Edmonton Junior High School Mathematics Competition 2016/17

Part 1

1. A 4-digit number uses each of the digits 3, 4, 5, and 6 exactly once. If the digits are placed randomly, what is the probability that the 4-digit number is a multiple of 6?

A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{2}{3}$ D. $\frac{1}{2}$ E. $\frac{5}{6}$

Answer: D

Solution:

There are 4 x 3 x 2 x 1 = 24 ways to write a 4-digital number. The 4-digital number is already divisible by 3 regardless of the positions of the digits. To be divisible by 6, the number must end with either a 4 or a 6. There are 3 x 2 x 1 = 6 ways to write the first three digits. This gives the probability of $\frac{2 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1} = \frac{1}{2}$

2. Two analog clocks run at the correct rate of speed. Both clocks show the correct time when it is 9:45 PM However, as the hands on one clock run forward, the hands on the other clock run backward. When will both clocks next show the same time?

A. 4:15 AM B. 3:45 AM C. 3:45 PM D. 4:15 PM E. 9:45 AM

Answer: B

Solution:

Since the two clocks run at the same speed, the two clocks would display the same time exactly 6 hours later. This gives 3:45 PM.

3. Cellphone company Apple has no monthly fee but charges:

- Local calls at $0.10/min, plus
- Long Distance calls at $0.50/min, plus
- Text Messages at $0.20/text beyond 75 texts, plus
- Data at $10/GB past 3 GB.

Cellphone company Banana charges $125/month for unlimited usage.

Jaime's typical use per month is:

- Local calls: 500 minutes, plus
- Long Distance calls: 10 minutes, plus
- Text Messages: 250
- Data: 5 GB

Based on Jaime's usage, which statement is true?

A. Jaime saves less than \$200/year using company Apple.
B. Jaime saves more than \$200/year using company Apple.
C. Jaime saves less than \$200/year using company Banana.
D. Jaime saves more than \$200/year using company Banana.
E. Both companies would charge Jaime the same amount.

Answer: A

Solution:

Using Jaime's data usage, we have $500(0.1) + 10(0.5) + 0.2(250 - 75) + 10(5 - 3) = 50 + 5 + 35 + 20 = \$110$. Each year, Jaime saves $12(125 - 110) = \$180$ using company Apple.

4. It will take me 2% of 8 hours to finish folding my laundry. It will take me 55% of 20 minutes to unload the dishwasher. Which task will take me longer to complete, and by how many more seconds?

   A. Folding laundry by 84 seconds.
   B. Folding laundry by 54 seconds.
   C. Unloading the dishwasher by 84 seconds.
   D. Unloading the dishwasher by 54 seconds.
   E. Both tasks take the same amount of time.

Answer: C

Solution:

Folding laundry requires $0.02(8)(60)(60) = 576$ seconds. Unloading the dishwasher requires $0.55(20)(60) = 660$ seconds. Unloading takes longer by $660 - 576 = 84$ seconds.

5. I started a game with an even number of points, and played 3 rounds. In the first round, I lost half of my points. In the second round, I won back twice the number of points that I had started the game with. I ended the third round with half the number of points that I had started that round with. I ended the game with 15 points. Which describes how the number of points I ended the game with compares to the number of points I started the game with?

   A. I ended the game with half the points that I started the game with.
   B. I ended the game with double the points that I started the game with.
   C. I ended the game with 3 more points than what I started the game with.
   D. I ended the game with 3 less points than what I started the game with.
   E. I ended the game with the same number of points that I started the game with.

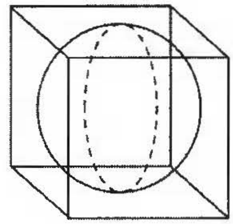
Answer: C

Solution:

Start of round	Win or lose	End of round
2n		
2n	n	n
n	4n	5n
5n	2.5n	2.5n = 15

Start of the game = 12 points. Therefore, Jaime earns 3 more points than the start of the game.

6. Given that the formula for the Volume of a Sphere is: $\mathbf{V} = \left(\frac{4}{3}\right)\pi r^3$



A cube has the same height as the diameter of a sphere. The Surface Area of the cube is 216 cm^2. Rounded to the nearest whole cm^3, how much larger is the volume of the cube compared to the volume of the sphere?

A. 96 B. 103 C. 108 D. 127 E. 216

Answer: B

Solution:

The length of one side of the cube is $\sqrt{\frac{216}{6}} = 6\ cm.$

The difference in volume is $(6 \times 6 \times 6) - \frac{4}{3}(\pi)(3^3) \cong 103cm^3$

7. A package contains 4 chocolate, 3 vanilla and 3 lemon cupcakes. How many chocolate cupcakes, represented by x, must be added to the package so that it will contain 60% chocolate cupcakes?

Which of the following equations could be used to solve this problem?

A. $\frac{x-10}{x-4} = \frac{60}{100}$ B. $\frac{x+10}{x+4} = \frac{60}{100}$ C. $\frac{x}{x+10} = \frac{0.6\,x}{1}$

D. $\frac{x+4}{x+10} = \frac{60}{100}$ E. $\frac{x}{0.6} = x + 10$

Answer: D

Solution:

The total number of chocolate cupcakes would increase by 4 while the total number of cupcakes also increases by 4. The proportional statement $\frac{number\ of\ chocolate\ cupcakes}{total\ number\ of\ cupcakes} = \frac{x+4}{x+10} = \frac{60}{100}$ gives the correct expression.

Part 2

8. Each person in a room shook hands once with each other person in the room. If the total number of handshakes was less than 1000, then what is the most number of people that could have been in the room?

Solution:

Total number of handshakes is best dealt with using the series (n – 1) + (n – 2) + (n – 3) + … + 3 + 2 + 1 where n is the number of people in the group. For example, if n = 5 people, there would be 4 + 3 + 2 + 1 handshakes in total. Pairing the front and back each time yield a sum of n. There are exactly $\frac{n-1}{2}$ pairs giving a sum of $n\left(\frac{n-1}{2}\right)$. Solving the inequality $n\left(\frac{n-1}{2}\right) < 1000$, we have n = 45.

9. The sum of two rational numbers is 1. Amy add the larger number to the square of the smaller number. Beth add the smaller number to the square of the larger number. What is the difference of the two values?

Solution:

Let the larger of the number be n and the smaller number be $(1-n)$.

We have $(n+(1-n)^2)-(n^2+(1-n)) = (n+1-2n+n^2)-\left(n^2+(1-n)\right) = 0.$

10. Although Jen has no savings, she wants to earn enough money in 4 months to buy a puppy. On the first month, Jen earns half of the total cost. On the second month, Jen earns one-third of the amount she still needs. On the third month, she earns \$80. After 3 months, she has earned 75% of the total cost of the puppy. How much money must Jen earn in the fourth month to have enough to buy the puppy?

Solution:

Let n be the cost of a puppy

We have the equation $\frac{3n}{4} = \frac{n}{2} + \frac{n}{6} + 80$. Solving for n yields $n = 960$. Jen needs to earn $\frac{1}{4} \times 960 = \$240$

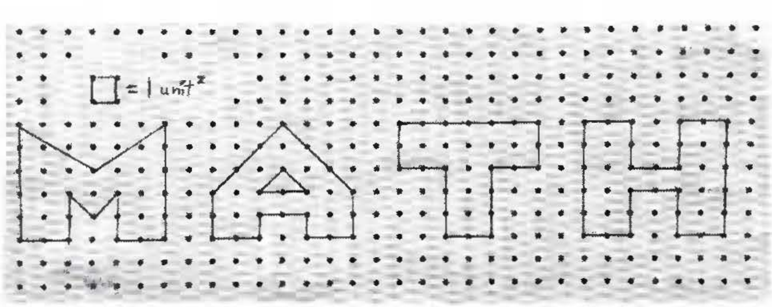
11. Xiang's age is 10 less than the sum of Yvonne's age and Zoe's age. The ratio of Xiang's age to Yvonne's age is 3:2. Zoe is 2 years older than Yvonne. What is the sum of the ages of the three people 4 years from now?

Solution:

Let Y be Yvonne's current age. It follows that $x = \frac{3y}{2}$ and z = y + 2.

Solving the equation $\frac{3y}{2} = y + (y+2) - 10$, we have $y = 16$, $x = 24$ and $x = 18$. In 4 years, we have 20 + 28 + 22 = 70.

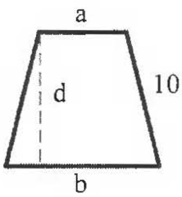
12. What is the sum of the interior areas, to the nearest unit^2, of the letters used to spell the word "MATH"?



Solution:
M = 21, A = T = 18, H = 22.
The total area = 79 unit^2

13. What is the area, in square centimeters, of an isosceles trapezoid, given the following clues?

- Its perimeter is 64 cm
- Each of the 2 congruent sides is 10 cm
- The difference in the lengths of the parallel sides is 12 cm



Solving the two equations $a + b + 20 = 64$ and $b - a = 12$, we have $a = 16$ and $b = 28$.

$d = \sqrt{10^2 - 6^2} = 8$. The area is $\frac{(16+28)(8)}{2} = 176\ cm^2$

Part 3

14. Mary divides by 5 each number from 1 to 2017, inclusive. She then adds together all the remainders she gets. Find the sum Mary obtains.

Solution:

When we divide the number 1, 2, 3, ... , 2017 by 5, the remainders have a repeating pattern: 1, 2, 3, 4, 0, 1, 2, 3, 4, 0, ... , 1, 2.

The pattern 1, 2, 3, 4, 0 repeats 403 times and ends with 1, 2. The sum is $403(10) + 1 + 2 = 4033$.

15. How many 4 digit palindromes are divisible by 7?

Solution:

A 4 digit palindrome has the form $abba = a(1001) + b(110)$. Since $7|1001$, we need $7|110b$. This is possible when $b = 0$ or 7. Since there is no restriction on a except $a \neq 0$, we have 9 choices for a and 2 choices for b. In total, there are $9 \times 2 = 18$ such numbers.

16. Nickels, dimes and quarters are to be used to make exactly $1.00. At least one of each type of coin must be used. In how many different ways can this be done if an even number of coins must be used?

Solution:

Using a table of values to organize the number of coins, we have 6 ways to make $1.00 using even number of coins.

25¢	10¢	5¢
1	2	11
1	4	7
1	6	3
2	2	6
2	4	2
3	2	1

17. A girl and a boy play the game Rock, Paper, Scissors ten times, where rock beats scissors, scissors beat paper and papers beat rock. The boy uses rock three times, scissors six times and paper once. The girl uses rock twice, scissors four times and paper four times. None of the ten games is a tie. How many games has the boy won?

Solution:

Scissors are used ten times altogether. Since there are no tied games, exactly one player uses scissors in each game. In the six games where the boy uses scissors, the girl wins two of them when she uses rock, and lose the other four games. In the four games where the girl uses scissors, the boy wins three of them when he uses rock, and lose the other one. Hence the boy wins seven games.

18. Of all the whole numbers N from 1 to 2017 inclusive, how many have the property that there exists a number M such that the sum of M and N is equal to the sum of the reciprocal of M and the reciprocal of N?

Solution:

Let *m, n* be the two numbers. We have

$$m+n=\frac{1}{m}+\frac{1}{n}$$
$$m+n=\frac{n+m}{mn}$$
$$1=\frac{1}{mn}$$
$$mn=1$$

Hence *m* and *n* are reciproal of one another. Of the numbers from 1 to 2017, there are 2017 reciprocals. Therefore, there are 2017 values for M.

19. Find a positive integer whose ones digit is 5, and when it is multiplied by 4, the 5 becomes the first digit while all other digits shift one place to the right.

Solution:

Create a pure repeating decimal x where the positive integer is the repeating block. Then $40x$ is the same as x except with an extra 5 in front of its decimal point. Hence $x=\frac{5}{39}=0.\overline{128205}$. Thus the desired positive integer is 128205.

Alternative solution:

Divide 5 by 4. The quotient is 1.25. Divide 51 by 4. The quotient is 12.75. Divide 512 by 4. The quotient is 128. The division is exact but the ones digit is not 5. So we continue. Divide 5120 by 4. The quotient is 1282.5. Divide 51282 by 4. The quotient is 12820.5. Divide 512820 by 4. The quotient is 128205. This is the positive integer we seek.