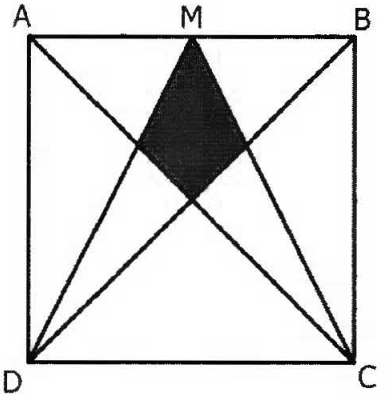
Kite in a Square

NRICH Mathematics

ABCD is a square. M is the midpoint of the side AB. By constructing the lines AC, MC, BD and MD, the shaded quadrilateral is formed.

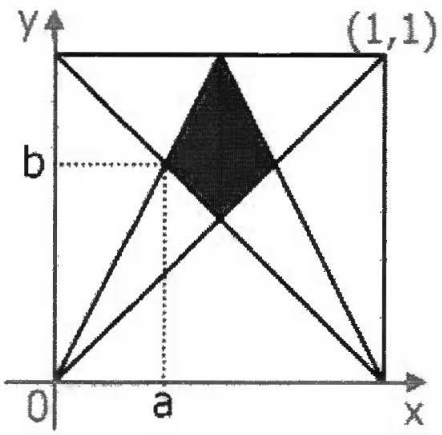
What fraction of the total area is shaded?



Below are three different methods for finding the shaded area. Unfortunately, the statements have been muddled up. Can you put them in the correct order?

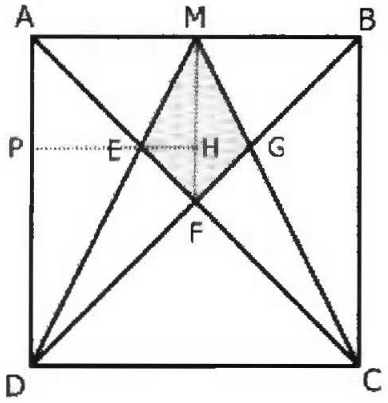
Coordinates

1. The shaded area is made up of two congruent triangles, one of which has vertices (1/3, 2/3), (1/2, 1/2), (1/2, 1).
2. The line joining (0, 0) to (1/2, 1) has equation $y = 2x$.
3. Area of the triangle = 1/2 (1/2 × 1/6) = 1/24.
4. The line joining (0, 1) to (1, 0) has equation $y = 1 - x$.
5. Therefore the shaded area is 2 × 1/24 = 1/12.
6. The point (a, b) is at the intersection of the lines $y = 2x$ and $y = 1 - x$.
7. Consider a unit square drawn on a coordinate grid.
8. The perpendicular height of the triangle is 1/2 – 1/3 = 1/6.
9. So $a = 1/3$, $b = 2/3$.
10. The line joining (0, 0) to (1, 1) has equation $y = x$.



Similar Figures

1. As line AC intersects line MD at point E, the two opposite angles $\angle MEF$ and $\angle AED$ are equal.
2. The line MF is half the length of AD.
3. Line AD is parallel to line MF, so $\angle EDA$ and $\angle$EMF are equal, and $\angle EAD$ and $\angle EFM$ are equal (alternate angles).
4. Therefore, ΔAED and ΔFEM are similar.
5. Therefore, the line EH is half the length of PE.
6. Let ABCD be a unit square.
7. Therefore, the shaded area MEFG = $1/24 \times 2 = 1/12$ sq units.
8. PH has length 1/2 units, so PE has length 1/3 units and EH has length 1/6 units.
9. ΔMEF has area $1/2\ (1/2 \times 1/6) = 1/24$ sq units.



Pythagoras

1. The area of ΔDMC = 2 sq units. The area of ΔDFC=1 sq unit. Thus the combined area of ΔDFE, ΔCFG and shaded area $MEFG$ is 1 sq unit.
2. $(EH)^2 + (HF)^2 = (EF)^2$
   $EH = HF$
   $(EH)^2 = 1/2\ (EF)^2$
   $EH = EF/2\sqrt{}$
3. Areas of ΔDFE, ΔCFG and shaded area $MEFG$ are equal, so each must have an area of 1/3 sq units.
4. Area of $\Delta MEF = 1/2\ (1 \times EH) = 1/2\ (EF/2\sqrt{})$
5. By Pythagoras, DF has length $2\sqrt{}$.
6. The total area of the square is 4 sq units, so the shaded area is 1/12 the area of the whole square.
7. Area of ΔDFE = DF × EF/2 = $2\sqrt{} \times EF/2 = EF/2\sqrt{}$
8. So the shaded area $MEFG$ is equal to the area of ΔDFE.
9. Assume that the sides of the square are each 2 units long. Thus, DJ and FJ are each 1 unit long.