Alberta High School Mathematics Competition 2016/17

Part 1

1. If it is 10:00 AM on a Tuesday, which day would it be 2016 hours later?

   (a) Tuesday (b) Wednesday (c) Thursday (d) Friday (e) Saturday

   Solution:
   Since $2016 = 32 \times 9 \times 7 = 12 \times 7 \times 24$, the answer is 10:00 AM on a Tuesday (12 weeks from the original day). The answer is (a).

2. If $x > 0$, $x \neq 1$, and $(\log_2 x)^2 = \log_4 x$, then:

   (a) $0 < x < 1$ (b) $1 < x < 2$ (c) $2 \leq x < 4$ (d) $4 \leq x < \infty$ (e) the situation is impossible

   Solution:
   The equation can be written as $(\log_2 x)^2 = \frac{1}{2}\log_2 x$, and since $x \neq 1$, this equation is equivalent to $\log_2 x = \frac{1}{2}$ with the solution $x = \sqrt{2} \in (1, 2)$. The answer is (b).

3. A ring of 10 grams is 60% gold and 40% silver. A jeweller wants to melt it down, add 2 grams of silver and add enough gold to make it 70% gold. How many grams of gold should be added?

   (a) 4 (b) 5 (c) 8 (d) 9 (e) more than 9

   Solution:
   Let x be the grams of gold which should be added. Then

$$\frac{7}{10} = \frac{10 \times \frac{6}{10} + x}{10 + 2 + x}$$

   Solving the equation one obtains $x = 8$.

   Alternative solution: The original ring contains 4 g of silver and 6 g of gold. The new ring will contain $4 + 2 = 6$ g of silver, which must account for 30% of the total. Thus the new ring must weigh $6 \times \frac{10}{3} = 20$ g, of which therefore 20-10-2=8 g must be added gold. The answer is (c).

4. A quadratic polynomial $f(x) = ax^2 + bx + c$, where a, b and c are integers, satisfies $f(2) = 4$ and $f(3) = 9$. The number of such polynomials is:

(a) 0 (b) 1 (c) 2 (d) 3 (e) more than 3

Solution:
Using the given condition we get $4a + 2b + c = 4, 9a + 3b + c = 9$ from which, solving for b, c in terms of a, one obtains $b = 5(1 - a), c = 6(a - 1)$, that is, infinitely many integer solutions. The answer is (e).

5. For any integer n, the expression $n^2 + 3n + 2$ cannot assume the value

(a) 0 (b) 2 (c) 110 (d) 375 (e) 420

Solution:
Since $n^2 + 3n + 2 = (n + 1)(n + 2)$ it must be even. Thus, 375 is not attainable. The other four numbers in the list can be attained using $n = -1$, $n = 0$, $n = 9$ and $n = 19$, respectively. The answer is (d).

6. Two straight lines with nonzero x and y-intercepts have the following property: the x-intercept of the first line equals the y-intercept of the second line, and the x-intercept of the second line equals the y-intercept of the first line. If the slope of the first line is m, then the slope of the second line is

(a) m (b) $-m$ (c) $\frac{1}{m}$ (d) $-\frac{1}{m}$ (e) none of these

Solution:
It is given that if $(a, 0)$ and $(0, b)$ lie on the first line then $(0, a)$ and $(b, 0)$ lie on the second line. The slopes of the lines are then $m = -b/a$ and $-a/b = 1/m$, respectively. The answer is (c).

7. The angles of a triangle when measured in degrees are all prime numbers. The smallest possible size of the largest angle is:

(a) 61° (b) 67° (c) 79° (d) 89° (e) the situation is impossible

Solution:
Let $A \le B \le C$ be the measures in degrees of the angles of $\triangle ABC$. Since $A + B + C = 180°$ one of the angles should be even and hence $A = 2°$. On the other hand, $178° = B + C \le 2C$, hence $C \ge 89°$. Since 89 is prime, we can take $B = C = 89°$, $A = 2°$. The answer is (d).

8. How many three-digit numbers can be written after 523 to yield a six-digit number which is divisible by each of 7, 8 and 9?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

Solution:
Since 7, 8 and 9 are pairwise relatively prime, the six-digit number must be a multiple of $7 \times 8 \times 9 = 504$. When 523999 is divided by 504, the remainder is 343. To get a multiple of 504, we subtract 343 from 999 to obtain 656. This is one of the answers, and we have $523656 = 7 \times 8 \times 9 \times 1039$. We can get another answer by subtracting 504 from 656 to obtain 152, and we have $523152 = 7 \times 8 \times 9 \times 1038$. This subtraction cannot be repeated without reducing the difference below 523000. Hence 656 and 152 are the only possible answers.

Alternative Solution: The six digit number should have the form $504n$ where n is a positive integer. The conditions of the problem lead to the inequality $523100 \le 504n \le 523999$ or equivalently $1038 \le n \le 1039$, and hence $n = 1038$ or $n = 1039$. With these two values of n, one obtains two three-digit numbers having the requested properties. The answer is (c).

9. When $x^4+x^5+x^{10}+x^{17}+x^{100}$ is divided by x^2-1 the remainder is

(a) $x+2$ (b) $x+3$ (c) $2x+3$ (d) $3x+1$ (e) $3x+2$

Solution:
The remainder must be a first degree polynomial $Ax+B$ and if $Q(x)$ is the quotient then

$$x^4+x^5+x^{10}+x^{17}+x^{100}=Q(x)(x^2-1)+Ax+B$$

for any real x. Taking $x=\pm1$ in the above equation we obtain $A+B=5$ and $-A+B=1$ hence $A=2, B=3$ and thus the remainder is $2x+3$. The answer is (c).

10. Let D be an arbitrary point on the side BC of the equilateral triangle ABC. Points E and F are on AB and AC respectively so that $DE\perp AB$ and $DF\perp AC$ and E_1,F_1 are points on BC such that $EE_1\perp BC$ and $FF_1\perp BC$. If E_1F_1 has length $\frac{1}{2}$ then the length of BC is

(a) $\frac{2}{3}$ (b) $\frac{4}{5}$ (c) 1 (d) $\frac{3}{2}$ (e) $\sqrt{3}$

Solution:
Let $BC=a$. We have $DE_1=DE\cos 30^\circ=\frac{\sqrt{3}}{2}DE$ and $DF_1=DF\cos 30^\circ=\frac{\sqrt{3}}{2}DF$, hence $E_1F_1=\frac{\sqrt{3}}{2}(DE+DF)$. On the other hand $DE\cdot AB+DF\cdot AC=2\text{Area}(ABC)$ that is, $a(DE+DF)=\frac{a^2\sqrt{3}}{2}$ and hence $DE+DF=\frac{a\sqrt{3}}{2}$. Therefore, $E_1F_1=\frac{3a}{4}$. Since $E_1F_1=\frac{1}{2}$ we must have $a=\frac{2}{3}$. The answer is (a).

11. A box contains two red balls, two green balls and two yellow balls. If you randomly remove three balls from the box without replacement, what is the probability that you have removed one of each colour?

(a) $\frac{1}{8}$ (b) $\frac{2}{5}$ (c) $\frac{1}{2}$ (d) $\frac{4}{5}$ (e) none of these

Solution:
There are a total of $\binom{6}{3}=20$ possibilities and only 8 are favourable. The requested probability is $\frac{8}{20}=\frac{2}{5}$. The answer is (b).

12. Let $f:\mathbb{R}\to\mathbb{R}$ be a function such $xf(x)+(1-x)f(-x)=x^2+x+1$ for any real number x. The greatest real number M for which $f(x)\geq M$ for all real numbers x, is

(a) $\frac{3}{4}$ (b) $\frac{5}{6}$ (c) $\frac{7}{8}$ (d) $\frac{9}{10}$ (e) $\frac{11}{12}$

Solution:
If x is replaced by $-x$ in the given equation then

$$-xf(-x)+(1+x)f(x)=x^2-x+1.$$

Using the given equation and the one that is obtained above, one obtains by subtraction that $f(-x)=f(x)+2x$, so $xf(x)+(1-x)(f(x)+2x)=x^2+x+1$ and thus

$$f(x)=x^2+x+1-2x(1-x)=3x^2-x+1=3\left(x-\frac{1}{6}\right)^2+\frac{11}{12}$$

and hence $f(x)\geq\frac{11}{12}$. If $x=\frac{1}{6}$ then we get $f\left(\frac{1}{6}\right)=\frac{11}{12}$. The answer is (e).

13. In a school's math club, the number of different 3-person committees that could be formed containing two girls and one boy is 2016 more than the number of different 3- person committees containing two boys and one girl. The number of girls in the club is:

(a) 1 (b) 63 (c) 64 (d) 2016 (e) not uniquely determined

Solution:
Let m, n denote the number of girls and respectively boys in the club. The condition of the problem is

$$n\binom{m}{2} - m\binom{n}{2} = 2016 \iff mn(m-n) = 2 \times 2016 = 63 \times 64 = 2^6 \times 3^2 \times 7$$

Let $k = (m, n)$ be the greatest common divisor of m, n. Since k is a divisor of $m, n, m-n$, then k^3 will be a divisor of $2^6 \times 3^2 \times 7 = mn(m-n)$ hence $k \in \{1,2,4\}$. If $k = 1$ the only convenient solutions are $m = 64, n = 1$ and $m = 64, n = 63$, otherwise $m > 64$, which is not possible. If $k = 2$ then $m = 2m_1, n = 2n_1$ with $(m_1, n_1) = 1$, and $m_1 n_1 (m_1 - n_1) = 8 \times 9 \times 7$. No convenient integer values for m_1, n_1 can be found in this case. Similarly, if $k = 4$ then $m = 4m_2, n = 4n_2$ with $(m_2, n_2) = 1$, and the equation can be simplified to $m_2 n_2 (m_2 - n_2) = 9 \times 7$, which does not have integer solutions.

Alternative Solution: The solutions of the equation $mn(m-n) = 2^6 \times 3^2 \times 7$ can be found using another approach. First one can remark by AGM inequality that

$$4032 = mn(m-n) \leq m\left(\frac{n + m - n}{2}\right)^2 = \frac{m^3}{4}$$

hence $m^3 \geq 16128$ and thus $m \geq 26$. On the other hand if $m > 64$ then

$$64 \cdot 63 = mn(m-n) > 64n(64-n))$$

hence

$$63 > n(64-n) \iff (n-1)(n-63) > 0$$

and thus $n > 63$, which is not possible since $mn(m-n) = 63 \cdot 64$.

We conclude that $m \in \{28, 32, 36, 42, 48, 56, 63, 64\}$. The only convenient value is $m = 64$ which gives the equation $n(64 - n) = 63$ and hence $n = 1, n = 63$.

The answer is (c).

14. The product of all real numbers x that are solutions of the equation $\sqrt[3]{x^2 + x + \sqrt[3]{x^2 + x + 3}} = 3$ is

(a) −26 (b) −24 (c) 4 (d) 20 (e) 26

Solution:
The function $f(t) = \sqrt[3]{x^2 + x + t}, t \in \mathbb{R}$ is increasing hence $f(f(t)) = t \iff f(t) = t$ that leads to the conclusion that the real solutions of the given equation and $\sqrt[3]{x^2 + x + 3} = 3$ are the same. The solutions of the last equation are just the solutions of the quadratic equation $x^2 + x - 24 = 0$, for which the product of the solutions is -24.

Alternative Solution: Let $y = \sqrt[3]{x^2 + x + 3}$ then the given equation can be written as

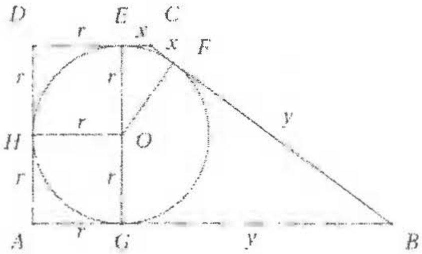
$$y^3 - 3 + y = 27 \iff (y-3)(y^2 + 3y + 10) = 0 \iff y = 3$$

and then $x^2 + x - 24 = 0$. The answer is (b).

15. The area of the trapezoid $ABCD$ with $AB \parallel CD$, $AD \perp AB$ and $AB = 3CD$ is equal to 4 . A circle inside the trapezoid is tangent to all of its sides. The radius of the circle is

(a) $\frac{\sqrt{3}}{5}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{\sqrt{3}}{2}$ (e) none of these

Solution:



Using the notations from the above diagram and the conditions from the problem one obtains:

$$3(r+x) = r+y \iff y = 3x+2r$$

and

$$(x+y)^2 = 4r^2 + \left((r+y)-(r+x)\right)^2 \iff xy = r^2.$$

Hence

$$x(3x+2r) = r^2 \iff x = \frac{r}{3}$$

and consequently $y = 3r$. On the other hand the area of the trapezoid $ABCD$ is 4, thus

$$(r+x+r+y)r = 4.$$

Substituting for $x = \frac{r}{3}$ and $y = 3r$ we get $r = \frac{\sqrt{3}}{2}$. The answer is (d).

16. A quadrilateral is called convex if its diagonals intersect inside the quadrilateral. A convex quadrilateral has side lengths 3, 3, 4, 4 not necessarily in this order, and its area is a positive integer. The number of non-congruent convex quadrilaterals having these properties is:

(a) 12 (b) 24 (c) 28 (d) 35 (e) none of these

Solution:

(i) Assume that the sides of the quadrilateral are of lengths 3,4,3,4, in this order. The quadrilateral is a parallelogram (so it is convex) . Let $\alpha \in (0, \frac{\pi}{2}]$ be the measure in radians of an angle of the parallelogram and S its area. Then we have

$$S = 12\sin\alpha \iff \sin\alpha = \frac{S}{12}.$$

Since $0 < \sin\alpha \leq 1$ there are twelve convenient integral values of S, namely $1, 2, \cdots 12$, and thus twelve distinct values of α. Therefore we get twelve non-congruent parallelograms with area a positive integer.

(ii) Assume that the sides of the quadrilateral are of lengths 4,4,3,3, in this order. Let α be the measure in radians of the angle between two sides of lengths 4 and 3. The quadrilateral is convex if $\alpha \in (\beta, \pi)$ with $\sin\beta = \frac{\sqrt{7}}{4}$. Note the the lower limit β for the angle α occurs when the sides of length 3 are both on the same line and hence, the quadrilateral degenerates to an isosceles triangle of sides 4,4,6.
As above, one obtains $\sin\alpha = \frac{S}{12}$. If $\alpha \in [\frac{\pi}{2}, \pi)$ there are 12 integer values for S for which we get twelve distinct values for α and hence one obtains twelve non-congruent quadrilaterals. If $\alpha \in (\beta, \frac{\pi}{2})$ then $\sin\beta < \sin\alpha < 1 \iff \frac{\sqrt{7}}{4} < \sin\alpha < 1 \iff \sqrt{63} < S < 12$. There are four integer values for S in $(\sqrt{63}, 12)$ and thus four distinct values for $\alpha \in (\beta, \frac{\pi}{2})$ such that $\sin\alpha = \frac{S}{12}$ for which one obtains four non-congruent convex quadrilaterals. The number of requested convex quadrilaterals is 12+12+4=28.

Alternative Approach: The largest parallelogram of sides in the order 3,4,3,4 is clearly the rectangle (as it has largest altitude) of area 12, and so other parallelograms in this family can have areas 1 to 11. Similarly the quadrilaterals with sides in the order 4,4,3,3 are all composed of two congruent triangles with two of the sides being 4 and 3, with area at most 6, so the largest such quadrilateral will also have area 12. As the angle between the sides 4 and 3 becomes greater than 90°, we get 11 more convex quadrilaterals of area 1 to 11. When this angle is less than 90°, we stay convex as long as the two sides of length 3 do not align, which happens when the quadrilateral becomes an isosceles triangle of sides 4,4,6 of area $\sqrt{63} < 8$. Thus we get four more convex quadrilaterals of areas 8,9,10 and 11 as well, for a total of 28. The answer is (c).

Part 2

Problem 1

Suppose for some real numbers x, y and z the following equation holds:

$$2x^2 + y^2 + z^2 = 2x(y + z).$$

Prove we must have $x = y = z$.

Solution:

Rewriting the equation gives $(x - y)^2 + (x - z)^2 = 0$ implying $x = y$ and $x = z$.

Alternative Solution: The given equation can be rewritten as $2x^2 - 2(y + z)x + (y^2 + z^2) = 0$, and hence

$$x = \frac{2(y+z) \pm \sqrt{4(y+z)^2 - 8(y^2 + z^2)}}{4} = \frac{y + z \pm \sqrt{-(y-z)^2}}{2}.$$

Since x must be real, $(y - z)^2 \le 0$ which means $y = z$, and then $x = \frac{2y+0}{2} = y$.

Problem 2

Two robots R2 and D2 are at a point O on an island. R2 can travel at a maximum 2 km/hr and D2 at a maximum of 1 km/hr. There are two treasures located on the island, and whichever robot gets to each treasure first gets to keep it (if both robots reach a treasure at the same time, neither one can keep it). One treasure is located at a point P which is 1 km west of O. Suppose that the second treasure is located at a point X which is somewhere on the straight line through P and O (but not at O). Find all such points X so that R2 can get both treasures, no matter what D2 does.

Solution:

Using Cartesian coordinates, we put $O = (0,0)$, $P = (-1,0)$ and $X = (x,0)$ for some real number $x \neq 0$. The treasure located at point P will be denoted P, and similarly for the treasure located at X. First note that if $x < 0$, then R2 can travel west in a straight line and get both treasures, one after the other, before D2. Now suppose that $x > 0$.

(a) If *R2 travels west at a maximum speed to pick up P and then returns east to pick up X*, it needs $\frac{1+1+x}{2} = \frac{2+x}{2}$ hours. D2 has no chance to get P, hence it should travel east for at least x hours and try to pick up X. If $\frac{2+x}{2} < x$ or equivalently $x > 2$, R2 will get both treasures.

(b) If *R2 travels east at a maximum speed to pick up X and then returns west to get P*, it needs a $\frac{x+x+1}{2} = \frac{2x+1}{2}$ hours. D2 should travel west to pick up P, for which it needs at least one hour. If $\frac{2x+1}{2} < 1$ or equivalently $x < \frac{1}{2}$, R2 will get both treasures.

(c) If $x \in (\frac{1}{2}, 2)$ there is no winning strategy for R2. This is equivalent to showing that always D2 can prevent R2 getting both treasures. Here is D2's strategy: *until R2 gets one treasure, D2 moves so that its position is always on the other side of the point O from R2's position, but at half the distance from O that R2 is.* Once R2 gets one of the treasures, R2 is at least twice as far from the other treasure than D2; then D2 heads straight to the other treasure and gets there before R2.

If $x = 2$ or $x = \frac{1}{2}$, *D2* can prevent *R2* to get both treasures by using the same strategy as above. In this case both robots reach one treasure at the same time, so neither one can keep it.

From (a),(b) and (c) we conclude that R2 has strategies to get both treasures no matter what D2 does if and only if $x \in (-\infty, 0) \cup (0, \frac{1}{2}) \cup (2, \infty)$.

Problem 3

One or more pieces of clothing are hanging on a clothesline. Each piece of clothing is held up by either 1, 2 or 3 clothespins. Let a_1 denote the number of clothespins holding up the first piece of clothing, a_2 the number of clothespins holding up the second piece of clothing, and so forth. You want to remove all the clothing from the line, obeying the following rules:

(i) you must remove the clothing in the order that they are hanging on the line;

(ii) you must remove either 2, 3 or 4 clothespins at a time, no more, no less;

(iii) all the pins holding up a piece of clothing must be removed at the same time.

Find all sequences $a_1, a_2, \ldots, a_n$ of any length for which all the clothing can be removed from the line.

Solution:

We claim that the clothing can be removed for all sequences *except* for 1, 131, 13131, and so on; that is, the exceptional sequences are of the form

$$a_1, a_2, \ldots, a_n = 1,3,1,3,\ldots,1,3,1,$$

where the 1's and 3's alternate, starting and ending with 1. Call such a sequence a *bad* sequence.

If $a_1, a_2, \ldots, a_n = 1,3,1,3,\ldots,1,3,1$, then in your first step you are forced to remove the first two pieces, using 1 and 3 pins respectively, because you cannot remove just one pin and cannot remove 5 pins at a time. This continues right to the end, till there is only one pin left, which you cannot remove. Thus all the bad sequences result in clothing left on the line.

Now we prove that any non-bad sequence $a_1, a_2, \ldots, a_n$ can be removed. Of course the 1-digit sequence 1(which is bad) cannot be removed, while the non-bad sequences 2 and 3 can be. We proceed by induction. Choose a non-bad sequence $a_1, a_2, \ldots, a_n$ of 1's, 2's and 3's, and suppose that all shorter non-bad sequences can be removed.

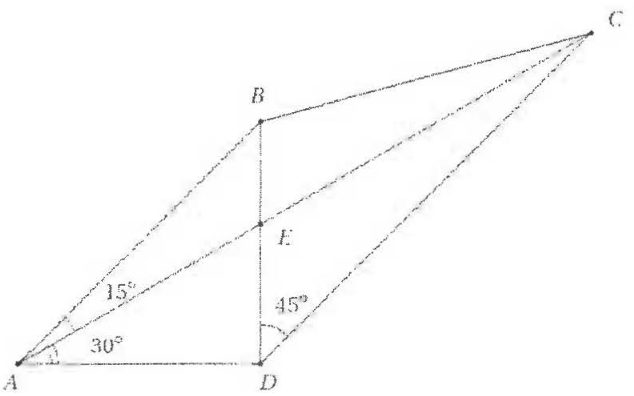
If $a_1 = 2$ or 3, and the sequence $a_2, a_3, \ldots, a_n$ is not bad, then we remove a_1 by itself, and the remaining sequence can be removed by induction. If $a_1 = 2$ or 3, and the remaining sequence $a_2, a_3, \ldots, a_n$ is bad, then we remove a_1 and $a_2 = 1$ (which add up to 3 or 4), and the remaining sequence $a_3, \ldots, a_n$ is not bad so can be removed by induction.

If $a_1 = 1$, and the sequence $a_3, a_4, \ldots, a_n$ is not bad, then we remove a_1 and a_2 (which add up to 2, 3 or 4), and the remaining sequence can be removed by induction. If $a_1 = 1$ and the sequence $a_3, a_4, \ldots, a_n$ is bad, and $a_2 = 1$ or 2, then we remove a_1, a_2 and $a_3 = 1$ (which add up to 3 or 4), and the remaining sequence can be removed by induction. Finally, if $a_1 = 1$ and the sequence $a_3, a_4, \ldots, a_n$ is bad, and $a_2 = 3$, then the sequence $a_1, a_2, \ldots, a_n$ is in fact bad, which is a contradiction.

Problem 4

$ABCD$ is a convex quadrilateral such that $\angle BAC = 15°$, $\angle CAD = 30°$, $\angle ADB = 90°$ and $\angle BDC = 45°$. Find $\angle ACB$.

Solution:



Let $AD = a$, then $DB = a, DE = \frac{a}{\sqrt{3}}, BE = a\left(1 - \frac{1}{\sqrt{3}}\right), AB = a\sqrt{2}$. $\triangle AEB$ is similar to $\triangle CED$ thus $\frac{AB}{DC} = \frac{EB}{ED}$, and hence $DC = a\frac{\sqrt{2}+\sqrt{6}}{2}$. In $\triangle BDC$, by using cosine law we get

$$BC^2 = DB^2 + DC^2 - 2DB \cdot DC\cos 45° = a^2 + \frac{a^2(8+4\sqrt{3})}{4} - 2a^2\frac{\sqrt{2}+\sqrt{6}}{2} \cdot \frac{\sqrt{2}}{2} = 2a^2$$

thus $BC = a\sqrt{2}$, which leads to $\triangle ABC$ is isosceles, hence $\angle BCA = \angle BAC = 15°$.

Problem 5

Find the minimum value of $|x + 4y + 7z|$ where x, y, z are **non-equal** integers satisfying the equation

$$(x - y)(y - z)(z - x) = x + 4y + 7z.$$

Solution:

If an integer m is a multiple of 3 let us write $m = \mathcal{M}3$. If x, y, z have different remainders when they are divided by 3, then $x = \mathcal{M}3 + r_1$, $y = \mathcal{M}3 + r_2$, $z = \mathcal{M}3 + r_3$ where $\{r_1, r_2, r_3\} = \{0, 1, 2\}$. One obtains that $x + 4y + 7z = \mathcal{M}3 + r_1 + r_2 + r_3 = \mathcal{M}3$ while $(x - y)(y - z)(z - x) = (\mathcal{M}3 + r_1 - r_2)(\mathcal{M}3 + r_2 - r_3)(\mathcal{M}3 + r_3 - r_1) = \mathcal{M}3 + (r_1 - r_2)(r_2 - r_3)(r_3 - r_1) \neq \mathcal{M}3$ which is a contradiction. Therefore at least two remainders are equal and hence

$$3|(x - y)(y - z)(z - x) \iff 3|(x + 4y + 7z) \iff 3|(x + y + z) \iff 3|(r_1 + r_2 + r_3).$$

Since two of the remainders are equal, $3|(r_1 + r_2 + r_3)$ if and only if all three remainders are equal. Therefore

$$27|(x - y)(y - z)(z - x) \implies 27|x + 4y + 7z.$$

Take $x = y + 3a$, $y = z + 3b$ and $x + 4y + 7z = 27k$ where a, b, k are integers. Since x, y, z are distinct, one obtains that none of the integers a, b or $a + b$ is equal to 0. By using these notations, the given equation $(x - y)(y - z)(z - x) = x + 4y + 7z$ can be written as $-ab(a + b) = k$. It is clear that $|k| \geq 2$. The value $|k| = 2$ could be obtained if we take $a = 1, b = 1$, for which we get $x = 0$, $y = -3$ and $z = -6$. We conclude that the minimum value of $|x + 4y + 7z|$ is $27 \cdot 2 = 54$.