Pol



Abstract

Automaticity is the ability of a student to automatically recall basic math facts without the use of strategies or manipulatives. It is an important component in the education of students and essential for student success inside and outside of the math classroom when it comes to complex problem solving. Using comparisons to reading instruction, this paper will investigate the importance of teachers helping students automate basic facts and the steps required for students to reach the mastery level of numeracy. The research for this paper was gathered through an extensive search of relevant scholarly literature using the University of Calgary's electronic library, along with the electronic databases Education Research Complete, Taylor Francis Online, ERIC, ProQuest and Google. After reviewing the information, it was found that students who were unable to automate in younger grades had difficulty with their math education as they continued in school. The automaticity of basic math facts is a foundational part of number sense and essential to helping scaffold students to higher levels of math numeracy.

Automaticity: Building the Foundation for Mathematical Mastery

One of my strongest school memories happened in a junior high math class. The teacher was working on an example math problem on the board for the rest of the class. He quickly pointed to one of the kids while working at the board and asked, "What is eight times four?" The boy didn't say anything and the teacher stopped what he was doing and asked again.

The boy began counting on his fingers and, after running out of fingers, just took a guess. The teacher then asked him again more sternly, "What is eight times four?" The student shrugged and gave a look which seemed to ask the rest of the class, how am I supposed to know that? Our math teacher then said with disappointment, "How can a student be in junior high and still not know their multiplication tables?"

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Most teachers who have taught math during their career have come across students who struggle with basic math facts. They struggle counting to 10 or they do not understand how to carry while adding. They look at their fingers while multiplying and always require two or three guesses before finally being told that was the right guess, all while hoping they were successful. They just seem to lack the basics of number sense.

The success of any student during his or her time in school is based upon many factors. In math class, one of these important factors is a student's capacity to recall basic facts quickly and accurately. The ability of a student to automatically recall facts without the use of any strategy is known as *automaticity* and is "fundamental to success in many areas of higher mathematics" (Woodward 2006, 1). By automating their basic math facts, students can have more cognitive resources free for understanding and performing increasingly complex mathematical chores (Caron 2007; Poncy, Skinner and Jaspers 2006; Ramos-Christian, Schleser and Varn 2008; Woodward 2006). It is important for teachers to ensure that students confidently know their basic math facts, to unpack the stages required to get them to the point of mastery and to fit memorizing basic math into their regular maintenance routines at all grade levels to provide students with the best chance to succeed.

The Stages of Mathematical Mastery and the Importance of Automaticity

Over the years, various researchers have agreed on three stages of understanding math facts and strategies that would help increase fluency in students (Ando and Ikeda 1971; Ashlock 1971; Carnine and Stein 1981; Garnett and Fleischner 1983; Isaacs and Carroll 1999). The first stage involves students getting a concrete or pictorial understanding of numbers. This can be seen as counting on fingers, numbering stuffed animals, or asking dad for two more scoops of ice cream. In literacy this can be understood as identifying letters and the phonemes attached to those letters. These activities are the building blocks for numeracy and literacy.

The second stage requires students to learn or figure out different methods or strategies to remember information and connect it to facts they already know or understand. To again use literacy as a comparison, this is similar to students sounding out longer words or using words which they have read in the past to help decipher the current word. Students cannot blindly go into math or reading by randomly guessing in order to get the answers, but must start at the beginning and use basic strategies in order to solve for the correct answer. Some of the most well-known strategies are "carrying the 1" or "borrowing" when it comes to addition and subtraction. When these strategies are used properly and with understanding, then students can move on to the final stage.

The final stage is, of course, mastery. Alberta Education explains that mastery occurs "when students understand and recall facts. This allows students to apply their knowledge to different and more complex computations and to be flexible in their thinking" (Alberta Education 2014a, 1). Students are expected to understand basic facts and apply that knowledge to solve problems. Students are also expected to recall facts. While recall expects students only to use a strategy to efficiently find an answer, automaticity requires students to instantly give a response. Students are not required in Alberta to automate basic math facts, but I would argue that it is very important in both math and reading.

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Students do not require time and energy to sound out common words when reading, but rather have these words automated to the point that they do not even have to think about them. How many times during this article have you stopped to sound out a word? You just know the average word after many years of regular practice. This is also true for math. There comes a point in time when recognizing 145 or adding 12 + 5 does not take any brain power at all, but is just an automatic reaction to seeing a specific combination of numbers.

Here is another example of automaticity, from Samuels and Flor as they write about the need to improve automaticity in students for reading.

> There are numerous examples of skills developed to automaticity with which readers will be familiar. Driving an automobile effortlessly through traffic is one example of automaticity. However, it was not always so easy. Recall your first ventures behind the wheel, when considerable attentional energy was consciously applied to mechanical aspects of driving such as avoiding accidents and shifting gears. For the beginning driver, so much attention is focused on the mechanical aspects of driving that holding a conversation with a passenger while driving is impossible. But with practice, the mechanical aspects of driving become less demanding, and the skilled driver can simultaneously listen to the radio, hold a conversation, and appreciate the scenery. Skills practiced and learned to the point where they are considered "automatic" demand less cognitive and attentional energy; thus the person with expertise is capable of performing multiple complex tasks at the same time. (Samuels and Flor 1997, 108).

As you can read in this example, automaticity does not come instantly. It needs to be worked on and practised over time in order to ensure that the students are learning the basics. In math this is true as well, and it is key that students progress through the first two stages before attempting automaticity in order to help achieve mastery. Students who are struggling are often asked to do the impossible and skip ahead to stages they are not prepared for. If you have a struggling student, ask yourself what stage the student is in and then work on how to best assist that student at his or her level. Sometimes the best way to move forward is to take a step back.

First Stage: Counting Strategies

Number sense begins long before a child enters kindergarten. It starts with how many more bites they need to finish on their dinner plate. It comes out with how many more books they want their parents to read for them. It progresses when they notice that siblings have more than they have. These quantitative concepts begin long before a child steps into a classroom and can greatly affect how children will do in the classroom (Gersten and Chard 1999). Gersten and Chard found, based on home visits, that the lack of knowledge in young students "reflected a lack of experiences with adults or siblings that would facilitate the association of quantity and numbers and would lead to the development of an abstract numerical understanding" (Gersten and Chard 1999, 23). This lack of experience leads to a poor number sense for those students, and it is only when they have a concrete idea of number that it is possible to move onto the next stage. Early educators and Division I teachers need to be especially conscious of this lack of concrete understanding because some students are stuck in this stage for a long time. Providing students with counting and mathematical situations is crucial to support their growth with numeracy.

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Second Stage: Reasoning Strategies

Once students have basic numeracy skills it is possible to teach them strategies for comparing and relating numbers or solving larger problems. These strategies are used to improve the accuracy of student answers. Some examples for addition and subtraction given by Isaacs and Carroll (1999) are

1. basic concepts of addition; direct modelling and counting all for addition (number lines);
2. the 0 and 1 addition facts; counting on; adding 2;
3. doubles (6 + 6, 8 + 8 and so on);
4. complements of 10 (9 + 1, 8 + 2 and so on);
5. basic concepts of subtraction; direct modelling for subtraction;
6. easy subtraction facts (-0, -1, and -2 facts); counting back to subtract;
7. harder addition facts; derived-fact strategies for addition (near doubles, over-10 facts);
8. counting up to subtract; and
9. harder subtraction facts; derived-fact strategies for subtraction (using addition facts, over-10 facts). (p 511)

These strategies are similar to the approach that the Government of Alberta has put into the Alberta program of studies in order to encourage mental mathematics—the ability of a student to use different math strategies reliably and accurately (Alberta Education 2014b, 5). These are not limited to just addition and subtraction, but exist throughout the math curriculum in all units. Learning different strategies is a key component of the second stage of mastery because it allows for students to find ways to accurately solve problems using reliable steps.

The most important thing to remember is that students learn numeracy skills through strategies that help them, as individuals, solve problems more fluently (Baroody 2006). For past generations, teachers relied on the strategies they were taught to teach their own students, which worked well for the majority of learners. The problem is that they did not work for every student. Most people reading this can think of a kid in their class growing up that did not seem to "get" math. The teacher tried teaching the strategy over and over again, but it did not stick. For some students, solving this issue may have required going back to the first stage and working on building counting skills, but for others, it just required adapting strategies to fit the needs of a particular student. There can be different strategies for different problems, but students need to find what works for them.

Third Stage: Mastery

The final step in mathematical understanding is mastery. Students are able to solve problems using their abilities to recall and understand number facts. Alberta Education explains to teachers that "Mastery is a progression of learning. For example, students work towards mastery of multiplication and related division facts, beginning in Grade 3" (Alberta Education 2015, 1). However, it is important to know that this happens at different times for different students. Students need to have spent an appropriate amount of time working on their counting and reasoning strategies before they can be expected to fully understand and use their basic fact knowledge. Practising automaticity while working on strategies is one way to help them achieve mastery.

The Importance of Automaticity

"Automaticity is an immediate and unconscious retrieval of answers, which suggests a rate faster than one answer per second" (DeMaioribus 2011, 6). Automaticity is different than just being fluent with numbers, because there is an expectation that this process happens without thought. Why is counting on fingers not good enough and why must a student be able to do these basic math calculations automatically? "Although correct answers can be obtained using procedural knowledge, these procedures are effortful and slow, and they appear to interfere with learning and understanding higher-order concepts" (Hasselbring, Goin and Bransford 1988, 2).

If a child is reading a novel and is struggling to pronounce and decipher every word, do you think he or she would have much luck comprehending what the novel is actually about? By automating certain words and reading without conscious thought, our brains are then more open to comprehending the text. If the words are confusing to the students, how do we expect them to understand that sentence or paragraph? If students are struggling with the basic calculations in a multiple-step problem, should we expect them to be able to successfully figure it out? In fact, "without this seemingly simple set of knowledge, by eighth grade, students are virtually denied anything but minimal growth in any serious use of mathematics or related subjects for the rest of their school years, and most likely, the rest of their lives" (Caron 2007, 279). Time spent practising basic math facts is as important as time spent practising reading. The goal is to improve automaticity and allow students to focus on the problem instead of just the numbers.

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"Researchers explored the devastating effects of the lack of automaticity in several ways. Essentially they argued that the human mind has a limited capacity to process information, and if too much energy goes into figuring out what 9 plus 8 equals, little is left over to understand the concepts underlying multidigit subtraction, long division, or complex multiplication" (Gersten and Chard 1999, 21). Automaticity is so important to students' numeracy skills because without it students' skills are extremely limited. We need students to automate as many of these basic facts as possible, so that as much attention as possible is freed up to deal with the most complex part of the problem.

Helping Students with Automaticity

In my own Grade 6 class we commit a few classes a month to improving student automaticity with basic facts. While it is not a part of the program of studies for Grade 6, Alberta Education (2014b) writes that teachers should work with students over time to "refine their strategies to increase their accuracy and efficiency" (p 9). I try to take time to go over basic addition strategies to help struggling students see success and already-successful students see even more growth. This past month we worked on adding doubles. We first went through all of the doubles that did not require regrouping and did a few math minutes using just those problems. We then went over doubles that did require regrouping and practised equations related to those numbers. Because what we were practising required remembering only small amounts of basic facts, most students had very little trouble with the problems. Those that did have some issues spent extra time practising only the equations they struggled with. Very quickly, all students were able to see success working with adding doubles.

The next time we worked on basic facts, we practised doubles + 1 without regrouping (4 + 5, 2 + 3). After students became proficient in those, we moved on to those that required regrouping. After that, doubles + 2 and then making 10s. Once most of the strategies had been shared and practised I provided students a chance to play adding games, like war with cards; adding dice; and rock, paper, numbers (two players stick out fingers like rock, paper, scissors and the first person to figure out the answer wins).

My class also has a set of Chromebooks. I, and researchers like Nelson et al (2013), have found success in using computer programs that can give students who struggle instant feedback as a way to improve fluency without the use of any mnemonic strategies, in the same way a parent reading with a child can give instant feedback and correction. For example, xtramath.org is a website that will assess a student's basic math facts abilities and then give the student questions on whatever needs the most help.

If a child is reading a novel and is struggling to pronounce and decipher every word, do you think he or she would have much luck comprehending what the novel is actually about?

At my school, a colleague uses a program that helps Grade 3 students scaffold their learning through different reasoning strategies. The class works on these strategies and practises automaticity through flashcards and math minutes. Students progress on a continuum moving from simple equations, such as adding 1, to more difficult facts once they have achieved a satisfactory level of recall and understanding. This practice is done daily for about 10 minutes per class. They focus only on addition before moving to subtraction and then multiplication and division facts. The program is modelled like a karate dojo and uses belts as a theme to encourage students, make it fun and keep them motivated. Daily practice gives students a chance to commit facts to memory and improve automaticity, which helps their problem-solving abilities in other outcomes and mathematical concepts.

While the practices mentioned above may seem like a lot of work for higher grades, it is a review of strategies they already know and requires just a little bit of time to practise. A few classes a month for older students is more than enough to maintain their current abilities and improve their speed and accuracy. Students in Grades 3 to 5 require more regular practice to learn strategies and also to play games to keep students engaged in working toward automaticity. The important thing to remember is that success in automaticity is often found in small, manageable bits. A little bit at a time is much better than everything all at once. For most students in younger grades (K–2), automaticity may be a bit early and it is probably best for them to stick to counting and simple reasoning strategies.

Conclusion

In Alberta, the program of studies clearly lays out the expectations for teaching the basic math facts from Grade 1 up to Grade 6. It is [important] for teachers during these years to ensure that they are not only creating a concrete understanding of numbers through manipulatives and strategies, but also promoting and practising automaticity to support students getting to mastery. If students are not ready to move on to the next stage, more and more practice may not be the right answer. However, when a stage is successfully realized, regular practice and feedback is a vital part of the process. The importance of automaticity is not just in helping students get the correct answer, but also about helping them achieve the correct answer while using less mental strain and allowing them to freely think about larger problems. Automaticity is about building a solid foundation on which other teachers are able to support students in understanding more and more complex problems. It is crucial that students have the automaticity needed to work through the large, complicated, real-world, multistep problems that they will encounter in daily life. And it is up to us as professionals to ensure that students are reaching this goal.

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Ronald Vanden Pol obtained a BA and a BEd from The King's University, in Edmonton, and an MEd from the University of Calgary. He currently teaches at Fort McMurray Christian School, in Fort McMurray, Alberta, along with his wife, Erica Vanden Pol, who contributed greatly to this article. He can be reached at ronald.vandenpol@fmpsd.ab.ca.