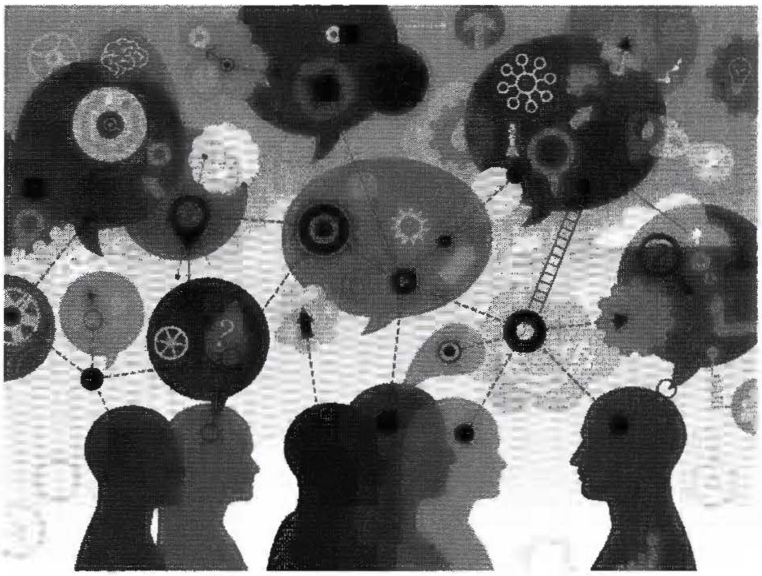
Research Article

Intersections in Reasoning Within Science and Mathematics

Ashley Pisesky, Janelle McFeetors and Mijung Kim

Elementary school classrooms are rich sites of children's mathematical and scientific thinking. As a preservice teacher (Ashley Pisesky) and researchers who have taught in schools (Janelle McFeetors and Mijung Kim), we are privileged to watch and listen to children's excitement as they make sense of a new mathematical idea or figure out a scientific way of problem solving. Observing colleagues in classrooms, teachers often plan in interdisciplinary ways knowing that children's learning is more meaningful when they connect ideas. With curricula packed with content, integrating content areas also helps to ensure that all outcomes are addressed in a school year. Teachers and students do not necessarily live out artificial distinctions between content areas in their classrooms.



With the advent of a STEM (science, technology, engineering and mathematics) approach, more resources are available for integrating science and mathematics. These resources contain activities students find engaging. However, a critical viewing reveals that much of the early implementation of STEM results in activities that prioritize one subject area over another where either mathematics serves the scientific ideas with technical skills or a mathematics idea is dressed up in a scientific context. This results in a coordinate approach (Babb et al 2016) being supported, rather than integration. Additionally, interdisciplinary teaching of science and mathematics is not assumed in curricular documents written for separate subject areas.

On one hand, teachers are balancing the expectations and realities of children's learning. While on the other hand, resources and curricula provide nominal support for integration of science and mathematics. We see an area with great potential for growth, given thoughtful design of opportunities for children to experience synchronicity in thinking across multiple subject areas to support integration. As there are no boundaries among disciplines in everyday problems, children as problem solvers do not experience separation or differences in mathematical and scientific reasoning; that is, children's reasoning processes intersect and integrate across disciplines, seeking answers and solutions to problems.

We hoped research-based literature would help us find intersections between mathematics and science learning. Our main intention was to move beyond tasks where mathematics and science coexist and to examine in finer detail how children think within the subject areas. As we reflected and discussed possible intersections, reasoning arose as an interesting site to explore. We framed our inquiry around the question: To what extent is the process of reasoning a possible intersection between mathematical thinking and scientific thinking in elementary school classrooms?

Because of the vast quantity of studies depicting children's reasoning both in mathematical and in scientific contexts in elementary school, we chose to first pursue this inquiry by understanding current research literature. The literature review would inform our understanding of how reasoning is referred to in mathematics and science in order to identify possible intersections.

Reasoning as Characterized in Curricula

To understand any intersections that may exist between science and mathematics, we needed to know how researchers were discussing reasoning in both subjects independently. The Alberta program of studies is a good place to look for working definitions regarding reasoning.

According to the mathematics program of studies, "mathematical reasoning helps students think logically and make sense of mathematics" (Alberta Education 2016, 6). While the benefits of students using reasoning are explicit, what defines reasoning is ambiguous. Reasoning, rather, is characterized by the actions students carry out in the process of reasoning and problem solving. For example, "analyze observations, make and test generalizations from patterns . . . . use a logical process to analyze a problem, reach a conclusion and justify or defend that conclusion" (2016, 6). Broad in nature, these actions could be woven throughout all of the content strands as children describe and support their mathematical thinking.

A commonality between both characterizations and emphases is that of problem solving.

Similarly, the science program of studies has no direct definition of reasoning, yet comparable language describes the qualities of reasoning. For example, the science "program provides a rich source of topics for developing questions, problems, and issues, that provide starting points for inquiry and problem solving" (Alberta Education 1996, A.2). As developing critical thinking skills is a main goal of science education, the science program of studies clearly emphasizes critical thinking with "evidence." The importance of evidence is shown in General Learner Expectations as follows: "critical-mindedness in examining evidence and determining what the evidence means" and "a willingness to use evidence as the basis for their conclusions and actions" (p B.24). The program of studies clearly emphasizes critical thinking and evidence-based reasoning as part of scientific thinking.

A commonality between both characterizations and emphases is that of problem solving. In the problem-solving process, children observe, collect data and information, analyze, and generalize with and for patterns. Interestingly, even though the science program of studies provides a similar characterization as to the definition of reasoning in the mathematics program of studies, the term *reasoning* is never formally defined. This might speak to some of the issues that arise when disciplines use different subsets of languages that have similar definitions.

Reflecting on the characterizations of reasoning from the respective programs of studies only gave us a general starting place. To continue in our inquiry on reasoning as a possible intersection between scientific thinking and mathematical thinking in children, we needed to locate more finely nuanced descriptions of reasoning. Framed by the curricular understandings of reasoning, we undertook the following inquiry.

Inquiry Process

Much has been written about reasoning in both mathematics education and science education. To begin, we scanned a few seminal readings in both mathematical thinking and reasoning (for example, English 1997; Mason, Burton and Stacey 2010; Polya 1954) and scientific reasoning and argumentation (for example, Erduran and Jimenez-Aleixandre 2007; Kuhn 2010; McNeill 2011; Osborne, Erduran and Simon 2004) to contextualize current research.

We then searched for current journal articles in databases, such as JSTOR, EBSCOHost, ProQuest, ERIC and the University of Alberta library catalogue. The search terms, in combination with either mathematics or science, included *elementary, reasoning, argumentation* and *proof*. The list of articles was substantial, and eventually searching with various keywords did not produce any new articles beyond what was already collected.

To collect a manageable group of readings in each discipline, we delineated the bounds for searching through the following selection criteria. Our selection focused on journal articles and excluded conference proceedings and books, as articles are usually the venue through which researchers share their most current findings. We looked for peer-reviewed reports of empirical studies published in academic and professional journals. To use the most recent research available, we used a date range of 2,000 to the present. In the end, we used about 40 papers for this literature review.

We did the initial analysis by reading all the papers to see how reasoning was defined and discussed within each discipline to ascertain the range of ideas. We found that researchers explained their understanding of reasoning through various examples that provided insight into characterizations initially outlined by them. We kept detailed notes on what type of reasoning the researchers explored, how they defined it, how they observed children developing reasoning and noteworthy findings. Throughout the reading and summary writing, prominent words began to emerge and were used to categorize articles. For each category, an overall analysis was written.

Major Themes of Mathematical Reasoning

After reading about 20 articles focused on mathematical reasoning, we identified 10 general themes regarding how researchers discuss reasoning in mathematics. These general themes can be sorted into two broader categories: processes of reasoning and forms of reasoning, depicted in Table 1.

Processes of Reasoning	Forms of Reasoning
Conjecturing	Deductive
Justifying	Inductive
Specializing	Plausible
Problem solving	By analogy and metaphor
Creating proofs	By contradiction

Table 1. *Ten themes within two categories for mathematical reasoning.*

Processes of reasoning encompass the ways in which children engage in acts of reasoning, also described as the verbs of mathematical reasoning (McFeetors and Palfy 2017). Conjecturing and justifying are integral processes often explored in literature. Forms of reasoning refers to logical chains of statements and their structural aspects that are conventions within mathematics leading to proofs. Interestingly, Polya's early work on deductive (demonstrative) and plausible reasoning has maintained high importance in recent literature. Rather than exploring all of the themes below, we describe two themes from each category that represent the best possibilities for intersection between mathematical reasoning and scientific reasoning in elementary school classrooms.

Processes of Reasoning

Conjecturing can be defined as offering "a statement which appears reasonable, but whose truth has not been established" (Mason, Burton and Stacey 2010, 58). Often children will express a conjecture based on a pattern that is emerging in their mathematical thinking, some initial sense they are making of a mathematical problem akin to a guess or hunch. Sharing a conjecture with others allows for investigation that could lead to justification or modification, where mathematical reasoning "often begins with explorations, conjectures" (NCTM 2009, 4). As a specific example for classrooms, Houssart and Sams (2008) had upper elementary school children play Lines, a game similar to Connect Four. One student pointed out a good starting place and conjectured about the value of the move, "because it's right in the middle and we could go up across, diagonal, loads of different ways" (p 62). Even though many students were not convinced initially, by the end of the sessions they had tested the conjecture sufficiently to show that they had a better chance of winning with a central start. Interestingly, Lane and Harkness (2012) noted that when students skip the process developing conjectures through exploring the problem context, they are unable to justify solutions convincingly. These examples demonstrate that it is important for children to form initial conjectures, evaluate the conjectures and continue to modify or offer new conjectures to lead toward convincing solutions to mathematical problems.

Justification is another key process in children's use of mathematical reasoning. In fact, many researchers refer to reasoning interchangeably with justification. They state, "mathematical reasoning . . . involves justifying" (Thom 2011, 234) or define reasoning as "the ability to justify choices and conclusion" (Johnsson et al 2014, 20). Staples, Bartlo and Thanheiser (2012, 448) see justification as "an argument that . . . uses . . . mathematical forms of reasoning," while Mason, Burton and Stacey (2010) see it as convincing yourself and others of why a conjecture or solution works all the time. As a specific example for in Grade 6 classrooms, Mueller and Maher (2009) used tasks with Cuisenaire rods, which focused on fractional relationships among the differing lengths. The researchers elicited justifications from students by asking, "How can you convince the whole class?" (p 112). In one instance, students defended their answers of why a rod of length 9 did not have any corresponding half lengths by lower and upper bounds: "The yellow is a little bit more than a half, and the purple is shorter than a half" (p 113). By contraction, "Here is not a rod that is half of the blue rod because there are nine little white rods; you can't really divide that into a half, so you can't really divide by two because you get a decimal or remainder" (p 113). This example demonstrates that elementary school children are capable of justifying their thinking and need their teachers' support through questioning to regularly express their reasoning in many ways. Additionally, the way justifications are constructed and expressed warrants more discussion in the following section.

Forms of Reasoning

Deductive reasoning is one of the defining forms of mathematical reasoning, typically described as being able to draw a conclusion from pre-established facts (Reid 2002a). The prominence deductive reasoning plays in mathematics as a discipline is not surprising as it is the primary form of constructing proofs (Flegas and Charalampos 2013; Reid and Zack 2009). Moving beyond a broad categorization, Reid (2002b) describes different types of deductive reasoning, such as "simple one-step deductive reasoning . . . multistep deductive reasoning . . . [and] hypothetical deductive reasoning" (pp 235–36). While the first two types refer to the complexity of chains of reasoning, the last type signals making inferences from the hypotheses generated during problem solving (Stylianides and Stylianides 2008). Furthermore, Komatsu (2016) emphasizes the importance of deductive thinking in students by explaining, "deductive guessing can be regarded as an authentic mathematical action because . . . it [can] overcome counter-examples" (p 159).

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Reasoning by counter-examples is not an exhaustive approach to proving, so the shift in students' use of deductive guessing in the reported research showed a shift in students' invocation of reasoning within problem solving. In other words, children show more sophistication in their reasoning as they move beyond using counter-examples to justify a conjecture toward creating chains of reasoning using established facts. The observable improvement in reasoning helps to further the idea that deductive reasoning is an essential skill that students should be developing. As a specific example for classrooms, Wanko (2009) introduced a variety of Japanese puzzles into his classroom to help foster deductive reasoning. He explains the value of using these puzzles in that "when students learn to provide deductive arguments for their puzzle-solving strategies, they are laying the foundation for good mathematical practices" (p 271). This statement emphasizes the essential nature of deductive reasoning in the mathematics classroom. Puzzles, like Sudoku, require students to use given information with completed cells and rules for placements to fill in the missing cell values.

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Plausible reasoning, as complementary to deductive reasoning, is important to solving mathematical problems and is a component of reasoning in daily life. Plausible reasoning (Polya 1954) is based on explorations that do not follow a prescribed pathway, is bound up with conjecturing through use of inferences, acknowledges personal knowing, coincides with mathematical thinking, and does not demand the same rigour and aim of absolute certainty as in deductive reasoning. Leading to developing mathematical ideas, plausible reasoning incorporates generalizing through pattern-noticing within inductive reasoning while relying on connections made to similar structures within analogic reasoning. Put in another way, Polya (1954) states that "it is reasonable to try the simplest case first" and how "even if we return eventually to a closer examination of more complex possibilities, the previous examination of the simplest case may serve as a useful preparation" (p 194). The following example further demonstrates this, wherein Sumpter and Hedefalk (2015) analyzed preschool children's reasoning through play. When a young child suggested measuring the height of a rock, the children collectively offer reasoning based on inferences. For example, "Yes, but the house is bigger than the rock" (p 5). Or where a conclusion is offered based on measuring as evidence, "It is bigger than me anyway [walks and stands next to the rock and looks up, using her own body as a measure]" (p 5). The informal reasoning implied by plausible reasoning is a wonderful starting place in the early years of elementary school, where children can be asked to provide defenses that are connected to their experiences and reasonable to the problem-solving context.

Major Themes of Science Reasoning

Forms and Skills of Scientific Reasoning

Several prominent themes emerged from the literature on science reasoning, and we have selected the most comprehensive descriptions and definitions. One major theme is deductive reasoning, which is also described as a means of reasoning in mathematics. Deduction, as a key skill for scientific reasoning (Van der Graaf, Segers and Verhoeven 2015), is often discussed with a hypothesis-based approach in science. For instance, researchers emphasized hypothetico-deductive reasoning whereby deduction is combined in an overall process of reasoning alongside hypothesizing (Chen and She 2015; Lawson 2008).

When students made a hypothesis, they were also challenged to give their reasoning and, where appropriate, to provide evidence to support their statements, that is, deductive reasoning.

The process of hypothetico-deductive reasoning in classrooms occurs when students make a hypothesis based on their experiences and knowledge to an unknown situation, deduce what would happen if their hypothesis was correct, design a test based on the deduced ideas and finally test it to verify or falsify it. If it is false, they will make another hypothesis. Lei et al (2009) explicitly states that "scientific reasoning ability . . . focuses on . . . reasoning skills such as the abilities to . . . formulate and test hypotheses" (p 586). The skills of scientific reasoning, such as hypothesizing and fair testing, are essential components of understanding scientific reasoning as an entirety, because they aid in describing the big picture of scientific problem solving and knowledge development. As a classroom example, Tytler and Peterson (2003) asked students to hypothesize which whirlybird would fall and spin faster. The whirlybirds had three different wingspans: short, medium and long. When students made a hypothesis, they were also challenged to give their reasoning and, where appropriate, to provide evidence to support their statements, that is, deductive reasoning. Deductive reasoning is also described as a reasoning skill that scientists often engage in (Wasserman and Rossi 2015).

Inductive reasoning is used to describe and discuss scientific reasoning and is often mentioned with reference to observed patterns. Lawson (2005) viewed it as a primary component of scientific reasoning. Wasserman and Rossi (2015) explain the significance of induction in scientific reasoning by describing how "one of the primary modes of reasoning in science is induction" (p 23). Wasserman and Rosi (2015) also found that "science teachers . . . were more prone to us[ing] inductive methods of reasoning" (p 32). Duschl (2003) further supports this by stating that "scientific inquiry . . . [is] an inductive process." A classroom example is an electric conductor and indicator activity. Students test various materials, such as a wood stick, metal spoon, nail, plastic pen, paper, rubber band and so on, in an electric circuit to determine that metal materials are conductors (induction). This approach is common in hands-on science inquiry. This science concept through inductive reasoning often continues to develop with deductive reasoning when teachers provide everyday materials, such as a key, a coin or a metal glass frame, and ask if the items would pass an electric current or if wearing rubber gloves would be safe during electricity repair. These further questions will help develop students' deductive reasoning (for example, the key is metal, metal is a conductor, conductors pass electricity, therefore, key passes electricity).

The collaboration of claims, evidence and justification in argumentation empowers students' scientific reasoning.

Another key theme to explain science reasoning is argumentation, which is a means through which scientific reasoning is developed. For example, it is seen as an essential aspect of "prompting scientific reasoning" (Driver, Newton and Osborne 2,000; Duschl and Osborne 2002; Roberts and Gott 2010). Argumentation is used to develop and evaluate claims based on data and evidence. When students encounter conflicting claims, they need to search for evidence to justify which claim is more convincing to reach an agreement or conclusion. For instance, when students propose two conflicting claims: (1) platypus is a mammal, and (2) platypus is an amphibian, they need to find sufficient evidence to justify their conclusion. The collaboration of claims, evidence and justification in argumentation empowers students' scientific reasoning (Osborne, Erduran and Simon 2004).

The Essence of Scientific Reasoning: Evidence

In the process of scientific reasoning, linking theory and evidence, that is, understanding the co-variation between theory and evidence is critical

(Kuhn and Pearsall 2,000). For instance, in hypothesis testing, students use scientific data or information as evidence to support or refute their hypothesis. In an inductive approach of scientific experiments, a conclusion must be drawn from data collected, that is, evidence-based data analysis. In the processes of argumentation, a claim must be justified with evidence to be persuasive and convincing. Thus, "argument[ation] in the science classroom . . . can help students develop science skills . . . [such as] using evidence to defend a point of view" (Thier 2010, 70). In any type of scientific reasoning and problem-solving process, students are challenged to connect their claims, explanations and conclusions to evidence to make their ideas scientific, justifiable and, thus, persuasive. So important is evidence in scientific reasoning that Tytler and Peterson (2004, 98) state, "A key aspect of scientific reasoning is the ability to suggest and make judgments about evidence." McNeill and Krajcik (2008) also explained the important role of evidence in science: "When scientists explain phenomena and construct new claims, they provide evidence and reasons to justify them or to convince other scientists of the validity of the claims" (p 121). This description of the importance of evidence and its role in science facilitates the concept that evidence-based thinking in science is critical.

Scientific reasoning can be broadly defined as *intentional* coordination of theory and evidence (Mayer et al 2014, italics added). As science reasoning requires one's intention, practice and skills to coordinate theory (claim) and evidence (data) in scientific explanation, for students to think and process material from a truly scientific perspective, we must provide the tools for this to become a reality. Helping students to learn evidence-based means of thinking will help to facilitate this into a reality. Hardy et al (2010) discuss the concept of evidence-based reasoning (EBR) and how it potentially "contribute[s] to the development of individual students' abilities in scientific reasoning" (p 198). They categorized evidence-based reasoning into three levels: (1) data-based reasoning—students' ideas (claims and statements) are supported by a single property or observation, (2) evidence-based reasoning—students' ideas are supported by a contextualized relationship between two or more data or evidence, and (3) rule-based reasoning—students' ideas are supported by a generalized relationship or principle (Hardy et al 2010). Evidence- and rule-based reasoning are higher and more sophisticated levels of reasoning than data-based reasoning in terms of evidence-claim evaluation and knowledge generalization and application. Another notion discussed in the literature is that of scientific literacy, viewed in relation to evidence. For example, Brown et al (2010, 124) state how "students who are scientifically literate should be able to make judgments based on the evidence supporting or refuting [an] assertion." This only further assists in demonstrating the critical nature of evidence-based thinking as it is viewed through this definition of scientific literacy as an essential component of it. The concept of scientific literacy is further backed by McNeill and Krajcik (2008), who claim that "students need to be able to critically read . . . by evaluating the evidence and reasoning presented . . . [this] allows students to make informed decisions" (p 121). That critical and evidence-based thinking are integral components to thinking scientifically is clearly a common theme throughout the literature.

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Discussion and Reflection

In elementary mathematics and science classrooms, reasoning is an important foundation for students to form a significant and thoughtful understanding of the processes that underlie these subjects and to apply and develop disciplinary content knowledge. For instance, claims and hypotheses are made, and data and evidence are evaluated as plausible or implausible based on children's current knowledge (Sadler and Zeidler 2005). When children's current knowledge does not support observed phenomena, such as discrepant events or cognitively conflicting situations, they need more plausible and fruitful knowledge to explain the phenomena in the justification process where teachers can expect conceptual change and development. Because of this significance, it is essential to understand how reasoning is understood within each discipline, as with that knowledge we can begin to develop stronger links between the two subjects that can facilitate increasing student understanding both in the individual subjects and between both subjects.

Reasoning as it was discussed in the mathematics literature primarily focused on the keywords that one typically may conjure up when thinking about reasoning from a more standard perspective—terms, definitions and examples of deductive, inductive and plausible reasoning were common themes in the realm of mathematics reasoning. Some of these key words and definitions were also demonstrated within the literature on scientific reasoning, in particular, deductive and inductive reasoning. In the discussion of deductive reasoning in science, hypothesis is a key idea whereby students' hypothesis testing often includes deductive reasoning. As a distinction within the commonality of deductive reasoning is that in mathematics constructing a proof is seen as the purpose of deductive reasoning. From the literature, we found conjecture in mathematics and hypothesis in science seem to share some degree of commonality where students make a claim based on their prior experiences, observation and knowledge to explain what is going to happen in an unknown situation.

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Interestingly, the prevalent theme of the topic of evidence and the essential role that a variety of authors viewed it to have in scientific reasoning, and how the understanding of reasoning with an emphasis on evidence was not prevalent in the literature on mathematics reasoning. However, although evidence was not necessarily a common theme that arose in the mathematics literature, other keywords were often referenced, which have similar meaning to evidence, such as justification through specific examples and specializing to convince with a smaller problem. We believe that even though the literature refers implicitly to the concept of evidence in the mathematics literature, the idea of evidence may be a commonality these two disciplines share about reasoning, and one that deserves further exploration to benefit future teachers and students.

Overall, commonalities of mathematical and scientific reasoning lie in the area of observing, analyzing and justifying in a problem-solving process. To understand and solve the problem, children observe, collect data (evidence) and analyze the observed data to come up with answers. In mathematics classrooms, teachers commonly use *conjecturing* and *justification* to explain this problem-solving process, and in science classrooms, teachers use the terms *making claims*, *seeking evidence* and *justification*. In this problem-solving process, inductive, deductive, hypothetico-deductive and plausible reasoning are complexly intertwined, yet whichever reasoning students call on, their solutions must be justified with evidence. Even though students' mathematics and science reasoning share many commonalities, in literature review, they are explained with different terms and language; thus, it seemed they were separate cognitive skills in children's thinking.

Reflection

In this section, we share our reflections on children's reasoning in elementary classrooms based on our own perspectives and experiences as a preservice teacher (Pisesky) and teacher educators (McFeetors and Kim).

Ashley Pisesky

These findings have been very helpful to me as a preservice teacher, and they would aid other elementary school preservice and current teachers. For example, the time-intensive lesson planning was a challenge while completing my practicums. Since elementary school generalist teachers are responsible for instructing about five subjects daily, lesson planning becomes overwhelming; few explicit cross-curricular connections between the subjects are taught in postsecondary preparation. Having more explicit connections specific to the school subjects demonstrated that this kind of preparation may have made lesson planning easier. Some of the mathematics and science lessons may have been linked together, using one lesson and one time block to instruct both sets of content.

The focus should be on the processing that students are engaging in.

Alongside this, students would benefit from having more of the subjects linked across the curriculum. I was a strong believer of this throughout my practicums, and I often looked for ways to link students' learning. However, many of the links that I found were more superficial in nature, such as how doing writing in science class links both language arts and science. Alternatively, linking content in subjects, such as a learning outcome in mathematics and in science, may also be viewed by some as more of an artificial connection. Although it is good to point out the two similarities and to reinforce one subject through another, a fundamental missing link between subjects at a deeper level in order to better understand and facilitate student processing is currently a deficit that should be included in preservice teacher training. A prime example of how this could be better integrated into preservice preparation is the research gathered through this literature review. With STEM being an increased focus in schools, both in the classroom and in extracurricular activities, it is essential that teachers know and understand the deeper meaning as to why and how these subjects are related to one another in order to better implement learning in the classroom. From my experiences, a better understanding of how students engage in the process of reasoning in both subjects will help to foster greater understanding in both. I therefore believe that linking the subjects of mathematics and science with students in the elementary classroom is something that not only could be but should be reasonably practised by preservice and practising teachers.

One revelation from this process was when I discussed the intersections between science and mathematics reasoning with my supervisors. Janelle and Mijung mentioned that in science reasoning we discuss the hypothesis-verification process to develop reasoning, but mathematics reasoning is developed through the use of conjectures. They proceeded to explain that conjectures and hypotheses essentially point to the same phenomenon; however, they are each used in their respective field. I think that this is something that should change in the future, as we look toward creating more cohesive and comprehensive learning opportunities for students. We should use both words interchangeably in both fields so that students do not get left behind in the language of the topic. The focus should be on the processing that students are engaging in. If we allow this to be the focus of teaching and learning, we will see increased student understanding in both domains. We will reduce the disparity that exists between students who excel in each domain but struggle in the other. All of these are important effects that students would benefit from.

Janelle McFeetors and Mijung Kim

Reasoning in general involves logic thinking. When children encounter a puzzling question, they try to find solutions by retrieving and reorganizing their thoughts, experiences and knowledge. We educators want to support students in constructing reasonable solutions developed through logical thinking processes. Through various pedagogical strategies, educators strive to enhance children's thinking and reasoning processes, which help them construct solutions, which also develops knowledge application. For instance, in mathematical problem solving, children learn to conjecture, specialize, justify and create proof, and in scientific problem solving, they learn to evaluate and justify claims with evidence to draw conclusions. In this process, children's knowledge is reflected, examined and developed to solve the current problem. However, often the particular terminologies for these cognitive actions are used in a way that teach children to see reasoning as if they were different and isolated within content areas. We seldom question what children do differently during *conjecturing* in mathematics class and *hypothesizing* in science classrooms. Children try to make sense of the current situation at hand (for example, a puzzle, question, discrepant event and so on) using their knowledge, experiences and creativity to come up with a possible explanation, which is *conjecture* in mathematics and *hypothesis* in science. We acknowledge these terminologies are unique in each disciplinary tradition, thus need to be acknowledged and respected. Yet when separately taught to preservice teachers and further to children in classrooms, they could become confusing and seemingly isolated cognitive processes.

Reasoning in general involves logic thinking.

In this study, we teacher educators looked at mathematics and science reasoning not from a subject disciplinary lens but from the perspectives of a child and a teacher who might not distinguish reasoning processes in two different subject areas. We believe there is a need for understanding how reasoning in mathematics and science could be integrated and taught, such as in STEM-oriented classrooms. In a STEM approach, students are engaged in problem solving, which requires integration of knowledge and skills among different disciplines and the boundaries of disciplines often disappear. Once the problems are identified and goals are shared in the problem-solving community, disciplinary traditions and knowledge and reasoning skills are all complexly intertwined and integrated in collective levels. Students create, justify, evaluate and negotiate their ideas to reach the best solutions to problems. Which mathematical reasoning and scientific thinking do students use in a STEM problem-solving process? One might find this question difficult and not necessary as children's reasoning and problem-solving process are intertwined and integrated without the boundaries of subjects, which motivated our interest in this study.

To illustrate, we offer a specific example of a STEM approach, where students are challenged to solve a problem, such as building a boat with material and time constraints. The boat needs to meet with certain criteria, such as (1) holding a certain weight, and (2) reaching a certain point as fast as possible when a fan is blowing. In this problem-solving situation, students must understand the relationship of density, buoyancy, geometrical shapes, friction of materials, measurement of distance and loading strategy. To prove their design, they would test their boat with a certain load and a fan blowing on water. When the load gets heavier, they would conjecture the maximum load before it sinks. In this problem process, children's reasoning is complexly intertwined with various types of reasoning. Thus, it is neither possible nor meaningful to indicate mathematical and science reasoning separately. An implication for classroom practice is that mathematics and science content be addressed simultaneously through intriguing problems for students, where reasoning is elicited in their actions and discourse. Rather than labelling these actions with discipline-specific terminology, teachers can celebrate the understandings students develop as they offer tentative explanations, explore the context and ultimately justify their ideas. This is where we feel the gap exists between theory of cognition and everyday practice.

During our reading and conversations, we questioned how we could develop more integrated ways of teaching. We reflected on our own classrooms in our teacher education program in subject-specific curriculum courses and our own teaching at the university. We recognized that it is also very isolated as we perpetuate distinctions using different terms for similar reasoning processes. This led us to examine the terminologies of reasoning that we use in each discipline and how we introduce them to preservice teachers. As we realize that students in schools and citizens in everyday life integrate knowledge and skills without disciplinary boundaries similar to a STEM approach, it was worthwhile questioning how reasoning is discussed in research, curriculum and in our own classes as an initiative of developing an integrated approach for mathematics and science teaching.

As a result of this inquiry, we have more questions and challenges as we start to reflect on our own classrooms at the university. The current teacher education program has perpetuated the separation between science and mathematics through its subject-based program design. Also, as the specific terms of reasoning, such as *conjecture* and *hypothesis*, are the means of communicating among educators and researchers within the subject disciplines, they will be continuously used in the communities of mathematics and science education. As we realize the need for an integrated approach in today's classrooms, how we introduce these terms without creating confusion and resistance becomes a challenge. Creative and collective efforts will be required in further conversations.

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References

Alberta Education. 1996. *Science Grade 1 to 6 Program of Studies*. Edmonton, Alta: Alberta Education.

——. 2016. *Mathematics Kindergarten to Grade 9 Program of Studies*. Edmonton, Alta: Alberta Education. (Originally published 2007).

Babb, A P P, M A Takeuchi, G A Yanez, K Francis, D Gereluk and S Friesen. 2016. "Pioneering STEM Education for Preservice Teachers." *International Journal of Engineering Pedagogy* 6, no 4: 4–11.

Brown, N, E Furtak, M Timms, S Nagashima and M Wilson. 2010. "The Evidence-Based Reasoning Framework: Assessing Scientific Reasoning." *Educational Assessment* 15, no 3/4: 123–41.

Chen, C-T, and H-C She. 2015. "The Effectiveness of Scientific Inquiry with/without Integration of Scientific Reasoning." *International Journal of Science and Mathematics Education* 1, no 1: 1–20.

Driver, R, P Newton and J Osborne. 2,000. "Establishing the Norms of Scientific Argumentation in Classrooms. *Science Education* 84, no 2: 287–312.

Duschl, R A. 2003. "Assessment of Inquiry." In *Everyday Assessment in the Science Classroom*, ed J M Atkin and J Coffey, 41–59. Arlington, Va: NSTA.

Duschl, R, and J Osborne. 2002. "Supporting and Promoting Argumentation Discourse in Science Education." *Studies in Science Education* 38, no 1: 39–72.

English, L D, ed. 1997. *Mathematical Reasoning: Analogies, Metaphors, and Images*. Mahwah, NJ: Lawrence Erlbaum.

Erduran, S, and M P Jimenez-Aleixandre, eds. 2007. *Argumentation in Science Education*. Netherlands: Springer.

Flegas, K, and L Charalampos. 2013. "Exploring Logical Reasoning and Mathematical Proof in Grade 6 Elementary School Students." *Canadian Journal of Science, Mathematics and Technology Education* 13, no 1: 70–89.

Hardy, I, B Kloetzer, K Moeller and B Sodian. 2010. "The Analysis of Classroom Discourse: Elementary School Science Curricula Advancing Reasoning with Evidence." *Educational Assessment* 15, no 3–4: 197–221.

Houssart, J, and C Sams. 2008. "Developing Mathematical Reasoning Through Games of Strategy Played Against the Computer." *International Journal for Technology in Mathematics Education* 15, no 2: 59–71.

Johnsson, B, M Norqvist, Y Liljekvist and J Lithner. 2014. "Learning Mathematics Through Algorithmic and Creative Reasoning." *Journal of Mathematical Behaviour* 36, 20–32.

Komatsu, K. 2016. "A Framework for Proofs and Refutations in School Mathematics: Increasing Content by Deductive Guessing." *Educational Studies in Mathematics* 92, no 2: 147–62.

Kuhn, D. 2010. "Teaching and Learning Science as Argument." *Science Education* 94, 810–24.

Kuhn, D, and S Pearsall. "Developmental Origins of Scientific Thinking." *Journal of Cognition and Development* 1, no 1: 113–29.

Lane, C, and S Harkness. 2012. "Game Show Mathematics: Specializing, Conjecturing, Generalizing, and Convincing." *Journal of Mathematical Behaviour* 31, no 2: 163–73.

Lawson, A E. 2005. "What Is the Role of Induction and Deduction in Reasoning and Scientific Inquiry?" *Journal of Research in Science Teaching* 42, no 6: 716–40.

——. 2008. "What Can Developmental Theory Contribute to Elementary Science Instruction?" *Journal of Elementary Science Education* 20, no 4: 1–14.

Lei, B, T Cai, K Koenig, K Fang, J Han, J Wang, Q Liu, L Ding, L Cui, Y Luo, Y Wang, L Li and N Wu. 2009. "Learning and Scientific Reasoning." *Science* 323, no 5914: 586–87.

Mason, J, L Burton and K Stacey. 2010. *Thinking Mathematically*. 2nd ed. Harlow, UK: Prentice Hall/Pearson.

Mayer, D. B Sodian, S Koerber and K Schwippert. 2014. "Scientific Reasoning in Elementary School Children: Assessment and Relations with Cognitive Abilities." *Learning and Instruction* 29, 43–55.

McFeetors, P J, and K Palfy. 2017. "We're in Math Class Playing Games, Not Playing Games in Math Class." *Mathematics Teaching in the Middle School* 22, no 9: 534–44.

McNeill, K. 2011. "Elementary Students' Views of Explanation, Argumentation, and Evidence, and Their Abilities to Construct Arguments Over the School Year." *Journal of Research in Science Teaching* 48, no 7: 793–823.

McNeill, K, and J Krajcik. 2008. "Inquiry and Scientific Explanations: Helping Students Use Evidence and Reasoning." In *Science as Inquiry in the Secondary Setting*, ed J Luft, R L Bell and J Gess-Newsome, 121–33. Arlington, Va: NSTA.

Mueller, M F, and C A Maher. 2009. "Convincing and Justifying Through Reasoning." *Mathematics Teaching in the Middle School* 15, no 2: 108–16.

National Council of Teachers of Mathematics (NCTM). 2009. *Focus in High School Mathematics: Reasoning and Sense Making*. Reston, Va: NCTM.

Osborne, J, S Erduran and S Simon. 2004. "Enhancing the Quality of Argumentation in School Science." *Journal of Research in Science Teaching* 41, 994–1020.

Polya, G. 1954. *Mathematics and Plausible Reasoning: Induction and Analogy in Mathematics*. Princeton, NJ: Princeton University Press.

Reid, D. 2002a. "Conjectures and Refutations in Grade 5 Mathematics." *Journal for Research in Mathematics Education* 33, no 1: 5–29.

——. 2002b. "Describing Reasoning in Early Elementary School Mathematics." *Teaching Children Mathematics* 9, no 4: 234–37.

Reid, D, and V Zack. 2009. "Aspects of Teaching Proving in Upper Elementary School." In *Teaching and Learning Proof Across the Grades*, ed D Stylianou, M Blanton and E Knuth, 133–46. New York: Routledge

Roberts, D, and S Gott. 2010. "Questioning the Evidence for a Claim in a Socio-Scientific Issue: An Aspect of Scientific Literacy." *Research in Science and Technological Education* 28, no 3: 203–26.

Sadler, T D, and D L Zeidler. 2005. "The Significance of Content Knowledge for Informal Reasoning Regarding Socioscientific Issues: Applying Genetics Knowledge to Genetic Engineering Issues." *Science Education* 89, 71–93.

Staples, M, J Bartlo and E Thanheiser. 2012. "Justification as a Teaching and Learning Practice: Its (Potential) Multifaceted Role in Middle Grades Mathematics Classrooms." *Journal of Mathematical Behaviour* 31, no 4: 447–62.

Stylianides, G, and A Stylianides. 2008. "Proof in School Mathematics: Insights from Psychological Research into Students' Ability for Deductive Reasoning." *Mathematical Thinking and Learning* 10, no 2: 103–33.

Sumpter, L, and M Hedefalk. 2015. "Preschool Children's Collective Mathematical Reasoning During Free Outdoor Play." *Journal of Mathematical Behavior* 39, 1–10.

Thier, M. 2010. "Developing Persuasive Voices in the Science Classroom." *Science and Children* 48, no 3: 70–74.

Thom, J. 2011. "Nurturing Mathematical Reasoning." *Teaching Children Mathematics* 18, no 4: 234–43.

Tytler, R, and S Peterson. 2003. "Tracing Young Children's Scientific Reasoning." *Research in Science Education* 33, 433–65.

Tytler, R, and S Peterson. 2004. "From "Try It and See" to Strategic Exploration: Characterizing Young Children's Scientific Reasoning." *Journal of Research in Science Teaching* 41, no 1: 94–118.

Van der Graaf, J, E Segers and L Verhoeven. 2015. "Scientific Reasoning Abilities in Kindergarten: Dynamic Assessment of the Control of Variables Strategy." *Instructional Science* 43, no 3: 381–400.

Wanko, J. 2009. "Japanese Logic Puzzles and Proof." *Mathematics Teacher* 103, no 4: 266–71.

Wasserman, N, and D Rossi. 2015. "Mathematics and Science Teachers' Use of and Confidence in Empirical Reasoning: Implications for STEM Teacher Preparation." *School Science and Mathematics* 115, no 1: 22–34.

Yankelewitz, D, M Mueller and C Maher. 2010. "A Task That Elicits Reasoning: A Dual Analysis." *Journal of Mathematical Behaviour* 29, no 2: 76–85.

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