Making Sense of Problem Solving and Productive Struggle

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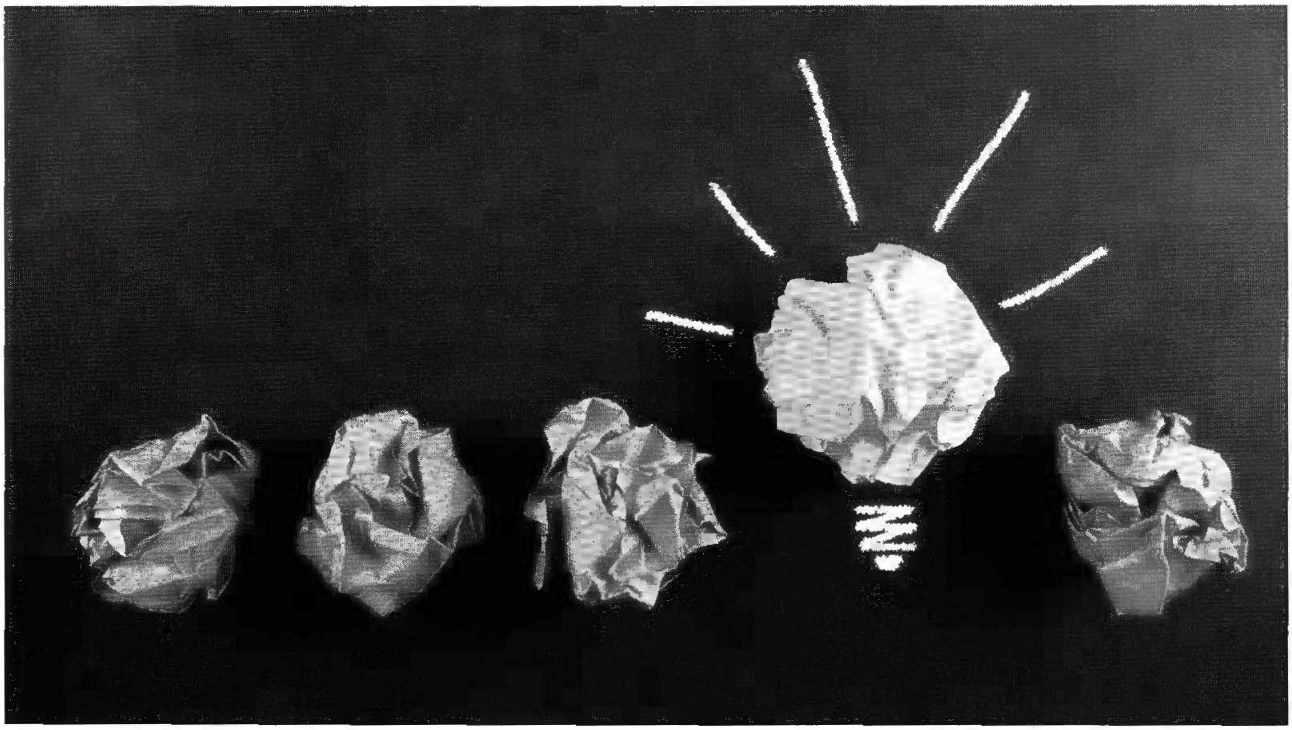

Introduction

How do we help children develop their abilities to solve problems? Can this problem-solving ability be developed so that it is also available in nonmathematical situations? These two questions inspired us (a group of teachers and a university professor) to embark on a professional learning journey concerning the teaching and learning of problem solving. In this paper, *problem solving* should be taken to mean the heuristics and metacognitive regulation available to a problem solver while navigating an initially unknown scenario (Schoenfeld 1992). We started with linear conceptions of the nature of problem solving and minimal conceptions of how it should be taught.

Can this problem-solving ability be developed so that it is also available in nonmathematical situations?

By watching children try to solve problems, and by adjusting our activities based on these observations, we have developed a rich understanding of problem-solving ability and how it can be developed. In this paper, we focus on productive struggle, which we believe is a fundamental component of problem-solving ability.

Context for a Professional Journey

Our professional learning journey was structured on principles of lesson study (Fernandez 2002), with school division support of teacher release time. We regularly developed and refined problem-solving activities by coplanning, coteaching and codebriefing, always focusing on our collective observations of children during these activities. As teachers, we deliberately decided to focus on teaching problem solving, embedded within our own practice and teaching concerns. All of us were concerned with how to help all children to successfully solve problems because we experienced minimal success in the past.

We wanted every child to be engaged and successful. We also knew that mathematics education organizations such as the National Council of Teachers of Mathematics recommend shifting instruction toward inquiry and scaffolding the learning of children, rather than only tightly directing the learning of children through modelling and practice (eg, NCTM 2014). But we didn't know how to accomplish these goals. As it turned out, our observations of children helped us to broaden our conceptions of problem-solving ability and to achieve these goals.

In particular, we developed the notion of *productive struggle*, which is the core idea unveiled throughout this paper. The idea starts with our initial belief that math teaching is successful when it makes learning simple. And yet, mathematics is not simple. We have also observed the debilitating effect of math anxiety on learning and students' lack of interest in mathematics. In the past, trying a word problem resulted in frustration for many children. Hence, we started our journey with an overall skepticism of problem-solving tasks because we feared that they were too difficult for the children. In the past, we tended to adopt easier word problems, if we tried them at all. "Keep it simple to ensure sucess for all" was our mantra, even though some children still became frustrated.

Productive struggle is a significant part of our problem-solving pedagogy because it is a fundamental component of problem-solving ability.

We faced our belief in the need to reject difficult problems very early in our journey. We decided to try the Neighbouring Numbers problem (see above) in a Grade 1/2 class, despite our concern that the problem was too difficult. Our planning session focused on what to do with specific students when they quickly gave up on the problem. We were surprised when our predictions of excessive frustration did not come to fruition. This started our journey to uncover why we needed to change our beliefs about problem-solving ability. We have learned to accept struggle and have developed our professional fluency in scaffolding struggle so that it is a space for learning about problem solving. Productive struggle is a significant part of our problem-solving pedagogy because it is a fundamental component of problem-solving ability.

Snapshots of a Journey to Productive Struggle

In what follows, we use two problem-solving activities to illustrate the idea of productive struggle, both as a fundamental component of problem-solving ability and as a pedagogic technique. The first problem, Neighbouring Numbers, involves arranging the digits 1 to 8 into a network so that no two neighbouring numbers are consecutive. We launched the problem with a short story about an apartment block (symbolized by the diagram in Figure 1) where the numbers 1 to 8 live. The premise of the story was that, when the numbers get home from work, they want a break from the order required at work and choose to live so that no neighbours are consecutive. Figure 1 is partially completed and illustrates two neighbours that are consecutive (3 and 4), which is not allowed. The students worked on the problem with a partner (see Figures 2 and 3—both arrangements are incorrect). We consolidated the activity by having students describe to the class their strategies to solve the problem.

When planning for the Neighbouring Numbers problem, many of us struggled to find a solution, hence our concern that it would be too difficult and cause frustration. So we developed the following scaffolds in anticipation of struggling students:

1. Based on their current work, we could ask students what would happen if a certain number was placed in a certain location, in order to illustrate allowed and not-allowed arrangements of numbers.
2. If students became frustrated to the point where they might give up, then we could suggest placing a 1 or an 8, or both, in one of the centre locations.

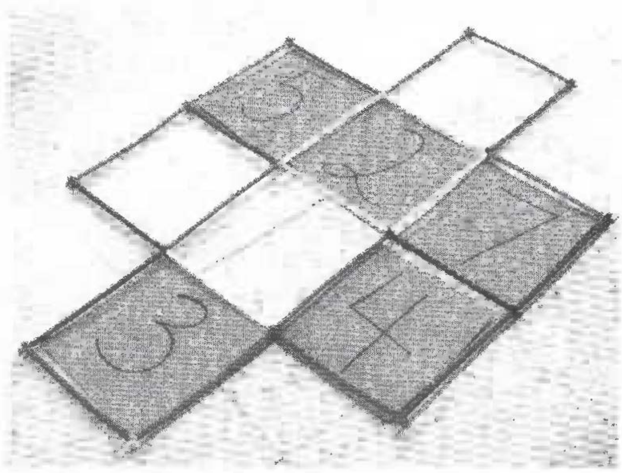

Figure 1: *Introducing the Neighbouring Numbers problem*

(The 1 and the 8 are located in the centre for any correct solution.)

We also anticipated that some students might quickly find a correct arrangement, and we could challenge them by suggesting they find a different solution. Finally, we concerned ourselves with engaging the children because we had noticed that students sometimes resist even starting to try a problem. We felt that when children find a problem interesting, they are naturally motivated to try to start solving the problem, which we hoped would be accomplished by the apartment-block story.

Despite our concerns about the problem's difficulty, the story motivated all the children, and we observed all the Grade 1/2 children sustain their interest in the problem for about 30 minutes. All children immediately started working on the problem, trying to find a correct arrangement of the numbers. They would excitedly raise their hand when they thought they had found a solution; in most cases, there was a problem with the solution and we would provide an "Are you sure?" scaffold. To our surprise, the children happily kept trying to find a solution. We observed some students struggling, but encouragement was enough to sustain their engagement. While consolidating the activity with the class, children

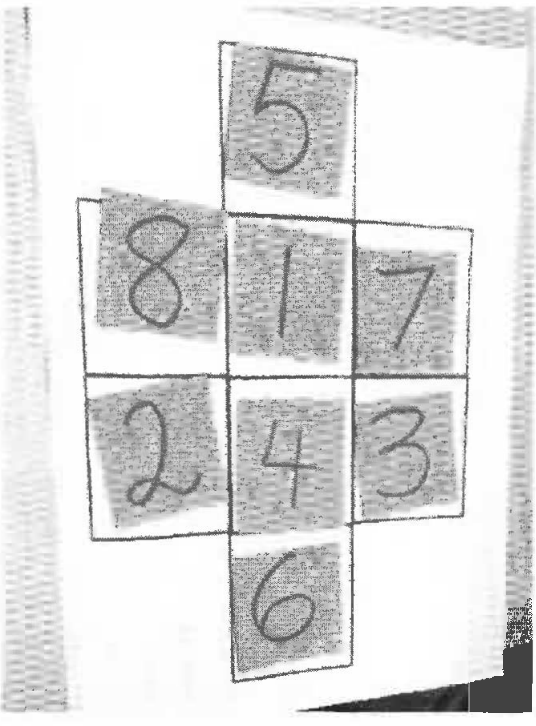

Figure 2: *The arrangement is not allowed because 1 and 2 are consecutive neighbours.*

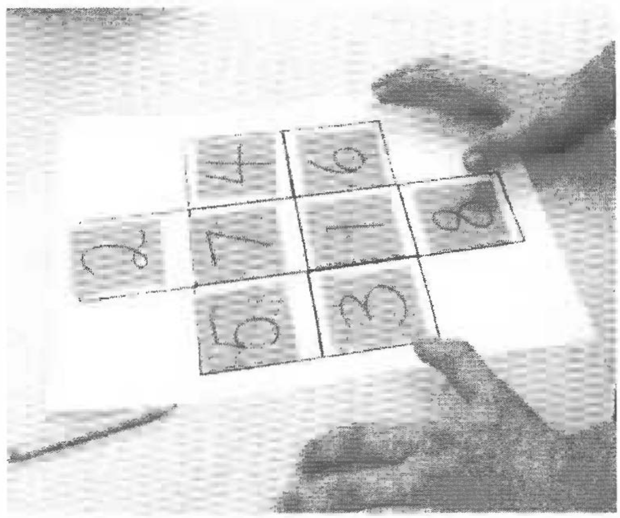

Figure 3: *The arrangement is not allowed because 6 and 7 are consecutive neighbours*

shared strategies such as "fixing problems" and "placing consecutive numbers far apart." During this discussion, we emphasized their problem-solving strategies and praised their effort and willingness to keep trying.

Even though many children struggled to find a solution, and some did not find a solution, the activity was a success for all. We were surprised, so our debriefing session as teachers focused on why a problem that seemed too hard was still a success. We noticed that all children naturally used trial and error as a problem-solving strategy. In part, the problem was successful because the children were engaged and the children quickly made progress by simply trying different arrangements of numbers.

We also noticed that children were willing to struggle. This contradicted our "make it easy" assumption. We had privileged protecting children from failure to such an extent that we couldn't see the

We praised their effort and willingness to keep trying.

benefit of temporary failure. At this moment, we began to realize the danger of overprotecting children from failure. Regulating frustration is an important goal in education, so some frustration was a reasonable event in school problem solving. Our challenge as teachers was now to develop our ability to support children through their frustration.

We tried the same problem again in a Grade 1 class, staying conscious of supporting frustration while scaffolding the students. Supporting frustration allowed us to notice how different students used the trial-and-error problem-solving strategy. Some tended to adjust the whole arrangement, which tended to make it more difficult to use the previous trial to inform their thinking. Other students focused on fixing an error to a given arrangement, which would usually generate an unexpected new problem with the arrangement. A few students tried to plan ahead by asking questions such as "If I fix this problem by switching these numbers, do I create a new problem?" These variations illustrate some of the nuance of problem-solving ability—in this case, how there can

Resist telling students what to do.

be differing degrees of sophistication in the use of an incorrect arrangement to support thinking. Our class consolidation focused on the different ways that children used trial and error. We reinforced how every child was problem solving, and also encouraged the students' willingness to keep trying.

When we debriefed as teachers after the activity, our observations of the differences between children's use of trial and error reiterated for us a significant problem of teaching problem solving: How much [should] we help children when they struggle? Understanding what children could do to solve the problem helps us to develop scaffolds for the next time. For example, a teacher could encourage a student to look more closely at how to fix a specific problem in an arrangement, without actually finding and fixing a problem for the student. It is in this scaffold that we realized an important pedagogic disposition: *Resist telling students what to do*. When we tell students what to do, we have minimized the potential for children to learn about problem solving. When we tell students what to do, we eliminate the potential for children to be frustrated but still succeed, and these kinds of experiences are a life skill. We decided that our response to frustration should be to provide the minimal amount of support needed for children to keep trying to solve the problem.

During our journey, we had been developing our professional ability to provide a minimal amount of support, in both planning an activity and in-the-moment teacher decision making. We are convinced of the need to resist telling students what to do, in favour of a minimal amount of support. The second problem we use to illustrate productive struggle was tried later in our journey. Our work with this problem, called Handshakes at a Party, illustrates our deepening understanding of scaffolding productive struggle as a fundamental problem-solving ability.

The Handshakes at a Party problem considers how many handshakes occur if everyone at a party shakes hands with everyone else exactly once. We launch the problem by acting out the problem with five students. While five students perform the handshakes, the rest of the class counts how many handshakes occurred. After several tries acting out the problem, the class agrees that there are 10 handshakes. We then ask the students to determine the number of handshakes for a larger number of people. In a Grade 2 class, we asked the students to try 10 people, with an extra challenge to try 20 people. In higher grades, we ask the students to find the number of handshakes if 20 people are at the party, and challenge students to determine a general method regardless of the number of people at the party. In what follows, we described what happened when we tried the problem for the first time, which was with a Grade 2/3 class.

Many students immediately guessed that there would be 20 handshakes among 10 people because it is double of the situation with 5 people. This is wrong, but we have learned to resist telling students what to do, so we used an "Are you sure?" scaffold. This immediately caused confusion for the students. Students often expect teachers to respond with "Right" or "Wrong" when a student gives an answer. We did neither, and the students struggled. For some students we suggested they try to produce a convincing argument without relying on their "doubling"

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observation, whereas for others we suggested they model or act out 10 handshakes to see if they were correct. With a few students who continued to struggle, we suggested they try to make a drawing or try the case of handshakes among 6 people. With patience, all students used blocks or a diagram to model the problem in order to count the number of handshakes. Some students used their model to recognize a pattern for adding up the number of handshakes: with 5 people, the number of handshakes is 4+3+2+1; and with 10 people, the number of handshakes is 9+8+ … +2+1.

When we debriefed as teachers after the activity, we saw a pattern in our scaffolding. The minimal amount of support depends on our knowledge of the children and on what type of progress they have made on the problem. Rather than tell the students they were wrong, we asked why. When struggle appeared to be at the point of excessive frustration, we provided further minimal guidance, to tell more without doing the problem for the student. We are looking for just the right amount of guidance, so that frustration is a learning opportunity and not interpreted by the student as complete failure. Our final analysis suggested that our scaffolding decision making is grounded in our professional relationships with each student, and with ensuring that the student experiences at least some problem solving.

Problems and struggling are intimately woven together.

Describing Productive Struggle as a Problem-Solving Ability and Pedagogy

Productive struggle occurs when a child learns something about problem solving when he or she cannot immediately solve a problem; productive struggle cannot happen when a child does not problem solve (Warshauer 2015). Struggling to make sense of mathematics is a necessary condition of learning mathematics (Hiebert and Grouws 2007). We concur: if a child does not struggle, then we believe that the experience was not problem solving for the student. Problems and struggling are intimately woven together. On the other hand, too much struggle, to the point of frustration, is not productive struggle because the process of problem solving is stopped. Negative emotions will mitigate against a child learning something about problem solving.

Given the foundational role of productive struggle as a problem-solving ability, we have developed a productive struggle pedagogy. Although we believe that this pedagogy is grounded in our professional decision making emerging from the context of a problem solving activity, there are still three recommendations we can make for other teachers.

First, cultivate an open-minded disposition for what students can do, and resist the desire to tell them what to do. If we are closed minded, we cannot see what students can do. If we are open minded, we are able to notice and tell students what they did. This is a metacognitive turn: when we label for students what they did as problem solvers, they become aware of what they can do, consistent with Schoenfeld's (1992) recommendations. Problem solving is no longer an "I can" or "I cannot" experience. Students can realize the problem-solving strategies they can use, even if they ultimately did not solve a problem.

Second, teachers should focus on problem-solving processes. If the focus is on the answer, then those children who do not find an answer are labelled as failing. On the other hand, if problem-solving processes are the goal, then children's problem-solving strategies and abilities can be developed. Given an open-minded disposition, teachers can notice and provide positive feedback for the strategies used by children. In our experience, all children are able to use at least one problem-solving strategy, although this ability differs based on the problem and the child. We are always able to find something in the activity of a child that can be labelled as problem solving. Here, finding the right scaffold is paramount. Just enough help is given so that a child is still challenged but not overly challenged. Our professional relationships with children guide our decision making to find the right balance. Every child is different, so even though the class works on the same problem, our sense of what problem-solving ability we could observe and the type of scaffolds needed varies for each child.

Finally, we plan for productive struggle by developing problem-solving activities that account for the diversity of learning needs found in a classroom. The core problem should always be rich in possibilities in how it could be solved, how far a problem could be explored and possible solutions. We always develop an engaging launch to the problem. We try to phrase the problem in an open-ended way, which allows for adaptations as children make progress on the problem. An open-ended and complex problem is a necessary condition for creating conditions in which productive struggle can occur.

While planning, we consider closely how much help to provide the whole class initially by asking two questions. Do we tell the students to model the problem using a manipulative? If we tell students which manipulative to use, the focus could switch from problem solving to the manipulative, and it is important for children to learn how to model a problem. Hence, it is valuable to provide opportunities for students to model the problem in a manner that they devise and are comfortable with. Further, do we provide the students with a preorganized recording sheet? Organization is a problem-solving strategy, and we may notice that some children lack an ability to organize their thinking and data while solving a problem. If we believe a problem has natural opportunities for students to recognize the need to be organized, then we do not provide a recording sheet. On the other hand, if we feel the problem has other problem-solving possibilities, and organization would be an extra factor that could cause too much frustration, then we do provide an organized recording sheet. There are no absolute rules for making these decisions, as it often depends on the observed abilities of the child and more specific goals for the activity.

Although planning for productive struggle is not a well-defined process, the process itself prepares us for in-the-moment decisions that effectively provide the minimum amount of support needed by each child. For example, when planning the Handshakes at a Party problem for a Grade 2 class, we wondered about both organization and modelling the problem. Recognizing an organized procedure for shaking hands is tantamount to counting the number of handshakes accurately. We decided to begin the launch of the problem, with five people, with a disorganized approach that made it difficult to track the handshakes Students were then motivated to come up with a better way to organize the handshakes. This led to also modelling an organized approach: person 1 shakes hands with persons 2, 3, 4, and 5; person 2 shakes hands with persons 3, 4, and 5; and so on. Each subsequent person shakes hands only with the remaining persons, and the last person will have nobody to shake hands with because this would repeat a handshake. During the launch, we tried to guide the class to this approach, but were more than willing to just tell the class by modelling this procedure. On the other hand, we decided that creating a model for solving the problem was rich in possibilities: students could use a diagram or manipulative to track people and handshakes, or they could act out the problem with smaller numbers of people and generate a pattern. Thus, we did not provide any help to the students by modelling the problem. One pair of students used a patterning approach with one to five persons at the party to start the pattern; the rest of the students used blocks or a diagram. We also noticed some students restart their model to help them keep track of counting the handshakes, which was an unexpected opportunity to label organization as a problem-solving strategy. Importantly for us, most students developed a model with little or no help from us. When students struggled and wanted help, we used prompts such as "Can you draw a picture?" rather than risking too much help by showing them how to draw a picture.

Our planning helped us decide how much help to provide when students did struggle. Our focus on an open-minded, open-ended and process disposition, use of planned and in-the-moment minimal-help scaffolds, and deliberate consideration of possible problem solving strategies and goals leads us to notice problem-solving ability that we would not have noticed otherwise. Our conversations with children while solving a problem and while consolidating the activity focus on the problem-solving processes and strategies we have noticed, so that every student can recognize the problem solving they did and feel success as a problem solver.

Conclusion

Fundamentally, with an engaging, well-structured yet open-ended problem-solving activity, we always observe children getting stuck and trying again. We are able to label this perseverance as a problem-solving strategy—students are problem solving. Productive struggle has transformed our sense of the nature of problem solving. It is a foundational component of problem-solving ability. It is a core principle of our problem-solving pedagogy.

References

Fernandez, C. 2002. "Learning from Japanese Approaches to Professional Development: The Case of Lesson Study." *Journal of Teacher Education* 53, no 5: 393–405.

Hiebert, J, and D A Grouws. 2007. 'The Effects of Classroom Mathematics Teaching on Students' Learning." *In Second Handbook of Research on Mathematics Teaching and Learning*, ed Frank K Lester Jr, 371–404. Charlotte, NC: Information Age.

National Council of Teachers of Mathematics (NCTM). 2014. *Principles To Actions: Ensuring Mathematical Success for All.* Reston, Va: NCTM.

Schoenfeld, A H. 1992. "Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics." In *Handbook of Research on Mathematics Teaching and Learning*, ed D A Grouws, 334–70. New York: Macmillan.

Warshauer, H K. 2015. "Strategies to Support Productive Struggle." *Mathematics Teaching in the Middle School* 20, no 7: 390–93.

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